Persistent Protests

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Abstract. A continuum of citizens with heterogeneous opportunity costs participate in a public protest, with well-defined demands. The government can concede at any time. As long as it does not, it shoulders a cost that is increasing in time and in participation rates. Apart from their collective demands, citizens enjoy a "merit reward" if the government concedes while they are actively participating. A protest equilibrium of the ensuing dynamic game must display: (a) a *build-up stage* during which citizens continuously join the protest, but the government ignores them, followed by (b) a *peak* at which the government concedes with some positive probability, failing which there is (c) a protracted *decay stage*, in which the government concedes with some density, and citizens continuously drop out. Citizens with higher opportunity costs enter later and exit earlier. While there are multiple equilibria, *every* equilibrium with protest has the above properties, and the set of all equilibria is fully described by a single pseudo-parameter, the protest peak time, which can vary within bounds that I characterize. Preliminary evidence from the *Black Lives Matter* movement support the features that I extract from this model.

1. Introduction

Public protests and social movements vary in size and duration. Static theories capture the essential multiplicity of "protest equilibria," giving us some idea of how people overcome coordination barriers. However, such theories do not capture the dynamics of protest: the entry and exit of citizens into the movement, the resulting path of the participant stock, and pattern of government concessions over time. The objective of my paper is to study the dynamics of participation in public protest in a context in which agents have heterogeneous opportunity costs of participating. I study how heterogeneity influences social behavior and shapes the overall contours of a persistent protest.

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I understand a protest event as the gathering of people to demonstrate against some authority about a given policy. The word *persistence* in this context refers to the duration of political unrest, in each of its potentially distinct phases. A protest may take time to build. It may take time to die out. The government may take more or less time to concede. Perhaps the most prominent example of a persistent protest is the Arab Spring, which began in Tunisia in the early 2010s, spreading to other countries in the Islamic world. More recent examples of protests include Chile and Iran in 2019, where again there was persistence of the protest in its different phases. The *Black Lives Matter* movement in the US is the most recent case of public protests characterized by persistent participation in all states, with different dynamics of participation and concessions.

In this paper, I build a model of protests to capture these dynamics, including buildups, sudden or slow concessions, and decays. The following assumed features are central to my theory. First, protests are costly to both parties. For citizens, the act of protest uses time and resources. For the government, facing down a protest is costly, both in terms of economic loss and political reputation. Second, the act of participation by an individual citizen is largely voluntary.[1] And finally, even if the goal of the protest is some non-excludable public good, citizens do have a separate individual incentive to participate, driven by a psychic or socially-conferred "merit reward" of being an active member of the movement.

Formally, I posit a continuum of small players — the citizens — and a single large player, the government. Time is continuous, and at any instant citizens face a binary choice: whether to participate in a protest or not. The cost of participating is the opportunity cost of the time spent in the protest, which is heterogeneous across citizens. The government decides at any instant whether to concede or not, but as long as it does not concede, it faces a cost that is increasing, both in the number of people protesting and in the duration of the protest. At the same time, concession is also costly to the government, because in that event it has to pay the cost of some public good: a new policy perhaps, or a regime change, or an expansion of rights. Everyone can enjoy this public good, whether or not they participated in the protest.

As already mentioned, citizens additionally enjoy a reward for being actively involved in the protest if and when the government concedes. To emphasize that the duration of involvement matters, we refer to this one-time victory payoff as a *veteran prize*. This formulation aims to combine an instrumental motive, i.e. obtaining the public good, with an intrinsic motive, i.e. personally contributing to the victory. The veteran reward increases with the time spent in the protest, but is only made available once the government concedes. The model works with the same qualitative features whether or not the reward

[1] There could be other settings in which an institutional affiliation enforces participation, but we do not study them here.

is fully contingent on being there at the moment of victory, but for concreteness we focus on this particular case.

This is a dynamic game, with its attendant possibilities of bootstrapped history-dependence in strategies and all sorts of complex, sometimes unrealistic, self-fulfilling equilibrium paths. However, one side of this game is populated by a continuum of agents, and every aggregate strategy that is the same barring a measure zero of agents will be taken to generate the same observed history from the point of view of the government. This is a standard *and* realistic assumption in the theory of large anonymous games: we cannot seriously entertain outcomes in which a monolithic government bases its actions on the behavior of any single anonymous citizen. That destroys the possibility of history-dependent strategies so crucial to the Folk theorem. (Matters would be different if there were a leader or a distinguished, non-anonymous agent, leading to the possibility of folk-theorem-like arguments. We do not consider that model here.)

That said, anonymity does not eliminate multiplicity, for multiplicity is a natural (and non-technical) consequence of any game with strategic complementarities. But it dramatically sharpens the set of equilibria. There is always an equilibrium with no protest and no government concession, but more remarkably, *every* equilibrium in which a protest occurs has exactly the same qualitative features. It is characterized by three stages: a build-up stage, a peak, and possibly a decay stage. The *build-up stage* corresponds to an initial period during which the protest grows as people continuously enter. It involves no concession at all on the part of the government. The second stage lasts but an instant, but is distinguished by the possibility of a government concession with positive probability — the protest is costly enough that the government can no longer ignore it. We call this stage the *peak*. If a concession does not occur, the third and final *decay phase* starts up. It is described by continuous dropout by the citizens, with the aggregate mass of protestors shrinking with time. All along, the government concedes with a continuous but changing hazard rate that we fully characterize.

The decay stage will be familiar to any economic theorist: it unfolds as a war of attrition, but the twist I add is that one side we have a *continuum* of players; namely, the citizens. Their cost heterogeneity allows me to purify their aggregate behavior, leading to ongoing dropouts in the decay phase. On the other side we have the government, which must randomize according to a continuous distribution over concession times. In particular, it must be indifferent at any time between conceding and waiting another instant. For this indifference condition to hold in equilibrium, the government will concede at some time-varying hazard rate that generates exactly the path of participation rates that guarantees this indifference. As far as citizens are concerned, they take as given the hazard path, and drop out as their expected gains from continuation become too low relative to their cost. Individual exits are deterministic, and aggregate to a smooth path of decay.

The peak stage is special because it involves a non-trivial probability of concession. Mathematically, that “initializes" the starting conditions of the war of attrition to follow, but it is also conceptually important because it suggests a sudden change in government attitudes that occurs precisely at the height of the protest.

In addition to these features, the build-up phase I describe is, to my knowledge, completely novel. It is not a part of any war of attrition, and stems from the assumption of varying opportunity costs of participation, along with the structure of the veteran reward. Individuals enter the protest in a spread-out way, leading to a swelling in unrest. During this entire period, I show that there cannot be a positive response from the government, because it must strictly prefer not to concede in this phase. Taken together, the three phases generate a rich but uniform prediction for the path of protests.

A central feature of equilibrium is that individual entry and exit decisions are monotone in their opportunity costs. I show that citizens enter at most once and exit at most once. The time at which an individual enters the protest increases with her opportunity cost, and the time at which she exits decreases in her cost. The resulting dynamics of entry and exit are therefore of the first-in-last-out form. The agent with the lowest opportunity cost is the first to enter, and will hold against the government forever. The last agent who joins the protest joins right before the peak, and exits just after it.

While build-up times, peak concession probabilities, decay rates and concession rates vary across equilibria, *all* equilibria share these qualitative features. Moreover, indiscriminate variation is not possible. I show that the set of all equilibria is fully described by a single “pseudo-parameter," the protest peak time, which can only vary within a range that I fully characterize.[2] This range is a bounded interval with a strictly positive lower bound. The veteran reward is responsible for the positive lower bound, as agents need time to build it, which means that every equilibrium with protests will involve a minimum delay before concessions are made. On the other hand, the peak is also bounded above, so that citizens with the lowest opportunity cost have incentives to begin the protest.

These predictions highlight the relevance of analyzing the dynamic shape of protests, not just theoretically but empirically. Specifically, my model provides a clear empirical prediction about the timing of participation: citizens with higher opportunity cost join protests later, and exit earlier. We explore this idea using county-level data from the recent *Black Lives Matter* protests. While individual participation is not directly observed, we can

[2]To be precise, there is also a second “pseudo-parameter" that could index equilibria, which is the start time of the protest, but without any loss of generality I normalize this to zero.

use county-level thresholds of participation, expressed as a share of the population, to map the notion of *individual* entry and exit to *aggregate* participation at the county level. Then, using these thresholds as our dependent variable, we study their relationship to different measures of opportunity costs. Higher income levels, related to larger opportunity costs of participation in protests, are connected to later entry times and earlier exit times. The same holds (controlling for income) for counties with a larger share of individuals with less than a college degree. Controlling for income, individuals with lower education levels will tend to work on less flexible jobs, so that their opportunity cost is expected to be higher.

Using variations in the incidence of COVID-19, I capture the idea of opportunity costs through two components. First, I exploit variation in the extent of re-opening by county, and in measures of job flexibility by county. As for the latter, I use Dingel & Neiman (2020)'s recent notion of *teleworkability* to obtain the share of workers in *flexible* jobs.[3] The idea of teleworkability measures how easy it is to do a job from home. I find that more flexibility is consistent with earlier entry times for those counties which had already reopened when the protests started nationwide. Second, to complement the analysis, I also evaluate the direct effect of the incidence of COVID-19 over the timing of entry. As there is a higher number of new cases, it is expected that the perceived risk of attending a demonstration increases. I provide preliminary evidence suggesting that higher incidence of COVID-19 is associated with later entry.

This paper is organized as follows. In the next subsection we briefly review the related literature and our main contribution. In Section 3, I develop the baseline model and provide some discussion of its main features. In Section 4, I characterize the dynamics of protests in equilibrium, which are then discussed in Section 5. I develop some extensions in Section 6, and the empirical analysis can be found in Section 7. All proofs can be found in Appendix A.

2. Related Literature

I contribute to the literature on persistent participation in public protests as a collective action problem. The literature most closed to this work is the one studying the coordination problem among citizens. Static models of coordination in protests have been studied by Shadmehr & Bernhardt (2011), Boix & Svolik (2013) and Morris & Shadmehr (2018).

The main contribution of this paper to the literature is to provide a full characterization of the dynamics of participation in equilibrium. The dynamics I obtain are intuitive, but

[3] As I explain in more detail later, I closely follow the methodology proposed by Glaeser et al. (2020). They study how effective restrictions on mobility are in limiting the spread of COVID-19, and use Dingel & Neiman (2020)'s teleworkability shares to construct an instrument for the level of mobility.

novel at the same time. A paper related to ours is the recent study by Chenoweth & Belgioioso (2019). As my paper, their work focuses on dynamics of participation instead of a static one-shot measure of the size of a protest. The authors propose to approximate the effect of social movements by the law of *momentum*: mass times velocity. The mass of a protest is the number of people participating, and the velocity is the frequency of events. They show empirical evidence of similar dynamics to the build-up stage in my framework. Moreover, their idea of momentum is based in the principle that social movements can compensate for low popular support, i.e. a low mass in their terminology, by concentrating their activities in time, i.e. high velocity. As I show in Section 4.2, in our dynamic model there is a similar trade-off, but it occurs between delay in the time at which the government starts conceding, and participation.[4]

The dynamics I focus on in this work also differ from those analyzed by Acemoglu & Wolitzky (2014). By developing an overlapping generations model, they study how incomplete information affects the dynamics of conflicts, understood as conflict spirals that generate more unrest in some periods than in others. In this paper, I do not focus on how a current protest affects the probability of occurrence of future events. On the contrary, I focus on one protest, and study the participation dynamics for that specific movement. Each equilibrium represents a unique protest with different stages of participation, concessional peaks, and decay.

This work is also related to the literature on the social psychology of public protests. In this literature, intrinsic motives for participation have been studied as a result of ideology or group identity (see Cohen (1985) and Jasper (1998)). Our assumption of a *veteran prize* constitutes a new explanation for persistent participation in protests that combines both an *intrinsic motivation*, i.e. the veteran reward, with an *instrumental motivation*: agents obtain this value only if the movement is successful, so that they take into account their expectations about the government conceding.[5] Intrinsic motives to protest coming from emotions have are also studied by Passarelli & Tabellini (2017), who study a game in which agents react to anger (see also Wood & Jean (2003) and Pearlman (2018) for the case of intrinsic motives and voting).

This work is also related to the literature on conflict (see Ray & Esteban (2017) for a detailed review). There is an extensive literature analyzing the relationship between conflict intensity and inequality, and the main idea is that inequality affects both grievances and opportunity costs (see Dal Bó & Dal Bó (2011) and Dube & Vargas (2013)). Although protests can be seen as a particular case of a conflict, the main forces driving the dynamics

[4]In particular, I show that among the set of equilibria, there is an inverse relation between the peak in participation, and the time at which the government makes the first probabilistic concession.

[5]See Feather & Newton (1982) and Klandermans (1984) for an analysis of instrumental motivations in protests.

are different when the "fight" is between a single large player facing an increasing participation cost (the government), and a continuum of small negligible players (the citizens).

From a methodological point of view, this work is related to the literature on wars of attrition. The decay stage of equilibria unfolds as a war of attrition with complete information, between a large player and a continuum of citizens. The seminal work of Hendricks et al. (1988) solves the war of attrition in a context with complete information for the case of two players. In my model, one of the sides is replaced by a continuum of anonymous citizens. When aggregated, their continuous dropouts resembles the behavior of a single opponent in the classic war of attrition.

There are, however, other works studying wars of attrition with more than two players. For instance, Bulow & Klemperer (1999) analyze a war of attrition with a finite number of firms competing for a set of prizes. More recently, Kambe (2019) studies a war of attrition with several agents, in which the exit of a single player is enough to end the game. The lack of anonymity in these cases changes the strategic problem in ways that are unrelated with the setup analyzed here.

Finally, this work also contributes to the recent literature studying the effects of COVID-19. I provide preliminary evidence for the effect of workers flexibility during the pandemic, over citizens decisions on the timing of protests. I follow the approach proposed by Glaeser et al. (2020), and use Dingel & Neiman (2020)'s notion of teleworkability of an occupation, to obtain a measure of job flexibility. I then explore how this flexibility affects the timing of protests at the county level. To the best of my knowledge, this is the first work exploring this relationship.

3. A Dynamic Model of Protest

In Section 3.1, I describe the baseline model, along with its main assumptions and the equilibrium concept. In Section 3.2, I comment on the assumptions and more generally on the model setup.

3.1. The Model. There is a single large player, the *government*, and a continuum of small players, the *citizens* or the *people*. Citizens are indexed by $i \in [0,1]$. Time is continuous, and at any instant $t \in [0,\infty]$, citizens decide whether to participate in a protest to ask the government for a public good. The choice for the government is also binary. At any moment in time the government can either concede, or keep waiting. The game ends when one of the two sides fully concedes: either the government provides the public good, or all citizens drop out.

Protests are costly to everyone. For citizens, participating in the protest requires an investment of time and resources, which is captured by an opportunity cost parameter θ. I assume the opportunity cost is heterogeneous, and drawn from a distribution F. In practice, this heterogeneity in opportunity costs may capture different levels of income, types of jobs, or even different residence locations, that make protesting more costly for some agents than for others. I assume F is continuously differentiable, with full support $[\underline{\theta}, \overline{\theta}]$, for some $\underline{\theta} > 0$. The maximum cost $\overline{\theta}$ might be unbounded.

For the government, staring down a protest is also costly. This cost might represent losses due to direct disruption caused by demonstrations, a loss in nationwide economic productivity, or a hit to the government's political reputation. We model this by presuming that the government pays a flow cost that is increasing in both the number of people participating in the protest at a given time, and in the duration of the protest. Concession is also costly, as once the government concedes, it pays the equivalent of a flow cost of q forever.

I make some natural assumptions regarding the cost function. First, if there is no one protesting, there is no cost to the government. Second, if the entire population is protesting, the flow cost of bearing the protest is higher than the flow cost of the public good. We summarize this and the above discussion in:

Assumption 1. *The cost function* $c : [0, 1] \times [0, \infty) \to \mathbb{R}_+$ *is continuously differentiable on both arguments, and satisfies:*

(i) $c(0, t) = 0$ *for all* t*, and* $c(1, 0) > q$.

(ii) $c(\pi, t)$ *is strictly increasing in* π*, and is strictly increasing in* t *if* $\pi > 0$.

Let $(\pi_t)_{t \geq 0}$ be a trajectory of participation. If the government concedes at some time τ, then its overall costs are given by:

$$\int_0^\tau e^{-rs} c(\pi_s, s) ds + e^{-r\tau} \frac{q}{r} \tag{1}$$

where $r > 0$ is the discount rate, which is the same as the citizens' discount rate.

Because the public good is non-excludable, citizens can enjoy it even if they were not involved in the protest. If the government concedes at a time τ, then from that time onward every citizen receives an extra flow payoff x from enjoying the public good. Notice that the value of the public good does not affect citizens' decision to protest, and it is without loss to assume that all of them value the public good equally.

In addition to the payoff from the public good, citizens get a reward for being active participants in the protest. This payoff increases with the time spent in the protest, and is only made available if the citizen is still in the protest by the time the government concedes. We call this prize the *veteran reward*. Formally, if the government concedes at time t, an agent who has been in the protest since time t_0, and is still in the protest when the government concedes, gets a one time reward of $v(t - t_0)$. I assume the veteran reward increases with the time spent in the protest, but at a decreasing rate. The following assumption formalizes this idea.

Assumption 2. *The veteran reward* $v : [0, \infty] \to \mathbb{R}_+$ *is continuously differentiable, and*

(i) $0 < v'(\Delta) < \infty$ *and* $v''(\Delta) \leq 0$ *for all* $\Delta \geq 0$*;*

(ii) $v(0) = 0$*.*

Part (i) ensures that v is increasing and concave. Part (ii) rules out opportunistic behavior, in the sense that it ensures that if citizens are not in the protest by the time the government concedes, they would rather free ride to get the public good.

Suppose the government concedes at some time τ, possibly random. Consider a citizen with opportunity cost θ who starts protesting at some time t_0, and is planning to exit at time t_1. Her expected payoff is given by the following expression.

$$E\left[-\theta \int_{t_0}^{t_1 \wedge \tau} e^{-rs} ds + e^{-r\tau}\left(\mathbb{1}_{\tau < t_1} v(\tau - t_0) + \frac{x}{r}\right)\right] \tag{2}$$

where the expectation is taken over τ. In words, the citizen will pay the cost of the protest for as long as she remains being an active participant. If by the time the citizen drops out the government has not conceded, then the citizen simply goes home and receives nothing at that time. Eventually, she will get to enjoy the public good if and when the government decides to provide it. If, on the contrary, the government concedes before the citizen drops out, then in addition to the public good she gets a one-time veteran reward of $v(\tau - t_0)$.

It remains to specify how the game is played at each instant. I assume that when the government decides whether to concede, it is already observing how many people are protesting. However, when citizens decide whether to protest or not, they only observe participation until *an instant before* they join. To help grasping better the interpretation for continuous time, we can build some intuition with a discrete time case. Imagine a game played repeatedly at times $\{0, 1, 2, ...\}$. At any time t, the stage game is such that first citizens make a protest decision, and then the government decides whether to concede or not. Thus, when citizens choose their actions, they only observe a history of participation up to $t - 1$, i.e. $\{\pi_0, \pi_1, ...\pi_{t-1}\}$. Once they take an action, the government gets to observe

π_t before deciding whether to concede. Hence, the relevant history for the government is given by $\{\pi_0, \pi_1, ..., \pi_t\}$.

Following this intuition, for any time t, define the histories $\pi^t = \{\pi_s : 0 \leq s < t\}$ and $\overline{\pi}^t = \{\pi_s : 0 \leq s \leq t\}$. Let $\Pi^t = \{\pi^t\}_{t\geq 0}$ be the set of all possible open histories at time t, and $\overline{\Pi}^t = \{\overline{\pi}^t\}_{t\geq 0}$ the set of all possible closed histories at time t. Also define $\pi^0 = \emptyset$. A strategy for the government is a process $\gamma = \{\gamma_t\}_{t\geq 0}$, with $\gamma_t : \overline{\Pi}^t \to \{0, 1\}$, where $\gamma_t = 1$ stands for *concede*, and $\gamma_t = 0$ for *not concede*. A strategy for a citizen with opportunity cost θ is a process $\sigma^\theta = \{\sigma^\theta_t\}_{t\geq 0}$ with $\sigma^\theta_t : \Pi^t \to \{0, 1\}$, where 1 stands for *participate*, and 0 for *not participate*. While the government decision is irreversible, citizens can reenter the protest after leaving. As we illustrated for the discrete time setting, the difference between citizens and the government is that the government is already observing the protest when it decides whether to concede, while citizens decide to protest simultaneously, and then they observe the open history π^t. We denote a strategy profile by (σ, γ), where $\sigma = \{\sigma^\theta\}_{\theta\in[\underline{\theta},\bar{\theta}]}$.

For any strategy profile (σ, γ), let $\pi^{\sigma t}$ be the trajectory up to time t, conditional on no concession, generated by the strategy σ. This can be defined recursively as follows:

$$\pi^\sigma_t = \int \sigma^\theta_t(\pi^{\sigma t}) dF(\theta) \qquad \forall t \geq 0 \tag{3}$$

In this game, citizens are non-atomic, as described by Schmeidler (1973). As each of them is negligible, their individual behavior is only significant to the opponent when is aggregated.[6] In this sense, this game is also anonymous in the sense of Mas-Colell (1984), since the only observable that matters in equilibrium is the aggregate behavior of protesters, and not their individual decisions. In large protests this assumption is natural, as it is very unlikely in that a government make decisions based on the behavior of negligible agents. I recognize that there may be situations in which some agents are distinguishable, which might be the case if there is a leader or some salient political group, but this lack of anonymity is not consider in this paper.

Given their anonymity, citizens individual deviations do not affect aggregate participation. Then, it is enough to describe government's strategies along the equilibrium path. This simplifies the problem to a great extent, as otherwise we would need to consider all possible history-dependent strategies. The focus on equilibrium strategies on the equilibrium path is equivalent to describe government strategies that are *open-loop*, in the sense that it is as if it commits to a sequence of actions at the beginning of the game. The idea of these

[6]These games have been widely studied in the context of both a monopolist with a continuum of buyers, or perfect competition with many firms. As in our model, in a perfectly competitive environment with non-atomic firms, the price is taken as exogenous to agents decisions, and then a single firm cannot have any influence on its competitors. See Green (1982) and Levine & Pesendorfer (1995).

open-loop strategies, is that players do not have to consider how their opponents would react to deviations form the equilibrium path.[7]

We allow the government to randomize over concession times. As we focus on the trajectory of participation the government expects in equilibrium, we can characterize its strategy as a mixed strategy: a distribution of concessions $G(t)$.[8] This distribution of government's concession corresponds to the probability of the government conceding in $[0,t]$, given a trajectory of participation up to time t, $\pi^{\sigma t}$. This function is weakly increasing and right continuous in t, and its support is defined as:

$$\mathcal{T} = \{t \geq 0 | G(t) - G(t-\epsilon) > 0 \;\; \forall \epsilon > 0\} \tag{4}$$

Define $\tau_0 = \inf \mathcal{T}$, i.e. the first time at which the government makes some concession, and $\tau_1 = \sup \mathcal{T}$. On the citizens side, their anonymous nature comes to play again, as it implies that we can obviate mixed strategies, and focus on pure strategies only.

I focus on the set of Nash Equilibria. An *equilibrium* is given by a distribution of government concessions $G(t)$, and a profile of citizens' strategies σ, such that given the outcome path $\{\pi_t^{\sigma}\}_{t\geq 0}$,

(i) The strategy for the government maximizes its expected total payoff.

(ii) Citizens' strategies maximize their expected total utility given the government's distribution of concession G.

Before moving to the equilibrium characterization, I discuss some features of the model.

3.2. *Key Features of the Model.*

3.2.1. Citizens' Motives. Citizens get utility from the public good and from the veteran reward to victory. The assumption of a common value for the public good is just a normalization, as in this setting it does not affect their decisions.

The problem of coordination in collective action problems lies in the fact that if collective action succeeds, it generates gains that can be enjoyed by all citizens, irrespective of their participation or merits in the victory (see Olson (2012)). Scholars have introduce the idea of some intrinsic rewards to explain participation in these collective action problems, such as voting and regime change models.

[7] Fudenberg & Levine (1988) compare the notions of *open-loop* and *closed-loop* equilibria for the case of games with non atomic players. In particular, they show that if there is a unique Nash equilibrium in every subgame, then both equilibria coincide.

[8] Without anonimity, a behavioral strategy in this context would specify for each possible history $\overline{\pi}^t$, a probability of concession.

In the context of protests, several studies recognized the relevance of intrinsic psychological motivations for citizens to participate (see for instance Wood & Jean (2003) and Pearlman (2018)), as well as group-based emotions (see Klandermans (1984) and Van Stekelenburg & Klandermans (2013)) . The literature on social psychology of public protests has identified four motives for protesting: (i) *Instrumental*, related to the expectation of reaching a goal; (ii) *Identity*, related to the identification with a group; (iii) *Emotions*, related to grievances and group-based anger; and (iv) *Ideology*, which is related to individual values and the perception of an illegitimate state of affairs. The latter three motives (namely, Identity, Emotions, and Ideology) operate through a similar mechanism, in the sense that they generate an inner obligation to contribute that prevents free riding.[9] However, as it is shown by Simon et al. (1998), in practice these three motives complement instrumental motives.

The veteran prize then aims to capture this complementarity between instrumental and intrinsic motives. People want to have merits in an eventual victory against the government, but they only obtain this rewarding feeling if they attain the goal. The necessity of goal attainment to obtain the reward is the instrumental part. As the protest in this case needs persistence to be successful, agents who contribute with more persistent participation get more merits. That is the rationale for the prize to be a function of the time the agent has been in the protest.

3.2.2. Conditional Nature of the Veteran Prize. As the model is currently described, citizens only get their veteran prize if they are actively participating at the time the government concedes. It might be natural, however, for citizens who make a relevant contribution to build-up the protest, to obtain some rewards even if they drop out before the government concedes. It is direct to extend the model to allow for citizens to obtain part of the veteran prize even if they retire before concession, provided that a part of the prize is only obtained by staying until government concession. If this wasn't the case, citizens' strategies would be completely independent of government behavior. This is consistent with the idea just described about protesters' motives. Intrinsic motives, such as identity, emotions and ideology, incentivize participation in ways that complement the instrumental motive of having the government to concede.

3.2.3. Cumulative Nature of the Veteran Prize. I formulated the model in a way that the veteran prize is a function of how long the citizen has been in the protest before government concession. This might seem restrictive in practice. For instance, a citizen that contributes by protesting every weekend does not need to feel less veteran than a citizen

[9]In a similar vein, in their book *Why Bother?*, Aytaç & Stokes (2019) develop the idea of a psychological *abstention cost*, i.e. a cost for not being in the protest, that encourages people's participation in protests.

that condense the same participation in a week. This assumption can be easily modified without changing the behavior of agents in this game. As citizens discount the future and the opportunity cost is constant, they will always prefer to push all their participation forward. In practice, there may be factors that differentiate the weekend-protester from the one that protest seven days a week that are not included in this model. For instance, some cyclical or non monotonic variation of the opportunity costs might make the weekdays being more costly than weekends, but that possibility is not considered here.[10]

3.2.4. Government Payoffs. For the government, the cost of the protest depends on two components: participation and duration. This aims to capture both the instantaneous effect of having a given number of people on the streets, and the cost that is accumulated over time. The effect of the size of a protest, can represent for instance a decrease in economic productivity, or destruction of infrastructure. It could also be that a larger protest is more costly because it has a detrimental effect over public opinion, and this affects government reputation. Including the duration of the protest as a separate argument represents more indirect costs, that are build through a persistent protest. For instance, as the protest captures media's attention, the reputation of the government is increasingly damaged as the protest persist.

The government then, when deciding whether to provide a public good or not, weights the costs of the protest with the costs of concession. This is a simplification of a possibly more complicated problem. For instance, another interpretation of this framework is that the government has its own agenda or policy preferences. In that case, the cost q would represent how far away from his preferred policy the government would be if it provides the public good. The protest is costly for government's reputation, and then it affects its probability of being reelected. As the protest lasts longer, the probability of being reelected decreases. Then, the government would weight the distance from its preferred policy, against its possibility of reelection.

4. The Dynamics of Protests

4.1. Equilibrium Characterization. In this section, I fully characterize the set of equilibria in which a protest occurs. We refer to an equilibrium as an *equilibrium with protests* if there is some (possibly probabilistic) concession by the government, i.e. $\mathcal{T} \neq \emptyset$. In addition to the set of equilibria with protests characterized below, there is always an equilibrium in pure strategies in which the government never concedes and nobody protests, i.e. $G(t) = 0$ and $\pi_t^\sigma = 0$ for every t. This equilibrium arises naturally in coordination games with complete

[10]However, if opportunity costs change monotonically, all the results of our model would still hold.

information, and in protest games it represents many situations in which protests simply do not occur.

Naturally, there is multiplicity of equilibria in this game. But what makes the results remarkable is that every equilibrium with protest has the same qualitative features. As we show in Theorem 1, any equilibrium with protests is characterized by three stages: a build-up stage, a peak, and possibly, a decay stage. The *build-up* stage corresponds to the initial period in which the protest grows as people continuously enter. However, in this initial stage the protest is still not costly enough to the government, and then the government does not make any concession. The *peak*, is the first time at which there is a possibility of concession by the government with positive probability. It coincides with the time at which participation reaches its peak (and then its name), and the protest becomes costly enough that the government can no longer ignore it. If concession occurs at the peak, the protest ends. If it does not occur, then the *decay* stage starts. In the decay stage, citizens continuously drop out and participation decreases. The government continues conceding, with a decreasing hazard rate.

This result is formalized in the following theorem.

Theorem 1. *Let $G : [0, \infty] \to [0, 1]$, $(\pi_t^\sigma)_{t \geq 0}$ be an equilibrium with protests. Then the following features obtain:*

(i) *There is always delay in government concession, i.e.* $\tau_0 > 0$.

(ii) π_t^σ *is continuous, increasing for* $t \leq \tau_0$, *and if* $G(\tau_0) < 1$, *decreasing for all* $t \geq \tau_0$.

(iii) *The distribution of concessions has at most one discrete jump at* τ_0, $G(\tau_0) \leq 1$.

(iv) *If* $G(\tau_0) < 1$, *then* $G(t)$ *is strictly increasing and continuous, and* $\tau_1 = \infty$.

(v) *In equilibrium, the government concedes with probability* 1, *i.e.* $\lim_{t \to \infty} G(t) = 1$.

Although we prove the result in Appendix A, I provide the main intuition here.

First, note that in any equilibrium with protests, the government's strategy is restricted to either a singleton support $\{\tau_0\}$, or an interval $[\tau_0, \tau_1]$ (see Lemma 2 in Appendix A). To see this, note that if the government stops conceding during some interval of time and resumes concession later on, citizens who are already in the protest would wait until the government starts conceding again. As the cost of the protest increases with time, this strategy cannot be optimal.

For the government to play a mixed strategy, then it must be that along the support, the following indifference condition holds:[11]

$$c(\pi_t, t) = q \text{ for all } t \in [\tau_0, \tau_1] \tag{5}$$

This indifference condition imposes a constraint to the number of people protesting that the government is willing to tolerate. Define the *indifference participation level* $\tilde{\pi}_t$, as the trajectory of participation that satisfies equation 5 for any time $t \in [0, \tau_1]$. By Assumption 1, this indifference participation path is continuous and strictly decreasing in t. Condition 5 implies that the trajectory of participation on the support of $G(t)$ must coincide with the function $\tilde{\pi}_t$, and then it is decreasing.

From the indifference condition, we also conclude that it must be that the interval must go all the way to infinity, i.e. $\tau_1 = \infty$. This result follows from government's incentives to randomize: it must be that at any time, the government is indifferent between conceding and waiting another instant. If the interval was finite, then there is a time at which the government is no indifferent anymore, and the equilibrium would unravel.

Citizens, on the other side, take the distribution of concession $G(t)$ as given, and decide when to protest. Even when they are allowed to exit and reenter many times, we show that in equilibrium they enter and exit at most once. Moreover, they only enter before the government starts conceding, and they only exit afterwards.

Consider the problem of a citizen with opportunity cost θ, who enters at t_0 and exits at t_1. Since the government makes the first probabilistic concession at time τ_0, the entry and exit times must be such that $t_0 < \tau_0 \leq t_1$. Let $\lambda_t = \frac{g(t)}{1-G(t)}$ to be the government's hazard rate of concession, i.e. the instantaneous probability of conceding at time t, given that it has not conceded yet. Once in the protest, this citizen keeps protesting as long as the benefit of staying another instant weakly exceeds the cost. In particular, she exits the protest if:

$$\theta \geq \lambda_{t_1} v(t_1 - t_0) \tag{6}$$

The left hand side corresponds to the opportunity cost of staying another instant. The right hand side corresponds to the expected gains: the veteran reward she can obtain, times the hazard rate at which the government is conceding.

Consider now the entry decision of the citizen who expects to exit at t_1. At any time $t < \tau_0$ she compares the expected payoff from entering at t, against the payoff from waiting an instant to enter. By entering at t instead of an instant later, the agent hast to pay the flow opportunity cost θ. However, the gains are given by the marginal increase in the veteran prize the agent might obtain during the time she remains in the protest. Then, an agent

[11] See Lemma 3 in Section A.

with opportunity cost θ enters the protest at t_0 if:

$$\theta \leq E\left[e^{-r(\tau-t_0)}\mathbb{1}_{\tau<t_1}v'(\tau-t_0)\right] \tag{7}$$

As I show in Lemma 4 in Appendix A, citizens' utilities satisfy a single-crossing property with respect to opportunity cost, and then these optimality conditions are both necessary and sufficient. Moreover, their strategies are monotone in the opportunity cost.

This monotonicity allows us to characterize their strategies by a pair of entry and exit thresholds that we denote by $\tilde{\theta}_0(t)$ and $\tilde{\theta}_1(t)$, respectively. At any time $t < \tau_0$, a citizen enters if $\theta \leq \tilde{\theta}_0(t)$. At any time $t \geq \tau_0$, she exits if $\theta \geq \tilde{\theta}_1(t)$. Then, equilibrium participation is given by:

$$\pi_t^\sigma = \begin{cases} F(\tilde{\theta}_0(t)) & t \leq \tau_0 \\ F(\tilde{\theta}_1(t)) & t > \tau_0 \end{cases} \tag{8}$$

The expected benefits from entry and exit depend on the government strategy $G(t)$, and this in turn determines the entry and exit thresholds, $\tilde{\theta}_0(t)$ and $\tilde{\theta}_1(t)$.

If the government concedes with probability one on its first concession, i.e. $\mathcal{T} = \{\tau_0\}$, then there is no relevant exit decision. In this case there is no decay stage either, as the protest ends at the peak. If, on the contrary, the support $\mathcal{T}$ is an interval, then the trajectory of participation in the decay stage must coincide with the indifference participation level $\tilde{\pi}_t$. Then, the following equilibrium condition must hold:

$$\pi_t^\sigma = F\left(\tilde{\theta}_1(t)\right) = \tilde{\pi}_t \tag{9}$$

This is, the participation level generated by citizens best responses, must coincide with the indifference participation level.

The equilibrium condition allows us to pin down a precise trajectory for the hazard rate of government concession. At any time $t \geq \tau_0$, there is a citizen who is at the margin between staying another instant, or dropping out. From the condition in equation 9, this citizen's opportunity cost must be such that $\tilde{\theta}_1(t) = F^{-1}(\tilde{\pi}_t)$. Then, citizens' exit times are determined in equilibrium by the trajectory of $\tilde{\pi}_t$. Given this exit time, citizens choose an entry time $t_0(t)$ according to the entry condition 7. Then the government hazard rate at time t is given by:

$$\lambda_t = \frac{\tilde{\theta}_1(t)}{v(t-t_0(t))} \tag{10}$$

which defines a unique distribution of concessions $G(t)$.

4.2. Equilibrium Multiplicity. So far we have shown that any equilibrium with protests can be parametrized by a time τ_0 at which the level of participation reaches, and at which the government makes the first concession. In this section, I show that the set of possible times τ_0 is bounded.

The bounds happen to be very intuitive. The lower bound, is given by the equilibrium in which the government concedes with probability 1 at the time that participation reaches its peak.

Let's call this lower bound $\underline{\tau}$. If the government is conceding with probability 1 at $\underline{\tau}$, the marginal benefit of the last agent entering is given by $v'(0)$, while the marginal cost is its opportunity cost, θ. As all the agents with lower opportunity cost have already entered, participation at the time of concession is given by $F(v'(0))$. Then, the lower bound $\underline{\tau}$ solves:

$$c\left(F(v'(0)), \underline{\tau}\right) = q \tag{11}$$

The upper bound is a bit more subtle. Recall that I normalize the time so that $t = 0$ is the time at which the first citizen enters the protest.[12] Given that entry is monotone in θ, the first citizen entering is the citizen with the lowest opportunity cost $\underline{\theta}$. Note that as the delay in the start of government concession increase, the payoff from entering at 0 decrease as well. But in order to have an equilibrium with protests, at least the agent with the lowest opportunity cost must be willing to enter. Then, the upper bound $\overline{\tau}$ must be such that:

$$\underline{\theta} = E\left[e^{-r\tau} v'(\tau)\right] \tag{12}$$

The left side is the lowest opportunity cost, and the right side is the expected marginal benefit of entering at 0 and staying in the protest forever, given the government's strategy $G(t)$. Note that as $\overline{\tau}$ increases, the right hand side decreases. Moreover, it is strictly greater than $\underline{\tau}$ if and only if $\underline{\theta} < e^{-r\underline{\tau}} v'(\underline{\tau})$. We formalize this in the following assumption.

Assumption 3. *Let $\underline{\tau}$ be such that $c(F(v'(0)), \underline{\tau}) = q$. Then, $\underline{\theta} < e^{-r\underline{\tau}} v'(\underline{\tau})$.*

Using these bounds we obtain the following existence result.

Theorem 2. *For every $\tau_0 \in [\underline{\tau}, \overline{\tau}]$, there exists a unique equilibrium $(G, (\pi_t^\sigma)_{t\geq 0})$ where the government concedes for the first time at τ_0.*

This result provides a strong characterization of the set of equilibria. Not only the possible delays are bounded, but also given any possible delay in government concession within this bounds, the equilibrium is unique.

[12] To be more precise, this normalization is an equilibrium selection. However, given that at any time protests can happen, and the objective of this work is to characterize its dynamics, if we did not set the starting time to 0, the predictions obtained with this normalization would be reproduced on any possible starting point.

To prove this result, we first show existence for the lower and upper bounds, $\tau_0 = \underline{\tau}$ and $\tau_0 = \overline{\tau}$. The lower bound is straightforward, and the latter follows from a fixed point argument that I explain below.

I define a modified problem, in which the peak time τ_0 is chosen by a fictitious player, and is given to both the citizens and the government. Suppose time τ_0 is given. Recall that in the decay stage, the trajectory of participation is fixed at $\tilde{\pi}_t$. Thus, in citizens' problem the equilibrium exit times are given: a citizen with opportunity cost θ exits at the time t at which $F(\theta) = \tilde{\pi}_t$. Then, their best reply is a sequence of entry times given the government distribution of concessions, and given their exit times.

The government in turn, given these entry times, for any $t \geq \tau_0$ must choose a hazard rate that makes the marginal agent indifferent between conceding and waiting another instant (in order to keep participation at the indifference level in the concession stage). To close things up, we introduce the fictitious player whose only role is to adjust τ_0 for equation 12 to be satisfied with equality, given the government strategy $G(t)$, for any $t \geq \tau_0$. This also allows us to get rid of discontinuities of the government strategy at τ_0, and then we can apply standard fixed point theorems. It is then straightforward to use the same fixed point argument to show that for any $\tau_0 \in [\underline{\tau}, \overline{\tau}]$ an equilibrium exists.

Figures 1 and 2 illustrate the continuum of equilibria. In both figures, Panel (a) shows the equilibrium with the shortest delay, $\underline{\tau}$; Panel (b) shows an equilibrium with an intermediate delay, $\tau_0 \in (\underline{\tau}, \overline{\tau})$; and Panel (c) shows an equilibrium with the maximum delay possible, $\overline{\tau}$.

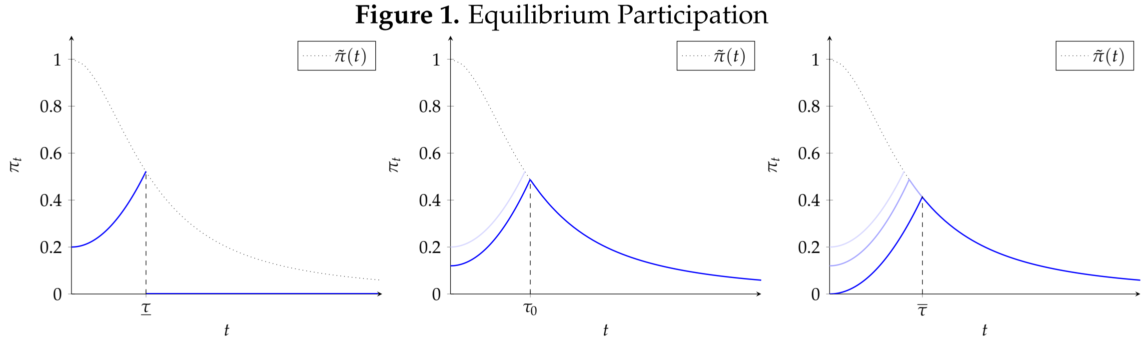
The three panels in Figure 1 illustrate the trajectory of participation for the three delays. The decreasing dotted line, $\tilde{\pi}_t$, corresponds to the indifference participation level. For any participation level π_t below this dotted line, the cost of the protest is still too low relative to the cost of the public good, and then the government is better off by ignoring protesters. Analogously, any participation level above this line is too costly, and then the government would rather concede.

The three panels in Figure 2 show the distributions of government concession corresponding to each delay τ_0. For any delay $\tau_0 \in [\underline{\tau}, \overline{\tau}]$, participation is increasing on $[0, \tau_0]$. This corresponds to the build-up stage. As in this stage participation is everywhere below the line $\tilde{\pi}_t$, the government is better off by waiting. Thus, in Figure 2 $G(t) = 0$ on $[0, \tau_0)$. Once participation hits the dotted line, then the protest becomes too costly and the government has to make some concession. The very precise moment at which this happens corresponds to the *peak*. The equilibrium with the shortest delay, $\underline{\tau}$ in panel (a), corresponds to the equilibrium in which the government concedes with probability 1 at the peak. Then, the distribution of government concessions jumps up to 1, and everyone drops out.

Now let's move to panel (b), with $\tau_0 \in (\underline{\tau}, \overline{\tau})$. Note that the government still makes a discrete concession, but with probability less than 1. Immediately after this concession, the government continues randomizing over time, and people continuously drop out. Participation then coincides with the dotted line in equilibrium.

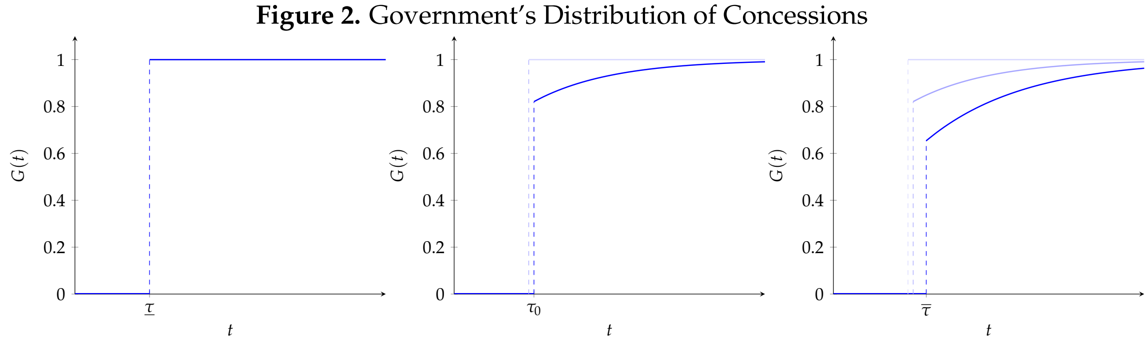
Note that as we increase delay (moving to panels (b) and (c)) participation decreases for every t on the build-up stage. I show that this is in fact a general feature of the equilibrium set, and it implies a very nice property: there exists a one-to-one relation between initial participation π_0, the peak in participation π_{τ_0}, and the time of the first government concession $\tau_0 \in [\underline{\tau}, \overline{\tau}]$.

Figure 1. Equilibrium Participation



(a) Lower bound $\underline{\tau}$ (b) $\tau_0 \in (\underline{\tau}, \overline{\tau})$ (c) Upper bound $\overline{\tau}$

Figure 2. Government's Distribution of Concessions



(a) Lower bound $\underline{\tau}$ (b) $\tau_0 \in (\underline{\tau}, \overline{\tau})$ (c) Upper bound $\overline{\tau}$

The relation between the peak in participation and delay is inverse: longer delay is consistent with a smaller participation at the peak. This suggests the existence of a trade-off between the mass of people that needs to get involved, π_{τ_0}, and the persistence required to make the government concede in equilibrium, τ_0. In particular, participation could grow and hit the constraint very quickly, and in that case it requires a very high participation peak. But it could also have a slow trend upwards, in which case the participation peak is smaller, as the protest can take advantage of the cost of time the government has to bear. We formalize the result in the following corollary.

Corollary 1. *Let $[\underline{\tau}, \overline{\tau}]$ be the set of equilibrium delays. Then, size of the protest at time $t = 0$ is inversely related with the time at which the government makes the first concession, τ_0.*

Moreover, conditional on the first event, there is a unique trajectory of participation. This result gives an idea of how informative it is the first event of a social movement respect to the future trajectory of participation. Fixing the fundamentals, the initial participation is enough to describe the full trajectory of participation, and then the expected duration until government concession.

5. Discussion

5.1. Equilibrium Characterization. A few remarks are in place regarding the characterization result in Theorem 1. First, note that any equilibrium with government concession involves delay. This is the main feature that separates this model from static models of protests and collective action.

I recognize there might be other motivations for this delay. One possibility is learning motives. It could be the case that citizens are uncertain about the value they might obtain from a given policy, and they learn about it from other participants in the protest. These types of situations, however, add another dimension of strategic behavior that is out of the scope of this paper.

The second relevant feature of this characterization, is that every equilibrium with protests has the same set of properties. All of them are characterized by the three stages already described: the build-up, the peak, and the decay stage.

In the equilibria in which the government randomizes, the decay stage unfolds as a war of attrition between the citizens and the government. In this war of attrition, instead of having two players, there is a single large player on one side (i.e. the government), and a continuum of small players on the other, whose aggregate behavior mimics the behavior of a single opponent. Thus, citizens drop out at some deterministic rate, that on aggregate keeps the government indifferent between waiting and conceding. This drop out rate will be determined by an exit threshold, which in turn depends on the hazard rate of government concession. In contrast with most classic wars of attrition, the hazard rate of government concession will not be constant. As for the government to randomize participation must be decreasing, it has to be the case that citizens' expected gains decrease as well, and then, government's hazard rate must decrease over time.

5.2. Multiplicity. As it is natural in a framework with complete information, there are multiple equilibria. However, the multiplicity in this framework provides key ideas about both (i) the dynamics that are common to all equilibria, and (ii) the trade-offs between persistence and participation across them.

The main common dynamic across equilibria is the existence of a building up stage: in all of them there exists a period in which participation is increasing and the government waits. Moreover, in all of them the peak in participation is reached at the time at which the government makes the first (probabilistic) concession.

The trade-off between persistence and participation is related to Corollary 1. Equilibria can be parametrized both in terms of delay in the first concession, $\tau_0 \in [\underline{\tau}, \overline{\tau}]$, and in terms of participation at the peak $\pi_{\tau_0} \in [\pi_{\overline{\tau}}, \pi_{\underline{\tau}}]$. The inverse relation between both suggests a novel trade-off across the equilibrium set. Equilibria in which it takes longer for the peak to be reached, are associated with lower levels of participation at the peak. The relevance of this trade-off lies on its empirical insights. In a dynamic setting, the characterization of a successful protest should combine a *critical mass*, with a *critical persistence*. This is in line with the intuition developed recently by Chenoweth & Belgioioso (2019), who propose that protests can be described by its *momentum*, which is defined as a function of mass (i.e. participation), and velocity (i.e. the frequency of events).

Besides the insights we can learn from this multiplicity, I also argue that multiplicity is a natural feature of the model. As it has been recognized in the literature, the spontaneous nature of mass uprisings gives them the features of a coordination problem which might, or might not, be successful (see Schelling (1960), Hardin (1997), and more recently, De Mesquita (2014)). This implies that in general in static models of collective action there are two equilibria in pure strategies: one in which a protest occurs, and one in which it does not occur. In our model, not only we observe equilibrium with and without protests, but there is a continuum of equilibria in which a protest occurs.

Consider, for instance, a society in which there are workers and students, with students having a lower opportunity cost than workers. It might be, as we observe in many countries, that frequently protests are initially activated entirely by students. As time goes by, workers join the movement. We can say that this society is stuck in the equilibrium with the longest delay in government concession, and the smallest possible initial participation. However, there are many societies in which citizens from all groups are politically active and it is easy for them to coordinate. It could be that protests naturally arise as the result of coordination among workers and students. Initial participation is high, and as more citizens join, the protest forces the government to concede with very high probability. We can say that this society lies in the equilibrium with the shortest delay in government concession. Whether a society focal point centers in one equilibrium or the other depends on features that, despite their relevance, this model does not aim to capture.

5.3. Refinements and Equilibrium Selection. For some problems it might be relevant to refine the set of equilibria. In order to do this, it is key to modify the model to include some sort of incomplete information. The three main approaches that can be applied as refinements are: (i) reputation, (ii) global games, and (iii) coalition proofness.

Reputation concerns in this model arise when there is some information about agents that is private. The attrition nature of the game makes behavioral types a la Abreu & Gul (2000) a natural candidate for refinement. Introducing a probability of the government being a behavioral type who never concedes, and a probability of citizens being some type who will protest forever, will pin down a unique equilibrium. The issue with this refinement is that, in our framework, is not very informative of the equilibrium selected.

Global games are a theoretical framework commonly used to study uprisings and regime change models. Since the seminal work of Morris & Shin (1998), their framework has been used to study public protests and revolutions in different institutional settings (see Edmond (2013), Egorov et al. (2009), Boix & Svolik (2013) and Morris & Shadmehr (2018)). The key component in these models is a coordination game with incomplete information, in which uncertainty in general is about the strength of the regime (although it might as well be uncertainty about preferences or other features of the game). Agents receive some private information, and in equilibrium they use threshold strategies: a player participates if her belief about the revolt being successful is high enough with respect to some threshold. This in general pins down a unique equilibrium. In dynamic setups is not direct to refine the set of equilibria in this way. For instance, Angeletos et al. (2007) study the role of learning in a framework in which agents can take actions many times and learn about the fundamentals. They show that the dynamic nature of the game introduces multiplicity even under conditions that guarantee uniqueness in static games.

Lastly, the possibility of coalition formation by citizens provides another rationale for equilibrium refinement. Naturally, political activism requires some organization that can be done before a protest begins. This could be done in a decentralized way (via social media, for instance) or through a political leader who is interested in fostering a particular equilibrium. If citizens could make pre-arrangements to decide their participation in the protest, it would be possible to coordinate in an equilibrium with short delay, by ensuring a share of the population high enough joins the protest at the beginning. If all citizens are better off with this outcome, we would expect no coalitions blocking that equilibrium, and then it would be coalition-proof in the sense of Bernheim et al. (1987) (see also Moreno & Wooders (1996) and Ray & Vohra (2001)).

Although in coalition-proofness agreements among agents are non-binding, sometimes leaders make some irreversible actions in order to obtain a specific outcome. For instance, Morris & Shadmehr (2018) construct a model in which citizens choose the level of effort to contribute to a regime change, and a leader designs reward schemes that assign psychological rewards to citizens' actions. In our model, a leader could target some sectors in the society in order to implement a particular equilibrium. For instance, when citizens in the protest have a higher opportunity cost, in equilibrium the government concedes at a higher hazard rate. Then the leader might want to design a veteran reward scheme to incentivize participation of people with higher opportunity costs. In practice, leaders make use of their charisma and targeted rhetoric to encourage specific groups of the population to get involved in a revolution. Another alternative is the existence of some organization that implements transfers among citizens, in order to subsidize the protest behavior of specific groups. For instance, in some countries protest organizers support protesters with food and supplies, which can be seen as a way to reduce participation costs to those citizens who attend demonstrations.

6. Extensions

6.1. Changes in the Distribution of Opportunity Costs. In this section I analyze how changes in the distribution of opportunity costs affect equilibrium set. An increase in citizens' opportunity costs has two effects. On one side, it has a direct effect over agents' entry decision, as a citizen with higher opportunity cost will want to wait for the marginal value of entry to increase. On the other side, it has an indirect effect over the government's best reply. In particular, as the opportunity cost of a citizen increases, the hazard rate that is required to make her drop out also increases. This is as if citizens with higher opportunity costs were stronger in front of the government, as they force it to concede faster. The second effect is not observed in the equilibrium with the shortest delay, but it affects the upper bound.

Consider first a general increase in agents' opportunity cost. When citizens' opportunity costs increase, it takes longer to reach the level of participation required to make the government concede with probability 1. This moves the lower bound of the equilibrium set to the right.

Now suppose that instead of a general increase, we apply a mean preserving spread to the distribution of opportunity costs. The effect in this case is ambiguous, as it depends on what happens with the agent who is at the margin when the government is going to concede for sure. We formalize these ideas in the following result.

Proposition 1. *Let F_1 and F_2 be two symmetric and unimodal distributions, with corresponding equilibrium sets $[\underline{\tau}_1, \overline{\tau}_1]$ and $[\underline{\tau}_2, \overline{\tau}_2]$.*

(i) *If F_1 first-order stochastically dominates F_2, then $\underline{\tau}_1 \geq \underline{\tau}_2$.*
(ii) *If F_2 is a mean preserving spread of F_1, and $v'(0) < \int \theta dF_1(\theta)$ then $\underline{\tau}_1 > \underline{\tau}_2$.*
(ii) *If F_2 is a mean preserving spread of F_1, and $v'(0) > \int \theta dF_1(\theta)$ then $\underline{\tau}_1 < \underline{\tau}_2$.*

The results above follow from the fact that the lower bound of the equilibrium set for a distribution F, depends uniquely on $F(v'(0))$. In any equilibrium at which the government concedes for sure, the amount of people who are willing to enter are those with opportunity cost $\theta \leq v'(0)$. Then, any changes to the distribution of opportunity costs that increases the amount of citizens that is willing to enter, forces the government to concede faster.

The effect of a change in opportunity costs over the upper bound is more subtle, as now the indirect effect through the government's hazard rate plays a role. Consider first a general increase in citizens' opportunity costs, so that protesting becomes more costly for every agent. Let F_1 be the initial distribution of opportunity costs, and F_2 be the distribution after the increase. Then, $F_1(\theta) > F_2(\theta)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, and the following result holds.

Proposition 2. *Suppose that citizens opportunity costs increase by the same proportion α, and let $[\underline{\tau}_\alpha, \overline{\tau}_\alpha]$ to be the new equilibrium set. Then, it must be that $\underline{\tau} < \underline{\tau}_\alpha < \overline{\tau}_\alpha < \overline{\tau}$.*

To see the intuition for the change in the upper bound, consider first a hypothetical situation in which the government's strategy remains constant after the change in the distribution of opportunity costs. In that case, agents delay their entry as protesting becomes more costly. But then, in order to give agents incentives to exit according to the indifference participation level, the government has to increase the hazard rate. This new situation cannot be an equilibrium, as the entry time for the lowest opportunity cost citizen would be such that $t_0(\underline{\theta}) > 0$.

6.2. Expected Duration across Equilibria. We would like to know in this setting how do different equilibria relate to each other in terms of welfare.

From the characterization of the equilibrium set in Theorem 2, it follows that the expected duration of a protest is increasing in τ_0. The following corollary formalizes this idea.

Corollary 2. *The expected duration of protests increases with τ_0, and decreases with π_0.*

This property follows from the fact that the distributions of government concessions do not cross. Then, the probability of survival of a protests is monotone in τ_0 for any $t > \tau_0$.

The same holds if we parametrize equilibria by initial participation. This result is intuitive, as when there are more people on the streets, the government concedes earlier.

As citizens care about the public good, if we do not consider the veteran prize, on aggregate they will be better off with the equilibria with the shortest duration and highest initial participation. Moreover, from agents optimality condition we learn that when the protest starts, on any equilibria but the upper bound $\overline{\tau}$, there is a positive mass of agents who are strictly better off by entering the protest. That mass increases as the peak is reached sooner.

Including the veteran prize in a welfare calculation has a non trivial effect. It could be the case that in some cases, after taking the utility obtained from activism into account, citizens are better off in an equilibrium with later concession, because then they get to enjoy their contribution to the social movement.

The government, is always better off with the equilibrium with the longest delay. Beyond τ_0, the government is indifferent between conceding and staring down the protest. However, during the build-up stage, the government is strictly better off staring down the protest than conceding. Then, by delaying concession the government minimizes its overall costs.

6.3. Unconditional Veteran Prize. In this section, I briefly develop an extension of the model, to allow for citizens to obtain rewards from their merits in the protest even if they do not remain protesting until the government ends.

Suppose that if an agent leaves before the government concedes, instead of throwing away the time invested in the protest, she still gets some rewards. In particular, suppose that if a citizen retires before the protest succeeds, she can retain a share $\alpha \in (0,1)$ of the veteran prize. Still, staying until the government concedes is preferable, because in that case agents receive the full veteran prize $v(\tau - t_0)$.

Now, given a random concession time τ for the government, the expected payoff of a citizen with opportunity cost θ is given by:

$$E\left[-\theta \int_{t_0}^{t_1 \wedge \tau} e^{-rs} ds + e^{-r\tau} v(t-t_0)\left[\mathbb{1}_{\tau<t_1} + \alpha \cdot \mathbb{1}_{\tau \geq t_1}\right]\right] \tag{13}$$

where the expectation is taken over τ. Solving for citizens' optimal conditions, a citizen with opportunity cost θ will exit at time t if the following condition holds:

$$\theta \geq (1-\alpha)\lambda_t v(t-t_0) + \alpha \cdot v'(t-t_0) \tag{14}$$

The left hand side is the cost of staying another instant, given by the opportunity cost θ. The right hand side is the expected benefit, which is now a convex combination of the payoff the agent gets by purely staying in the protest, and the payoff the agent gets once the government concede.

Everything else in the game remains the same, but in equilibrium, the government hazard rate in this setup is lower than in the baseline case.

6.4. Unsuccessful Protests. In our baseline model, every protest that occurs in equilibrium will eventually succeed. As information is complete, citizens do not have incentives to enter a protest that will fail, and then in equilibrium those protests do not occur. In reality, however, we see many protests that end before they succeed in reaching their ultimate goal.

In order to study situations with protests that end before the government concedes, we need to be very precise in the definition of the *ending* of a protest. This is crucial to understand how to associate our main equilibrium results with empirical observations about protests failure. However, identifying the ending of a social movement is not an easy task.

One possibility, is to define the *end* of a persistent protest as a decay in participation sufficiently large, so that the movement becomes small, but still some fraction of the population remains participating. My model can perfectly accommodate these situations. To see this, note that our "always concession" result does not state that the protest will be successful in a precise finite time. On the contrary, it states that in the limit the government continues randomizing until eventually concedes. Then, many persistent protests that we interpret as failures, might be part of an equilibrium on a late decay stage, in which still the probability of future government's concession is, although small, positive.

Another possibility, is to understand the end of a protest as a situation in which all protesters give up with no government concession. Unfortunately, with the current specification, this type of behavior cannot be observed as an equilibrium of this model. Citizens know that once the government is randomizing it is just a matter of keep going, and then eventually concession will occur.

Klandermans & van Stekelenburg (2013) study people disengagement from social movements as the result of two possible effects, i.e. *insufficient gratification* and *declining commitment*, that together with a *precipitating event* make the agent leave. In general, these precipitating events are some exogenous shocks that precipitate the exit of agents who already have the intention to leave. I borrow the intuition of this precipitating events, and introduce the possibility of an exogenous shift in agents preferences. The shift is captured by a shock that makes the value of winning to the government become vanishingly small, so that agents no longer have incentives to remain active in the protest. This shock can represent, for instance, news events that shift citizens' attention away from the goals of the social movement, or any possible small event (as in Klandermans & van Stekelenburg (2013)) that just precipitate agents exit.

Suppose with a small constant probability $\delta > 0$ citizens receive a shock that brings the veteran prize to zero. In practice, agents take into account the possibility that they might not enjoy it if the shock is realized, and then it is as if the effective veteran prize was smaller than in the baseline case. Then, citizens' behavior is the same as without the shock. The exit decision for an agent with opportunity cost θ is now given by the following expression:

$$\theta \geq \lambda_t(1-\delta)v(t-t_0(t)) \tag{15}$$

which is just a re-normalization of the original exit condition. The same will happen with entry.

We can show that the equilibrium will have the same features as the baseline model. However, with some small probability the shock realizes and, as soon as this happens, citizens give up and the protest ends immediately. Even when in expectation the behavior is the same, empirically we might observe some scenarios in which people give up with no government concession.

6.5. Income and Opportunity Cost. So far we have characterized agents opportunity cost of the time spent in the protest by a parameter θ. This parameter captures the utility that agents give up by spending time in the protest instead of spending that time on other activities. In general, those other activities are often related to productive activities, and then the opportunity cost can be associated to labor income.

In order to set ideas, consider the following situation. As in our baseline framework, there is a protest and citizens have to decide whether to join, and when to join. A citizen who joins the protest, attends every day a demonstration which lasts one hour (every day is discrete, but consider this just an illustration). There is no physical cost of protesting, and then the only cost to the citizen is the alternative use of time that she can give to this hour, which is equivalent to one hour-wage.

Suppose now agents have heterogeneous income, ω. Let ϵ be the fraction of time an agent spends in the protest (i.e. if the agent works 8 hours each day, and the demonstration lasts one hour, then $\epsilon = 1/8$). In addition, suppose there is a minimum level of consumption that citizens must satisfy, which corresponds to a subsistence level. We can think of this consumption as basic needs that the agent must fulfill, and then she can only afford to join a protest once these basic needs are covered. We represent the subsistence level by a minimum income $\underline{\omega}$, such that any agent with income $\omega < \underline{\omega}$ cannot afford to become an activist.

The cost of attending the protest for a citizen with $\omega \geq \underline{\omega}$ is be equivalent to:

$$\theta = u(\omega) - u\left(\omega\left(1-\epsilon\right)\right) \tag{16}$$

where $u(\cdot)$ is the agent's utility of income (consumption). Then, the relation between citizens' income and opportunity costs depends on the shape of the utility function.

Consider, for instance, the following CRRA utility function:

$$u(\omega) = \frac{\omega^{1-\sigma}}{1-\sigma} \tag{17}$$

for some $\sigma \geq 0$, $\sigma \neq 1$. In this case, the relation between income and opportunity cost depends on the curvature of the utility function, captured by σ. If $\sigma < 1$, then the marginal utility of income is increasing, which implies that for citizens with higher income the hour spent in the demonstration is more costly than for citizens with lower income. In the extreme case with $\sigma = 0$, utility is linear and then $\theta = \epsilon\omega$. In this case there is a one-to-one relation between the distribution of opportunity costs, and the distribution of income. In general, when the marginal utility of income is increasing, high income citizens have incentives to delay their entry compared to those with lower income.[13]

If the opposite holds, i.e. $\sigma > 1$, then the marginal utility of income is decreasing. In this case, high income citizens are able to enter earlier, as the forgone utility for them is lower.

There are other factors that might affect the relation between income and opportunity cost. For instance, job flexibility might affect how workers can make use of their own time. This in general is also related with education and the type of industries under analysis. Moreover, income might affect other components in the propensity of an agent to protest that might not be related with opportunity costs. Education, for instance, is key in how knowledgeable about the political environment citizens are. This implies that when comparing citizens with different income levels we need to also take into account the effect of their income over education levels.

There might be also groups of protesters whose opportunity cost does not have a mapping to income at all. Students are an example of that case. Students' participation in protests is in general very high, which of course reflects many factors such as energy levels. But a relevant factor comes from the fact that the opportunity cost of their time tend to be lower than workers, as they have more freedom to allocate their time across activities.

6.6. Government Partial Concessions. In many situations, the decision to provide a public good is not discrete. Authorities might make some concessions that do not completely fulfill protesters' demands, but that dissuade some of them, and then they alleviate the

[13] In Section 7 I explore these predictions for the case of *Black Lives Matter* protests.

cost burden of the protest. As an example, suppose protesters' demand is a stimulus package to provide economic assistance. The government, instead of agreeing to the full package, might instead decide to provide the stimulus package, but in a smaller amount to what protesters demand. This might be enough for some agents, who then decide to leave the protest, whereas others continue protesting to exert pressure for the government to provide the full stimulus package. In this section, I illustrate how the baseline model can be modified in order to allow for such concessions.

Suppose the government can concede a fraction of the public good. Conceding a fraction α of the public good has a cost αq, where q is the cost of the entire public good. Every time the government concedes a fraction α of the public good, agents receive a flow utility $\alpha v(t - t_0)$, corresponding to their veteran payoff. Other than this, citizens' and government's payoffs remain the same as in the baseline case. The protest ends when either all citizens have dropped out, or the government has fully provided the public good.

Following the same reasoning as in the baseline case, we can define the government's strategy as a function $h : [0, \infty) \to [0, 1]$ that determines, for any time t, the additional share of public good that the government provides at time t. We also denote by $H(t)$ the share of public good that has been provided at time t.

A citizen's payoff from entering at a time t_0 and exiting at t_1 is given by:

$$U(t_0, t_1; \theta) = -\theta \left[\frac{e^{-rt_0} - e^{-rt_1}}{r} \right] + \int_{t_0}^{t_1} e^{-rs} v(s - t_0) dH(s) \tag{18}$$

As in the main model, citizens' utility functions satisfy a single-crossing property, and then their strategies are monotone in opportunity cost. In particular, entry and exit conditions for a citizen with opportunity cost θ are given by:

$$\theta = \int_0^{t_1} e^{-rs} v'(s - t_0) dH(s) \tag{19}$$

$$\theta = v(t_1 - t_0) h(t_1) \tag{20}$$

Then, we obtain analogous characterization results as those in the baseline model. For any equilibrium $H : [0, \infty) \to [0, 1]$, $(\pi_t^\sigma)_{t \geq 0}$ with $\tau_0 = \inf\{t \in [0, \infty] : h(t) > 0\}$, the following conditions hold:

(i) There is always delay in government concession, i.e. $\tau_0 > 0$.
(ii) π_t^σ is continuous, increasing for $t \leq \tau_0$, and if $H(\tau_0) < 1$, decreasing for all $t \geq \tau_0$.
(iii) The government makes at most one discrete concession at τ_0, $H(\tau_0) \leq 1$.
(iv) If $H(\tau_0) < 1$, then $H(t)$ is strictly increasing, concave, and for $t > \tau_0$ $H(t) < 1$.

6.7. Support for the Public Good. In our baseline model we assume the unit mass of citizens is willing to consider participation in the protest, as the only constraint is the heterogeneity in opportunity costs. In this section we illustrate the case in which only a subset of agents is willing to consider participating in the protest. Suppose the value of the public good x is now a random variable that can take two values, 0 or 1, and let p be the probability of $x = 1$. Each citizen's value for the public good is independent of her opportunity cost.

Moreover, assume citizens only value the veteran prize if they value the public good. Thus, we modify the veteran prize function to be $x \cdot v(t - t_0)$. With this new framework, agents who don't value the public good do not have incentives to participate in the protest. It is clear to see that the baseline case is equivalent to setting $p = 1$, and then as p decreases, the mass of citizens willing to enter the protest also decreases.

What is interesting about this perturbation, is that all the dynamics of the model remain the same, but the set of equilibria is reduced. For the government, the properties shown for the baseline case still hold: (i) the government concedes according to a distribution $G(t)$ with support $\mathcal{T}$, with $\tau_0 = \inf \mathcal{T}$; (ii) the support might be either a singleton, or an interval $[\tau_0, \infty)$; (iii) $G(\tau_0) < 1$ then $\mathcal{T} = [\tau_0, \infty)$, and $G(t)$ is continuous, strictly increasing, and differentiable in (τ_0, ∞).

Any citizen with opportunity cost θ solves the exact problem as in the baseline model. Citizens' strategies can be characterized by thresholds $\tilde{\theta}_0(t), \tilde{\theta}_1(t)$ such that a citizen enters if $\theta \leq \tilde{\theta}_0(t)$ and exits if $\theta > \tilde{\theta}_1(t)$. For any possible entry threshold $\tilde{\theta}_0(t)$, participation is just a re-scaling of the original problem, and is given by $\pi_t = p \cdot F(\tilde{\theta}_0(t))$. Since the hypothesis are analogous to the baseline case, both Theorem 1 and Theorem 2 hold in this framework.

We highlight some of the main features that differentiate this case with respect to the baseline case. Let $[\underline{\tau}, \overline{\tau}]$ be the equilibrium set in the baseline case, and denote by $[\underline{\tau}_p, \overline{\tau}_p]$ the equilibrium set with $p < 1$. First, note that it has to be that $\underline{\tau} < \underline{\tau}_p$ and $\overline{\tau}_p \leq \overline{\tau}$. The intuition is analogous to the comparative statics in Proposition 1. To see why the lower bound is delayed with $p < 1$ (i.e. $\underline{\tau} < \underline{\tau}_p$), recall that this corresponds to the equilibrium in which the government concedes with probability 1. When not all agents are willing to participate, it takes more time to make the government concede.[14] The upper bound it does not necessarily decreases as it depends on the citizen with the lowest opportunity cost.

Consider now an intermediate equilibrium, with delay $\tau_0 \in (\underline{\tau}_p, \overline{\tau}_p)$. Let $G(t)$ be the government distribution of concessions with $p = 1$, and $G_p(t)$ for $p < 1$. Note that

[14]Formally, concession occurs when $\tilde{\pi}_{\underline{\tau}} = F(v'(0))$. When not all agents are willing to participate, then$\tilde{\pi}_{\underline{\tau}_p} = p \cdot F(v'(0)) < \tilde{\pi}_{\underline{\tau}}$.

both $G_p(t)$ and $G(t)$ have support $[\tau_0, \infty)$. Moreover, in both equilibria participation must coincide on $[\tau_0, \infty)$. Then, the initial government concession is such that $G(\tau_0) < G_p(\tau_0)$.

The main idea of these differences, is that now the universe of citizens is smaller, as not all of them are willing to enter the protest. However, conditional on reaching some participation level, it has to be that the ones who are protesting have (a weakly) higher opportunity costs (with respect to the baseline case), and that makes them stronger in front of the government.

7. Empirical Predictions: Entry, Exit, and Opportunity Costs

The equilibrium characterization developed in Section 4 provides us with some testable empirical predictions. First, participation is hump-shaped. We should always observe, at least, an initial stage with increasing participation. If the protest does not end when it reaches its peak, participation then should decay over time. Second, the time at which agents join the protest increases with opportunity cost. And third, the time at which agents leave the protest decreases with opportunity costs.

In this section, we focus on the predictions regarding the timing of protests and the opportunity cost. These predictions are informative of some regularities in agents behavior that, to the best of my knowledge, have not been documented. The relevance of these regularities, lies in the fact that monotonic agents' behavior, meaning their decisions can be ranked according to some variable, allows us to also better understand the dynamics of the characteristics of people on the streets. If people with higher opportunity costs enter later, that implies that as participation increases, agents who are protesting are paying a higher cost for being there, and then it is going to be more costly to make them go home.

The *Black Lives Matter* protests occurring this year provide a framework to study these ideas. There are many reasons for why focusing on this movement. First, note that this movement gathers the support of a broad sector of the population. Most protests were pacific for the period analyzed. Also, the protests were detonated by an event whose date of occurrence is exogenous, in contrast with other movements that are initiated as a response of changes in government policies and political reforms. In this case, the murder of Mr. George Floyd was the tipping point that woke up the outrage of masses against systemic racism.

Ideally, to test these predictions we would need data on individual participation over time and opportunity costs. However, as individual participation is not directly observed, I begin by mapping our framework with individual decisions, to a framework with aggregate data.

7.1. Mapping Individual Decisions to Aggregate Level. Consider a unit mass of citizens, distributed over a set of J counties. Let μ_j to be the mass of citizens living at county j. As in the baseline model, each citizen is characterized by an opportunity cost of protesting. The opportunity cost of citizen i at county j is the sum of an aggregate component θ_j, which is common to all citizens living at county j, and an idyosincratic component that we do not observe, ϵ_i. We assume this unobserved component is *iid* across the population.

Given that citizens' entry decisions are monotone in opportunity cost, for any time t there exists an entry threshold $\tilde{\theta}_0(t)$ such that a citizen protests if $\theta_i \leq \tilde{\theta}_0(t)$, and waits otherwise. Thus, participation at county j is given by:

$$\pi_{j,t} = P\left(\theta_j + \epsilon_i < \tilde{\theta}_0(t)\right) \cdot \mu_j \tag{21}$$

This is, the size of the county times the probability of participation.

Let $\overline{\pi}_{j,t} = \frac{\pi_{j,t}}{\mu_j}$ be the adjusted participation level at time t in county j. From equation 21, we obtain the following observation.

Claim 1. *If citizens' decisions are monotone in opportunity cost, for every time t, adjusted participation $\overline{\pi}_{j,t}$ is decreasing in θ_j.*

To take this to an *entry time*, define for any $p \in [0,1]$, the time $t_0^j(p)$ as the first time at which the adjusted participation crosses the level p. Analogously, define the *exit time* $t_1^j(p)$, as the last time at which the adjusted participation crosses the level p. Formally,

$$t_0^j(p) = \min\{t : \pi_{jt} \geq p\}; \qquad t_1^j(p) = \max\{t : \pi_{jt} \geq p\} \tag{22}$$

We can then state our empirical predictions as follows.

Prediction 1. For any $p \in [0,1]$, the entry time $t_0^j(p)$ is increasing in θ_j.

Prediction 2. For any $p \in [0,1]$, the exit time $t_1^j(p)$ is decreasing in θ_j.

Given that the distribution of entry and exit times is skewed, we will use the logarithm of these times as our dependent variables.

7.2. Data. To measure participation, we use data from the Crowd Counting Consortium. This consortium collects publicly available data on political crowds reported in the United States, including marches, protests, strikes, demonstrations, riots, and other actions.[15] In particular, it also includes data from the CountLove.org initiative,[16] which gathers events and participation records from media sources.

[15] The CCC data can be found here: CCC Website.

[16] The CountLove.org data can be found here: CountLove Website.

To analyze entry times, we use events occurring between May 26, and June 30 2020. There is a record of $7,707$ events across the US in favor of the civil rights movement on the period of analysis.[17]

Our main measure of opportunity cost uses Dingel & Neiman (2020)'s teleworkability shares by 2-digit NAICS. In order to construct the measure, we combine their estimation of teleworkability, with county-level employment data from the Quarterly Census of Employment and Wages, from the Bureau of Labor Statistics.

For Covid-19, we use data on new cases by county from USAFacts.org. In addition, to account for predicted cases by county, we use forecasts for the number of cases from the Shaman Group, Columbia University.[18] Finally, we include county level demographics from the MIT Election Data and Science Lab.

7.3. Timing of Protests and Opportunity Costs. It is not an easy task to come up with a measure of opportunity cost that captures its essence in this context. The alternative use of the time spent in a demonstration depends on many factors, including the organization of a city over the space, and different lifestyles. However, it is very likely that the opportunity cost of time is related with the type of occupation. More flexible occupations give workers the chance to manage their time more freely, whereas tight schedules tend to make it harder to fit other activities on a daily basis.

To set ideas, we begin by studying the relation between the timing of entry, and two factors affecting opportunity costs: income and education. Both are imperfect measures of opportunity costs, and in the next section we make efforts to capture this more precisely.

The effect of income over opportunity costs will depend, as we show in Section 6.5, on the shape of the utility function. If the marginal utility of income is increasing, we expect higher income to be consistent with a higher opportunity cost. Thus, we would expect higher income counties to enter later, and exit earlier.

The effect of education is more subtle. It is, of course, not the case that for less educated citizens is more costly to attend a demonstration per se. But, it is often the case that people with lower education levels have jobs that rely more on physical activities and that tend to be less flexible in terms of time management. Also, lower formality of the job makes it harder to realize activities during work days, such as protesting. We would then expect

[17]Although the data entry for the Crowd Counting Consortium keeps going, in this draft we use data downloaded on October 1st 2020, as we plan to keep updating. The partial entries might generate lower estimates of participation than what we would get using the finalized data.

[18]The Shaman Group provides forecasts for daily new confirmed cases and daily new infections. See Pei & Shaman (2020), and their website Shaman Group Website.

that people with lower education levels have a higher opportunity cost, and then enter later.

In Table 1 we explore the relation between these two variables, and entry and exit times. In the first three regressions, the dependent variable is the (logarithm of the) time at which the respective entry threshold is crossed, whereas in the last three the dependent variable is the time of exit. As we can see, both the share of the population with less than a college degree, and the median household income are positively correlated with entry times for every threshold level p.

What is even more interesting is that with exit these effects are reversed for every threshold. Not only counties with a higher income take more time to join the protest but they also exit faster.[19]

Table 1. Entry and Exit (OLS)

	Entry $\log(t_0(p))$			Exit $\log(t_1(p))$		
Threshold p	0.001%	0.005%	0.1%	0.001%	0.005%	0.1%
UNDER COLLEGE	0.0125***	0.0150***	0.0142***	-0.0113***	-0.0106***	-0.00899***
	(0.00329)	(0.00341)	(0.00359)	(0.00185)	(0.00196)	(0.00193)
INCOME (10K)	0.135***	0.137***	0.107***	-0.0426***	-0.0326**	-0.0253*
	(0.0254)	(0.0259)	(0.0264)	(0.0137)	(0.0143)	(0.0138)
Other Controls	X	X	X	X	X	X
State FE	X	X	X	X	X	X
Observations	1108	1083	1024	1129	1093	1030

Standard errors in parentheses;* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Controls: CLINTON, LABOR, UNDER 29, POPULATION (LOG), DENSITY (LOG), RURAL, BLACK.

A natural concern it is the fact that these estimates might be biased due to omitted variables. There are many other variables related to a county's income and educational level that are also related with a propensity to protest. But note that we are not estimating the effect of income and education over *the level of participation* in protests, our dependent variables are the *timing* of entry and exit. Even when income might be highly correlated with other intrinsic qualities that determine a propensity to protest, these other factors would also have to be related with agents delaying entry and exiting earlier.

In the next section, we explore different ways to capture the level of opportunity costs of protesting.

7.3.1. Entry and Job Flexibility. Following the intuition developed above, I conjecture that a higher share of workers in flexible occupations is consistent with a lower opportunity cost. To measure flexibility, I first use Dingel & Neiman (2020)'s teleworkability shares. In

[19] In the Appendix we illustrate the relation with a scatter plot with the same controls as in Table 1.

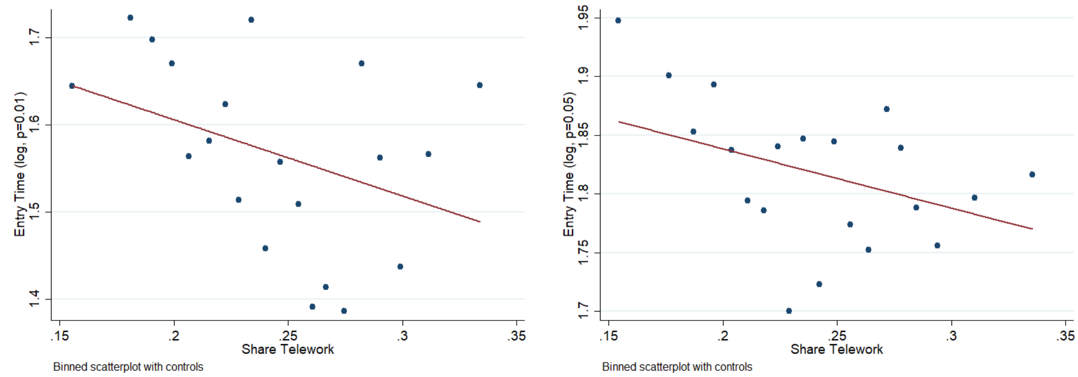
their recent study, the authors determine, for each occupation, the feasibility of working from home. To do this, they take into account different features of the work, as the use of physical activities, level of risk, or required machinery. We follow the same ideas than Glaeser et al. (2020), and combine Dingel & Neiman (2020)'s teleworkability shares with county level employment data at the 2-digit NAICS. As a result we obtain the variable TELEWORK, that corresponds to the share of workers in occupations that can be done from home. The following figure shows the distribution of the variable for all counties.

Figure 3. Histogram of Telework

Figures 4 and 6 show the relation between TELEWORK, and the dependent variables for entry and exit for two thresholds, $p = 0.01\%$ and $p = 0.05\%$. The scatterplots include demographic controls (see Table 4 for a full list of controls). A higher share of workers in occupations more *teleworkable*, is consistent with more job flexibility and then it lowers the opportunity cost. Figure 4 shows that lower opportunity cost (higher share of telework) is consistent with entering earlier (lower threshold on the vertical axis). The opposite holds for exit, although the relation seems weaker. Figure 6 shows that lower opportunity cost (higher share of telework) is consistent with exiting later (higher threshold on the vertical axis).

As this movement started in the middle of a pandemic, counties economies were affected by lockdowns and economic restrictions. I exploit the variation in counties reopening status, to capture the differential effect of job flexibility for open and closed counties. The variable OPEN is a dummy that takes value 1 if the county had reopened by May 25. We expect having more flexible positions makes entry happen earlier if the county is open.

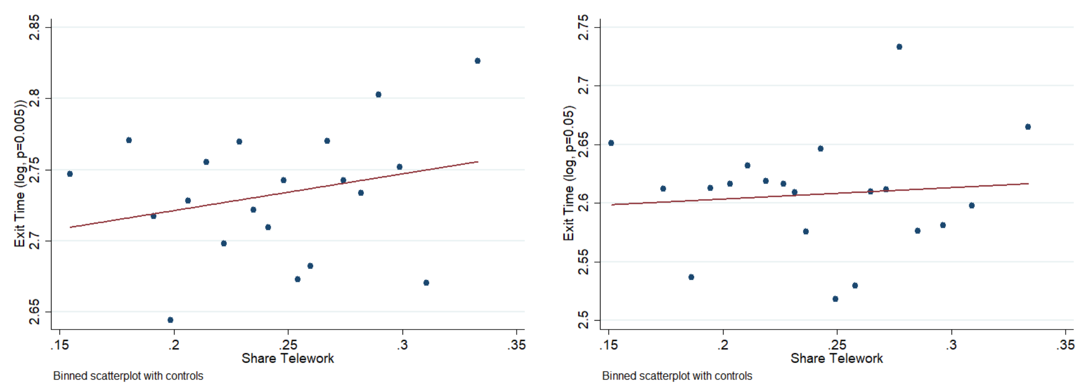
Figure 4. Entry and Share of Teleworkability Jobs with Controls



(a) Entry $t_0(0.01\%)$ **(b)** Entry $t_0(0.05\%)$

Controls: INCOME (10K), UNDER COLLEGE, CLINTON, LABOR FORCE, UNDER 29, POPULATION (LOG), DENSITY (LOG), RURAL, BLACK.

Figure 6. Exit and Share of Teleworkability Jobs with Controls



(a) Exit $t_1(0.001\%)$ **(b)** Exit $t_1(0.01\%)$

Controls: INCOME (10K), UNDER COLLEGE, CLINTON, LABOR FORCE, UNDER 29, POPULATION (LOG), DENSITY (LOG), RURAL, BLACK.

The intuition is that if the county is closed, the difference in job flexibility is less relevant as most activities are constrained.

We estimate the following regression for entry

$$\log(t_0(p\%)) = \beta_0 + \beta_1 \text{OPEN} + \beta_2 \text{TELEWORK} + \beta_3 \text{OPEN} \times \text{TELEWORK} + \beta_4 X_j + \beta_5 \text{STATE} + \epsilon_j \tag{23}$$

The results can be found in Table 2, and the description of controls in Table 4. As it can be seen, a higher share of workers in teleworkable occupations implies earlier entry for counties that were open by May 25. This suggests that having more flexibility only makes a difference if the county is open. Intuitively, a county being closed makes workers condition more equal, as workers see themselves in the same condition regardless of their type of occupation. Moreover, overall open counties enter later, which is consistent with the cost of time being higher when citizens have to go to work.

It is worth mentioning the relation between the findings shown in Table 2 and 1. Citizens with higher income tend to have works that rely less on physical activities and working with other people. However, the effect of having a higher share of workers on teleworkable occupations goes in the opposite direction of the effect of income. The intuition for this is that both capture different components of the opportunity cost: the value of time, and the flexibility of it.

To take care of the possibility of omitted variable bias, we add a set of county controls and fixed effects by state. As Dingel & Neiman (2020) report, the extent to which an activity can be done from home is related with demographics, such as age, income, and education. Activities who are not possible done from home are those that involve physical efforts and higher risks, which tend to be the ones with lower wages.

In Table 5 in the Appendix, I progressively add controls to evaluate the sensitivity of the coefficient and the R^2. this procedure has been done in the context of conflicts by Bellows & Miguel (2009), and has been recently formalized by Oster (2016). I do this specifically for the case of $p = 0.01\%$. The main reasons to expect the estimate to be biased is its correlation with education and income. The regression including only state fixed effects is shown in the first column of table 5. As we can see, the coefficients' significance remain. The R^2 is low, but it is reasonable as there are many other features of the timing of protests that job flexibility does not explain. In the second column we add a set of controls. In the third column, we add the share of young people in the county and the share of people with less than a college degree. As is expected, this variable makes entry happens earlier, but it does not affect the significance of the first two variables. Both the coefficients and the R^2 remain relatively stable as we add education and rural population in column 4, and finally income in column 5.

7.4. Entry and Incidence of COVID-19. In order to control for the risk of infection associated to attending a demonstration, I introduce controls for the level of COVID-19 before the start of the protests. The evidence suggests the risk of infection affects people's behavior even when there are no restrictions imposed. Recently, de Bruin & Bennett (2020) conducted a national survey, and find evidence suggesting that citizens adopt social distancing behavior, such as avoiding crowds, when they perceive risk of COVID-19 infection. We expect then a higher level of COVID-19 incidence to delay entry. In order to

Table 2. Entry: Telework and Reopening

	Dependent Variable: $\log(t_0(p))$				
Threshold p:	(1) 0.001%	(2) 0.005%	(3) 0.01%	(4) 0.03%	(5) 0.05%
OPEN×TELEWORK	-0.823	-1.998**	-2.616***	-2.365**	-1.956*
	(0.964)	(0.964)	(1.008)	(0.944)	(1.115)
OPEN	0.267	0.646**	0.849***	0.816**	0.796**
	(0.292)	(0.297)	(0.311)	(0.322)	(0.374)
TELEWORK	-0.0580	0.911	1.435	0.682	1.220
	(0.951)	(0.944)	(0.995)	(0.966)	(1.095)
County Controls	X	X	X	X	X
State FE	X	X	X	X	X
Observations	1108	1083	1024	769	555

Standard errors in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Controls: INCOME (10K), UNDER COLLEGE, CLINTON, LABOR FORCE, UNDER 29, POPULATION (LOG), DENSITY (LOG), RURAL, BLACK.

measure the incidence of the virus, we use the daily cases forecast for May 25, reported on May 14 by the Shaman Group (variable COVID in the regression below).

We estimate the following regression for entry

$$\log(t_0(p\%)) = \beta_0 + \beta_1 \text{OPEN} + \beta_2 \text{TELEWORK} + \beta_3 \text{OPEN} \times \text{TELEWORK} + \beta_4 \text{COVID} + \beta_5 X_j + \beta_6 \text{STATE} + \epsilon_j \tag{24}$$

The results can be found in table 3. The incidence of COVID-19 delays entry, and although coefficients are small, the effect is significant for thresholds above 0.01%.

Figure 8 shows the relation between COVID, and the dependent variable for entry, for two thresholds, $p = 0.01\%$ and $p = 0.05\%$. The scatterplots include demographic controls (see Table 4 for a full list of controls).

A. Appendix: Proofs

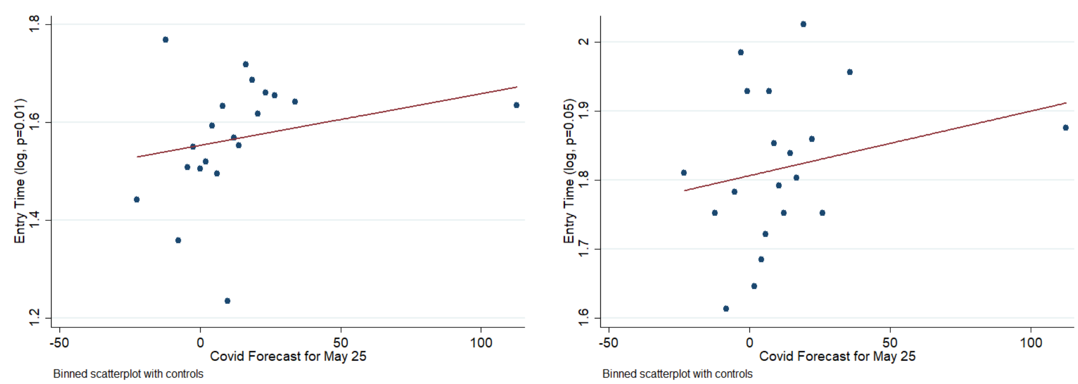
A.1. Proof of Theorem 1: Equilibrium Characterization. This section is devoted to prove the equilibrium characterization described in Section 2. We begin by proving some properties of the government's equilibrium strategy, and then we use these to fully characterize the set of equilibria. In the first lemma, we show that if there is an interval after τ_0 in which the government does not concede (i.e. the distribution G is constant in that interval), then no agent who is in the protest drops out during that interval. More precisely, we say that an agent with opportunity cost θ is participating at a time t if $\sigma_t^\theta = 1$.

Table 3. Entry: Telework and Reopening

	Dependent Variable: $\log(t_0(p))$				
	(1)	(2)	(3)	(4)	(5)
Threshold p:	0.001%	0.005%	0.01%	0.03%	0.05%
COVID	0.000568	0.000266	0.00102*	0.00110*	0.00147**
	(0.000507)	(0.000504)	(0.000529)	(0.000565)	(0.000654)
OPEN×TELEWORK	-0.787	-1.982**	-2.564**	-2.285**	-1.744
	(0.959)	(0.965)	(1.015)	(0.941)	(1.074)
OPEN	0.258	0.641**	0.834***	0.790**	0.734**
	(0.291)	(0.298)	(0.313)	(0.323)	(0.363)
TELEWORK	-0.0734	0.905	1.417	0.634	1.051
	(0.948)	(0.945)	(1.001)	(0.963)	(1.063)
County Controls	X	X	X	X	X
State FE	X	X	X	X	X
Observations	1108	1083	1024	769	555

Standard errors in parentheses;* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Controls: INCOME (10K), UNDER COLLEGE, CLINTON, LABOR FORCE, UNDER 29, POPULATION (LOG), DENSITY (LOG), RURAL, BLACK.

Figure 8. Entry and COVID-19, with Controls



(a) Entry $t_0(0.01\%)$ **(b)** Entry $t_0(0.05\%)$

Controls: INCOME (10K), UNDER COLLEGE, CLINTON, LABOR FORCE, UNDER 29, POPULATION (LOG), DENSITY (LOG), RURAL, BLACK.

Lemma 1. *Assume $\tau_0 < \tau_1$ and take t_1, t_2 such that $\tau_0 \leq t_1 < t_2 \leq \tau_1$. If G is constant in (t_1, t_2), then no agent participating at t_1 drops out in $(t_1, t_2]$.*

Proof. For any citizen that is participating at t_1, she is strictly better off quitting at t_1, than at any $t \in (t_1, t_2]$. □

Lemma 2. *The support of G is either a singleton, or a connected interval $\mathcal{T} = [\tau_0, \tau_1]$.*

Proof. By contradiction, suppose there exists $t \in [\tau_0, \tau_1]$ such that $t \notin \mathcal{T}$. Then, $t > \tau_0$, and there exists $\epsilon \in (0, t - \tau_0]$ such that $G(t) - G(t - \epsilon) = 0$. But then $[t - \epsilon/2, t] \cap \mathcal{T} = \emptyset$, so if there is $t \notin \mathcal{T}$, there is an interval which does not belong to $\mathcal{T}$. Then take t_0, t_1, with $\tau_0 \leq t_0 < t_1 \leq \tau_1$ such that $G(s) = G(t_0)$ $\forall s \in [t_0, t_1)$.

Assume $[t_0, t_1)$ is maximal, i.e. there is no interval $[t'_0, t'_1)$ such that $[t_0, t_1) \subsetneq [t'_0, t'_1)$ and $G(s) = G(t'_0)$ for every $s \in [t'_0, t'_1)$. Maximality of the interval implies that $t_0 \in \mathcal{T}$. If not, there exists $\epsilon_1 > 0$ such that $G(t_0) - G(t_0 - \epsilon_1) = 0$, but then $G(s) = G(t_0 - \epsilon)$ $\forall s \in [t_0 - \frac{\epsilon}{2}, t_1]$. By maximality, for every $\epsilon > 0$ $[t_1, t_1 + \epsilon) \cap \mathcal{T} \neq \emptyset$. Then, it is optimal for the government to concede at t_0 and at t_1.

Note that for the government to concede at t_0 the cost of conceding must less than or equal than the cost of waiting. The cost of conceding at t_0 is $\frac{q}{r}$, while the cost of waiting to concede at some $t_0 + \delta$ for $\delta > 0$ is given by

$$\int_0^\delta e^{-rs} c(\pi^\sigma_{t_0+s}, t_0 + s) ds + e^{-r\delta} \frac{q}{r} \tag{25}$$

Then, we have:

$$\int_0^\delta e^{-rs} c(\pi^\sigma_{t_0+s}, t_0 + s) ds + e^{-r\delta} \frac{q}{r} \quad \geq \quad \frac{q}{r} \qquad\qquad \forall \delta > 0 \tag{26}$$

or, equivalently

$$\int_0^\delta e^{-rs} \left(c(\pi^\sigma_{t_0+s}, t_0 + s) - q \right) ds \quad \geq \quad 0 \qquad\qquad \forall \delta > 0. \tag{27}$$

Define $\bar{t} = \frac{t_0 + t_1}{2}$. Note that as 27 holds for every $\delta > 0$, it must also hold for $\bar{\delta} = \bar{t} - t_0$.

Since $t_0 \in \mathcal{T}$, then it must be that $\pi^\sigma_{t_0} > 0$, as otherwise the cost of the protest is zero. Moreover, by lemma 1, no citizen drops out at $(t_0, t_1]$, so $\pi^\sigma_{\bar{t}+s} \geq \pi^\sigma_{t_0+s}$ for all $s \in (0, \bar{\delta}]$. As, $\pi_{t_0} > 0$, then the cost is strictly increasing in time, and we have:

$$c(\pi^\sigma_{t_0+s}, t_0 + s) < c(\pi^\sigma_{\bar{t}+s}, \bar{t} + s) \qquad\qquad \forall s \in (0, \bar{\delta}] \tag{28}$$

Then, we can compute:

$$\begin{aligned} \int_{t_0}^{t_1} e^{-r(s-t_0)} \left(c(\pi^\sigma_s, s) - q \right) ds \quad &= \quad \int_{t_0}^{\bar{t}} e^{-r(s-t_0)} \left(c(\pi^\sigma_s, s) - q \right) ds \\ &+ \quad e^{-r(\bar{t}-t_0)} \int_{\bar{t}}^{t_1} e^{-r(s-\bar{t})} \left(c(\pi^\sigma_s, s) - q \right) ds \end{aligned} \tag{29}$$

The first term on the right hand side is weakly greater than 0. By 28 the second term must then be strictly greater than zero, which implies $\int_{t_0}^{t_1} e^{-rs}\left(c(\pi^{\sigma}_{t+s}, t+s) - q\right) ds > 0$. But then the government strictly prefers to concede at t_0 than at t_1, which is a contradiction. □

Lemma 3. *If $\mathcal{T}$ is not a singleton, then it must be that $c(\pi^{\sigma}_s, s) = q$ and $\pi^{\sigma}_s = \tilde{\pi}_s$ for every $s \in [\tau_0, \tau_1)$.*

Proof. For the government to be randomizing over concession times $\tau \in [\tau_0, \tau_1]$, it must be that:

$$\int_0^{\tau} e^{-rs} c(\pi^{\sigma}_s, s) + e^{-r\tau}\frac{q}{r} = a \qquad \forall \tau \in [\tau_0, \tau_1] \tag{30}$$

for some constant a. Taking first order conditions with respect to τ, we obtain $c(\pi^{\sigma}_{\tau}, \tau) - q = 0$, which proves the result. □

Lemmas 2 and 3 provide a characterization of the regions over which the government concedes. The problem for the government is a stopping time problem, in which we allow the government to randomize. For citizens the problem is a little different. Given that we do not impose restrictions on the actions that citizens can take, they could enter and exit the protest many times. So far there is nothing that prevents a citizen to protest over a time interval, then drop out to spend some time outside the protest, and then protesting again. However, we show that in equilibrium citizens enter and exit at most once. In particular, their optimality conditions satisfy a monotonicity property with respect to opportunity cost, that ensures that citizens' strategies can be characterized by opportunity cost thresholds. In lemma 4 we give some sufficient conditions for these entry and exit times to be optimal. Optimality conditions are stated in terms of the hazard rate of government concession, $\lambda_t = \frac{g(t)}{1-G(t)}$, which corresponds to the instantaneous probability of government concession conditional on the it being still in the game.

Lemma 4. *In equilibrium, citizens enter and exit at most once. For a person with opportunity cost θ who does enter, the optimal entry and exit times, $t_0(\theta), t_1(\theta)$ are a solution to the following sufficient conditions:*

$$\theta = \lambda_{t_1} v(t_1 - t_0) \tag{31}$$

$$\theta = \frac{1}{1-G(t_0)} \int_{t_0}^{t_1} e^{-r(s-t_0)} v'(s-t_0) dG(s) \tag{32}$$

Moreover, optimal entry and exit times satisfy $t_0'(\theta) > 0$ and $t_1'(\theta) < 0$, respectively.

Proof. Consider a citizen with opportunity cost θ who is planning to enter, on the equilibrium path, at some time t_0 and exit at t_1, i.e. $\sigma^{\theta}_t = 1$ for $t \in [t_0, t_1)$. Given a random concession time τ for the government, the citizen solves the following problem:

$$\max_{(t_0,t_1)\in[0,\infty]^2} E\left[-\theta\int_{t_0}^{t_1\wedge\tau} e^{-rs}ds + e^{-r\tau}\mathbb{1}_{\tau<t_1}v(t-t_0)\right] \tag{33}$$

where the expectation is taken over τ, and where we have omitted additive payoffs that are not under the agent's control. Plugging in the distribution of government concessions G the objective function can be rewritten as:

$$U(t_0,t_1;\theta) = \int_{t_0}^{t_1}\left[-\frac{\theta}{r}(e^{-rt_0}-e^{-rs}) + e^{-rs}v(s-t_0)\right]dG(s) - (1-G(t_1))\frac{\theta}{r}(e^{-rt_0}-e^{-rt_1}) \tag{34}$$

As long as an agent is in the protest she has to pay the cost of the protest. If the government concedes before the time she drops out, the citizen gets the veteran reward. If the government has not conceded by the time the agent drops (which happens with probability $(1-G(t_1))$), then the agent only pays the cost of the protest and does not get any prize. Taking first order conditions with respect to t_0 and t_1, we have:

$$\frac{\partial U}{\partial t_0} = -(1-G(t_0))\theta + g(t_0)v(0) + \int_{t_0}^{t_1} e^{-r(s-t_0)}v'(s-t_0)dG(s) \tag{35}$$

$$\frac{\partial U}{\partial t_1} = -\theta e^{-rt_1}(1-G(t_1)) + g(t_1)e^{-rt_1}v(t_1-t_0) \tag{36}$$

Reorganizing, we obtain equations 31 and 32 from the lemma. Note that these equations have a unique solution.

The fact that first order conditions are also sufficient follows from a single-crossing property of agents utility with respect to opportunity cost. In particular, the marginal utilities of agents' strategies are monotone in θ, i.e.

$$\frac{\partial^2 U}{\partial t_0\partial\theta} = e^{-rt_0}(1-G(t_0)) \geq 0 \qquad \frac{\partial^2 U}{\partial t_1\partial\theta} = -e^{-rt_1}(1-G(t_1)) \leq 0 \tag{37}$$

Thus, citizens follow monotone strategies satisfying $t_0'(\theta) > 0$, $t_1'(\theta) < 0$.

Now, suppose an agent is considering to reenter. Note that once the agent exits, her problem becomes the same from equation 33, as the veteran payoff goes back to zero. But then by the single crossing property we just proved reentry cannot be optimal. This concludes the proof. □

From equation 31 we see that an agent will exit when the marginal cost of staying another instant, i.e. θ, exceeds the marginal benefit, i.e. the prize times the instantaneous probability of government concession conditional on the government being still in the game. Equation 32 has a similar interpretation: the agent enters if the marginal cost is smaller than the marginal benefit. The marginal benefit now has two components. The first term in the right hand side captures the probability of obtaining the prize immediately, while the second one corresponds to the marginal benefit obtained from increasing the prize for all future periods that the agent plans to protest.

From Lemma 4, at any time agents' decision can be characterized by opportunity cost thresholds. More precisely, define $\tilde{\theta}_0(t) = t_0^{-1}(t)$, and note that this corresponds to the agent who is indifferent between entering at time t or waiting (i.e. equation 32 holds with equality). Any citizen with opportunity cost $\theta < \tilde{\theta}_0(t)$ is strictly better off by being in the protest. Analogously define $\tilde{\theta}_1(t) = t_1^{-1}(\theta)$, and note that it corresponds to the agent who is indifferent between staying in the protest another instant or exit immediately. Any citizen with $\theta > \tilde{\theta}_1(t)$ is strictly better off by dropping out.

We now put this ingredients together to prove Proposition 1 using the following steps.

Step 1: If $\tau_0 < \tau_1$, then π_t^σ is strictly decreasing in t, for every $t \in [\tau_0, \tau_1)$. From lemma 3, it must be that $c(\pi_t^\sigma, t) = q$ at every $t \in [\tau_0, \tau_1)$. Then, $\pi_t = \tilde{\pi}(t)$ for every $t \in [\tau_0, \tau_1)$. This function is well-defined, continuous and decreasing by assumption 1.

Step 2: The distribution has at most one discrete jump at τ_0. Suppose there is $t > \tau_0$ such that the distribution G jumps at t, i.e. there is $\epsilon > 0$ such that $G(t) > G(s)$ for all $s \in [t - \epsilon, t)$. But then there is an interval over which citizens will not drop, contradicting the previous step.

Step 3: If $\tau_0 < \tau_1$, then at every $t \in [\tau_0, \tau_1)$ the distribution of concessions G has decreasing hazard rate. From equation 31 in Lemma 4, for citizens' decision to be optimal the exit threshold must satisfy:

$$\tilde{\theta}_1(t) = \lambda_t v(t - t_0(\tilde{\theta}_1(t))) \tag{38}$$

From the previous step, we have that the threshold must satisfy $F(\tilde{\theta}_1(t)) = \tilde{\pi}(t)$, and then it is decreasing over time. Then the left-hand side of equation 38 is decreasing, while the prize function increases over time, so it has to be that λ_t is decreasing.

Step 4: If $\tau_0 < \tau_1$, then $\tau_1 = \infty$. Suppose $\tau_1 < \infty$. First, it must be that $G(\tau_1) = 1$. Suppose that this is is not the case and the government stops conceding at some τ with $G(\tau) < 1$. Using the same arguments as in the proof of lemma 2, it must be $c(\pi_\tau, \tau) \geq q$. But then $\pi_\tau > 0$, as otherwise $c(\pi_\tau, \tau) = 0$ by assumption 1. By lemma 1 no citizen drops after τ, but then as the cost is increasing in time, eventually the cost of the protest would be higher than the cost of waiting, contradicting the optimality of the government's strategy. Thus, it must be that $G(\tau_1) = 1$. If this is the case, it must be that $\int_0^{\tau_1} \lambda_s ds = \infty$, which cannot happen in finite time as λ_t is decreasing in t. So, $\tau_1 = \infty$.

Step 5: If a citizen with opportunity cost θ ever enters the protest (i.e. $\exists t$ such that $\sigma_t^\theta = 1$), then $t_0(\theta) \leq \tau_0 \leq t_1(\theta)$. $t_1(\theta) \geq \tau_0$ follows directly from optimality, as otherwise the expected prize is zero with probability 1. Now consider an agent with opportunity cost θ entering at $t_0 > \tau_0$. From lemma 4, the marginal benefit of entering is given by:

$$\lambda_{t_0} v(0) + \frac{1}{1 - G(t_0)} \int_{t_0}^{t_1} e^{-r(s-t_0)} v'(s - t_0) dG(s) \tag{39}$$

By step 3, the expression above is decreasing in t_0 for any $t_0 \geq \tau_0$, and the marginal cost is constant. Then, the agent is strictly better off entering earlier.

Step 6: At any $t < \tau_0$, π_t^σ is increasing. From the previous claim, $\pi_t^\sigma = F(\tilde{\theta}_0(t))$, which is increasing.

Step 7: π_t^σ is continuous at every $t \in [0, \infty]$. We know that π_t^σ is continuous on $[\tau_0, \infty]$, and by the entry condition we also know it is continuous in $[0, \tau_0)$. It remains to show that it is also continuous at τ_0. In particular, we rule out cases in which there is a positive mass of people entering at a given time t (see figure 10). Take two agents entering at a given time $\bar{t}_0$. Note that as $\tilde{\pi}_t$ is strictly decreasing, these two agents cannot exit at the same time. Suppose they exit at some times $t_1 < t_1'$. Thus, from the exit condition their opportunity costs are given by $\tilde{\theta}_1(t_1) > \tilde{\theta}_1(t_1')$. But from the entry condition, we have:

$$\tilde{\theta}_1(t_1) = \int_{\bar{t}_0}^{t_1} e^{-r(s-\bar{t}_0)} v'(s - \bar{t}_0) dG(s) < \int_{\bar{t}_0}^{t_1'} e^{-r(s-\bar{t}_0)} v'(s - \bar{t}_0) dG(s) = \tilde{\theta}_1(t_1') \tag{40}$$

a contradiction.

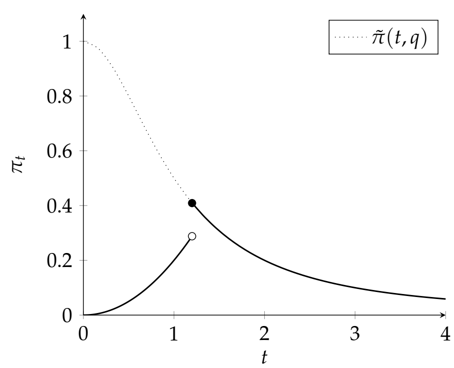


Figure 10. Continuity of π_t.

Step 8: $\tau_0 > 0$. We begin by showing that if $G(\tau_0) = 1$, then $\tau_0 > 0$. Consider first the case in which the government concedes with probability 1 at τ_0, i.e. $G(\tau_0) = 1$. Note that the payoff from entering at τ_0 is zero, so at τ_0 nobody enters anymore. But for $\tau_0 \in \mathcal{T}$ it must be that $c(\pi_{\tau_0}, \tau_0) \geq q$ (see the proof of lemma 2). The benefit of the last citizen entering is given by $G(\tau_0) \cdot v'(0)$, and then, for this to be an equilibrium, it must be that $F(v'(0)) = \tilde{\pi}_{\tau_0}$. This pins down τ_0 as the time at which $c(F(v'(0)), \tau_0) = q$. This time must be strictly positive, as otherwise there is no entry.

Denote by $\underline{\tau} = \tau_0$ the time at which the government concedes with probability 1. Then, we prove that if $G(\tau_0) < 1$, then it must be that $\tau_0 > \underline{\tau}$. Note that if $G(\tau_0) < 1$ then by lemma 3 it must be that $c(\pi_{\tau_0}, \tau_0) = q$. The payoff to the last agent entering is given by $G(\tau_0)v'(0)$, and then it must be that at $F\left(G(\tau_0)v'(0)\right) = \tilde{\pi}_{\tau_0}$. But $\tilde{\pi}_{\tau_0} < \tilde{\pi}_{\underline{\tau}}$, so by assumption 1 it must be $\tau_0 < \underline{\tau}$.

Step 9: In equilibrium the government concedes in finite time, i.e. $\lim_{t\to\infty} G(t) = 1$. From step 4, $\tau_1 = \infty$. Denote by $\underline{\lambda}_t = \frac{\theta}{v(t)}$ the hazard rate that makes the lowest opportunity cost citizen indifferent between dropping out and protesting at any time t. Note that by assumption 2, $\underline{\lambda}_t > 0$ for all t. Moreover, $\lambda_t \geq \underline{\lambda}_t$ for all t, and then $\int_0^\infty \lambda_t dt \to \infty$. So we have:

$$\lim_{t\to\infty} G(t) = 1 - \lim_{t\to\infty}\left[(1 - G(\tau_0))\exp\left(-\int_0^t \lambda_s ds\right)\right] = 1 \tag{41}$$

With this, we complete the proof of Theorem 1. □

Lemma 5. *Government initial concession $G(\tau_0)$ is decreasing in τ_0.*

A.2. Proof of Lemma 5. Using Lemma 5, the enttry threshold can be written as:

$$\tilde{\theta}_0(t) = \begin{cases} \int_t^{t_1} e^{-r(s-t)} v'(s-t)dG(s) & t \in [0, \tau_0) \\ \tilde{\theta}_1(t) & t = \tau_0 \end{cases} \tag{42}$$

Using continuity of π_t, it has to be that $\tilde{\theta}_0(t)$ is also continuous, i.e. $\lim_{t\to\tau_0^-} \tilde{\theta}_0(t) = \tilde{\theta}_1(\tau_0)$. Thus, at τ_0 the following condition holds:

$$\tilde{\theta}_0(\tau_0) = v'(0)G(\tau_0) \quad \Rightarrow \quad G(\tau_0) = \frac{\tilde{\theta}_1(\tau_0)}{v'(0)}$$

□

A.3. Proof of Theorem 2: A Continuum of Equilibria. It is direct to see that there is an equilibrium with $\tau_0 = \underline{\tau}$. We prove next that there exists an equilibrium satisfying $\tau_0 = \overline{\tau}$. Then we show that for any τ_0 in between this thresholds, an equilibrium exists.

Lemma 6. *There exists an equilibrium $(G, (\pi_t^\sigma)_{t\geq 0})$ with $\tau_0 = \overline{\tau}$ satisfying*

$$\underline{\theta} = \int_{\overline{\tau}}^\infty e^{-rs} v'(s)dG(s) \tag{43}$$

Proof. In order to prove existence of this equilibrium with the longest delay, we show that there exists a fixed point satisfying condition 43.

As we describe in Section 4.2, in equilibrium citizens' exit times are determined by the government indifference condition.[20] Then, given their exit times, and the government distribution of concessions $G(t)$, their best reply associates each exit time $t \in [\tau_0, \infty)$, with an entry time $t_0(t)$. The government, given these entry times, and a delay τ_0, chooses a distribution of concessions $G(t)$. Moreover, we introduce a fictitious player who chooses the delay τ_0, in such a way that given $G(t)$, condition 43 is satisfied.

Define the best reply correspondence: $\Psi : \mathbf{Z} \twoheadrightarrow \mathbf{Z}$ with tipical element $\mathbf{z} = (G, t_0, \tau_0)$ as:

$$\Psi = (\Gamma(t_0, \tau_0), \Phi(G, \tau_0), \Theta(G, t_0)) \tag{44}$$

where $\Gamma(t_0, \tau_0)$ is the government's best reply, $\Phi(G, \tau_0)$ is citizens' best reply, and $\Theta(G, t_0))$ is the best reply of the fictitious player.

The space $\mathbf{Z} = [0, T] \times \mathcal{S} \times \mathcal{C}$ is such that $\mathcal{S}$ corresponds to the space of probability distributions,[21] and $\mathcal{C}$ corresponds to the space of continuous functions. T is the upper bound on the maximum concession time $\overline{\tau}$. We use Kakutani-Fan-Glicksberg theorem to prove that an equilibrium exists. This theorem states that if $\mathbf{Z}$ is a nonempty compact convex subset of a locally convex Hausdorff space, and the correspondence $\Psi : \mathbf{Z} \twoheadrightarrow \mathbf{Z}$ has closed graph and nonempty convex values, then the set of fixed points is compact and nonempty (Aliprantis & Border (2013), Corollary 17.55).

Step 1: Define Citizens' Best Response $\Phi : [0, T] \times \mathcal{S} \to \mathcal{C}$. In equilibrium, given a distribution $G \in \mathcal{S}$ with support $[\tau_0, \infty)$, for each possible exit time $t \in [\tau, \infty]$, $t_0(t)$ is the optimal entry time that solves the following equation:

$$\tilde{\theta}_1(t) = \int_{t_0}^{t} e^{-r(s-t_0)} v'(s - t_0) dGs \tag{45}$$

where $\tilde{\theta}_1(t) = F^{-1}(\tilde{\pi}(t))$. Figure 11 illustrates citizens' best reply function.

Step 2: Define Government's Best Response $\Gamma : \mathcal{C} \times [0, T] \to \mathcal{S}$. In equilibrium, given citizens' best reply $t_0 \in \mathcal{C}$ and the delay time τ_0, we consider a modified problem for the government of choosing a distribution of concessions over $[\tau_0, \infty)$, i.e. $G : [\tau_0, \infty) \to [0, 1]$ such that:

$$G(t) = 1 - (1 - G(\tau_0)) \exp\left(- \int_{\tau_0}^{t} \lambda_s ds \right) \tag{46}$$

[20] More precisely, on the support $\mathcal{T}$, it has to be the case that $\pi_t = \tilde{\pi}_t$. Thus, there exist a unique exit threshold $\tilde{\theta}_1(t)$ such that $\tilde{\pi}_t = F(\tilde{\theta}_1(t))$ for every $t \in \mathcal{T}$.

[21] Space of functions that are increasing, right-continuous, and such that $\lim\limits_{t \to -\infty} G(t) = 0$ and $\lim\limits_{t \to \infty} G(t) = 1$.

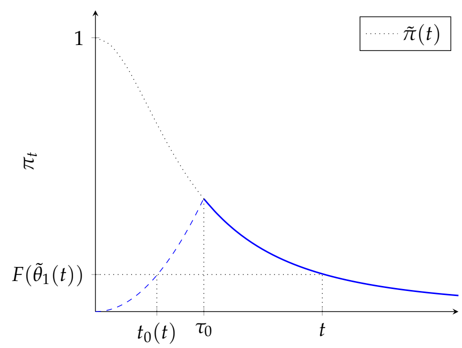


Figure 11. Citizens' exit is determined by $\tilde{\pi}_t = F(\tilde{\theta}_1(t))\ \forall t \in [\tau_0, \infty)$. A citizen with opportunity cost $\theta = \tilde{\theta}_1(t)$ exits at t, and given this exit time, equation 45 defines the entry time $t_0(t)$.

with $G(\tau_0) = \frac{\tilde{\theta}_1(\tau_0)}{v'(0)}$, and $\lambda_t = \frac{\tilde{\theta}_1(t)}{v(t-t_0(t))}$.

Step 3: Fictitious Player Best Response. The fictitious player best response $\Theta : \mathcal{C} \times \mathcal{S} \to [0, T]$ chooses a time $\tau_0 \in [0, T]$, that solves

$$\underline{\theta} = \int_{\tau_0}^{\infty} e^{-rs} v'(s) dG(s) \tag{47}$$

Step 4: **Z** *is a non-empty, convex and compact subset of a locally convex Haussdorf space.*

Let T be the time at which $\underline{\theta} = e^{-rT} v'(T)$. This upper bound corresponds to the time that makes the lowest opportunity cost citizen indifferent of entry even if the government concedes for sure, which satisfies $T > \overline{\tau}$. Thus, $[0, T]$ is well defined, and it is compact and convex.

For the space of government's distribution of concession, since both $G(\tau_0)$ and λ_t are continuous and well defined, it is non-empty. Moreover, note that the function G constrained to $[\tau, \infty)$ is continuous and bounded. Moreover, they are monotone by Proposition 1, and have bounded variation. By Helly's selection theorem, it is also compact.

Similarly, note that for citizens t_0 is a monotone continuous function with values in $[0, \tau_0)$, and then it has bounded variation. Moreover, it is uniformly bounded, and then we can apply Helly's selection theorem to obtain compactness. To see that it is non-empty, fix t, and note that $t_0(t)$ solves the following equation:

$$\tilde{\theta}_1(t) = \int_{t_0}^{t} e^{-r(s-t_0)} v'(s - t_0) dGs \tag{48}$$

which has always a unique solution for every $t \in [\tau, \infty]$.

Finally, by Tycohnoff Product Theorem (see Aliprantis & Border (2013), Theorem 2.61), the space $\mathbf{Z}$ is compact in the product topology.

Step 5: Ψ has closed graph. Take a sequence $(t_0^n, G^n, \tau^n) \in Graph(\Psi)$ such that $(t_0^n, G^n, \tau_0^n) \to (\bar{t}_0, \overline{G}, \overline{\tau}_0)$. We want to show $(\bar{t}_0, \overline{G}, \overline{\tau}_0) \in Graph(\Psi)$.

Claim 1. Γ has closed graph. We show that for any sequence $(\tau_0^n, G^n, t_0^n) \to (\overline{\tau}_0, \overline{G}, \bar{t}_0)$, with $(\tau_0^n, G^n) \in \Gamma(t_0^n)$ for all n, then $(\overline{\tau}_0, \overline{G}) \in \Gamma(\bar{t}_0)$.

Note that by continuity of $\tilde{\theta}_1(t)$, $G^n(\tau_0^n) \to \overline{G}(\overline{\tau}_0)$.

Moreover, by continuity of v and F^{-1} the hazard rate $\lambda_n(t)$ converges uniformly to:

$$\overline{\lambda}(t) = \frac{F^{-1}(\tilde{\pi}(t))\epsilon}{v(t - \bar{t}_0(t))} \tag{49}$$

which proves the graph is closed.

Claim 2. Φ has closed graph. We show that for any sequence $(\tau_0^n, G^n, t_0^n) \to (\overline{\tau}_0, \overline{G}, \bar{t}_0)$, with $t_0^n \in \Phi(\tau_0^n, G^n)$ for all n, then $\bar{t}_0 \in \Phi(\overline{\tau}_0, \overline{G})$.

Rewrite t_0 as the solution to a fixed point problem to the following equation:

$$H(t_0; G, \tau_0) = \frac{1}{r}\left[\ln F^{-1}(\tilde{\pi}(t)) - \ln\left(\int_{t_0}^{t} e^{-rs} v'(s - t_0(t)) dG(s)\right)\right] \tag{50}$$

Thus, it is enough to prove that $\|\bar{t}_0 - H(\bar{t}_0)\| = 0$. Note that:

$$\|\bar{t}_0 - H(\bar{t}_0; \overline{G}, \overline{\tau})\| \leq \|\bar{t}_0 - t_0^n\| + \|t_0^n - H(t_0^n)\| + \|H(t_0^n; G^n, \tau^n) - H(\bar{t}_0; \overline{G}, \overline{\tau})\| \tag{51}$$

the first two terms in the right-hand side converge to 0 by hypothesis. The third one also converges pointwise to 0 as $\int\limits_{t_0^n}^{t} e^{-rs} v'(t - t_0^n(t)) dG^n(s) \to \int\limits_{\bar{t}_0}^{t} e^{-rs} v'(t - \bar{t}_0(t)) d\overline{G}(s)$ for all t.

Claim 3. Θ has closed graph. We show that for any sequence $(\tau_0^n, G^n, t_0^n) \to (\overline{\tau}_0, \overline{G}, \bar{t}_0)$, with $(\tau_0^n) \in \Theta(t_0^n, G^n)$ for all n, then $(\overline{\tau}_0) \in \Gamma(\bar{t}_0, \overline{G})$. Note that G^n converges to $\overline{G}$ in distribution, and then applying Continuous Mapping Theorem we obtain

$$\int_{\tau_0^n}^{\infty} e^{-rs} v'(s) dG^n(s) \to \int_{\overline{\tau}_0}^{\infty} e^{-rs} v'(s) d\overline{G}(s) \tag{52}$$

Then, using claims 1, 2 and 3, we have that Ψ has closed-graph, and therefore is upper-hemicontinuous. By Kakutani-Fan-Glicksberg theorem it has a fixed point. □

Lemma 7. *Let $(G^1, (\pi_t^1)_{t \geq 0})$ and $(G^2, (\pi_t^2)_{t \geq 0})$ be two distinct equilibria with delays τ_0^1, τ_0^2, such that $\tau_0^1 < \tau_0^2$. Then, the distributions of concessions G^1, G^2 do not cross at any $t \in [\tau_1, \infty]$.*

Proof. From agents entry condition, we have

$$\frac{\partial\tilde{\theta}_0(t)}{\partial\tau_0} = -e^{-r(\tau_0-t)}v'(\tau_0-t)g(\tau) + \int_t^{t_1(t)} e^{-r(s-t)}v'(\tau_0-t)g'(\tau_0)ds < 0 \tag{53}$$

Given that this holds for all $t \in [0,\tau_0)$, we have that the functions $t_0(t)$ do not cross, and this ensures the hazard rates do not cross, and then the distributions of concessions do not cross either. □

We are now in a place to show that any $\tau_0 \in [\underline{\tau},\overline{\tau}]$ generates an equilibrium. Fix an arbitrary $\tau^* \in (\underline{\tau},\overline{\tau})$ and let $(G,(\pi_t)_{t\geq 0})$ be the equilibrium consistent with it. We know from lemma 7 that

$$\underline{\theta} < \int_{\overline{\tau}}^{\infty} e^{-rs}v'(s)dG(s) \tag{54}$$

and $G(\tau) < 1$. Then we can solve the same fixed point problem we solved in the previous claim fixing the fictitious player strategy to choosing τ^*. Using the same arguments, a fixed point exists. As τ^* was arbitrary, this completes the proof of the theorem. □

A.4. Proof of Proposition 1. Recall that for any distribution of opportunity costs F_j, the lower bound $\underline{\tau}_j$ is given by the equilibrium in which the government concedes with probability 1, and then it is such that

$$\tilde{\pi}_{\underline{\tau}_j} = F_j\left(v'(0)\right) \tag{55}$$

Then, statement (i) follows from the fact that F_1 first order stochastically dominates F_2, and then $F_1(v'(0)) < F_2(v'(0))$.

To prove statements (ii) and (iii), note that as F_1 is symmetric and unimodal and F_2 is obtained from a mean preserving spread, then $F_2(\theta) < F_1(\theta)$ for every $\theta < \int \theta dF_1(\theta)$, and $F_2(\theta) > F_1(\theta)$ otherwise.

B. Empirical Appendix

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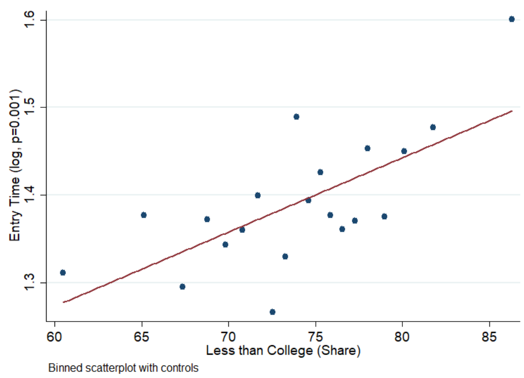
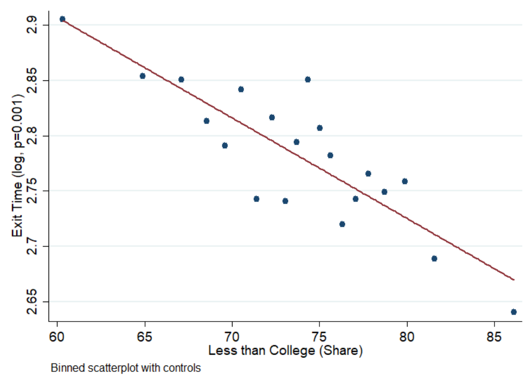
Table 4. Description of Main Variables

Variable	Description
TELEWORK	Share of workers on teleworkable occupations.
OPEN	Dummy to indicate if the county was open by May 25.
RESIDENTIAL	Average Google Mobility Index, week prior to May 25.
CLINTON	Share of votes for Clinton, as a total of votes.
LABOR	Labor Force over Total Population
POPULATION (LOG)	Population (Log)
DENSITY (LOG)	Population by Squared Mile (log)
INCOME (10K)	Median Household Income (10k)
RURAL	Share of population living in rural areas
BLACK	Black Population (Share)
UNDER 29	Population under 29 years old
UNDER COLLEGE	Population with less than college degree

Figure 12. Education and Timing of Protests, with Controls



(a) Entry $t_0(0.001\%)$



(b) Exit $t_0(0.001\%)$

Controls: INCOME (10K), UNDER COLLEGE, CLINTON, LABOR FORCE, UNDER 29, POPULATION (LOG), DENSITY (LOG), RURAL, BLACK.

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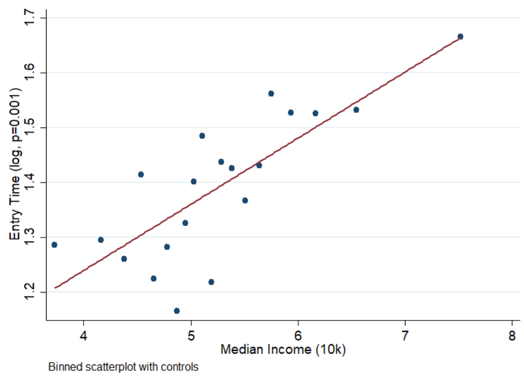
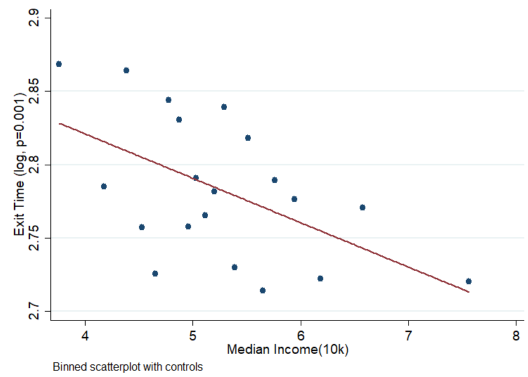
Figure 14. Income and Timing of Protests, with Controls



(a) Entry $t_0(0.001\%)$



(b) Exit $t_0(0.001\%)$

Controls: INCOME (10K), UNDER COLLEGE, CLINTON, LABOR FORCE, UNDER 29, POPULATION (LOG), DENSITY (LOG), RURAL, BLACK.

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Table 5. Sensitivity Analysis: Telework and Reopening

	Dependent Variable: $log(t_0(0.01\%))$				
	(1)	(2)	(3)	(4)	(5)
OPEN×TELEWORK	-2.814***	-2.661***	-2.733***	-2.627**	-2.616***
	(1.013)	(0.997)	(1.019)	(1.022)	(1.008)
OPEN	1.042***	0.809***	0.848***	0.804**	0.849***
	(0.318)	(0.307)	(0.313)	(0.315)	(0.311)
TELEWORK	-0.447	1.539	1.555	1.473	1.435
	(0.948)	(0.987)	(1.006)	(1.002)	(0.995)
CLINTON		-1.017***	-0.634**	-0.616**	-0.299
		(0.221)	(0.267)	(0.263)	(0.275)
POPULATION (LOG)		-0.107***	-0.0812**	-0.0652*	-0.0749**
		(0.0328)	(0.0337)	(0.0345)	(0.0346)
DENSITY (LOG)		-0.00995	-0.00541	0.0139	0.000956
		(0.0304)	(0.0310)	(0.0335)	(0.0335)
BLACK		0.00305	0.000530	0.000562	0.000152
		(0.00285)	(0.00311)	(0.00313)	(0.00310)
UNDER COLLEGE			0.00583*	0.00561*	0.0132***
			(0.00306)	(0.00327)	(0.00363)
UNDER 29			-0.0140***	-0.0119***	-0.00688
			(0.00427)	(0.00439)	(0.00472)
LABOR SHARE				0.00328	-0.673
				(0.436)	(0.480)
RURAL				0.00253*	0.00277**
				(0.00133)	(0.00133)
INCOME (10K)					0.108***
					(0.0262)
CONSTANT	1.114***	2.475***	2.167***	1.746***	0.817
	(0.363)	(0.424)	(0.502)	(0.601)	(0.622)
State FE	X	X	X	X	X
Observations	1024	1024	1024	1024	1024
R^2	0.144	0.209	0.222	0.225	0.238

t statistics in parentheses; * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

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