Persistent Protests

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Abstract

I study a dynamic model of public protests in continuous time. There is a continuum of citizens with heterogeneous opportunity costs, who can participate in a protest demanding a public good. The government can concede at any time, and as long as it does not, it pays a flow cost for the protest that is increasing in time and participation rates. Citizens are motivated to participate in the protest because they enjoy a merit reward if the government concedes while they are active. I show that there is always delay in government concession. Citizens entry times are increasing in opportunity costs, and their exit times are decreasing. Any equilibrium with protests can be decomposed into two stages: a building-up stage, in which citizens continuously join the protest and the government ignores them, and a concession stage, in which there is a war of attrition between the citizens and the government. There is multiplicity of equilibria with protests: for each time τ_0 within a bounded interval, there is a unique equilibrium where the concession stage starts at τ_0.

1 Introduction

Public protests and social movements are heterogeneous in terms of their size and duration. Although we have some idea of how people overcome coordination barriers, the persistence

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of people's participation in social movements in situations in which they could free ride, still remains a puzzle. In this work, we study the dynamics of participation in public protest in a context in which agents have heterogeneous opportunity cost of participating, in order to understand how this heterogeneity shapes agents behavior and equilibrium outcomes.

The word *persistence* in this context refers to the duration of a particular event associated with political unrest, i.e. how long a particular protest event or uprising might last over time. We understand a protest event as the gathering of people to demonstrate against some authority about a given policy. The persistence of this event is then the number of days it takes until it vanishes. The most prominent example of a persistent protest is the Arab Spring, which began in Tunisia in the early 2010s, spreading to other countries in the Islamic world. The unfolding of events vary across countries, with some countries undergoing a regime change in the face of pressure from protests, and others ending up in civil wars. More recently, protests erupted in Chile and Iran in 2019, and participation persisted over time. The Black Lives Matter movement in the US is the most recent case of public protests characterized by persistent participation in all states, with different dynamics of participation and government concession.

We model protests as a game between a continuum of small players, the citizens, and a single large player, the government. Time is continuous, and at any instant, citizens decide whether to participate in a protest to ask the government for a public good. The cost of participating is the opportunity cost of the time spent in the protest, which is heterogeneous across citizens. The government decides at any time whether to concede or not, but as long as it does not concede, it faces a cost that is increasing in both the number of people protesting and how long the protest lasts. Concession is also costly to the government, because once the government concedes, it has to pay a flow cost of a public good forever. After concession, everyone can enjoy the public good, regardless of whether they participated in the protest.

In addition to obtaining the public good, at the time the government concedes, citizens who are in the protest receive a one-time victory payoff, which we refer to as the *seniority prize*. This prize aims to capture not only the value that agents obtain from winning against the government, but also from their merits in the victory. To capture the idea of *merits*, we assume the seniority prize is increasing in the time they have spent in the protest. Thus, despite agents being able to enjoy the public good even if they did not participate in the protest, they still have incentives to participate, because they want to be part of the "victory team."

In this context, we characterize the set of equilibria in which a protest occurs.[1] First, we show that in any equilibrium with protests there is delay in government concession. As citizens need time to become veteran enough for protesting to be worthwhile, the seniority prize imposes a lower bound on the set of equilibrium delays. If the government concedes too soon, citizens' expected payoff from entering the protest would be too low relative to the benefits, and then citizens would rather stay home and free ride on the public good. This prediction highlights the relevance of analyzing the dynamics of protests.

Second, we have that any equilibrium with protests can be characterized by two stages: a building-up stage, and a concession stage. The building-up stage corresponds to the initial period in which the protest grows as people continuously enter, but it is still not costly enough to make the government concede. The second stage starts when the protest becomes so costly the government can no longer ignore it. At this point, the government makes some concession, which can be either immediate (with probability 1) or can take the form of a continuous probability distribution over concession times. If the government does not concede immediately, it continues conceding forever as people continuously drop out.

What makes the concession stage particularly interesting is that it unfolds as a war of attrition between the government and the continuum of citizens. To see this, note that on one side we have the citizens, whose behavior is characterized by the aggregate trajectory of participation. On the other side we have the government, who randomizes according to a continuous distribution over concession times. For the government to be randomizing, it must be indifferent at any time between conceding and waiting another instant. For this indifference condition to hold in equilibrium, the government will concede at some hazard

[1] In addition to this set, there is always an equilibrium in which the government never concedes and nobody protests. Given that we are interested in characterizing the dynamics of participation, we do not include this type of equilibrium in the analysis.

rate that generates exactly the participation level that makes him indifferent. Given this hazard rate of concession, citizens will drop out as the expected gains from the protest becomes too low relative to the cost. Although their exit is deterministic, on aggregate, their behavior mimics a single player who is standing against the government. The equilibrium then follows from a fixed-point argument. The government concession rate determines the value of protesting to citizens, and then it determines participation. Participation, in turn, determines the expected value to the government, and then it determines the government concession rate.

A third feature of equilibrium behavior in this context is that citizens' participation decisions are monotone in their opportunity costs. The time at which citizens join the protest increases with opportunity costs, and the time at which they exit is decreasing. Then, the dynamics of entry and exit are of the first-in-last-out form: the agent with the lowest opportunity cost is the first entering, and will hold against the government forever. The last agent who joins the protest joins right before the government starts conceding, and exits immediately. This monotonicity in citizens' participation decisions predicts that the aggregate level of opportunity costs among protesters should increase over time at the beginning, and then decrease. This prediction has not been documented, although we provide some empirical evidence to test its validity.

Finally, we parametrize the set of equilibria by the time at which the government starts conceding for the first time, i.e. the time at which the concession stage starts. Any government delay on a certain range can generate an equilibrium with the features described above. Even more, we show that the range of delays that can generate an equilibrium is a bounded interval. On one hand, the seniority reward puts a positive lower bound to the time at which the government starts conceding, as agents need time to build it. On the other hand, the delay cannot be too long, as it has to be such that at least the lowest opportunity cost citizen has incentives to start the protest.

Although we have multiple equilibria, we study some comparative statics of the set of equilibria with respect to the distribution of opportunity costs. A general increase in opportunity costs has two opposite effects: on one side, it makes protesting more costly and then provides incentives for citizens to delay their entry. On the other, once they enter they are stronger in front of the government, as the government is forced to concede faster for making them drop out.

This paper is organized as follows. In the next subsection we briefly review the related literature and our main contribution. In Section 2 we develop the baseline model and characterize equilibria and in Section 3 we obtain the existence result. In Section **??** we analyze the comparative statics of the equilibrium set. We develop some extensions in Section **??**. All the proofs can be found in Section 6.

1.1 Related Literature

This work aims to contribute to the literature on persistent participation in public protests as a collective action problem (see Van Stekelenburg & Klandermans (2013) for more details about the paradox of persistence participation). The literature most closed to our work is that studying the coordination problem among citizens. Static models of coordination in protests has been studied by Shadmehr & Bernhardt (2011),Boix & Svolik (2013) and Morris & Shadmehr (2018).

The main contribution of this paper is to add dynamics to analyze the coordination problem in protests. These dynamics come from citizens enjoying having merits in an eventual victory against the government. This is related to traditional models of voting and protests, which introduce some psychological utility in order to get rid of the free-riding problem (see for instance Wood & Jean (2003) and Pearlman (2018)).

Our work is also related with the literature on the social psychology of public protests. In this literature, intrinsic motives for participation have been studied as a result of ideology or group identity (see Cohen (1985) and Jasper (1998)). Our assumption of a *seniority prize*, constitute a new explanation for persistent participation in protests that combines both an *intrinsic motivation*, i.e. the emotion, with an *instrumental motivation*: agents obtain this value only if the movement is successful, and then they take into account expectations about the government conceding.[2]

[2]see Feather & Newton (1982) and Klandermans (1984) for an analysis of instrumental motivations in

Multiple equilibria in protests have been studied since the seminal works of Schelling (1960), Hardin (1997), and more recently, De Mesquita (2014).

This work is also related with the literature on conflicts (see Ray & Esteban (2017) for a detailed review). There is an extensive literature analyzing the relationship between conflicts intensity and inequality, and the main idea is that inequality affects both grievances and opportunity costs (see Dal Bó & Dal Bó (2011) and Dube & Vargas (2013)). Although protests can be seen as a particular case of a conflict, the main forces driving the dynamics are different when the "fight" is between a single large player facing an increasing participation cost (the government), and a continuum of small players (the citizens) behaving myopically.

From a methodological point of view, this work is related to the literature in bargaining and wars of attrition. We develop a model that resembles a war of attrition with complete information, between a large player and a continuum of citizens. The seminal work of Hendricks et al. (1988) solves the war of attrition in a context with complete information for the case of two players. In our model with a continuum of agents on one side, the aggregate behavior of small players resembles the behavior of a single opponent in a war of attrition. Moreover, as the cost of delaying increases in time, the hazard rate of government concession in equilibrium is decreasing.

2 Model and Equilibrium Characterization

In this section we describe the baseline model and obtain the characterization result. A more detailed discussion on the main assumptions and results can be found in section 2.1.

We construct a model of public protests in continuous time. There is a large player, the government, and a continuum of small players, which we refer to indistinctly as the *citizens* or the *people*. We index citizens by $i \in [0, 1]$. At any instant $t \in [0, \infty]$ citizens decide whether to participate in a protest to ask the government for a public good. We use π_t to denote the mass of citizens protesting at time t.

protests.

The government can concede at any moment in time, and once it concedes the protest ends. While it has not conceded, it pays a flow cost that is increasing in both the size of the protest, and the duration of it. If the government concedes, it pays a flow cost q forever.

Let $t = 0$ be the time at which a protest begins, and let $c(\pi_t, t)$ be the flow cost of the protest. We make some natural assumptions about the cost function. First, we assume that protests get more costly as they last longer, and as there are more people participating. If there is no one protesting then there is no cost to the government, and if the whole population is protesting, it is less costly to the government to concede immediately.

Assumption 1 *The cost function $c : [0, 1] \times [0, \infty) \to \mathbb{R}_+$ is continuously differentiable on both arguments, and satisfies:*

(i) $c(0, t) = 0$ for all t, and $c(1, 0) > q$.

(ii) $c(\pi, t)$ is strictly increasing in π, and is strictly increasing in t if $\pi > 0$.

Let $(\pi_t)_{t \geq 0}$ be a trajectory of participation. Thus, if the government concedes at some time τ, its total costs are given by:

$$\int_0^\tau e^{-rs} c(\pi_s, s) ds + e^{-r\tau} \frac{q}{r} \tag{2.1}$$

where $r > 0$ is the discount rate, which is the same as the citizens' discount rate.

Citizens have heterogeneous opportunity costs of protesting, which are drawn from a distribution F with full support on $[\underline{\theta}, \overline{\theta}]$. We assume F is continuously differentiable and $\underline{\theta} > 0$. The maximum cost $\overline{\theta}$ might be unbounded. We denote by θ_i the opportunity cost of participating of citizen i.

If the government concedes at a time τ, then every citizen receives a flow payoff from enjoying the public good forever. We assume the public good is non-excludable and citizens can enjoy it even if they were not involved in the protest. Then, the value of the public good does not affect their decision to protest, and it is without loss to assume that all citizens value the public good equally. We denote this flow payoff by v.

In addition, we assume citizens get utility from being in the protest when the government concedes. If the government concedes at time t, an agent protesting at t who has been in the protest since time t_0, gets a one time reward $\xi(t-t_0)$. We call this the *seniority reward*. This reward can be motivated as agents intrinsic motivation to participate in the protest, which represent the value they attached to their own merits in an eventual victory (see section 2.1 for discussion and references).

Consider a citizen with opportunity cost θ who starts protesting at some time t_0, and is planning to exit at time t_1. Suppose the government will concede at some time τ, possibly random. Then, her expected payoff is given by the following expression.

$$E\left[-\theta\int_{t_0}^{t_1\wedge\tau} e^{-rs}ds + e^{-r\tau}\left(\mathbb{1}_{\tau<t_1}\xi(\tau-t_0)+\frac{v}{r}\right)\right] \tag{2.2}$$

where the expectation is taken over τ. If the government concedes at time $\tau < t_1$, then the citizen bears the cost of the protest between t_0 and τ, and when the government concedes she gets: a one time payoff $\xi(\tau - t_0)$, and a flow payoff v forever. If, on the contrary, the government concedes at $\tau \geq t_1$, then the citizen bears the cost of the protest between t_0 and t_1, but given that when the government concedes she is no longer in the protest, she only gets the value of the public good.

We assume the seniority reward is an increasing function of the time agents spend in the protest, so that activists who have been a longer time in the protest get a higher payoff than the ones who join closer to the end. In addition, we assume that the reward is increasing and concave in the time spent in the protest.

Assumption 2 *The seniority reward $\xi : [0,\infty] \to \mathbb{R}_+$ is continuously differentiable, and it satisfies:*

(i) $0 < \xi'(\Delta) < \infty$ *and* $\xi''(\Delta) \leq 0$ *for all* $\Delta \geq 0$*;*

(ii) $\xi(0) = 0$*, and* $\xi'(0) < \overline{\theta}$*.*

Part (i) ensures that the function is increasing and concave. Part (ii) rules out opportunistic behavior, in the sense that it ensures that if citizens are not in the protest by the time the government concedes, they would rather free ride to get the public good. Finally, we assume that both the government and citizens discount the future with the same rate r.

As citizens are small, at the individual level each citizen's action is insignificant to the other players, and it can only affect their payoffs when aggregated. Their aggregate behavior is described by the trajectory of participation $(\pi_t)_{t\geq 0}$.

It remains to specify how the game is played at each instant. We assume that when the government decides whether to concede, it is already observing how many people are protesting. However, when a citizen decides whether to protest or not, she only observes participation until *an instant before* she joins. To help grasping better the interpretation for continuous time, we can build some intuition in the discrete time case. Imagine a game played repeatedly at times $\{0, 1, 2, ...\}$. At any time t, the stage game is such that first citizens make a protest decision, and then the government decides whether to concede or not. Thus, when citizens choose their actions, they only observe a history of participation up to $t-1$, i.e. $\{\pi_0, \pi_1, ...\pi_{t-1}\}$. Then, once π_t is observed the government decides whether to concede or not, and then the relevant history for the government is given by $\{\pi_0, \pi_1, ...\pi_{t-1}, \pi_t\}$.

Following this intuition, for any time t we define the histories $\pi^t = \{\pi_s : 0 \leq s < t\}$ and $\overline{\pi}^t = \{\pi_s : 0 \leq s \leq t\}$. Let $\Pi^t = \left\{\pi^t\right\}_{t\geq 0}$ be the set of all possible open histories at time t, and $\overline{\Pi}^t = \left\{\overline{\pi}^t\right\}_{t\geq 0}$ the set of all possible closed histories at time t. Also define $\pi^0 = \emptyset$. A strategy for the government is a process $\gamma = \{\gamma_t\}_{t\geq 0}$, with $\gamma_t : \overline{\Pi}^t \to \{0, 1\}$, where $\gamma_t = 1$ stands for *concede*, and $\gamma_t = 0$ for *not concede*. A strategy for a citizen with opportunity cost θ is a process $\sigma^\theta = \{\sigma_t^\theta\}_{t\geq 0}$ with $\sigma_t^\theta : \Pi^t \to \{0, 1\}$, where 1 stands for participating in the protest, and 0 for not participating. While the government decision is irreversible, citizens can reenter the protest after leaving. As we illustrated for a discrete time setting, the difference between citizens and the government is that the government is observing the protest when it decides whether to concede, while all citizens decide to protest simultaneously, and then they observe the open history π^t. We denote a strategy profile by (σ, γ), where $\sigma = \{\sigma^\theta\}_{\theta\in[\underline{\theta},\overline{\theta}]}$.

For any strategy profile (σ, γ), let $\pi^{\sigma t}$ be the trajectory up to time t, conditional on no concession, generated by the strategy σ. This can be defined recursively as follows:

$$\pi_t^\sigma = \int \sigma_t^\theta(\pi^{\sigma t})dF(\theta) \qquad \forall t \geq 0 \tag{2.3}$$

We focus on the set of Nash equilibria of the game. We take advantage of the fact that citizens' individual deviations do not affect aggregate participation, hence we only need to describe the strategy of the government along the equilibrium path. This is equivalent to focusing on government strategies that are *open-loop*, in the sense that it is as if it commits to a sequence of actions at the beginning of the game.[3] In our model, it is sufficient to look at open-loop strategies as we only observe participation outcome on path.

We allow the government to randomize over concession times. A behavioral strategy in this context would specify for each possible history $\overline{\pi}^t$, a probability of concession. Instead, we focus on the trajectory of participation the government expects in equilibrium, and then we can characterize its strategy as a mixed strategy: a distribution of concessions $G(t)$. This distribution corresponds to the probability of the government conceding in $[0, t]$, given a trajectory of participation up to time t, $\pi^{\sigma t}$. This function is weakly increasing and right continuous in t, and its support is defined as:

$$\mathcal{T} = \{t \geq 0 | G(t) - G(t - \epsilon) > 0 \ \ \forall \epsilon > 0\} \tag{2.4}$$

Define $\tau_0 = \inf \mathcal{T}$, i.e. the first time at which the government makes some concession, and $\tau_1 = \sup \mathcal{T}$.

An equilibrium is given by a distribution of government concessions $G(t)$, and a profile of citizens' strategies σ, such that given the outcome path $\{\pi_t^\sigma\}_{t \geq 0}$,

(i) The strategy for the government maximizes its expected total payoff.

(ii) Citizens' strategies maximize their expected total utility given the government's distribution of concession G.

This game has equilibria in both pure and mixed strategies. We focus on the equilibria in which protests occur. We refer to an equilibrium as an *equilibrium with protests* if there is some (possibly probabilistic) concession by the government, i.e. $\mathcal{T} \neq \emptyset$. Note that in addition to the set of equilibria with protests characterized below, as it is the case

[3] Fudenberg & Levine (1988) compare the notions of *open-loop* and *closed-loop* equilibria for the case of games with non atomic players. In particular, they show that if there is a unique Nash equilibrium in every subgame, then both equilibria coincide.

in coordination games with complete information, there is always an equilibrium in pure strategies in which the government never concedes and nobody protests, i.e. $G(t) = 0$ and $\pi_t^\sigma = 0$ for every t. In this scenario, citizens can only affect the outcome if they manage to shift society from one equilibrium to another. As they are small, none of them has incentives to do so by herself, and then protests occur only when citizens are sufficiently coordinated for this to happen.[4]

We leave the formal proofs to Section 6, but we outline the main features of equilibria here. Note that in any equilibrium with protests the support of the government's strategy must be either a singleton $\{\tau_0\}$, or an interval $[\tau_0, \infty)$ (see Lemma 3 in Section 6). The intuition for the support being unbounded follows from the fact that if the government randomizes, it does so at a hazard rate that is continuous and decreasing. Then, there can not be a subsequent time at which the government concedes for sure (see Step 5 in Section 6.1). Then, either the government concedes with probability 1 at some time τ_0, or it begins conceding at τ_0 and it continues conceding forever.

For the government to play a mixed strategy in equilibrium, at any time $t \in [\tau_0, \infty)$ it has to be indifferent between conceding immediately, or waiting another instant. This implies that the following condition holds:[5]

$$c(\pi_t, t) = q \qquad\qquad \forall t \in [\tau_0, \infty) \tag{2.5}$$

Define $\tilde{\pi}_t$ as the trajectory of participation that makes the government indifferent, at any time $t \in [0, \infty)$, between conceding and waiting another instant, i.e. $c(\tilde{\pi}_t, t) = q$ for all $t \in [0, \infty)$. As the cost function is continuously differentiable by Assumption 1, this indifference participation level is continuous and strictly decreasing in t. Condition 2.5 implies that the trajectory of participation on the support of $G(t)$ must coincide with the function $\tilde{\pi}_t$, and then it is decreasing.

Citizens, on the other side, take the distribution of concessions $G(t)$ as given, and decide

[4] Note that this equilibrium with no protests happens to be unstable to perturbations. In particular, by allowing a positive mass of citizens to have zero opportunity cost, then any equilibrium of the game has some concession by the government.

[5] See Lemma 4 in Section 6.

when to protest. Even when we allow them to exit and reenter, we show that in equilibrium they enter and exit at most once. Moreover, they only enter before the government starts conceding, and they only exit afterwards. As we show in Lemma 5 in Section 6, citizens' utilities satisfy a single-crossing property with respect to opportunity cost, and then their strategies are monotone in θ. Thus, entry and exit can be characterized by thresholds $\tilde{\theta}_0(t)$ and $\tilde{\theta}_1(t)$, respectively, such that a citizen enters at $t < \tau_0$ if $\theta \leq \tilde{\theta}_0(t)$, and exits at $t \geq \tau_0$ if $\theta \geq \tilde{\theta}_1(t)$. Then, equilibrium participation is given by:

$$\pi_t^\sigma = \begin{cases} F(\tilde{\theta}_0(t)) & t \leq \tau_0 \\ F(\tilde{\theta}_1(t)) & t > \tau_0 \end{cases} \tag{2.6}$$

Using these properties of the equilibrium strategies, we characterize an equilibrium with protests in Proposition 1. First, note that in any equilibrium with protests there is delay in government concession. The intuition for this is that citizens need time to build their seniority in the protest, and then the government has incentives to delay concession. Before this time τ_0, participation increases as people enter the protest. At time τ_0, either the government concedes with probability 1, which is the case in which $\mathcal{T}$ is a singleton, or it makes a discrete concession $G(\tau_0) < 1$ and continues conceding forever.

Thus, any equilibrium with protests is characterized by two stages: building up stage and a concession stage. The first stage corresponds to a period in which the protest is growing as people continuously enter, but it is still not costly enough to make the government concede. The second stage starts when protest becomes so costly the government can no longer ignore it. As the government concedes, people continuously drop out.

Interestingly, for any τ_0 in a certain range we can construct an equilibrium in which participation is single-peaked at τ_0, and the government concedes at this point for the first time. As we show in Section 3, τ_0 belongs to a bounded interval. It cannot be too small, as agents need time to build their seniority reward. And it cannot be too large either, as it has to be such that at least the smallest opportunity cost citizen has incentives to start the protest (at time 0).

Proposition 1 *Let $G : [0, \infty] \to [0, 1], (\pi_t^\sigma)_{t \geq 0}$ be an equilibrium with protests. Then the following conditions hold:*

(i) There is always delay in government concession, i.e. $\tau_0 > 0$.

(ii) π_t^σ *is continuous, increasing for* $t \leq \tau_0$*, and if* $G(\tau_0) < 1$*, decreasing for all* $t \geq \tau_0$.

(iii) The distribution of concessions has at most one discrete jump at τ_0, $G(\tau_0) \leq 1$.

(iv) If $G(\tau_0) < 1$*, then* $G(t)$ *is strictly increasing and continuous, and* $\tau_1 = \infty$.

(v) In equilibrium, the government with probability 1*, i.e.* $\lim_{t\to\infty} G(t) = 1$.

Although we prove the result at the end of the paper, there are some key ingredients that are worth mentioning. First, consider the problem of a citizen with opportunity cost θ, who enters at t_0 and exits at t_1. As we mention above, it must be that $t_0 < \tau_0 \leq t_1$. Let $\lambda_t = \frac{g(t)}{1-G(t)}$ to be the government's hazard rate, i.e. the instantaneous probability of concession given that it has not conceded yet. Once in the protest, this citizen keeps protesting as long as the benefit of staying another instant weakly exceeds the cost. A citizen with opportunity cost θ, exits the protest if:

$$\theta \geq \lambda_{t_1} \xi(t_1 - t_0) \tag{2.7}$$

The left hand side corresponds to the opportunity cost of staying another instant, and the right hand side corresponds to the expected gains.

Consider now the entry decision of the citizen when she expects to exit at t_1. At any time $t < \tau_0$ she compares the marginal cost and benefit of entering at t versus entering an instant later. The marginal cost is given by the opportunity cost θ, and the benefit is the marginal increase in the seniority prize the agent might obtain during the time she remains in the protest. An agent with opportunity cost θ enters the protest if:

$$\theta \leq E\left[e^{-r(\tau-t_0)} \mathbb{1}_{\tau<t_1} \xi'(\tau - t_0)\right] \tag{2.8}$$

The expected benefits of entry and exit depend on the government strategy $G(t)$, and this in turn determines the entry and exit thresholds $\tilde{\theta}_0(t)$ and $\tilde{\theta}_1(t)$.

If $\mathcal{T} = \{\tau_0\}$, then there is no relevant exit decision as the government concedes with probability 1. If $\mathcal{T}$ is an interval, the government's optimal strategy is to randomize over concession times in $[\tau_0, \infty)$. Then equilibrium participation, which is determined by the exit threshold, must coincide with the indifference level along the support:

$$\pi_t^{\sigma} = F\left(\tilde{\theta}_1(t)\right) = \tilde{\pi}_t \tag{2.9}$$

In equilibrium the government chooses a hazard rate of concession λ_t such that citizens' exit threshold satisfies condition 2.9.

Fixing participation at $\tilde{\pi}_t$ uniquely identifies the exit time of any citizen in the protest. In particular, take a citizen exiting at t. From condition 2.9, this citizen's opportunity cost must be $\tilde{\theta}_1(t) = F^{-1}(\tilde{\pi}_t)$. Given this exit time, the citizen choose an entry time $t_0(t)$ according to entry condition 2.8. Then the government hazard rate at time t is given by:

$$\lambda_t = \frac{\tilde{\theta}_1(t)}{\xi(t - t_0(t))} \tag{2.10}$$

which defines a unique distribution of concessions $G(t)$.

2.1 Discussion

Model. Let's begin by discussing some key features of the model. Citizens get utility from the public good and from winning to the government. The assumption of a common value for the public good is just a normalization, and as it does not affect citizens' decision in this setting it is without loss. The seniority prize is a victory reward that is increasing in the time spent in the protest, and it does not depends on citizens' identity.

The idea of agents obtaining pleasure from participation is common in both regime change models, and voting models, in which otherwise citizens would free ride. In elections, the idea of expressive voting is motivated by either intrinsic values, or by group identity. In the context of protests we have the analogous problem: if collective action succeed, it generates gains that can be enjoyed by all citizens, irrespective of their participation or merits in the victory (see Olson (2012)). Several studies recognized the relevance of intrinsic psychological motivations for citizens to participate in a protest (see for instance Wood & Jean (2003) and Pearlman (2018)), as well as group-based emotions (see Van Stekelenburg & Klandermans (2013)). In this paper, the seniority payoff is not only intrinsic, but instrumental, in the sense that agents obtain this reward only if the protest is successful.

There are several other ways in which this payoff can be motivated. A first interpretation resonates with the social psychology literature mentioned above: agents not only enjoy the victory, but they enjoy having merits in the victory. As the protest in this case needs persistence to be successful, agents who contribute with more persistent participation get more merits. When compared to static models of collective action (and global games) this assumption is the dynamic analogous of assuming that if a protest is successful, agents would rather be there. But there could also be materialistic motives. From an organizational point of view, there is evidence that after a successful social movement there are gains that are distributed among activists. For instance, there could be the promise of future seats in the Congress, or some leadership positions. All these possible gains depend, in practice, in how involved agents are in the organization, which in our model is an increasing function of the time spend in the revolt.

For the government, the cost of the protest has two components, π_t and t. This aims to capture both the instantaneous effect of having given number of people on the streets, and the cost that is accumulated over time. The effect of the size of a protest, can represent a productivity decrease, destruction of infrastructure, injuries, etc. It could also be that a larger protest is more costly because it has a detrimental effect over opinion polls and public opinion in general. Including the duration of the protest as a separate argument represents more indirect costs, that are build through a persistent protest. For instance, it takes time for protesters to capture media's attention, and then at the beginning the popularity of the government is not very affected by a demonstration. Once protests occur in a repeated way, they capture media's attention, affect public opinion, and even give rise to international attention that increases the cost to the government even if participation was constant over time.

The government then, when deciding whether to provide a public good or not, weights the costs of the protest with the costs of concession. This is a simplification of a possibly more complicated problem, in which the government might include other factors on its decision. One possible interpretation of this framework is that the government has its own agenda or policy preferences. The cost q represents how far away from his preferred policy the government would be if it provides the public good. The cost of the protest might represent the effect over public opinion, which affects the probability of being reelected. As the protest lasts longer, the probability of being reelected decreases. Then, concession would occur at a time when the government is willing to get further away from its preferred policy, so that it can be reelected in the future.

Equilibrium characterization. A few remarks are in place about the characterization result stated above. First, note that any equilibrium with government concession involves delay. This is the main feature that separates this model from static models of protests and collective action. It is also a key feature of social movements, which make use of many techniques to ensure persistent participation in order to be successful.

We recognize there could be other motivations for this delay. For instance, there could be learning motives from both sides. On one side, citizens could take time to learn how strong they are, or how much they value the prize. On the other, the government might be interested in getting some information from the protest before deciding whether to concede. This case is particularly interesting because the government would want to choose a concession strategy that allows it to extract as much information as possible with the minimum cost.

The second relevant feature of this characterization, is that all equilibrium with partial government concession have the same shape. In all of them there is a period in which the protest builds up as people continuously join. In this initial stage the protest is still not too costly and then the government ignores it. Once the protest becomes costly enough (which happens when π_t^σ reaches the constraint $\tilde{\pi}_t$ for the first time), the government stops ignoring the protest and makes some concession.

In the equilibria in which the government randomizes, the concession stage unfolds as a war of attrition between the government and citizens. In this war of attrition, instead of having two players, there is a large player on one side (i.e. the government), and a continuum of small players on the other, whose aggregate behavior mimics the behavior of a single opponent. Thus, citizens drop out at some deterministic rate, that on aggregate keeps the government indifferent between waiting and conceding. This drop out rate will be determined by an exit threshold: i.e. an agent who is at the margin between staying a bit longer in the protest, or going home. As this threshold depends on the probability of getting the seniority prize, the government concedes at a hazard rate that makes this marginal agent indifferent.

The driving forces are similar to a two player war of attrition, but behind the mass of participation that decreases over time, there is a set of citizens who are making a deterministic exit decision. Moreover, in contrast with most classic wars of attrition, the hazard rate of government concession will not be constant. As for the government to randomize participation must be decreasing, it has to be the case that citizens' expected gains decrease as well. Then, government's hazard rate must decrease over time.

3 A continuum of equilibria

So far we have shown that any equilibrium in which a protest occur can be parametrized by a time τ_0 at which the protest reaches its maximum size, and at which the government starts conceding. In this section we study how big the set of such delays is. In particular, we show that the set of equilibria it is in fact big, as there is a continuum of them. But it is not too big, as we are able to characterize the upper and lower bounds of it.

Before obtaining the bounds we state the following result, which follows from Proposition 1.

Lemma 1 *Government initial concession $G(\tau_0)$ is decreasing in τ_0.*

What drives this result is the fact that the peak of participation must decrease as the equilibrium is delayed, as the cost of it is increasing in time. But then, as the government concession is delayed, incentives for citizens entering must decrease. This forces the government to decrease initial concession $G(\tau_0)$.

The previous lemma implies the lower bound in a straightforward way. In particular, the initial concession cannot be greater than 1, and then, the set of equilibria is bounded below by the equilibria in which the government concedes for sure at time τ_0. Let's call this lower bound $\underline{\tau}$. If the government is conceding with probability 1 at $\underline{\tau}$, the marginal benefit of the last agent entering is given by $\xi'(0)$, while the marginal cost is its opportunity cost θ. As all the agents with lower opportunity cost have already entered, participation at the time of concession is given by $F(\xi'(0))$. Then, $\underline{\tau}$ solves:

$$c\left(F(\xi'(0)), \underline{\tau}\right) = q \tag{3.1}$$

The upper bound is a bit more subtle. We are normalizing the time, so that $t = 0$ is the time at which the first citizen enters the protest. Given that entry is monotone in θ, the first citizen entering is the citizen with the lowest opportunity cost $\underline{\theta}$. Note that as the delay in the start of government concession increase, the cost of entering increase as well. For this agent to be willing to enter it must be that her cost equal her benefits from the protest, and then $\overline{\tau}$ must be such that:

$$\underline{\theta} = \int_{\overline{\tau}}^{\infty} e^{-rs}\xi'(s)dG(s) \tag{3.2}$$

The left side is the lowest opportunity cost, and the right side is the marginal benefit of entering at 0 and staying in the protest forever, given the government's strategy $G(t)$. Note that as $\overline{\tau}$ increases, the right hand side decreases. Moreover, it is strictly greater than $\underline{\tau}$ if and only if $\underline{\theta} < e^{-r\underline{\tau}}\xi'(\underline{\tau})$. We formalize this in the following assumption.

Assumption 3 *Let $\underline{\tau}$ be such that $c(F(\xi'(0)), \underline{\tau}) = q$. Then, $\underline{\theta} < e^{-r\underline{\tau}}\xi'(\underline{\tau})$.*

Using these bounds we obtain the following existence result.

Theorem 1 *For every $\tau_0 \in [\underline{\tau}, \overline{\tau}]$, there exists a unique equilibrium $(G, (\pi_t^\sigma)_{t\geq 0})$ where the government concedes for the first time at τ_0.*

To prove this result, we first show that there exist equilibria with $\tau_0 = \underline{\tau}$ and $\tau_0 = \overline{\tau}$. The lower bound is straightforward, and the latter follows from a fixed point argument. The key point to find the fixed point is that on the concession stage the trajectory of participation is fixed at $\tilde{\pi}_t$. Thus, in citizens' problem the equilibrium exit times are given: a citizen with opportunity cost θ exits at the time t at which $F(\theta) = \tilde{\pi}_t$. Then, their best reply is a sequence of entry times given the government distribution of concessions, and taking their exit times as given. The government in turn, gives these entry times, chooses a hazard rate that makes the marginal agent indifferent between conceding and waiting another instant (in order to keep participation at the indifference level in the concession stage). To close things up, we introduce a fictitious player whose only role is to adjust $\underline{\tau}$ for equation 3.2 to be satisfied with equality, given the government strategy $G(t)$. This also allows us to get rid of discontinuities of the government strategy at τ_0, and then we can apply standard fixed point theorems. It is then straightforward to use the same fixed point argument to show that for any $\tau_0 \in [\underline{\tau}, \overline{\tau}]$ an equilibrium exists.

Figures 1 and 2 illustrate the continuum of equilibria. The three panels in figure 1 illustrate the trajectory of participation for three possible delays τ_0. The decreasing dotted line, $\tilde{\pi}_t$, shows the participation levels that makes the government indifferent, at any time t, between conceding and waiting another instant. For any participation level π_t below this dotted line, $c(\pi_t, t) < q$, and then the government is better off by ignoring the protest. Analogously, any participation level above this line it is too costly and the government would rather concede.

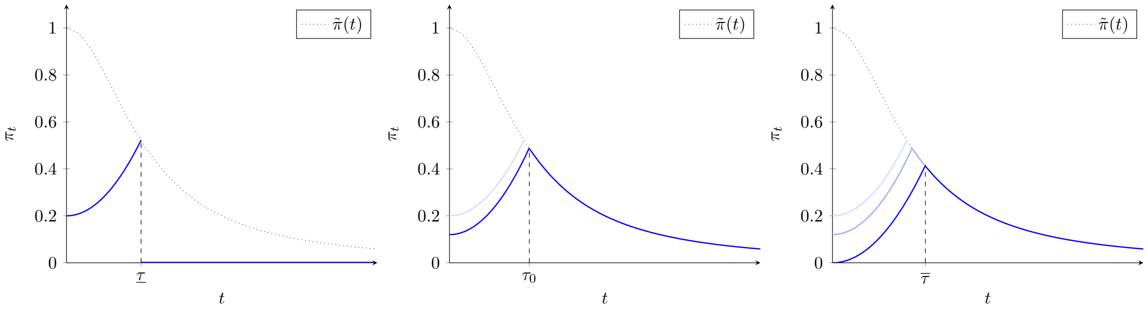
The three panels in figure 2 show the distributions of government concession for the same delays τ_0. Panel (a) in both figures shows the equilibrium with the shortest delay, $\underline{\tau}$. Panel (b) shows an equilibrium with intermediate delay, $\tau_0 \in (\underline{\tau}, \overline{\tau})$, and panel (c) shows an equilibrium with the maximum delay possible, $\overline{\tau}$.

For any delay τ_0, participation is increasing on $[0, \tau_0]$. This corresponds to the building-up stage. As in this stage participation is everywhere below the line $\tilde{\pi}_t$, the government is better off by waiting. Thus, $G(t) = 0$ on $[0, \tau_0]$ in figure 2. Once participation hits the dotted line, then the protest has become costly enough and the government has to make some concession. The equilibrium with the shortest delay, $\underline{\tau}$ in panel (a), corresponds to the equilibrium in which the government concedes with probability 1. Then, the distribution of government concessions jumps up to 1, and everyone drops out.

Now let's move to panel (b), with $\tau_0 \in (\underline{\tau}, \overline{\tau})$. Note that the initial government concession decreases (i.e. $G(\tau_0) < 1$), which is consistent with Lemma 1. Then the government randomizes over time, and people continuously drop out. Participation then coincides with the dotted line in equilibrium.

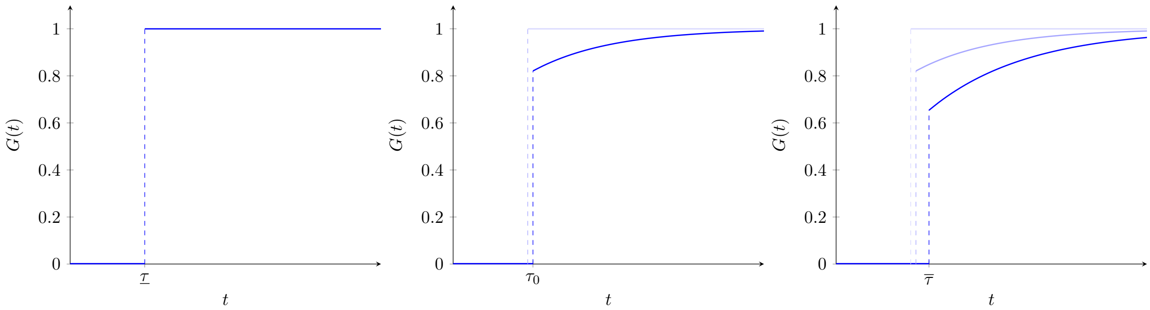
Note that as we increase delay (moving to panels (b) and (c)) participation decreases for every t on the building-up stage. We can show that this is in fact a general feature of the equilibrium set, and it implies a very nice property: there exists a one-to-one relation between initial participation π_0, the peak in participation π_{τ_0}, and the delay in the concession stage $\tau_0 \in [\underline{\tau}, \overline{\tau}]$.

Figure 1: Equilibrium Participation



(a) Lower bound $\underline{\tau}$ (b) $\tau_0 \in (\underline{\tau}, \overline{\tau})$ (c) Upper bound $\overline{\tau}$

Figure 2: Government's Distribution of Concessions



(a) Lower bound $\underline{\tau}$ (b) $\tau_0 \in (\underline{\tau}, \overline{\tau})$ (c) Upper bound $\overline{\tau}$

The one-to-one relationship between the peak in participation and delay shows the existence of a trade-off between the mass of people that needs to get involved, π_{τ_0}, and the persistence, τ_0, required to make the government concede in equilibrium. In particular, participation could grow and hit the constraint very quickly, and in that case it requires a very high participation peak. But it could also have a slow trend upwards, in which case the participation peak is lower, as the protest can take advantage of the cost of time the government has to bear. This is related with the idea of *momentum* studied by Chenoweth & Belgioioso (2019), according to which protests need a combination of mass (participation) and velocity (frequency of events) in order to succeed. We formalize the result in the following corollary.

Corollary 1 *Let $[\underline{\tau}, \overline{\tau}]$ be the set of equilibrium delays. Then, $\tilde{\pi}_{\underline{\tau}} > \tilde{\pi}_{\overline{\tau}}$, and the mapping $\tau_0 \mapsto \pi_{\tau_0}$ is bijective and strictly decreasing.*

The key to obtain the result above lies in the fact that for any two possible equilibria with participation outcomes $(\pi'_t)_{t\geq 0}$ and $(\pi''_t)_{t\geq 0}$, and different delays τ'_0 and τ''_0, participation outcomes do not cross on the building up stage. In particular, if $\tau'_0 < \tau''_0$, then $\pi''_t < \pi'_t$ for every $t \in [0, \tau''_0)$. This implies that not only we can rank the peak in participation, but participation between any two equilibria in their building-up stage.

Then, we have that conditional on the first event, there is a unique trajectory of participation. This result gives an idea of how informative it is the first event of a social movement. Fixing the fundamentals, the initial participation is enough to describe the complete trajectory of participation, and then the expected duration until government concession.

3.1 Discussion

Multiplicity: As it is natural in a framework with complete information, there are multiple equilibria. However, the multiplicity in this framework provides key ideas about both (i) the dynamics that are common to all equilibria, and (ii) the trade-offs between persistence and participation across them. The main common dynamic across equilibria is the existence of a building up stage: in all of them there exists a period in which participation is increasing and the government waits. Moreover, in all of them the peak in participation is reached when the government starts conceding. And finally, they can be also parametrized according to initial participation π_0. This last feature is insightful in the sense that conditional on initial participation, the equilibrium is unique.

The second point is related to Corollary 1. In particular, equilibria can be ranked both in terms of delay, $\tau_0 \in [\underline{\tau}, \overline{\tau}]$, and in terms of participation peak $\pi_{\tau_0} \in [\pi_{\overline{\tau}}, \pi_{\underline{\tau}}]$. What's interesting is that the relation is inverse: equilibria with longer delay in government concession have a lower peak in participation. Then, there is a trade off across equilibria regarding *persistence*, i.e. the time until the government starts conceding, and citizens' *involvement*, i.e. maximum participation required for the government to start conceding.

Besides any insights we can learn from multiplicity, we also argue that multiplicity is a natural feature of the model. As it has been recognized in the literature, the spontaneous nature of uprisings gives them the features of a coordination problem which might, or might not, be successful (see Schelling (1960), Hardin (1997), and more recently, De Mesquita (2014)). This implies that in general in static models of collective action there are two equilibria in pure strategies: one in which a protest occurs, and one in which it does not occur . In our model, the dynamics of equilibrium imply that not only we observe equilibrium with and without protests, but there is a continuum of equilibria with protests.

Consider, for instance, a society in which there are workers and students. Workers are not very politically involved and they coordination capacity is low. This might be because of the type of institutions, culture, or the country's history. Students are the group with the lowest opportunity cost, and they have better coordination technologies than workers. Thus, every time there is a general revolt, it begins with only students as participants, and then as time goes by workers join the movement. We can say that this society is stuck in the equilibrium with the longest delay in government concession.

However, consider another society in which citizens are politically active and it is very easy for them to coordinate. In this society, every time a protest begins it includes not only students but also workers. Initial participation is high, and as more citizens join, the protest forces the government to concede with very high probability. We can say that this society is in the equilibrium with the shortest delay in government concession. Whether a society focal point centers in one equilibrium or the other depends on features that, despite their relevance, this model does not aim to capture.

Refinements and Equilibrium Selection: Even when multiplicity is insightful, for some problems it might be relevant to refine the set of equilibria. In order to do this it will be key to modify the model to include some sort of incomplete information. The three main approaches that can be applied as refinements are: (i) reputation, (ii) global games, and (iii) coalition proofness. Reputation concerns in this model arise when there is some information about agents that is private. The attrition nature of the game makes behavioral types a la Abreu & Gul (2000) a natural candidate for refinement. Introducing a probability of the government being a behavioral type who never concedes, and a probability of citizens being some type who will protest forever, will pin down a unique equilibrium. The only issue with this refinement is that, in our framework, is not very informative of the equilibrium selected.

Global games are a theoretical framework commonly used to study uprisings and regime change models. Since the seminal work of Morris & Shin (1998), their framework has been used to study public protests and revolutions in different institutional settings (see Edmond (2013), Egorov et al. (2009), Boix & Svolik (2013) and Morris & Shadmehr (2018)). The key component is in these models is a coordination game with incomplete information, in which uncertainty in general is about the strength of the regime (although it could as well be uncertainty about preferences or other features of the game). Agents in this setup receive some private information, and in equilibrium they use threshold strategies, so that a player participates if her belief about the revolt being successful is high enough with respect to the threshold. This pins down a unique equilibrium. In dynamic setups is not direct to refine the set of equilibria in this way. For instance, Angeletos et al. (2007) study the role of learning in a framework in which agents can take actions many times and learn about the fundamentals. In this context, they show that the dynamic nature of the game introduces multiplicity even under conditions that guarantee uniqueness in static games.

Lastly, the possibility of coalition formation by citizens provides another rationale for equilibrium refinement. Naturally, political activism requires some organization that can be done before a protest begins. This could be done in a decentralized way (via social media, for instance) or through a political leader who is interested in fostering a particular equilibrium. If citizens could make prearrangements to decide their involvement in the protest, they could coordinate in the equilibria with the shortest delay by making sure a fraction of the population high enough joins the protest since the beginning. As all citizens are better off with this outcome, we would expect no coalitions blocking that equilibrium, and then it would be coalition-proof in the sense of Bernheim et al. (1987) (see also Moreno & Wooders (1996) and Ray & Vohra (2001)).

Although in coalition-proofness agreements among agents are non-binding, sometimes leaders make some irreversible actions to try to obtain a specific outcome. For instance, Morris & Shadmehr (2018) construct a model in which citizens choose the level of effort to contribute to a regime change, and a leader designs reward schemes that assign psychological rewards to their actions. In our model a leader could target some sectors in the society in order to implement different equilibria. For instance, citizens with higher opportunity costs make the government concede faster, and then the leader might want to design a seniority reward scheme to incentivize their participation. In practice, leaders make use of their charisma and targeted rhetoric to encourage specific groups of the population to get involved in a revolution. Another alternative is that a strong organization may want to implement transfers among citizens to subsidize the protest behavior of specific groups to keep them in the fight. For instance, in some countries protest organizers support protesters with food and supplies, which can be seen as a way to reduce participation costs to citizens who attend demonstrations.

3.2 Expected duration across equilibria

Even when the equilibria can be ranked in terms of the delay in government concession, the expected duration of a protest depends also on the hazard rate at which the government concedes over time.

Consider two different equilibria with delays $\tau_0^1 < \tau_0^2$. For any time $t \geq \tau_0^2$, the hazard rates are such that $\lambda_t^1 < \lambda_t^2$. Thus, we have that the life expectancy of the two protests from time $t \geq \tau_0^2$ is shorter for the equilibrium with delay τ_0^2 than for the one with shorter delay τ_0^1. The intuition for this is that if we compare the same citizen in both equilibria, she has spend a shorter time in the protest in the equilibria with more delay, and then for her to be protesting in equilibrium the government must concede at a higher hazard rate.

4 Comparative statics: Equilibrium set

In this section we analyze some comparative statics with respect to the equilibrium set. An increase in a citizen's opportunity costs has two effects. On one side, it has a direct effect over agents' entry decision, as a citizen with higher opportunity cost will want to wait for the marginal value of entry to increase. On the other side, it has an indirect effect over the government's best reply. In particular, citizens with higher opportunity costs are stronger in front of the government, as the government has to concede at a higher hazard rate to make them drop out. The second effect is not observed in the equilibrium with the shortest delay, but it affects the upper bound.

In particular, we have that a general increase in agents opportunity cost moves the lower bound of the equilibrium set to the right. When agents' participation costs are higher, it takes longer to reach the level of participation required to make the government concede with probability 1. This is a general result that in fact does not depend on what happens with agents that would not protest even if the government concedes with probability one.

In addition, the comparison with weaker types of dominance is ambiguous, as it depends on what happens with the agent who is at the margin when the government is going to concede for sure. We formalize these ideas in the following result.

Proposition 2 *Let F_1 and F_2 be two symmetric and unimodal distributions, with corresponding equilibrium sets $[\underline{\tau}_1, \overline{\tau}_1]$ and $[\underline{\tau}_2, \overline{\tau}_2]$.*

(i) If F_1 first-order stochastically dominates F_2, then $\underline{\tau}_1 \geq \underline{\tau}_2$.

(ii) If F_2 is a mean preserving spread of F_1, and $\xi'(0) < \int \theta dF_1(\theta)$ then $\underline{\tau}_1 > \underline{\tau}_2$.

(ii) If F_2 is a mean preserving spread of F_1, and $\xi'(0) > \int \theta dF_1(\theta)$ then $\underline{\tau}_1 < \underline{\tau}_2$.

The results above rely on the fact that the lower bound of the equilibrium set for a distribution F, depends uniquely on $F(\xi'(0))$. In any equilibrium at which the government concedes for sure, the amount of people who are willing to enter are those with opportunity cost $\theta \leq \xi'(0)$. Then, any changes to the distribution of opportunity costs that increases the amount of citizens that is willing to enter, push the government towards conceding faster.

The analysis for the upper bound is more subtle, as changes to the distribution of opportunity costs have a secondary effect through the government's hazard rate. Consider first a general increase in citizens' opportunity costs, so that protesting becomes more costly for every agent. Let F_1 be the initial distribution of opportunity costs, and let F_2 be the distribution after the increase. Then, $F_1(\theta) > F_2(\theta)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. Two things are going to happen. First, keeping the government strategy constant, agents will delay their entry as protesting becomes more costly. For the exit to be an equilibrium, the government will increase the hazard rate. But note that this new situation cannot be an equilibrium as the entry time for the lowest opportunity cost citizen is such that $t_0(\underline{\theta}) > 0$. Then it has to be that agents enter earlier. We show in the next result that this must imply that $\overline{\tau}$ decreases under F_2.

Proposition 3 *Suppose that citizens opportunity costs increase by the same proportion α, and let $[\underline{\tau}_\alpha, \overline{\tau}_\alpha]$ to be the new equilibrium set. Then, it must be that $\underline{\tau} < \underline{\tau}_\alpha < \overline{\tau}_\alpha < \overline{\tau}$.*

5 Some extensions

5.1 Income and Opportunity cost

So far we have characterized agents opportunity cost of the time spent in the protest by a parameter θ. What this parameter is measuring is the utility that agents give up by spending time in the protest instead of spending time on other activities. In reality, those other activities are related to productive activities, and then the opportunity cost can be associated to labor income. In this section we analyze the relation between this opportunity cost and citizens' income.

In order to set ideas, consider the following situation. As in our baseline framework, there is a protest and citizens have to decide whether to join, and when to join. A citizen who joins a protest, attends every day a demonstration which lasts one hour. There is no physical cost of protesting, and then the only cost to the citizen is the alternative use of time that she can give to this hour, which is equivalent to one hour-wage. Let ϵ be the fraction of time the agent spends in the protest (i.e. if the agent works 8 hours and the demonstration lasts one hour, then $\epsilon = 1/8$).

In addition, suppose there is a minimum level of consumption that citizens must satisfy, which corresponds to a subsistence level. We can think of this as basic needs that the agent must fulfill, and then she can only afford to join a protest if she has satisfied her basic needs. We represent the subsistence level by a minimum income $\underline{\omega}$, such that any agent with income $\omega < \underline{\omega}$ cannot afford to become an activist.

The cost of attending the protest for a citizen with $\omega \geq \underline{\omega}$ would be equivalent to:

$$\theta = u(\omega) - u\left(\omega\left(1-\epsilon\right)\right) \tag{5.1}$$

where $u(\cdot)$ is the agent's utility of income (consumption). Then, the relation between citizens' income and opportunity costs will depend on the shape of the utility function.

Consider, for instance, the following CRRA utility function:

$$u(\omega) = \frac{\omega^{1-\sigma}}{1-\sigma} \tag{5.2}$$

for some $\sigma \geq 0$, $\sigma \neq 1$. In this case the relation between the opportunity cost of protesting and income will depend on the curvature of the utility function, captured by σ. If $\sigma < 1$, then the marginal utility of income is increasing, and then for citizens with higher income the hour spent in the demonstration is more costly than for citizens with lower income. In the extreme case with $\sigma = 0$, utility is linear and then $\theta = \epsilon\omega$. In this case there is a one-to-one relation between the distribution of opportunity costs, and the distribution of income. In general, when the marginal utility of income is increasing, agents with higher income have incentives to delay their entry compared to lower income citizens.

If the opposite holds, i.e. $\sigma > 1$, then the marginal utility of income decreases with income. In this case, high income citizens are able to protest more as the forgone marginal utility for them is lower.

There are other factors that might affect the relation between income and opportunity costs. For instance, job formality might affect how workers can make use of their own time to join a demonstration. This in general will also be related with education and the type of industries under analysis. Moreover, income might affect other components in the propensity of an agent to protest that might not be related with opportunity costs. Education, for instance, is key in how knowledgeable about the political environment citizens are. This implies that when comparing citizens with different income levels we need to also take into account the effect of their income over education levels.

There might be also groups of protesters whose opportunity cost does not have a mapping to income at all. Students are an example of that case. Students' participation in protests is in general very high, which of course reflects many factors that have been extensively studied by the sociologists and social psychologists. But a relevant factor comes from the fact that the opportunity cost of their time tend to be lower than workers, as they have more freedom to allocate their time across activities.

5.2 Government partial concessions

In many situations, the decision to provide a public good is not discrete. Governments might make some concessions that do not completely fulfill the demands, but that dissuade some protesters, and then alleviate the cost burden of the protest. In this section, we illustrate how the baseline model can be modified in order to allow for such concessions.

In this new framework, the government can concede a fraction of the public good. Conceding a fraction α of the public good has a cost αq, where q is the cost of the entire public good. Every time the government concedes a fraction α of the public good, agents receive a utility $\alpha\xi(t-t_0)$, corresponding to their seniority payoff. Other than this, citizens' and government's payoffs remain the same as in the baseline case. The protest ends when either all citizens have dropped out, or the government has fully provided the public good.

In this context, following the same reasoning as in the baseline case, we can define the government's strategy as a function $h : [0, \infty) \to [0, 1]$ that determines, for any t, additional proportion of public good that the government provides at time t. We also denote by $H(t)$ the share of public good that has been provided at time t.

A citizen's payoff from entering at a time t_0 and exiting at t_1 is given by:

$$U(t_0, t_1; \theta) = -\theta \left[\frac{e^{-rt_0} - e^{-rt_1}}{r} \right] + \int_{t_0}^{t_1} e^{-rs} \xi(s - t_0) dH(s) \tag{5.3}$$

Citizens' utility functions satisfy a single-crossing property, and then their strategies will be monotone in opportunity cost. In particular, entry and exit conditions for a citizen with opportunity cost θ are given by:

$$\theta = \int_0^{t_1} e^{-rs} \xi'(s - t_0) dH(s) \tag{5.4}$$

$$\theta = \xi(t_1 - t_0) h(t_1) \tag{5.5}$$

Then, we obtain analogous characterization results as those in the baseline model. For any equilibrium $H : [0, \infty) \to [0, 1], (\pi_t^\sigma)_{t \geq 0}$ with $\tau_0 = \inf\{t \in [0, \infty] : h(t) > 0\}$, the following conditions hold:

(i) There is always delay in government concession, i.e. $\tau_0 > 0$.

(ii) π_t^σ is continuous, increasing for $t \leq \tau_0$, and if $H(\tau_0) < 1$, decreasing for all $t \geq \tau_0$.

(iii) The government makes at most one discrete concession at τ_0, $H(\tau_0) \leq 1$.

(iv) If $H(\tau_0) < 1$, then $H(t)$ is strictly increasing, concave, and for $t > \tau_0$ $H(t) < 1$.

5.3 Support for the public good

In our baseline model we assume the unit mass of citizens is willing to consider participation in the protest, as the only constraint is the heterogeneity in opportunity costs. In this section we consider a situation in which only a subset of agents is willing to consider participating in the protest. Suppose the value of the public good v is now a random variable that can take two values: either $v_l = 0$, or $v_h > 0$, and let p be the probability of $v = v_h$. Each citizen's value for the public good is independent of her opportunity cost.

Moreover, assume citizens only value the seniority prize if they value the public good. Thus, we modify the seniority prize function to be $v \cdot \xi(t - t_0)$. Without loss of generality we set $v_h = 1$. With this new framework, agents who don't value the public good do not have incentives to participate in the protest. It is clear to see that the baseline case is equivalent to setting $p = 1$, and then as p decreases, the mass of citizens willing to enter the protest also decreases. Thus, for any possible entry threshold $\tilde{\theta}_0(t)$, participation is given by $\pi_t = p \cdot F(\tilde{\theta}_0(t))$.

What is interesting about this perturbation, is that all the dynamics of the model remain the same, but the set of equilibria is reduced. For the government, the same properties shown for the baseline case still hold: (i) the government concedes according to a distribution $G(t)$ with support $\mathcal{T}$, with $\tau_0 = \inf \mathcal{T}$; (ii) the support might be either a singleton, or an interval $[\tau_0, \infty)$; (iii) $G(\tau_0) < 1$ then $\mathcal{T} = [\tau_0, \infty)$, and $G(t)$ is continuous, strictly increasing, and differentiable in (τ_0, ∞).

For citizens, the optimization problem remains the same. A citizen with opportunity cost θ who values the public good, the entry and exit times $(t_0(\theta), t_1(\theta))$ solve the following conditions:

$$\theta = \lambda_{t_1} \xi(t_1 - t_0) \tag{5.6}$$

$$\theta = \int_{t_0}^{t_1} e^{-r(s-t_0)} \xi'(s - t_0) dG(s) \tag{5.7}$$

As citizens utility still satisfy the single-crossing property, their optimal strategies will be monotone in θ. Then, they can be characterized by thresholds $\tilde{\theta}_0(t), \tilde{\theta}_1(t)$ such that a citizen enters if $\theta \leq \tilde{\theta}_0(t)$ and exits if $\theta > \tilde{\theta}_1(t)$.

Since the hypothesis are analogous to the baseline case, both Proposition 1 and Theorem 1 hold in this framework.

We highlight some of the main features that differentiate this case with respect to the baseline case. Let $[\underline{\tau}, \overline{\tau}]$ be the equilibrium set in the baseline case, and denote by $[\underline{\tau}_p, \overline{\tau}_p]$ the equilibrium set with $p < 1$. First, note that it has to be that $\underline{\tau} < \underline{\tau}_p$ and $\overline{\tau}_p \leq \overline{\tau}$. The intuition is analogous to the comparative statics in proposition 2. To see why the lower bound is more delayed with $p < 1$, i.e. $\underline{\tau} < \underline{\tau}_p$, recall that this corresponds to the equilibrium in which the government concedes with probability 1. In this equilibrium, concession occurs when $\tilde{\pi}_{\underline{\tau}} = F(\xi'(0))$. When not all agents are willing to participate, it must be that $\tilde{\pi}_{\underline{\tau}_p} = p \cdot F(\xi'(0)) < \tilde{\pi}_{\underline{\tau}}$. The upper bound it does not necessarily decreases as it depends on the citizen with the lowest opportunity cost.

Consider now an intermediate equilibrium, with delay $\tau_0 \in (\underline{\tau}_p, \overline{\tau}_p)$. Let $G(t)$ be the government distribution of concessions with $p = 1$, and $G_p(t)$ for $p < 1$. Note that both $G_p(t)$ and $G(t)$ have support $[\tau_0, \infty)$. Moreover, in both equilibria participation must coincide on $[\tau_0, \infty)$. Then, the initial government concession is such that $G(\tau_0) < G_p(\tau_0)$.

The main idea of these differences, is that now the universe of citizens is smaller, as not all of them are willing to enter the protest. However, conditional on reaching some participation level, it has to be that the ones who are protesting have higher opportunity costs (with respect to the baseline case), and that makes them stronger in front of the government.

6 Proofs

6.1 Proof of Proposition 1: Equilibrium Characterization

This section is devoted to prove the equilibrium characterization described in Section 2. We begin by proving some properties of the government's equilibrium strategy, and then we use these to fully characterize the set of equilibria. In the first lemma, we show that if there is an interval after τ_0 in which the government does not concede (i.e. the distribution G is constant in that interval), then no agent who is in the protest drops out during that interval. More precisely, we say that an agent with opportunity cost θ is participating at a time t if $\sigma_t^\theta = 1$.

Lemma 2 *Assume $\tau_0 < \tau_1$ and take t_1, t_2 such that $\tau_0 \leq t_1 < t_2 \leq \tau_1$. If G is constant in (t_1, t_2), then no agent participating at t_1 drops out in $(t_1, t_2]$.*

Proof. For any citizen that is participating at t_1, she is strictly better off quitting at t_1, than at any $t \in (t_1, t_2]$. □

Lemma 3 *The support of G is either a singleton, or a connected interval $\mathcal{T} = [\tau_0, \tau_1]$.*

Proof. By contradiction, suppose there exists $t \in [\tau_0, \tau_1]$ such that $t \notin \mathcal{T}$. Then, $t > \tau_0$, and there exists $\epsilon \in (0, t - \tau_0]$ such that $G(t) - G(t - \epsilon) = 0$. But then $[t - \epsilon/2, t] \cap \mathcal{T} = \emptyset$, so if there is $t \notin \mathcal{T}$, there is an interval which does not belong to $\mathcal{T}$. Then take t_0, t_1, with $\tau_0 \le t_0 < t_1 \le \tau_1$ such that $G(s) = G(t_0)$ $\forall s \in [t_0, t_1)$.

Assume $[t_0, t_1)$ is maximal, i.e. there is no interval $[t_0', t_1')$ such that $[t_0, t_1) \subsetneq [t_0', t_1')$ and $G(s) = G(t_0')$ for every $s \in [t_0', t_1')$. Maximality of the interval implies that $t_0 \in \mathcal{T}$. If not, there exists $\epsilon_1 > 0$ such that $G(t_0) - G(t_0 - \epsilon_1) = 0$, but then $G(s) = G(t_0 - \epsilon)$ $\forall s \in [t_0 - \frac{\epsilon}{2}, t_1]$. By maximality, for every $\epsilon > 0$ $[t_1, t_1 + \epsilon) \cap \mathcal{T} \neq \emptyset$. Then, it is optimal for the government to concede at t_0 and at t_1.

Note that for the government to concede at t_0 the cost of conceding must less than or equal than the cost of waiting. The cost of conceding at t_0 is $\frac{q}{r}$, while the cost of waiting to concede at some $t_0 + \delta$ for $\delta > 0$ is given by

$$\int_0^\delta e^{-rs} c(\pi^\sigma_{t_0+s}, t_0 + s) ds + e^{-r\delta} \frac{q}{r} \tag{6.1}$$

Then, we have:

$$\int_0^\delta e^{-rs} c(\pi^\sigma_{t_0+s}, t_0 + s) ds + e^{-r\delta} \frac{q}{r} \quad \geq \quad \frac{q}{r} \qquad \forall \delta > 0 \tag{6.2}$$

or, equivalently

$$\int_0^\delta e^{-rs} \big(c(\pi^\sigma_{t_0+s}, t_0 + s) - q\big)\, ds \quad \geq \quad 0 \qquad \forall \delta > 0. \tag{6.3}$$

Define $\bar{t} = \frac{t_0 + t_1}{2}$. Note that as 6.3 holds for every $\delta > 0$, it must also hold for $\overline{\delta} = \bar{t} - t_0$.

Since $t_0 \in \mathcal{T}$, then it must be that $\pi^\sigma_{t_0} > 0$, as otherwise the cost of the protest is zero. Moreover, by lemma 2, no citizen drops out at $(t_0, t_1]$, so $\pi^\sigma_{\bar{t}+s} \ge \pi^\sigma_{t_0+s}$ for all $s \in (0, \overline{\delta}]$. As, $\pi_{t_0} > 0$, then the cost is strictly increasing in time, and we have:

$$c(\pi^\sigma_{t_0+s}, t_0 + s) < c(\pi^\sigma_{\bar{t}+s}, \bar{t} + s) \qquad \forall s \in (0, \overline{\delta}] \tag{6.4}$$

Then, we can compute:

$$\begin{aligned} \int_{t_0}^{t_1} e^{-r(s-t_0)} \left(c(\pi^\sigma_s, s) - q\right) ds \quad &= \quad \int_{t_0}^{\bar{t}} e^{-r(s-t_0)} \left(c(\pi^\sigma_s, s) - q\right) ds \\ &\quad + \quad e^{-r(\bar{t}-t_0)} \int_{\bar{t}}^{t_1} e^{-r(s-\bar{t})} \left(c(\pi^\sigma_s, s) - q\right) ds \end{aligned} \tag{6.5}$$

The first term on the right hand side is weakly greater than 0. By 6.4 the second term must then be strictly greater than zero, which implies $\int_{t_0}^{t_1} e^{-rs}\left(c(\pi^\sigma_{t+s}, t+s) - q\right) ds > 0$. But then the government strictly prefers to concede at t_0 than at t_1, which is a contradiction. $\square$

Lemma 4 *If $\mathcal{T}$ is not a singleton, then it must be that $c(\pi^\sigma_s, s) = q$ and $\pi^\sigma_s = \tilde{\pi}_s$ for every $s \in [\tau_0, \tau_1)$.*

Proof. For the government to be randomizing over concession times $\tau \in [\tau_0, \tau_1]$, it must be that:

$$\int_0^\tau e^{-rs} c(\pi^\sigma_s, s) + e^{-r\tau}\frac{q}{r} = a \qquad \forall \tau \in [\tau_0, \tau_1] \tag{6.6}$$

for some constant a. Taking first order conditions with respect to τ, we obtain $c(\pi^\sigma_\tau, \tau) - q = 0$, which proves the result. $\square$

Lemmas 3 and 4 provide a characterization of the regions over which the government concedes. The problem for the government is a stopping time problem, in which we allow the government to randomize. For citizens the problem is a little different. Given that we do not impose restrictions on the actions that citizens can take, they could enter and exit the protest many times. So far there is nothing that prevents a citizen to protest over a time interval, then drop out to spend some time outside the protest, and then protesting again. However, we show that in equilibrium citizens enter and exit at most once. In particular, their optimality conditions satisfy a monotonicity property with respect to opportunity cost, that ensures that citizens' strategies can be characterized by opportunity cost thresholds. In lemma 5 we give some sufficient conditions for these entry and exit times to be optimal. Optimality conditions are stated in terms of the hazard rate of government concession, $\lambda_t = \frac{g(t)}{1-G(t)}$, which corresponds to the instantaneous probability of government concession conditional on the it being still in the game.

Lemma 5 *In equilibrium, citizens enter and exit at most once. For a person with opportunity cost θ who does enter, the optimal entry and exit times, $t_0(\theta), t_1(\theta)$ are a solution to* *the following sufficient conditions:*

$$\theta = \lambda_{t_1}\xi(t_1 - t_0) \tag{6.7}$$

$$\theta = \frac{1}{1-G(t_0)}\int_{t_0}^{t_1} e^{-r(s-t_0)}\xi'(s-t_0)dG(s) \tag{6.8}$$

Moreover, optimal entry and exit times satisfy $t_0'(\theta) > 0$ *and* $t_1'(\theta) < 0$*, respectively.*

Proof. Consider a citizen with opportunity cost θ who is planning to enter, on the equilibrium path, at some time t_0 and exit at t_1, i.e. $\sigma_t^\theta = 1$ for $t \in [t_0, t_1)$. Given a random concession time τ for the government, the citizen solves the following problem:

$$\max_{(t_0,t_1)\in[0,\infty]^2} E\left[-\theta\int_{t_0}^{t_1\wedge\tau} e^{-rs}ds + e^{-r\tau}\mathbb{1}_{\tau<t_1}\xi(t-t_0)\right] \tag{6.9}$$

where the expectation is taken over τ, and where we have omitted additive payoffs that are not under the agent's control. Plugging in the distribution of government concessions G the objective function can be rewritten as:

$$U(t_0,t_1;\theta) = \int_{t_0}^{t_1}\left[-\frac{\theta}{r}(e^{-rt_0} - e^{-rs}) + e^{-rs}\xi(s-t_0)\right]dG(s) - (1-G(t_1))\frac{\theta}{r}(e^{-rt_0} - e^{-rt_1}) \tag{6.10}$$

As long as an agent is in the protest she has to pay the cost of the protest. If the government concedes before the time she drops out, the citizen gets the seniority reward. If the government has not conceded by the time the agent drops (which happens with probability $(1 - G(t_1))$), then the agent only pays the cost of the protest and does not get any prize. Taking first order conditions with respect to t_0 and t_1, we have:

$$\frac{\partial U}{\partial t_0} = -(1-G(t_0))\theta + g(t_0)\xi(0) + \int_{t_0}^{t_1} e^{-r(s-t_0)}\xi'(s-t_0)dG(s) \tag{6.11}$$

$$\frac{\partial U}{\partial t_1} = -\theta e^{-rt_1}(1-G(t_1)) + g(t_1)e^{-rt_1}\xi(t_1-t_0) \tag{6.12}$$

Reorganizing, we obtain equations 6.7 and 6.8 from the lemma. Note that these equations have a unique solution.

The fact that first order conditions are also sufficient follows from a single-crossing property of agents utility with respect to opportunity cost. In particular, the marginal utilities of agents' strategies are monotone in θ, i.e.

$$\frac{\partial^2 U}{\partial t_0 \partial\theta} = e^{-rt_0}(1-G(t_0)) \geq 0 \qquad \frac{\partial^2 U}{\partial t_1 \partial\theta} = -e^{-rt_1}(1-G(t_1)) \leq 0 \tag{6.13}$$

Thus, citizens follow monotone strategies satisfying $t_0'(\theta) > 0$, $t_1'(\theta) < 0$.

Now, suppose an agent is considering to reenter. Note that once the agent exits, her problem becomes the same from equation 6.9, as the seniority payoff goes back to zero. But then by the single crossing property we just proved reentry cannot be optimal. This concludes the proof. $\square$

From equation 6.7 we see that an agent will exit when the marginal cost of staying another instant, i.e. θ, exceeds the marginal benefit, i.e. the prize times the instantaneous probability of government concession conditional on the government being still in the game. Equation 6.8 has a similar interpretation: the agent enters if the marginal cost is smaller than the marginal benefit. The marginal benefit now has two components. The first term in the right hand side captures the probability of obtaining the prize immediately, while the second one corresponds to the marginal benefit obtained from increasing the prize for all future periods that the agent plans to protest.

From Lemma 5, at any time agents' decision can be characterized by opportunity cost thresholds. More precisely, define $\tilde{\theta}_0(t) = t_0^{-1}(t)$, and note that this corresponds to the agent who is indifferent between entering at time t or waiting (i.e. equation 6.8 holds with equality). Any citizen with opportunity cost $\theta < \tilde{\theta}_0(t)$ is strictly better off by being in the protest. Analogously define $\tilde{\theta}_1(t) = t_1^{-1}(\theta)$, and note that it corresponds to the agent who is indifferent between staying in the protest another instant or exit immediately. Any citizen with $\theta > \tilde{\theta}_1(t)$ is strictly better off by dropping out.

We now put this ingredients together to prove Proposition 1 using the following steps.
Step 1: If $\tau_0 < \tau_1$, then π_t^σ is strictly decreasing in t, for every $t \in [\tau_0, \tau_1)$. From lemma 3, it must be that $c(\pi_t^\sigma, t) = q$ at every $t \in [\tau_0, \tau_1)$. Then, $\pi_t = \tilde{\pi}(t)$ for every $t \in [\tau_0, \tau_1)$. This function is well-defined, continuous and decreasing by assumption 1.

Step 2: The distribution has at most one discrete jump at τ_0. Suppose there is $t > \tau_0$ such that the distribution G jumps at t, i.e. there is $\epsilon > 0$ such that $G(t) > G(s)$ for all $s \in [t-\epsilon, t)$. But then there is an interval over which citizens will not drop, contradicting the previous step.

Step 3: If $\tau_0 < \tau_1$, then at every $t \in [\tau_0, \tau_1)$ the distribution of concessions G has decreasing *hazard rate.* From equation 6.7 in Lemma 5, for citizens' decision to be optimal the exit threshold must satisfy:

$$\tilde{\theta}_1(t) = \lambda_t \xi(t - t_0(\tilde{\theta}_1(t))) \tag{6.14}$$

From the previous step, we have that the threshold must satisfy $F(\tilde{\theta}_1(t)) = \tilde{\pi}(t)$, and then it is decreasing over time. Then the left-hand side of equation 6.14 is decreasing, while the prize function increases over time, so it has to be that λ_t is decreasing.

Step 4: If $\tau_0 < \tau_1$, then $\tau_1 = \infty$. Suppose $\tau_1 < \infty$. First, it must be that $G(\tau_1) = 1$. Suppose that this is is not the case and the government stops conceding at some τ with $G(\tau) < 1$. Using the same arguments as in the proof of lemma 3, it must be $c(\pi_\tau, \tau) \geq q$. But then $\pi_\tau > 0$, as otherwise $c(\pi_\tau, \tau) = 0$ by assumption 1. By lemma 2 no citizen drops after τ, but then as the cost is increasing in time, eventually the cost of the protest would be higher than the cost of waiting, contradicting the optimality of the government's strategy. Thus, it must be that $G(\tau_1) = 1$. If this is the case, it must be that $\int_0^{\tau_1} \lambda_s ds = \infty$, which cannot happen in finite time as λ_t is decreasing in t. So, $\tau_1 = \infty$.

Step 5: If a citizen with opportunity cost θ ever enters the protest (i.e. $\exists t$ such that $\sigma_t^\theta = 1$), then $t_0(\theta) \leq \tau_0 \leq t_1(\theta)$. $t_1(\theta) \geq \tau_0$ follows directly from optimality, as otherwise the expected prize is zero with probability 1. Now consider an agent with opportunity cost θ entering at $t_0 > \tau_0$. From lemma 5, the marginal benefit of entering is given by:

$$\lambda_{t_0} \xi(0) + \frac{1}{1 - G(t_0)} \int_{t_0}^{t_1} e^{-r(s-t_0)} \xi'(s - t_0) dG(s) \tag{6.15}$$

By step 3, the expression above is decreasing in t_0 for any $t_0 \geq \tau_0$, and the marginal cost is constant. Then, the agent is strictly better off entering earlier.

Step 6: At any $t < \tau_0$, π_t^σ is increasing. From the previous claim, $\pi_t^\sigma = F(\tilde{\theta}_0(t))$, which is increasing.

Step 7: π_t^σ is continuous at every $t \in [0, \infty]$. We know that π_t^σ is continuous on $[\tau_0, \infty]$, and by the entry condition we also know it is continuous in $[0, \tau_0)$. It remains to show that it is also continuous at τ_0. In particular, we rule out cases in which there is a positive mass of people entering at a given time t (see figure 3). Take two agents entering at a given time

$\bar{t}_0$. Note that as $\tilde{\pi}_t$ is strictly decreasing, these two agents cannot exit at the same time. Suppose they exit at some times $t_1 < t_1'$. Thus, from the exit condition their opportunity costs are given by $\tilde{\theta}_1(t_1) > \tilde{\theta}_1(t_1')$. But from the entry condition, we have:

$$\tilde{\theta}_1(t_1) = \int_{\bar{t}_0}^{t_1} e^{-r(s-\bar{t}_0)}\xi'(s-\bar{t}_0)dG(s) < \int_{\bar{t}_0}^{t_1'} e^{-r(s-\bar{t}_0)}\xi'(s-\bar{t}_0)dG(s) = \tilde{\theta}_1(t_1') \tag{6.16}$$

a contradiction.

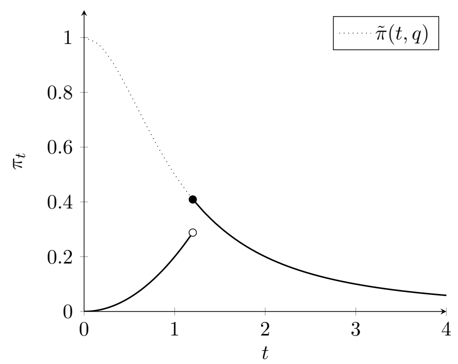


Figure 3: Continuity of π_t.

Step 8: $\tau_0 > 0$. We begin by showing that if $G(\tau_0) = 1$, then $\tau_0 > 0$. Consider first the case in which the government concedes with probability 1 at τ_0, i.e. $G(\tau_0) = 1$. Note that the payoff from entering at τ_0 is zero, so at τ_0 nobody enters anymore. But for $\tau_0 \in \mathcal{T}$ it must be that $c(\pi_{\tau_0}, \tau_0) \geq q$ (see the proof of lemma 3). The benefit of the last citizen entering is given by $G(\tau_0) \cdot \xi'(0)$, and then, for this to be an equilibrium, it must be that $F(\xi'(0)) = \tilde{\pi}_{\tau_0}$. This pins down τ_0 as the time at which $c(F(\xi'(0)), \tau_0) = q$. This time must be strictly positive, as otherwise there is no entry.

Denote by $\underline{\tau} = \tau_0$ the time at which the government concedes with probability 1. Then, we prove that if $G(\tau_0) < 1$, then it must be that $\tau_0 > \underline{\tau}$. Note that if $G(\tau_0) < 1$ then by lemma 4 it must be that $c(\pi_{\tau_0}, \tau_0) = q$. The payoff to the last agent entering is given by $G(\tau_0)\xi'(0)$, and then it must be that at $F(G(\tau_0)\xi'(0)) = \tilde{\pi}_{\tau_0}$. But $\tilde{\pi}_{\tau_0} < \tilde{\pi}_{\underline{\tau}}$, so by assumption 1 it must be $\tau_0 < \underline{\tau}$.

Step 9: In equilibrium the government concedes in finite time, i.e. $\lim_{t\to\infty} G(t) = 1$. From step 4, $\tau_1 = \infty$. Denote by $\underline{\lambda}_t = \frac{\theta}{\xi(t)}$ the hazard rate that makes the lowest opportunity cost citizen indifferent between dropping out and protesting at any time t. Note that by assumption 2, $\underline{\lambda}_t > 0$ for all t. Moreover, $\lambda_t \geq \underline{\lambda}_t$ for all t, and then $\int_0^\infty \lambda_t dt \to \infty$. So we have:

$$\lim_{t\to\infty} G(t) = 1 - \lim_{t\to\infty} \left[(1 - G(\tau_0)) \exp\left(- \int_0^t \lambda_s ds \right) \right] = 1 \tag{6.17}$$

With this, we complete the proof of Proposition 1. $\square$

6.2 Proof of Lemma 1

Using Lemma 5, the enttry threshold can be written as:

$$\tilde{\theta}_0(t) = \begin{cases} \int\limits_t^{t_1} e^{-r(s-t)} \xi'(s-t) dG(s) & t \in [0, \tau_0) \\ \tilde{\theta}_1(t) & t = \tau_0 \end{cases} \tag{6.18}$$

Using continuity of π_t, it has to be that $\tilde{\theta}_0(t)$ is also continuous, i.e. $\lim\limits_{t\to\tau_0^-} \tilde{\theta}_0(t) = \tilde{\theta}_1(\tau_0)$. Thus, at τ_0 the following condition holds:

$$\tilde{\theta}_0(\tau_0) = \xi'(0) G(\tau_0) \quad \Rightarrow \quad G(\tau_0) = \frac{\tilde{\theta}_1(\tau_0)}{\xi'(0)}$$

$\square$

6.3 Proof of Theorem 1: A Continuum of Equilibria

It is direct to see that there is an equilibrium with $\tau_0 = \underline{\tau}$. We prove next that there exists an equilibrium satisfying $\tau_0 = \overline{\tau}$. Then we show that for any τ_0 in between this thresholds, an equilibrium exists.

Lemma 6 *There exists an equilibrium $(G, (\pi_t^\sigma)_{t\geq 0})$ with $\tau_0 = \overline{\tau}$ satisfying*

$$\underline{\theta} = \int_{\overline{\tau}}^\infty e^{-rs} \xi'(s) dG(s) \tag{6.19}$$

Proof. In order to prove existence of this equilibrium with the longest delay, we show that there exists a fixed point satisfying condition 6.19.

As we describe in Section 3, in equilibrium citizens' exit times are determined by the government indifference condition.[6] Then, given their exit times, and the government distribution of concessions $G(t)$, their best reply associates each exit time $t \in [\tau_0, \infty)$, with an entry time $t_0(t)$. The government, given these entry times, and a delay τ_0, chooses a distribution of concessions $G(t)$. Moreover, we introduce a fictitious player who chooses the delay τ_0, in such a way that given $G(t)$, condition 6.19 is satisfied.

Define the best reply correspondence: $\Psi : \mathbf{Z} \twoheadrightarrow \mathbf{Z}$ with tipical element $\mathbf{z} = (G, t_0, \tau_0)$ as:

$$\Psi = (\Gamma(t_0, \tau_0), \Phi(G, \tau_0), \Theta(G, t_0)) \tag{6.20}$$

where $\Gamma(t_0, \tau_0)$ is the government's best reply, $\Phi(G, \tau_0)$ is citizens' best reply, and $\Theta(G, t_0))$ is the best reply of the fictitious player.

The space $\mathbf{Z} = [0, T] \times \mathcal{S} \times \mathcal{C}$ is such that $\mathcal{S}$ corresponds to the space of probability distributions,[7] and $\mathcal{C}$ corresponds to the space of continuous functions. T is the upper bound on the maximum concession time $\overline{\tau}$. We use Kakutani-Fan-Glicksberg theorem to prove that an equilibrium exists. This theorem states that if $\mathbf{Z}$ is a nonempty compact convex subset of a locally convex Hausdorff space, and the correspondence $\Psi : \mathbf{Z} \twoheadrightarrow \mathbf{Z}$ has closed graph and nonempty convex values, then the set of fixed points is compact and nonempty (Aliprantis & Border (2013), Corollary 17.55).

Step 1: Define Citizens' Best Response $\Phi : [0, T] \times \mathcal{S} \to \mathcal{C}$. In equilibrium, given a distribution $G \in \mathcal{S}$ with support $[\tau_0, \infty)$, for each possible exit time $t \in [\tau, \infty]$, $t_0(t)$ is the optimal entry time that solves the following equation:

$$\tilde{\theta}_1(t) = \int_{t_0}^{t} e^{-r(s-t_0)} \xi'(s - t_0) dGs \tag{6.21}$$

where $\tilde{\theta}_1(t) = F^{-1}(\tilde{\pi}(t))$. Figure 4 illustrates citizens' best reply function.

[6] More precisely, on the support $\mathcal{T}$, it has to be the case that $\pi_t = \tilde{\pi}_t$. Thus, there exist a unique exit threshold $\tilde{\theta}_1(t)$ such that $\tilde{\pi}_t = F(\tilde{\theta}_1(t))$ for every $t \in \mathcal{T}$.

[7] Space of functions that are increasing, right-continuous, and such that $\lim_{t \to -\infty} G(t) = 0$ and $\lim_{t \to \infty} G(t) = 1$.

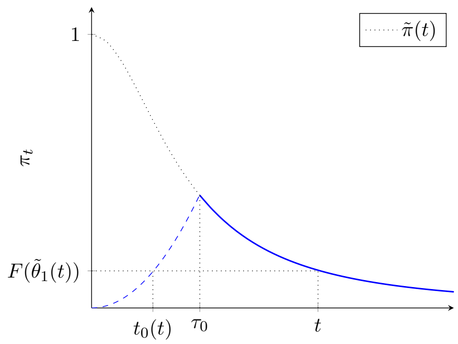


Figure 4: Citizens' exit is determined by $\tilde{\pi}_t = F(\tilde{\theta}_1(t))\ \forall t \in [\tau_0, \infty)$. A citizen with opportunity cost $\theta = \tilde{\theta}_1(t)$ exits at t, and given this exit time, equation 6.21 defines the entry time $t_0(t)$.

Step 2: Define Government's Best Response $\Gamma : \mathcal{C} \times [0,T] \to \mathcal{S}$. In equilibrium, given citizens' best reply $t_0 \in \mathcal{C}$ and the delay time τ_0, we consider a modified problem for the government of choosing a distribution of concessions over $[\tau_0, \infty)$, i.e. $G : [\tau_0, \infty) \to [0,1]$ such that:

$$G(t) = 1 - (1 - G(\tau_0)) \exp\left(- \int_{\tau_0}^{t} \lambda_s ds \right) \tag{6.22}$$

with $G(\tau_0) = \frac{\tilde{\theta}_1(\tau_0)}{\xi'(0)}$, and $\lambda_t = \frac{\tilde{\theta}_1(t)}{\xi(t - t_0(t))}$.

Step 3: Fictitious Player Best Response. The fictitious player best response $\Theta : \mathcal{C} \times \mathcal{S} \to [0,T]$ chooses a time $\tau_0 \in [0,T]$, that solves

$$\underline{\theta} = \int_{\tau_0}^{\infty} e^{-rs} \xi'(s) dG(s) \tag{6.23}$$

Step 4: **Z** *is a non-empty, convex and compact subset of a locally convex Haussdorf space.*

Let T be the time at which $\underline{\theta} = e^{-rT}\xi'(T)$. This upper bound corresponds to the time that makes the lowest opportunity cost citizen indifferent of entry even if the government concedes for sure, which satisfies $T > \overline{\tau}$. Thus, $[0,T]$ is well defined, and it is compact and convex.

For the space of government's distribution of concession, since both $G(\tau_0)$ and λ_t are continuous and well defined, it is non-empty. Moreover, note that the function G constrained to $[\tau, \infty)$ is continuous and bounded. Moreover, they are monotone by Proposition 1, and have bounded variation. By Helly's selection theorem, it is also compact.

Similarly, note that for citizens t_0 is a monotone continuous function with values in $[0, \tau_0)$, and then it has bounded variation. Moreover, it is uniformly bounded, and then we can apply Helly's selection theorem to obtain compactness. To see that it is non-empty, fix t, and note that $t_0(t)$ solves the following equation:

$$\tilde{\theta}_1(t) = \int_{t_0}^{t} e^{-r(s-t_0)} \xi'(s - t_0) dGs \tag{6.24}$$

which has always a unique solution for every $t \in [\tau, \infty]$.

Finally, by Tycohnoff Product Theorem (see Aliprantis & Border (2013), Theorem 2.61), the space $\mathbf{Z}$ is compact in the product topology.

Step 5: Ψ has closed graph. Take a sequence $(t_0^n, G^n, \tau^n) \in Graph(\Psi)$ such that $(t_0^n, G^n, \tau_0^n) \to (\bar{t}_0, \overline{G}, \overline{\tau}_0)$. We want to show $(\bar{t}_0, \overline{G}, \overline{\tau}_0) \in Graph(\Psi)$.

Claim 1. Γ has closed graph. We show that for any sequence $(\tau_0^n, G^n, t_0^n) \to (\overline{\tau}_0, \overline{G}, \bar{t}_0)$, with $(\tau_0^n, G^n) \in \Gamma(t_0^n)$ for all n, then $(\overline{\tau}_0, \overline{G}) \in \Gamma(\bar{t}_0)$.

Note that by continuity of $\tilde{\theta}_1(t)$, $G^n(\tau_0^n) \to \overline{G}(\overline{\tau}_0)$.

Moreover, by continuity of ξ and F^{-1} the hazard rate $\lambda_n(t)$ converges uniformly to:

$$\overline{\lambda}(t) = \frac{F^{-1}(\tilde{\pi}(t))\epsilon}{\xi(t - \bar{t}_0(t))} \tag{6.25}$$

which proves the graph is closed.

Claim 2. Φ has closed graph. We show that for any sequence $(\tau_0^n, G^n, t_0^n) \to (\overline{\tau}_0, \overline{G}, \bar{t}_0)$, with $t_0^n \in \Phi(\tau_0^n, G^n)$ for all n, then $\bar{t}_0 \in \Phi(\overline{\tau}_0, \overline{G})$.

Rewrite t_0 as the solution to a fixed point problem to the following equation:

$$H(t_0; G, \tau_0) = \frac{1}{r}\left[\ln F^{-1}(\tilde{\pi}(t)) - \ln\left(\int_{t_0}^{t} e^{-rs}\xi'(s - t_0(t))dG(s)\right)\right] \tag{6.26}$$

Thus, it is enough to prove that $\|\bar{t}_0 - H(\bar{t}_0)\| = 0$. Note that:

$$\|\bar{t}_0 - H(\bar{t}_0; \overline{G}, \overline{\tau})\| \leq \|\bar{t}_0 - t_0^n\| + \|t_0^n - H(t_0^n)\| + \|H(t_0^n; G^n, \tau^n) - H(\bar{t}_0; \overline{G}, \overline{\tau})\| \quad (6.27)$$

the first two terms in the right-hand side converge to 0 by hypothesis. The third one also converges pointwise to 0 as $\int_{t_0^n}^{t} e^{-rs}\xi'(t - t_0^n(t))dG^n(s) \to \int_{\bar{t}_0}^{t} e^{-rs}\xi'(t - \bar{t}_0(t))d\overline{G}(s)$ for all t.

Claim 3. Θ has closed graph. We show that for any sequence $(\tau_0^n, G^n, t_0^n) \to (\overline{\tau}_0, \overline{G}, \bar{t}_0)$, with $(\tau_0^n) \in \Theta(t_0^n, G^n)$ for all n, then $(\overline{\tau}_0) \in \Gamma(\bar{t}_0, \overline{G})$. Note that G^n converges to $\overline{G}$ in distribution, and then applying Continuous Mapping Theorem we obtain

$$\int_{\tau_0^n}^{\infty} e^{-rs}\xi'(s)dG^n(s) \to \int_{\overline{\tau}_0}^{\infty} e^{-rs}\xi'(s)d\overline{G}(s) \quad (6.28)$$

Then, using claims $1, 2$ and 3, we have that Ψ has closed-graph, and therefore is upper-hemicontinuous. By Kakutani-Fan-Glicksberg theorem it has a fixed point. □

Lemma 7 *Let $(G^1, (\pi_t^1)_{t\geq 0})$ and $(G^2, (\pi_t^2)_{t\geq 0})$ be two distinct equilibria with delays τ_0^1, τ_0^2, such that $\tau_0^1 < \tau_0^2$. Then, the distributions of concessions G^1, G^2 do not cross at any $t \in [\tau_1, \infty]$.*

Proof. From agents entry condition, we have

$$\frac{\partial \tilde{\theta}_0(t)}{\partial \tau_0} = -e^{-r(\tau_0 - t)}\xi'(\tau_0 - t)g(\tau) + \int_t^{t_1(t)} e^{-r(s-t)}\xi'(\tau_0 - t)g'(\tau_0)ds < 0 \quad (6.29)$$

Given that this holds for all $t \in [0, \tau_0)$, we have that the functions $t_0(t)$ do not cross, and this ensures the hazard rates do not cross, and then the distributions of concessions do not cross either. □

We are now in a place to show that any $\tau_0 \in [\underline{\tau}, \overline{\tau}]$ generates an equilibrium. Fix an arbitrary $\tau^* \in (\underline{\tau}, \overline{\tau})$ and let $(G, (\pi_t)_{t\geq 0})$ be the equilibrium consistent with it. We know from lemma 7 that

$$\underline{\theta} < \int_{\overline{\tau}}^{\infty} e^{-rs}\xi'(s)dG(s) \quad (6.30)$$

and $G(\tau) < 1$. Then we can solve the same fixed point problem we solved in the previous claim fixing the fictitious player strategy to choosing τ^*. Using the same arguments, a fixed point exists. As τ^* was arbitrary, this completes the proof of the theorem. □

6.4 Proof of Proposition 2

Recall that for any distribution of opportunity costs F_j, the lower bound $\underline{\tau}_j$ is given by the equilibrium in which the government concedes with probability 1, and then it is such that

$$\tilde{\pi}_{\underline{\tau}_j} = F_j\left(\xi'(0)\right) \tag{6.31}$$

Then, statement (i) follows from the fact that F_1 first order stochastically dominates F_2, and then $F_1(\xi'(0)) < F_2(\xi'(0)$.

To prove statements (ii) and (iii), note that as F_1 is symmetric and unimodal and F_2 is obtained from a mean preserving spread, then $F_2(\theta) < F_1(\theta)$ for every $\theta < \int \theta dF_1(\theta)$, and $F_2(\theta) > F_1(\theta)$ otherwise.

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