Crises, Catharses, and Boiling Frogs: Path Dependence in Collective Action

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August 2021

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Abstract

We show a strong form of path-dependence in collective action. For a given distribution of anti-regime grievances and sentiments in the society, the size of the protest is larger when this distribution of grievances is the result of a sudden large change rather than a series of smaller unexpected changes. Society as a whole behaves like the legendary boiling frog, even though each individual does not. Large grievance shocks (crises) coordinate behavior far more effectively into revolts than a sequence of small shocks that generate the same final distribution of grievances. Our analysis applies advances in incomplete information coordination games (Morris and Yildiz 2019), deviating from the literature by relying only on the notion of rationalizability (as opposed to Nash equilibrium) and assuming heavy-tailed distributions of grievance shocks. We explore the unexpected link between this theory and Davies's (1962) classic J-curve theory of revolution.

Keywords: Protest, Revolution, Path Dependence, Coordination, Heavy Tails

Word Count: 9,819

1 Introduction

Consider two hypothetical countries with identical distributions of anti-regime grievances and sentiments in the population. Further, suppose that resources, capacities, and cultures of the state, society, and oppositions are also identical across these countries. They may have had different histories and distributions of anti-regime grievances in the past. However, at the moment, these distributions of grievances—encompassing all individual and social memories, emotions and cultures, and available alternatives to the status quo—are identical. The literature suggests that these countries will be very similar regarding the size and likelihood of protest in the current period. The "Gurrian approach" (Davies, 1962; Gurr, 1968, 1970; Muller, 1985; Useem, 1998; Buechler, 2004) and its precedents (Kornhauser, 1959; Smelser, 1962; Geschwender, 1971) emphasize current grievances. The "Tillyan approach" (Tilly, 1978), which includes resource mobilization (McCarthy and Zald, 1977) and political process theories (Gamson, 1975; McAdam, 1999; Tarrow, 2011; Tilly and Wood, 2009), emphasizes the cost-benefit calculus of protest that hinges on material and cultural resources of the opposition and the structure of political opportunities—see Shadmehr (2014) for an overview. Wood (2003) and the subsequent literature combined these branches and identified critical mechanisms through which grievances emerge and translate into collective action (Snow et al., 1986; Lohmann, 1994; Rasler, 1996; Petersen, 2001; Chenoweth and Stephan, 2011; Lawrence, 2017; Pearlman, 2018; Aytaç and Stokes, 2019). This new generation of theories emphasizes how histories and memories influence the nature of grievances and their transformation into collective action. However, if we fix the distribution of anti-regime grievances and sentiments and their nature, as well as resources, opportunities, and cultures, all theories suggest that outcomes should be similar.

In this paper, we argue that there is a stronger form of path-dependence in protests and revolts. Fixing the current distribution of anti-regime grievances and sentiments—as well as all of the other factors discussed above—the size of the protest (and its likelihood of success) is larger when this distribution of anti-regime grievances results from a sudden large change, rather than a series of smaller unexpected changes. In short, we show that *how* the society arrives at a distribution of grievances matters.

To highlight its import, we first discuss what this result does not say or rely on. When the final anti-regime grievances increase, all else equal, one expects larger protests and higher chances of success, at least weakly. This is not our point. Instead, our theory compares the outcomes of arriving at a given distribution of grievances via different paths: through a sudden significant change versus through a series of small unexpected changes.

If each individual, by assumption, somehow cared about the path through which their grievances have increased, then their collective behavior would naturally reflect this. We do not make such an assumption because we take the general approach of directly looking at grievances and sentiments. If instead, we had focused on some *source* of such sentiments as the critical variable (e.g., inequality, economic downturn, or state repression), then it would be more plausible to assume that the speed of change has a different impact based on how changes translate into grievances. However, with grievances themselves as the key variable, such a hypothesis means that individual humans act like the mythical boiling frog, who jumps out if thrown into a pan of boiling water, but remains in the pan if the temperature increases slowly from temperate to boiling. Neither a typical frog nor a human would act like the mythical boiling frog. Instead, we argue that even though individuals are not like that boiling frog, society as a whole is. Such stark differences between the characteristics of individuals and the collective famously appear in the Condorcet Paradox: it is in the nature of collective decision-making that even when all individual preferences are transitive, collective preferences need not be. Our result has a similar flavor: when acting against the state at a given time, individuals care about their final grievance level and not about whether they arrive at those grievances in one sharp increase or a sequence of small increases in grievances. However, society as a whole will act *as if* it cares about the path of grievance accumulation.

If path-dependence is not somehow assumed in the preferences of individuals that constitute a society or in how its institutions function, then what is its source? Endogenous individual beliefs about others' grievances and likely actions. Protests and revolts involve coordination motives (Hardin, 1982; DeNardo, 2014; Lohmann, 1994; McAdam et al., 2001; Wood, 2003; Tarrow, 2011; Chwe, 2013; Hager et al., 2020; Correa, 2021): all else equal, an individual has more incentives to participate when they believe that others are more likely to participate, because, for example, greater participation reduces the expected costs of repression for an individual, or one may derive more satisfaction from joining a larger movement. Thus, an individual's decision to protest depends both on their own grievances and on their expectation about the participation of others, which, in turn, depends on their own grievances and expectations. Naturally, individual grievances are correlated and distributed around the average grievances in the society, which we call *aggregate grievances*: individual grievances are the sum of aggregate and idiosyncratic grievances. Thus, an increase in aggregate grievances raises individual grievances across the board, with a more considerable increase in the aggregate implying a more significant increase in individual grievances.

We show that this familiar setting has a surprising feature: when individuals believe that aggregate grievances have heavy tails, a large shock to aggregate grievances has a very different implication for the distribution of individuals' *beliefs about others' grievances* than a sequence of small shocks. In particular, after a large shock to aggregate grievances, an individual who feels a very high level of anti-regime grievances and sentiments will *not* believe that they are outliers. In fact, they will believe that their anti-regime grievances and feelings are close to the median of the population: they think that about half of the people feel even more resentment toward the state and hence are even more inclined to act. They believe this because, as we show, they cannot make any inferences about the aggregate grievance level beyond that it is very likely within some particular range. It follows that any value of the aggregate grievances within that range is almost equally likely, so they cannot infer whether their idiosyncratic grievances are large or small relative to others, i.e., their grievance level is equally likely to have any rank in the population. Consequently, they believe that, in expectation, about half the population has even more grievances.

The effect of large shocks on belief formation implies that a large shock to aggregate grievances will result in larger protests than a sequence of small shocks that generate the identical final distribution of grievances. To see the logic, consider the citizen M with the median grievance level. Suppose M will protest if they believe that at least 45% of others protest. When the aggregate grievance shock is very large, as discussed above, M will believe that about half the population, say, 49%, has higher grievances. Will M believe that some or all of them protest? Citizen M knows that those with higher grievances also believe that their grievance levels are about the population median. Moreover, all of them know that a (perhaps tiny) fraction of the population, say, 0.1% has such extreme grievances that they will protest regardless of others. Now, consider citizens with grievances slightly below these extremes, say, the next top 0.6% of the population. To protest, they only need to believe that a small fraction of the population will protest. But they believe that about half the population has higher grievances than themselves, and those with higher grievances are sure to protest. Thus, they will protest. This contagion-like logic now continues to those with slightly lower grievances, and so on. The contagion stops at some percentile of grievance levels below the median citizen M: by the above logic, M is reasonably sure that all those 49% with higher grievances will protest, because (i) they require even less than 45% threshold to protest, and (ii) they also believe that about half the population has more grievances than themselves. Thus, citizen M will protest.[1] Small shocks do not sustain the contagion because far fewer citizens will believe that about half the population has higher grievances than themselves.

This contagion logic is present even in standard settings where the distribution of aggregate grievances is thin-tailed (e.g., Normal) so that the surprising citizen belief property does not hold. However, the contagion stops quickly in those settings. For example, when we get to the 3rd percentile of grievances, they protest if they believe that 10% of the population protest, but they can only be reasonably sure that those above them—i.e., a small fraction—will protest. In contrast, when aggregate grievances have a heavy-tailed distribution so that the "I'm-about-the-median" property holds, the citizens around the top 3rd percentile of grievances believe that about half the population (e.g., 49.5%) has even higher grievances. As a result, the contagion continues. Thus, it is essential for our results that the distribution of shocks to aggregate grievances (equivalently, the prior belief about the distribution of aggregate grievances) is heavy-tailed. We discuss the emergence of heavy-tailed distributions and their appropriateness for our contexts following the model, arguing that they better describe the distribution of aggregate grievances. This is a crucial methodological departure from the extensive formal literature on protests and revolutions, in which either aggregate grievances are known or have light-tailed distributions (e.g., Normal or other log-concave distributions) (Bueno De Mesquita, 2010; Shadmehr and Bernhardt, 2011; Edmond, 2013; Boix and Svolik, 2013; Casper and Tyson, 2014; Chen and Suen, 2016, 2017; Morris and Shadmehr, 2018, 2021; Tyson and Smith, 2018; Shadmehr, 2019; Bueno de Mesquita and Shadmehr, 2020a; Nandong, 2020).

The second methodological departure is that, as in Morris and Yildiz (2019), we use the notion of *rationalizability* of different actions. As we will discuss in our analysis, an action

[1] This logic is very different from the classic Granovetter (1978)'s model, which will have multiple equilibria in general. As we will discuss below, this contagion-like argument is common in many models. The key difference here is the structure of endogenous beliefs.

is rationalizable for an individual whenever that individual can hold some plausible belief about the behavior of other individuals that justifies that action. In any Nash equilibrium, all equilibrium actions are rationalizable; however, not all rationalizable actions constitute a Nash equilibrium. Our formal results characterize and compare the share of individuals for whom protesting is the unique rationalizable action in different settings. We will show that large shocks to aggregate grievances make protesting the unique rationalizable action for a larger fraction of individuals than a sequence of small shocks that generate the same final distribution of grievances in the population. When interpreting the results, we take a conservative approach to protest, so that individuals protest if and only if protesting is the sole rationalizable action for them. An alternative interpretation is that we compare lower bounds on the size of protests in different environments.

Our analysis applies the new advances in incomplete information coordination games developed in Morris and Yildiz (2019).[2] However, there is a surprising link between our theory and the J-curve theory of revolution in Davies (1962). In his classical paper, Davies argued that "Revolutions are most likely to occur when a prolonged period of objective economic and social development is followed by a short period of sharp reversal" (Davies, 1962, 5). The critical immediate cause of revolt in the J-curve theory is an unexpected, large negative change. If the *same* final condition emerges through *gradual* change, people do not revolt. Davies's core explanation is based on individual human psychology: people "subjectively fear that ground gained with great effort will be quite lost; their mood becomes revolutionary" (p. 5). Due to unspecified properties of human physiology and psychology, a slow reversal of fortunes does not generate enough frustration, fear, anger, and, more generally, grievances to cause protest. In contrast, a sharp reversal of fortunes does: in the J-curve theory, individual humans act like the mythical boiling frog. Moreover, like various relative deprivation theories that followed, Davies's approach focused on "motives," ignoring "opportunities." As such, it belongs to the "classical theories" that viewed protests as a cathartic process that relieved psychological strains, e.g., alienation in Kornhauser (1959), or cognitive dissonance in Geschwender (1971). These theories put little weight on the individuals' assessments of the costs of action or strategic interactions. Instead, they treated unrest and violence as a therapy (McAdam, 1999) that released individuals' psychological tensions. Crisis led to catharsis in the form of protest and revolt with little human agency.

In our theory, individuals are not automatons that respond to environmental stimuli with no agency of their own—with no strategic considerations or conscious concerns for costs and benefits of actions beyond the relief of their psychological strains. Thus, we contrast sharply with Davies's assumptions and explanation of the J-curve theory. But our approach agrees with a critical element of the J-curve theory: large unexpected changes (shocks) can be

[2]We interpret and apply Morris and Yildiz (2019)'s model in protest settings. Naturally, parts of our analysis closely follow that seminal paper, while other parts provide new theoretical results and intuitions. In particular, the contents of our Lemma 1 and Proposition 2 are in Lemma 1, Propositions 1 and 3, and Corollary 1 of that paper. However, much of our intuition for and discussion of Lemma 1 is new, including the use of asymptotic scale invariance. Our Proposition 1 extends the content of Proposition 2 and Corollary 1 of that paper. Our Propositions 3, 4, and 5 are new, and Proposition 5 is our main theoretical contribution.

fundamentally different from a sequence of small unexpected changes (shocks). However, our explanation is based on the emergence of individuals' beliefs about each others' behavior, and our unexpected changes are shocks to aggregate grievances, encompassing cultural, social, or economic dimensions.

We next present the model and discuss our crucial assumption about the heavy-tailed distribution of aggregate grievances. We then discuss the concept of rationalizability and present the analysis. A conclusion follows. Proof are in the Appendix.

2 Model

A continuum 1 of citizens, indexed by $i \in [0,1]$, simultaneously decide whether to revolt. A citizen's payoff from not revolting is normalized to 1. A citizen i's payoff from revolting is $x_i + A$, where A is the fraction of other citizens who revolt and x_i is that citizen's expressive payoff from revolting. Thus, a citizen i's payoff from revolting versus not revolting is:

$$u_i = x_i + A - 1.$$

We will refer to x_i as citizen i's anti-regime grievance level or sentiments.[3] Naturally, these grievances are heterogeneous, but correlated among citizens. In particular,

$$x_i = \theta_0 + \sigma(\eta + \epsilon_i),$$

where θ_0 is commonly known, η is an unknown common shock, and ϵ_i is an unknown idiosyncratic shock. The parameter σ captures the sensitivity of grievances to these shocks.

A citizen observes her own grievance level x_i, but she remains uncertain about other citizens' grievance levels: for a given θ_0, a large x_i could be due to her large idiosyncratic shock ϵ_i, or due to a large common shock η to all citizens' grievance levels. Citizens share common priors that $\epsilon_i \sim iid\, F$ and $\eta \sim G$, independently from each other and other parameters, with corresponding cdfs f and g, and full support on $\mathbb{R}$. We make the following assumption.

Assumption 1 *G and F are single-peaked and symmetric around 0. Moreover, f is log-concave, and g is a regularly varying distribution (e.g., Student's t distribution).*

An interpretation is that citizens have correlated, heterogeneous grievances, $x_i = \theta + \sigma\epsilon_i$, and there is aggregate uncertainty about the average level of grievances. In particular, citizens share a prior that $\theta \sim \theta_0 + \sigma\eta$, where θ_0 is the expected aggregate grievances in the society. Assumption 1 then implies that this common prior has heavy tails.

The timing of the game is as follows. Nature draws the common shock η and idiosyncratic shocks ϵ_is. Citizens observe their private signals x_is and simultaneously decide whether to revolt.

[3] Various interpretations fit this formulation. For example, normalize a citizen's payoff from not revolting to 0 and suppose the expected costs of participation is $(1-A)c$. Thus, a citizen i's net payoff from revolting versus not revolting becomes $x_i - (1-A)c$. Normalizing c to 1 yields the same net payoff as above.

2.1 Heavy Tails in Aggregate Grievances

A key assumption in our analysis is that the distribution of the aggregate grievance level in the society has heavy tails: it is a regularly varying distribution. A random variable η has a regularly varying distribution when $Pr(\eta > a) = L(a)/a^{\rho}$, where $\rho > 0$ and $\lim_{a\to\infty} L(ab)/L(a) = 1$ for all $b > 0$. That is, regularly varying distributions behave asymptotically like power law distributions and are scale-invariant, so that the shape of the tail does not change, up to a constant, when we change the unit of measurement (Nair et al., 2020, Ch. 2). The class of regularly varying distributions include Pareto (power law), Student's t, and Cauchy, as well as any distribution that has power law tails (Nair et al., 2020, 38). The familiarity of the Normal distribution may lead one to initially think that regularly varying distributions are odd or unusual. We now show that they arise naturally in our setting.

One source of heavy tails in the distribution of grievances is the people's uncertainty about the exact model of the world. Consider a simple scenario. Suppose $\eta \sim N(\mu, \sigma^2)$, but people are uncertain about its variance σ^2, a very natural assumption—a mean is easier to estimate than the expected squared deviation from the mean. For example, if $1/\sigma^2 \sim \chi^2$, then people will believe that η is distributed according to the Student's t distribution, which is a regularly varying distribution. This is an example of "model uncertainty", which Morris and Yildiz (2019) highlight as a mechanism for the emergence of heavy-tailed distributions.[4]

A second source is the relationship between aggregate grievances and aggregate economic variables such as GDP growth rate—"food riots" and protests in response to price changes have been a common feature of societies (Tilly, 1975, 1995). Aggregate grievances can inherit heavy tails from aggregate economic variables. Critically, as Acemoglu et al. (2017) show, even in the large U.S. economy with various sectors, the aggregate growth rate has heavy tails. In a similar vein, Weitzman (2007) shows that seemingly puzzling established patterns in macroeconomic data (e.g., the infamous equity premium puzzle) are resolved if one recognizes that economic agents have model uncertainty about economic growth—see Warusawitharana (2018) for an empirical study. In particular, while standard models assume that consumption growth ($\log(C_{t+1}/C_t)$) is distributed Normally with known variance, Weitzman observes that, when there are shocks to that variance (it is hard to imagine why variance must be constant), even with large data, agents will believe that the growth rate has heavy tails. Given Weitzman's assumptions, the distribution is the Student's t. Finally, turning to stock markets, a long tradition of empirical literature establishes that the change in stock prices ($\log(p_{t+1}/p_t)$) has a power law distribution (Fama, 1963; Gabaix et al., 2003).

A third source is multiplicative processes. Regularly varying distributions arise routinely in multiplicative processes such as proportional growth processes. Suppose the aggregate grievance level θ_t changes both proportionally and additively, so that $\theta_{t+1} = a_t\theta_t + \gamma b_t$, where

[4] Model uncertainty also plays a key role in Chen and Suen's models of revolt (Chen and Suen, 2016, 2017). For example, in Chen and Suen (2016), revolution is far less likely in one worldview than another. Thus, if revolution happens in another country, it will greatly impact those who believe in the "tranquil world", contributing to a contagion and clustering of revolts. However, distributions in these papers are thin-tailed as not all forms of model uncertainty lead to heavy tails.

$a_t, b_t \in \mathbb{R}$ are random variables, capturing various random shocks, and $\gamma > 0$ captures the weight of additive shocks. Under quite general conditions,[5] the steady state distribution of aggregate grievances has power law tails, and hence is regularly varying (Kesten, 1973, Theorem 5) (Goldie, 1991, Theorem 4.1)—such processes underlie heavy tails in many networks.

What about additive processes that are the subject of Central Limit Theorems (CLTs)? Familiar CLTs state that if a random variable has finite mean and variance, a normalized sum of infinite iid random samples will converge to the Normal distribution. However, even with additive processes, if the finite mean and variance conditions are relaxed, then a normalized sum of infinite random samples can converge to a regularly varying distribution (Nair et al., 2020, Theorems 5.8 and 5.9)—a result known as the Generalized Central Limit Theorem.

3 Analysis

3.1 Rationalizability

We will employ the concept of rationalizability developed in Bernheim (1984) and Pearce (1984). We say that an agent's strategy is rationalizable when it is optimal given some belief about other agents' behavior, with the minimal restriction that belief is consistent with agents not using (iteratively) dominated strategies. For example, in a one-shot Prisoner's Dilemma game, it is not reasonable that an agent holds the belief that his opponent will play the dominated strategy of cooperation. Critically, all Nash equilibrium actions are rationalizable. However, not all rationalizable actions are part of a Nash equilibrium. For example, in the matching pennies game, there is no pure strategy Nash equilibrium, whereas all pure strategies are rationalizable.

We first find conditions under which revolting is the *unique* rationalizable action for at least a fraction p of citizens. In general, this is a difficult task. However, our game belongs to the class of Bayesian games of strategic complementarities (Van Zandt and Vives, 2007), which greatly simplifies the task of finding rationalizable actions. Van Zandt and Vives (2007) show that the largest and the smallest Bayesian Nash equilibria of such games are in monotone strategies. Moreover, in such (supermodular) games, all rationalizable strategies are within the bounds of these largest and smallest equilibria (Milgrom and Roberts, 1990). For example, in our setting, order strategies so that a larger strategy prescribes revolt after a larger set of signals, and take a signal for which the largest Bayesian Nash equilibrium of the game prescribes "no revolt". Then, no rationalizable strategy prescribes "revolt" for that signal. Conversely, no rationalizable strategy prescribes "no revolt" for a signal for which the smallest Bayesian Nash equilibrium of the game prescribes "revolt". As we will describe in detail below, these two results allow us to fully identify rationalizable actions.

The first step to find rationalizable actions is to find the largest and smallest Bayesian Nash equilibria of the game in monotone strategies; not because we aim to use Bayes Nash

[5] A key condition is the existence of a $\kappa > 0$ such that $E[|a_t|^\kappa] = 1$.

solution concept, but, instead, to characterize the citizens' rationalizable actions.

In an equilibrium monotone strategy, a citizen revolts if and only if his signal is above a threshold. Because the game is symmetric, the largest and smallest equilibria are also symmetric. Letting $z_i = \eta + \epsilon_i$, a citizen revolts if and only if $z_i > z$ for some $z \in \mathbb{R}$. Given the strategy of other citizens z and his private signal z_i, the citizen i's expected net payoff from revolting versus not revolting is given by:

$$E[u_i|z_i] = \theta_0 + \sigma z_i + E[A|z_i] - 1 = \theta_0 + \sigma z_i - Pr(z_j \leq z|z_i),$$

which is increasing in z_i (Morris and Yildiz, 2019, Lemma 2).

Denote by $R(z) = Pr(z_j \leq z|z_i = z)$ citizen i's belief of his rank in the population. In equilibrium, a citizen must be indifferent between revolting and not revolting. Thus, the equilibria are characterized by the following indifference condition:

$$\theta_0 + \sigma z = R(z).$$

Let $\underline{z}$ be the smallest solution to this indifference condition and let $\overline{z}$ be the largest solution. These will characterize the largest and smallest equilibria of the game. As we described above, all rationalizable strategies are bounded between these smallest and largest equilibria. This means, in *any* rationalizable strategy, a citizen whose signal is above $\overline{z}$ will revolt, and a citizen whose signal is below $\underline{z}$ will not revolt. In contrast, when a citizen's signal is in between $\underline{z}$ and $\overline{z}$, there is a Bayesian Nash equilibrium in which that citizen revolts (and hence revolting is rationalizable), and there is a Bayesian Nash equilibrium in which that citizen does not revolt (and therefore not revolting is rationalizable). Moreover, rather obviously, if revolting is uniquely rationalizable for a citizen with a signal z_i, then it is also uniquely rationalizable for all citizens j with higher signals $z_j > z_i$.

3.2 Citizen Beliefs

How a citizen perceives her grievance level relative to others is key in assessing what fraction of other citizens will revolt. For example, when a citizen has a very high grievance level, she will be more inclined to revolt if she believes that many others have even more grievances. But if she believes that she is an outlier and her high grievance level is due to her unusual idiosyncratic situation, she will be less inclined to revolt, because she believes that most others have less grievances than her and are less inclined to revolt. Indeed, the indifference condition, $\theta_0 + \sigma z = R(z)$, shows that the rank function $R(z)$ plays a critical role in identifying rationalizable actions. We now examine the key properties of the rank function. The full support of z_i implies that $R(z) \in (0, 1)$. Moreover,

Lemma 1 *The rank function $R(k) = Pr(z_j < k|z_i = k)$ that identifies the fraction of citizens with less grievances than a citizen with a grievance level $\theta_0 + \sigma k$ has the following properties: (1) $R(z) = 1 - R(-z)$, so that $R(0) = 1/2$. (2) If $R(z) > 1/2$, then $R(z') > 1/2$ for all $z' > z$. (3) $\lim_{z \to \infty} R(z) = 1/2$.*

The following two examples illustrate. In these examples, in addition to the above properties, the rank function is also unimodal on $z \geq 0$.

Example 1. Suppose $\epsilon_i \sim iidN(0,1)$, and citizens share a common prior that $\eta \sim Cauchy(0, 0.5)$. The rank function $R(z)$ is illustrated in Figure 1.

Example 2. Suppose $\epsilon_i \sim iidN(0,1)$, and citizens share a common prior that $\eta \sim N(0, \sigma_\eta^2)$, but they are also uncertain about the variance of the distribution. In particular, citizens believe that its precision, $1/\sigma_\eta^2$, is distributed according to a Gamma distribution, which includes χ^2 as a special case. Then, η has a Student's t distribution, which is a regularly varying distribution. The rank function $R(z)$ satisfies the properties in Lemma 1. It is also single peaked on $z \geq 0$.

In standard models with a thin-tailed distribution of aggregate shocks (e.g., a log-concave distribution such as Normal), $R(z)$ has the first two features. However, it is monotone increasing and $\lim_{z\to\infty} R(z) > 1/2$. For example, if $\eta, \epsilon_i \sim iidN(0,1)$, then $R(z) = \Phi(\alpha z)$, for some $\alpha > 0$. Thus, the key consequence of the heavy-tailed distribution of aggregate shocks is that a citizen with a very high grievance level (i.e., a very high signal z) believes that her grievance level is about the median of the population, so that about half of the population has even higher grievance levels (signals). Because this feature of citizen beliefs is a key building block of our arguments, we now discuss its intuition in detail.

The logic is intuitive but subtle. To assess other citizens' grievances a citizen must infer what part of her grievances is due to a common problem (and hence is shared by others) and what part of her grievances is due to her idiosyncratic situation (and hence is unique to her own). For example, if she believes that her grievances are due to an unusually large idiosyncratic shock, then she knows that few people will have larger grievances than her. In the Normal setting, when a citizen's grievances are higher, she also believes fewer people have higher grievances than her. In particular, a citizen with a very high grievance level will believe that she is an outlier: $R(z) = \Phi(\alpha z) \approx 1$ for large z.

In contrast, when the distribution of common (aggregate) grievance shocks has heavy tails and the distribution of idiosyncratic grievance shocks has thin tails, a very large grievance level is far more likely due to a very large common shock. More importantly, a citizen with a very high grievance level will not make much inferences about the *relative* size of her idiosyncratic grievances in the population: she has little information about her rank in the population, so she believes that she is equally likely to be in any percentile of grievance levels in the society. To see the intuition, suppose $\theta_0 = 0$ and $\sigma = 1$, so that a citizen i's grievance level becomes $z_i = \eta + \epsilon_i$. Let the tail of idiosyncratic grievance shocks be Normal and that of common aggregate shocks be a power law: $f(\epsilon_i) \propto e^{-(\epsilon_i)^2}$ and $g(\eta) \propto \eta^{-\rho}$, for some $\rho > 1$. When a citizen feels a very high grievance level (i.e., z_i is very high), what does she learn about the relative location of her idiosyncratic grievances ϵ_i in the population?

$$\frac{pdf(\epsilon'|z)}{pdf(\epsilon|z)} = \frac{pdf(z|\epsilon')}{pdf(z|\epsilon)}\frac{f(\epsilon')}{f(\epsilon)} = \frac{g(z-\epsilon')}{g(z-\epsilon)}\frac{f(\epsilon')}{f(\epsilon)} \propto \begin{cases} e^{(z-\epsilon)^2-(z-\epsilon')^2}\frac{f(\epsilon')}{f(\epsilon)} & ;\text{Normal tail} \\ \left(\frac{z-\epsilon}{z-\epsilon'}\right)^{\rho}\frac{f(\epsilon')}{f(\epsilon)} & ;\text{power law tail.} \end{cases}$$

Thus, when z is very large,

$$\frac{pdf(\epsilon'|z)}{pdf(\epsilon|z)} \propto \begin{cases} e^{(\epsilon'-\epsilon)(2z-\epsilon'-\epsilon)}\ \frac{f(\epsilon')}{f(\epsilon)} & ;\text{Normal tail} \\ \frac{f(\epsilon')}{f(\epsilon)} & ;\text{power law tail.} \end{cases}$$

This simple calculation shows that when the distribution of aggregate grievances has power law tails, a citizen with a very high grievance level will learn very little from her grievance level about her *relative* grievance level in the society. From her perspective, her grievance level provides almost no information about the relative likelihoods of different idiosyncratic grievance shocks. Thus, she believes the chances that her particular idiosyncratic situation is better or worse than another random citizen is the same. With power law tails, the likelihood ratio behaves as if the distribution of aggregate grievances $g(\eta)$ is uniform in the tails, i.e., $\frac{g(z-\epsilon')}{g(z-\epsilon)} \approx 1$ for large z. The underlying reason is the scale invariance property. We say that $g(\cdot)$ is asymptotically scale-invariant whenever $\lim_{x\to\infty} g(kx)/g(x) = h(k)$ for some continuous function $h(\cdot) > 0$. When $g(\cdot)$ is asymptotically scale-invariant, from the perspective of a citizen with a very high grievance level,

$$\frac{pdf(\epsilon'|z)}{pdf(\epsilon|z)} = \frac{g(z-\epsilon')}{g(z-\epsilon)}\ \frac{f(\epsilon')}{f(\epsilon)} \approx \frac{h(z)g(1-\epsilon'/z)}{h(z)g(1-\epsilon/z)}\ \frac{f(\epsilon')}{f(\epsilon)} = \frac{g(1-\epsilon'/z)}{g(1-\epsilon/z)}\ \frac{f(\epsilon')}{f(\epsilon)} \approx \frac{f(\epsilon')}{f(\epsilon)}.$$

This intuition is in line with our common sense: when a very large common shock is added to small idiosyncratic shocks, it should wipe out the effects of those small shocks. But this common sense misses a key link between this observation and its consequences for citizen beliefs about their rank—recall that $R(z) \approx 1$ for large z in Normal settings. The key link is scale invariance: when we change the scale to a very large z, the effect of small shocks disappear, because we know that ϵ/η is very large, so that $\epsilon/z \approx 0$; if, in addition, we have scale invariance, this re-scaling does not change the shape of the distribution.

A related intuition builds on beliefs about the aggregate grievance shock. In particular, a citizen with a very high grievance level will believe that the distribution of the aggregate grievance shock is almost uniform in the vicinity of that high level—moving very far from that vicinity is irrelevant from the citizen's perspective because idiosyncratic shocks are log-concave and vanish at a rate faster than exponential. Such a citizen will have very little information about the aggregate grievance shock in a range that is relevant from her perspective. But if the aggregate grievance shock (η) is distributed uniformly so that a citizen has no prior information about it, then she believes that her grievance level is at the median of the population ($R(z) = 1/2$). To see the intuition, suppose the distribution of idiosyncratic shocks F is uniform on $[-1, 1]$. What does a citizen with a very high grievance level k believe about the distribution of aggregate grievance shock? It is straightforward to show

$$Pr(\eta \leq a | z_i = k) = \begin{cases} 1 & ; a \geq k+1 \\ \frac{G(a)-G(k-1)}{G(k+1)-G(k-1)} & ; k-1 \leq a \leq k+1 \\ 0 & ; a \leq k-1. \end{cases}$$

Thus, from the perspective of a citizen with grievance shock $z_i = k$, the "relevant" range of the aggregate grievance shock is $[k-1, k+1]$. Although this stark range is the result of the uniform distribution of idiosyncratic shocks, it is easy to see that a similar argument will hold when the tails of the distribution of those shocks vanish fast enough, e.g., are faster than exponential as in log-concave distributions. Critically,

Remark. Suppose $F = U[-1, 1]$ and G is asymptotically scale-invariant, i.e., is a regularly varying distribution. For large k, the distribution of the aggregate grievance shock η conditional on a grievance level $z_i = k$ is uniform on $[k-1, k+1]$:

$$\lim_{k \to \infty} \frac{G(a) - G(k-1)}{G(k+1) - G(k-1)} = \frac{a - (k-1)}{2}, \quad k-1 \leq a \leq k+1.$$

In contrast, we do not obtain this uniform distribution when the aggregate shock does not have power law tails. For example, when $\eta \sim N(0, 1)$, a citizen with a very large grievance level will believe that η is about $k-1$. In terms of conditional expectations, for large k,

$$E[\eta | k-1 \leq \eta \leq k+1] \approx \begin{cases} k-1 & ; G = N(0,1) \\ k & ; G = Cauchy(0,1). \end{cases}$$

We now use this key property of citizen beliefs to understand citizen behavior.

3.3 Citizen Behavior After A Large Unexpected Change

Our goal is to establish that, fixing the current distribution of anti-regime grievances, the size of the protest is higher when this final distribution of anti-regime grievances is the result of a large sudden change rather than a series of smaller unexpected changes. We first show that when the final distribution of grievances is the result of a large shock, revolt is the unique rationalizable action for a fraction of citizens.

Proposition 1 *Fix a current aggregate grievance level $\theta = \theta_0 + \sigma\eta > 1/2$, so that the current distribution of grievances in the population is also fixed. Let $p_\theta = 1 - F(\frac{1/2-\theta}{\sigma})$, so that $p_\theta > 1/2$ and p_θ is strictly increasing in θ. For any $p \in [0, p_\theta)$, if the common shock η is sufficiently large, then revolting is the unique rationalizable action for at least a fraction p of citizens.*

The proof exploits the relationship between rationalizable actions and Bayesian Nash equilibria in supermodular games. Given an aggregate grievance level θ, the citizen with the grievance level $\theta + \sigma F^{-1}(1-p)$ is exactly at the pth highest percentile of the distribution of grievances in the population. Moreover, as we will discuss shortly, when $\theta > 1/2$ and the common shock η is very large, the smallest Bayesian Nash equilibrium cutoff on grievances (not grievance shocks) will be very close to 1/2: all those with grievances larger than about 1/2 will revolt in this equilibrium. Thus, from the relationship between rationalizability and Bayesian Nash equilibria discussed before, the question becomes: the marginal citizen in which percentile will have grievances larger than this cutoff? Clearly, any citizen in a percentile $p < 1 - F(\frac{1/2-\theta}{\sigma})$. It remains to argue why large common shocks cause the smallest Bayesian Nash equilibrium cutoff on grievances to converge to 1/2. When analyzing citizen beliefs, we showed that a large common shock causes citizens to believe that their grievance level is about the median of the population. But then the marginal citizen who is indifferent between revolting and not revolting will have a grievance level about $1/2 \approx R(\bar{z}) = \theta_0 + \sigma\bar{z}$.

In the Introduction, we provided an intuition without reference to any technical term, showing how this result follows from the structure of citizen beliefs. We now offer an intuition based on the material that we have discussed. For concreteness, consider the citizen who has the median grievance level in the population, so that her grievance level is exactly the aggregate grievance level: $med(x_i) = \theta_0 + \sigma\ med(z_i) = \theta_0 + \sigma\eta = \theta > 1/2$. This citizen revolts if she believes that more than half of the population does so, because $med(x_i) = \theta > 1/2 > 1 - A$. If this is the only belief that she can "reasonably" hold, then revolting will be uniquely rationalizable for her. What prevents her from believing, e.g., that no one else will revolt? She knows that some citizens will have such extremely high grievances that they will revolt regardless of what others do: these "extremists", trivially, have a unique rationalizable action to revolt. But then, some other citizens with grievances just below those "extremists" will also have a unique rationalizable action to revolt. How far does this contagion logic continue? It covers all citizens with grievance levels higher than the smallest monotone Bayesian Nash equilibrium: if i has a unique rationalizable action to revolt, then she must revolt in any Nash equilibrium. Thus, this reasoning leads our median citizen to conclude that at least all those with grievances larger than the smallest Bayesian Nash equilibrium cutoff will have a uniquely rationalizable action to revolt. How many are these citizens from the perspective of our median citizen? Her grievance level is $med(x_i) = \theta > 1/2$ and (when the aggregate grievance shock is very large) she believes that almost half of the population has even larger grievances. But she has a strict incentive to revolt even if she believes that a fraction $1 - \theta < 1/2$ will revolt. Thus, the marginal citizen who is indifferent between revolting and not revolting should have an even lower grievance level than her. As a result, she believes that at least half the population will have a unique rationalizable action to revolt.

Proposition 1 does not state that a large shock is always necessary for the result. Rather, it states its sufficiency. Indeed, when the current aggregate grievance level θ is very high, a majority of citizens will always have a unique rationalizable action to revolt regardless of the magnitude of the aggregate grievance shock. Moreover, given any current aggregate grievance level larger than 1/2, when the old aggregate grievance level is sufficiently high, again, a majority of citizens will always have a unique rationalizable action to revolt. The following Proposition formalizes these observations.

Proposition 2 *Let $\overline{R}$ be the maximum value of the rank function, $\overline{R} = \max_z R(z)$.*

1. *If $\theta > \overline{R}$, then revolt is the unique rationalizable action for a majority of citizens.*
2. *There exists a $\overline{\theta}_0$ such that if $\theta_0 \geq \overline{\theta}_0$ and $\theta > 1/2$, then revolt is the unique rationalizable action for a majority of citizens.*

These features are shared between our model and the standard setting with thin tails. Although Proposition 2 may have limited substantive import, it reveals the importance of large shocks. In particular, even when $\theta < \overline{R}$ and $\theta_0 < \overline{\theta}_0$, Proposition 1 shows that large shocks alone suffice to make revolt the unique rationalizable action for at least a majority as long as the aggregate grievances $\theta > 1/2$. Figures 1 to 4 illustrate.

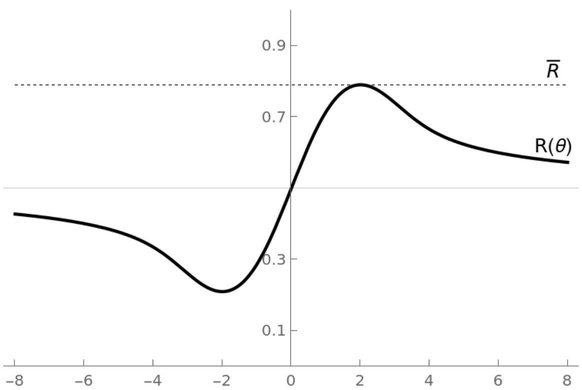

Figure 1: $R(z)$ and $\overline{R}$. When $\theta > \overline{R}$, the unique rationalizable action is to revolt.

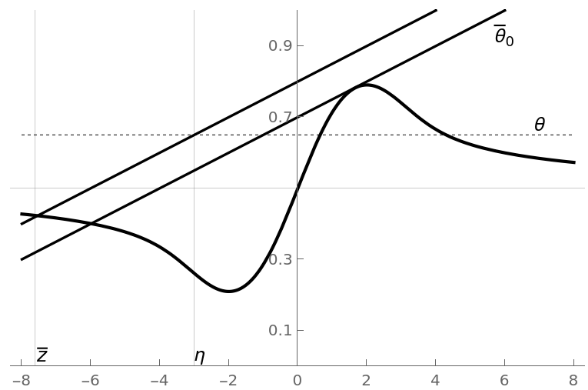

Figure 2: When $\theta_0 > \bar{\theta}_0$, the majority has a unique rationalizable action to revolt.

In Proposition 1, we fixed the final distribution of grievances and studied the effect of arriving at that distribution via a large aggregate grievance shock—we will soon compare this large aggregate shock with a series of small aggregate shocks. But we first state another stark implication of our arguments.

Proposition 3 *Suppose $R(z)$ is single-peaked on $z \geq 0$. Fix $\theta, \theta' \in (1/2, \overline{R})$, with $\theta' > \theta$. If we arrive at the aggregate grievance level θ through a sufficiently large aggregate grievance shock η, then revolt is the unique rationalizable action for a fraction $p \in (1/2, p_\theta)$ of the population. However, there is a smaller aggregate grievance shock η' such that if we arrive at the larger aggregate grievance level θ' through this smaller aggregate grievance shock, then the fraction of citizens for whom revolt is the unique rationalizable action is smaller than p.*

Consider two societies A and B with the distribution of grievances $x_A \sim N(\theta_A, 1)$ and $x_B \sim N(\theta_B, 1)$, where $\theta_A < \theta_B$, so that grievances are higher in society B in the first order

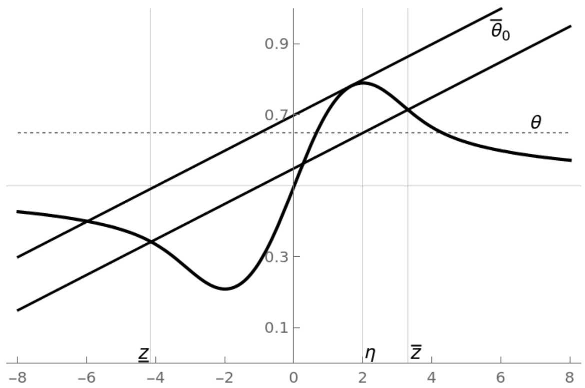

Figure 3: When $\theta_0 < \bar{\theta}_0$, but the common shock η is small, revolting and not revolting are rationalizable for the majority.

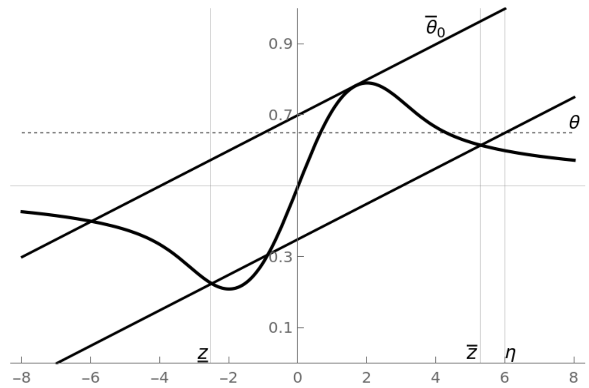

Figure 4: When $\theta_0 < \bar{\theta}_0$, but the common shock η is large, revolting is the unique rationalizable action for the majority.

stochastic dominance sense. One may expect that more people revolt in society B than in society A. Our paper shows that this seemingly common sense is inaccurate—because it does not consider the coordination aspects of revolt. How we arrive at the distribution of grievances matter. Proposition 3 identifies conditions under which if we arrive at the relatively low grievances of society A via a large aggregate grievance shock, but arrive at the relatively high grievances of society B via a smaller aggregate shock, the fraction of citizens who will have a uniquely rationalizable action to revolt will be higher in society A.

Given the banality of Normal distributions in the literature, one may be tempted to think that similar results must also hold when the distribution of aggregate shocks is Normal. In fact, the results will be akin to the opposite. Recall that when both idiosyncratic and aggregate grievance shocks have Normal distributions, the rank function takes the simple form $R(z) = \Phi(\alpha z)$ for some $\alpha > 0$, where $\Phi(\cdot)$ is the cdf of the standard Normal distribution.

Proposition 4 *Suppose aggregate and idiosyncratic grievance shocks are normally distributed, $G = N(0, \sigma_\eta)$ and $F = N(0, \sigma_\epsilon)$, and consider $\theta \in (1/2, 1)$. Then increases in the aggregate grievance shock always reduce the fraction of citizens, $p(\eta)$, for whom revolt is the unique rationalizable action. Moreover,* $\lim_{\eta \to -\infty} p(\eta) = \Phi(\theta/\sigma\sigma_\epsilon) > \lim_{\eta \to +\infty} p(\eta) = 1 - \Phi((1-\theta)/\sigma\sigma_\epsilon)$.

3.4 Citizen Behavior After A Series of Small Unexpected Changes

In the previous section, we showed that when the final distribution of grievances is the result of a large shock to aggregate grievances, revolt is the unique rationalizable action for some fraction of citizens. We now identify sufficient conditions under which when the same final distribution of grievances is the result of a consequence of smaller shocks, revolt is the unique rationalizable action for a strictly smaller fraction of citizens.

As we discussed in the Introduction, our goal is to establish a strong form of path-dependence in protest and revolt. For example, if one path allows for endogenous or exogenous "first-movers", but the other path does not, naturally, the outcomes will be different. More generally, if two distinct paths have different dynamic incentives, different outcomes would not be surprising. Thus, to keep everything else except the size of the shocks constant, we consider the following setup.

Suppose we start at the aggregate grievance level θ_0. In one scenario, this aggregate grievance level increases from θ_0 to θ in one period. This is the game that we studied in previous sections. In the second scenario, the aggregate grievance level increases over $N > 1$ periods. Suppose $\theta_t = \theta_{t-1} + \sigma\eta_t$, $\eta_t \sim iid\ G$, $t = 1, \cdots, N$, and consider the following realization of aggregate grievance levels $\theta_0 < \theta_1 < \cdots < \theta_N = \theta$. Thus, along this path, in period t, the aggregate grievance level increases from θ_{t-1} to θ_t. In period t, citizens observe the previous period's aggregate grievance level, θ_{t-1}, and engage in the same game that we analyzed in previous sections.Thus, the only distinguishing feature of these two scenarios is the size of the aggregate grievance shocks.

The results will be analogous in an infinite horizon game in which the state evolves according to $\theta_t = \theta_{t-1} + \sigma\eta_t$, $t = 1, \cdots$, and $\eta_t \sim iid\ G$. In each period t, citizens observe the last period's state, θ_{t-1} (as in, e.g., Bueno de Mesquita and Shadmehr (2020b) and Angeletos and La'O (2010)), and their private signals $x_{it} = \theta_{t-1} + \sigma(\eta_t + \epsilon_{it})$, where $\epsilon_{it} \sim iid\ F$ and independent of η_ts. Note that because there is continuum of citizens, a citizen's action has negligible effect on current or future outcomes, so that the only link between periods is information (see Angeletos et al. (2007)). In this setting, we would be comparing the change along two different finite sequence of the realizations of η_ts.

Proposition 5 *Fix a $p \in (1/2, p_\theta)$, and a current aggregate grievance level $\theta = \theta_0 + \sigma\eta \in (1/2, \overline{R})$, so that the current distribution of grievances in the population is also fixed. Then there exists $\theta_0 < \overline{\theta_0}$ such that when aggregate grievances increase suddenly from θ_0 to θ, revolt is the unique rationalizable action for a fraction p of citizens. However, there is a more gradual increase in aggregate grievances from θ_0 to θ over $N > 1$ periods such that the fraction of citizen for whom revolt is the unique rationalizable action always remains smaller than p.*

Figure 5 illustrates. We emphasize that Proposition 5 does not say that small aggregate grievance shocks always generate smaller protests (a smaller fraction of citizens for whom revolt is the unique rationalizable action) than large shocks. For example, suppose $\theta \in (\overline{\theta}_0, \overline{R})$ and we arrive at θ from $\theta_0 = \theta - \epsilon$ for a small ϵ. Then the difference between $\bar{z}$, which is necessarily negative, and η, which is necessarily positive, could be quite large. In fact, it could be larger than $p_\theta - \epsilon$, the fraction of citizens with uniquely rationalizable action to revolt, which is obtained when the aggregate grievance shock is very large. This example also highlights the challenges of proving this result. However, Proposition 5 does imply as a corollary that, under the general conditions specified above, for every large aggregate grievance shock that makes revolt the unique rationalizable action for a fraction $p > 1/2$ of citizens, we can find a smaller aggregate grievance shock that generates the exact grievance distribution in the population, but makes revolt the unique rationalizable action for a strictly smaller fraction of citizens.

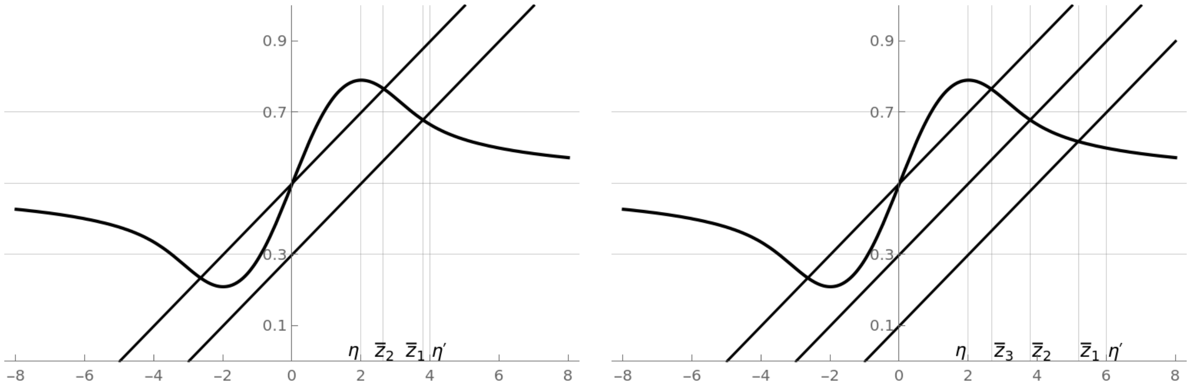

Figure 5: The left graph demonstrates the difference in the majority's behavior for two paths of increase in grievances. In one path, grievances suddenly and sharply increase from $\theta_0 = 0.3$ to $\theta = 0.7$, so that the common shock is large: $\eta' = 4$. Because $\eta' > \bar{z}_1$, the majority has a unique rationalizable action to protest. In another path, grievances increase more gradually, first from 0.3 to 0.5, and then from 0.5 to 0.7. In each step the common shock is smaller: $\eta = 2$. Because $\eta < \bar{z}_1, \bar{z}_2$, the majority also has a rationalizable action not to protest. The right panel demonstrates a similar pattern, but in three steps: from 0.1 to 0.3, from 0.3 to 0.5, and from 0.5 to 0.7. Parameters: $F = N(0, 1)$, $G = Cauchy(0, 0.5)$, $\sigma = 0.1$, $\theta = 0.7$.

4 Conclusion

In the Introduction, we discussed how even though individual humans do not act like the legendary boiling frog, the society as a whole does. Large grievance shocks have a way of coordinating behavior that often cannot be reproduced by a sequence of small grievance shocks that add up to the exact same final distribution of grievances in the society. We showed that the difference lies in the different effects of large and small shocks on individual beliefs about each others' behavior. We end with another analogy. Psychological shocks cause fight, flight, or freeze responses in individuals through physiological processes that work through sympathetic and parasympathetic nervous systems. In a sense, aggregate trauma causes societies to mount large or small protests, corresponding to fight or freeze responses, through collective action processes that work through higher-order belief systems—beliefs about the state of the world and other people's beliefs, and hence, their actions. Seen through this lens, we show that when people believe that the social trauma can be very large (heavy-tailed aggregate shocks), the society's response to trauma is to fight by mounting large protests. Even though individual humans may be shocked into inaction (freeze response) by psychological trauma, the society as whole is shocked into action by social trauma.

5 Appendix: Proofs

Proof of Lemma 1: Part 1. Using Bayes rule,

$$R(z) = Pr(z_j < z | z_i = z) = \int_{-\infty}^{\infty} Pr(z_j < z|\eta) pdf(\eta|z_i = z) d\eta = \frac{\int_{-\infty}^{\infty} F(z-\eta) f(z-\eta) g(\eta) d\eta}{\int_{-\infty}^{\infty} f(z-\eta) g(\eta) d\eta}.$$

Thus,

$$R(-z) = \frac{\int_{-\infty}^{\infty} (1 - F(z+\eta)) f(z+\eta) g(-\eta) d\eta}{\int_{-\infty}^{\infty} f(z+\eta) g(-\eta) d\eta} = 1 - R(z),$$

where we used symmetry: $F(-z-\eta) = 1 - F(z+\eta)$, $f(-z-\eta) = f(z+\eta)$, and $g(\eta) = g(-\eta)$.

Part 2. Let H be the cdf of z_i, with the corresponding pdf h, and observe that $h(z) = h(-z)$ by the symmetry of f and g.

$$\begin{aligned}
R(z) - R(-z) &= \frac{\int_{-\infty}^{\infty} F(z-\eta) f(z-\eta) g(\eta) d\eta - \int_{-\infty}^{\infty} F(-z-\eta) f(-z-\eta) g(\eta) d\eta}{h(z)} \\
&= \frac{\int_{-\infty}^{\infty} F(\gamma) f(\gamma) g(z-\gamma) d\eta - \int_{-\infty}^{\infty} F(\gamma) f(\gamma) g(-z-\gamma) d\gamma}{h(z)} \quad \text{(change of variables)} \\
&= \frac{\int_{-\infty}^{\infty} F(\gamma) f(\gamma) (g(z-\gamma) - g(z+\gamma)) d\gamma}{h(z)} \quad \text{(symmetry of } g\text{)} \\
&= \frac{\int_{-\infty}^{0} F(\gamma) f(\gamma) (g(z-\gamma) - g(z+\gamma)) d\gamma + \int_{0}^{\infty} F(\gamma) f(\gamma) (g(z-\gamma) - g(z+\gamma)) d\gamma}{h(z)} \\
&= \frac{\int_{0}^{\infty} F(-\gamma) f(-\gamma) (g(z+\gamma) - g(z-\gamma)) d\gamma + \int_{0}^{\infty} F(\gamma) f(\gamma) (g(z-\gamma) - g(z+\gamma)) d\gamma}{h(z)} \\
&= \frac{\int_{0}^{\infty} (F(\gamma) - 1) f(\gamma) (g(z-\gamma) - g(z+\gamma)) d\gamma + \int_{0}^{\infty} F(\gamma) f(\gamma) (g(z-\gamma) - g(z+\gamma)) d\gamma}{h(z)} \\
&= \frac{\int_{0}^{\infty} (2F(\gamma) - 1) f(\gamma) (g(z-\gamma) - g(z+\gamma)) d\gamma}{h(z)}.
\end{aligned}$$

Thus, for $z > 0$, $R(z) - R(-z) > 0$, because (1) $2F(\gamma) - 1 > 0$ by the symmetry of F around 0, and (2) $g(\cdot)$ is symmetric and single-peaked. Thus, for any $z > 0$, $R(z) - R(-z) = R(z) - (1 - R(z)) = 2R(z) - 1 > 0$ and hence $R(z) > 1/2 > R(-z)$.

Part 3. Using a change of variables $y = z - \eta$, for any $\gamma \in (0, 1)$, we have:

$$\begin{aligned}
R(z) &= \frac{\int_{-\infty}^{\infty} F(y) f(y) g(z-y) dy}{\int_{-\infty}^{\infty} f(y) g(z-y) dy} \\
&\leq \frac{\int_{-\infty}^{-\gamma z} F(y) f(y) g(z-y) dy + \int_{-\gamma z}^{\gamma z} F(y) f(y) g(z-y) dy + \int_{\gamma z}^{\infty} F(y) f(y) g(z-y) dy}{\int_{-\gamma z}^{\gamma z} f(y) g(z-y) dy}.
\end{aligned}$$

Thus, for any $z > 0$, we have:

$$\begin{aligned}
R(z) &\leq \frac{g((1+\gamma)z)\int_{-\infty}^{-\gamma z} F(y)f(y)dy + g((1-\gamma)z)\int_{-\gamma z}^{\gamma z} F(y)f(y)dy + g((1-\gamma)z)\int_{\gamma z}^{\infty} F(y)f(y)dy}{g((1+\gamma)z)(F(\gamma z) - F(-\gamma z))} \\
&= \frac{g((1+\gamma)z)\ (F(-\gamma z))^2}{2g((1+\gamma)z)(F(\gamma z) - F(-\gamma z))} + \frac{g((1-\gamma)z)\ (F(\gamma z) - F(-\gamma z))(F(\gamma z) + F(-\gamma z))}{2g((1+\gamma)z)(F(\gamma z) - F(-\gamma z))} \\
&\quad + \frac{g((1-\gamma)z)(1 - F(\gamma z))(1 + F(\gamma z))}{2g((1+\gamma)z)(F(\gamma z) - F(-\gamma z))} \\
&= \frac{(F(-\gamma z))^2}{2(F(\gamma z) - F(-\gamma z))} + \frac{1}{2}\ \frac{g((1-\gamma)z)}{g((1+\gamma)z)}\ \left((F(\gamma z) + F(-\gamma z)) + \frac{(1 - F(\gamma z))(1 + F(\gamma z))}{(F(\gamma z) - F(-\gamma z))}\right).
\end{aligned}$$

Thus,

$$\lim_{z\to\infty} R(z) = \frac{1}{2}\ \frac{g((1-\gamma)z)}{g((1+\gamma)z)}. \tag{1}$$

We recognize that if $g(\cdot)$ was, for example, the pdf of the Normal distribution, then the right hand side would be infinity, and the upper bound on $R(z)$ would be trivial. However, because $g(\cdot)$ is a regularly varying function,

$$\lim_{z\to\infty} \frac{g((1-\gamma)z)}{g((1+\gamma)z)} = \left(\frac{1-\gamma}{1+\gamma}\right)^{-\beta},\ \text{for some } \beta > 0,\ \text{independent of } \gamma. \tag{2}$$

Combining (1) and (2),

$$\lim_{z\to\infty} R(z) \leq \frac{1}{2}\left(\frac{1-\gamma}{1+\gamma}\right)^{-\beta},\ \text{for some } \beta > 0,\ \text{independent of } \gamma. \tag{3}$$

Because this is true for any $\gamma \in (0, 1)$, inequality (3) implies

$$\lim_{z\to\infty} R(z) \leq \frac{1}{2}.$$

But, by part 2 of the Lemma, we know that $R(z) > 1/2$ for all $z > 0$. This together with (4) implies $\lim_{z\to\infty} R(z) = 1/2$. □

Proof of Remark. Let $a(k) = k - 1 + 2\delta$ for $\delta \in (0, 1)$,

$$\begin{aligned}
\lim_{k\to\infty} \frac{G(a(k)) - G(k-1)}{G(k+1) - G(k-1)} &= \lim_{k\to\infty} \frac{g(a(k))G(k-1) - g(k-1)G(a(k))}{g(k+1)G(k-1) - g(k-1)G(k+1)} \quad \text{(by the L'Hopital rule)} \\
&= \lim_{k\to\infty} \frac{\frac{g(a(k))}{G(a(k))}\frac{G(k-1)}{g(k-1)} - 1}{\frac{g(k+1)}{G(k+1)}\frac{G(k-1)}{g(k-1)} - 1} \frac{G(a(k))}{G(k+1)} \quad \text{(factoring out } g(k-1)G(a(k))\text{)} \\
&= \lim_{k\to\infty} \frac{\frac{k-1}{a(k)}\frac{g(1)}{G(1)}\frac{G(1)}{g(1)} - 1}{\frac{k-1}{k+1}\frac{g(1)}{G(1)}\frac{G(1)}{g(1)} - 1} \frac{G(a(k))}{G(k+1)} \quad \text{(by scale invariance)} \\
&= \lim_{k\to\infty} \frac{-2\delta}{k-1+2\delta} \frac{k+1}{-2} \frac{G(k-1+2\delta)}{G(k+1)} \quad \text{(substituting for } a(k)\text{)} \\
&= \delta = \frac{a-(k-1)}{2},
\end{aligned}$$

for $k - 1 \leq a \leq k + 1$. □

Proof of Proposition 1: Recall that θ and σ are fixed. For a given η, let $z_{p,\eta}$ be the signal at the pth percentile of signals $(z_i)_{i\in[0,1]}$, so that $p = 1 - F(z_{p,\eta} - \eta)$, i.e., $z_{p,\eta} = \eta + F^{-1}(1-p)$. Because $R(z) > 1/2$ for $z > 0$ and $\lim_{z\to\infty} R(z) = 1/2$, we have $\lim_{\eta\to\infty} R(\overline{z}_\eta) = 1/2$. Thus,

$$\lim_{\eta\to\infty} R(\overline{z}_\eta) = \lim_{\eta\to\infty} \theta_0(\eta) + \sigma\overline{z}_\eta = 1/2. \tag{4}$$

In contrast,

$$\begin{aligned}
\lim_{\eta\to\infty} \theta_0(\eta) + \sigma z_{p,\eta} &= \lim_{\eta\to\infty} \theta_0(\eta) + \sigma\eta + \sigma F^{-1}(1-p) = \theta + \sigma F^{-1}(1-p) \\
&> \theta + \sigma F^{-1}(1-p_\theta) = \theta + \sigma F^{-1}\left(F\left(\frac{1/2-\theta}{\sigma}\right)\right) \\
&= 1/2.
\end{aligned} \tag{5}$$

Combining (4) and (5) implies that for sufficiently large η, we have: $z_{p,\eta} > \overline{z}_\eta$. □

Proof of Proposition 2: Part 1. The median's signal is $med(z_i) = \eta$. We show if $\theta > \overline{R}$, then $\eta > \overline{z}$. Suppose not, so that $\eta < \overline{z}$. Then, $\theta_0 + \sigma\eta < \theta_0 + \sigma\overline{z} = R(\overline{z})$. But the assumption that $\theta > \overline{R}$ means $\theta_0 + \sigma\eta > \overline{R} \geq R(z) \geq R(\overline{z})$. A contradiction. Thus, $med(z_i) = \eta > \overline{z}$, which implies that half of citizens have signals that are above $\overline{z}$.

Part 2. Let $\overline{\theta}_0$ be the maximum θ_0 such that $\theta_0 + \sigma z = R(z)$ has a solution in $[0, \infty)$. If $\theta_0 \geq \overline{\theta}_0$, then $\theta_0 + \sigma z \geq R(z)$ for all $z \geq 0$. Thus, $\theta_0 + \sigma\overline{z} = R(\overline{z}) \leq 1/2 < \theta = \theta_0 + \eta$, which implies $\overline{z} < \eta$. □

Proof of Proposition 3: Define $M_\theta = \max\{z \ s.t. \ R(z) = \theta\}$. Because $R(z)$ is single-peaked, $M_{\theta'} < M_\theta$. Choose $\eta' = M_{\theta'}$, so that $\overline{z}(\theta', \eta') = M_{\theta'} = \eta'$. Thus, exactly half of the population has a uniquely rationalizable action to revolt.

From Proposition 1, there exists a $\bar{\eta}_\theta$ such that for all $\eta > \bar{\eta}_\theta$, a fraction $p > 1/2$ of the population has a uniquely rationalizable action to revolt. Choose any such η. Moreover, we show that $\bar{\eta}_\theta \geq M_\theta$. Suppose not, so that $\bar{\eta}_\theta < M_\theta$. Then we could choose $\eta = M_\theta$, in which case exactly a fraction $p = 1/2$ of the population would have a uniquely rationalizable action to revolt. Thus, $M_\theta \leq \bar{\eta}_\theta < \eta$. Thus, $\eta' = M_{\theta'} < M_\theta < \eta$. $\square$

Proof of Proposition 4: The first part is immediate. For the second part, fix $\theta \in (1/2, \overline{R})$. For a given η, let $\overline{z}_\eta$ be the largest solution to $R(z) = \theta_0 + \sigma z$, and let $p(\eta)$ be the proportion of agents for whom revolt is the uniquely rationalizable action. Note that $1 - p(\eta) = \Phi\left(\frac{\overline{z}_\eta - \eta}{\sigma_\epsilon}\right)$.

Recall the rank function is monotone, with $\lim_{z\to\infty} R(z) = 1$ and $\lim_{z\to-\infty} R(z) = 0$. Thus, $\lim_{\eta\to\infty} \theta_0(\eta) + \sigma\overline{z}_\eta = 1$. Substituting from $\overline{z}_\eta$ from above, we have:

$$\lim_{\eta\to\infty} \theta_0(\eta) + \sigma(\eta + \sigma_\epsilon \Phi^{-1}(1 - p(\eta))) = 1, \text{ so that } \lim_{\eta\to\infty} p(\eta) = 1 - \Phi\left(\frac{1-\theta}{\sigma\sigma_\epsilon}\right).$$

Similarly, $\lim_{\eta\to-\infty} \theta_0(\eta) + \sigma\overline{z}_\eta = 0$. Thus, $\lim_{\eta\to-\infty} p(\eta) = 1 - \Phi(-\theta/\sigma\sigma_\epsilon) = \Phi(\theta/\sigma\sigma_\epsilon)$. $\square$

Proof of Proposition 5: Fix $\theta \in (1/2, \overline{R})$ and $p \in (1/2, p_\theta)$. From Proposition 1, there is a threshold on $\hat{\theta}_0$ such that if $\theta_0 < \hat{\theta}_0$, then going from θ_0 to θ will make revolt the unique rationalizable action for a fraction $p > 1/2$ of the population. Pick one such $\theta_0 < \min\{\hat{\theta}_0, 1/2\}$, so that $\theta_0 < 1/2 < \bar{\theta}_0$.

We will show there exists $\{\theta'_i\}_{i=1}^N$, $N > 1$, with $\theta_0 < \theta'_1 < \cdots < \theta'_N = \theta$, such that going from θ'_{i-1} to θ'_i, $i = 1, \cdots, N$, will make revolt the unique rationalizable action for *at most* a fraction $p = 1/2$ of the population.

Let $m_\theta = \min\{z : R(z) = \theta\}$, and note that m_θ is increasing in θ for $\theta \in (1/2, \overline{R})$. Let $\theta_1 = \theta - \sigma m_\theta$, i.e., the intercept of a line with slope σ that goes through $(m_\theta, \theta) = (m_\theta, R(m_\theta))$. Now, consider θ_1 as the initial aggregate grievance level in the one-period game. The largest equilibrium threshold will be $\bar{z}_{\theta_1} \geq m_\theta$, with equality only if the above line is tangential to $R(z)$.

Going from θ_1 to θ corresponds to the aggregate shock $\eta_{\theta_1,\theta} = \frac{\theta-\theta_1}{\sigma} = m_\theta$. Because $\eta_{\theta_1,\theta} = m_\theta \leq \bar{z}_{\theta_1}$, revolt is the uniquely rationalizable action for at most a majority. This also implies that $\theta_1 \geq \hat{\theta}_0 > \theta_0$.

Suppose $\theta_1 \leq 1/2$. Clearly, going from θ_0 to θ_1 will make revolt the uniquely rationalizable action for at most a fraction $p < 1/2$. Thus, neither going θ_0 to θ_1, nor going from θ_1 to θ will make revolt the uniquely rationalizable action for a fraction $p > 1/2$ of the population.

Next, suppose $\theta_1 > 1/2$. Repeat the above process until $\theta_n \leq 1/2$ for some $n > 1$, if such n exists, so that $\theta_{i+1} = \theta_i - \sigma m_{\theta_i}$, $i = 1, 2, \cdots$. The path then will be $\theta_0 < \theta_n < \cdots < \theta_1 < \theta$. If such n does not exists, so that for all n, $\theta_n > 1/2$, then we must have (i) $\{\theta_i\}_{i=1}^\infty$ will converge to $1/2$ from above, and (ii) $R'(0) > \sigma$. As above, clearly, going from $\theta_0 < 1/2$ to $1/2$ will make revolt the uniquely rationalizable action for at most a fraction $p < 1/2$ of the population. Moreover, (i) and (ii) imply that there exists a large enough i, which we call n, such that going from $1/2$ to θ_n will make revolt the uniquely rationalizable action for at most a fraction $p < 1/2$ of the population. To see this, let $\eta_{1/2,\theta_n}$ be the aggregate shock that corresponds to moving from $1/2$ to θ_n, so that $\eta_{1/2,\theta_n} = \frac{\theta_n - 1/2}{\sigma}$. From (i) and (ii), by choosing θ_n close enough to $1/2$, we will have $1/2 + \sigma\eta_{1/2,\theta_n} < R(\eta_{1/2,\theta_n})$. Thus, $1/2 + \sigma z = R(z)$ has a solution strictly larger than $\eta_{1/2,\theta_n}$.W have constructed a desired path, $\theta_0 < 1/2 < \theta_n < \cdots < \theta_1 < \theta$. $\square$

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