Sofia Guo Econ 142 PSET 4

Sofia Guo

2/28/2019

0. Question

Suppose that x is a positive random variable and that $E[y|x] = g(x)$, where g is an increasing function, so $x_1 \geq x_2 \Rightarrow g(x_1) \geq g(x_2)$. Prove that $cov[y, x] \geq 0$.

0. Proof:

We know that $x_1 \geq x_2 \Rightarrow g(x_1) \geq g(x_2)$ so $g(x_1) - g(x_2) \geq 0$
$\Rightarrow E[g(x_1) - g(x_2)] \geq 0$ by taking expectations of both sides
$\Rightarrow E[g(x_1)] - E[g(x_2)] \geq 0$ distributing expectations. Given that $E[y|x] = g(x)$ we get the inequality:

$$E[E[y|x_1]] - E[E[y|x_2]] \geq 0 \tag{1}$$

Now, we look at the definition of $cov[x, y] = E[y(x - \mu)]$ given in class. We can substitute $y = g(x)$ by LIE where $E[y|x] = g(x)$.
$\Rightarrow cov[x, y] = E[g(x)(x - \mu)]$
$\Rightarrow cov[x, y] = E[xg(x) - \mu g(x))]$
$\Rightarrow cov[x, y] = E[xg(x)] - E[\mu g(x))]$
$\Rightarrow cov[x, y] = E[xE[y|x]] - \mu E[E[y|x]]$ from $E[y|x] = g(x)$. Assuming that x and $E[y|x]$ are independent random variables, we can distribute the expectation in the first term
$\Rightarrow cov[x, y] = E[x]E[E[y|x]] - \mu E[E[y|x]]$. Since we know that $E[x] = \mu$ we can substitute in the first term
$\Rightarrow cov[x, y] = \mu E[E[y|x]] - \mu E[E[y|x]]$. Knowing that since $g(x)$ is an increasing function, then $\mu \geq 0$. We can set the $cov[x, y] \geq 0$ and divide each side by μ. We then get the inequality:

$$E[E[y|x]] - E[E[y|x]] \geq 0 \tag{2}$$

Equation (1) matches equation (2) in that the expressions are almost identical; if we take $cov[x, y|x_1, x_2]$ from the second equation, we get equation (1).
$\therefore cov[x, y] \geq 0$ $\blacksquare$.

0(i). Question

Show that $cov[x, y] = E[g(x)(x - \mu)]$ using LIE.

0(i). Proof:

We are given the definition of $cov[x, y] = E[y(x - \mu)]$ from lecture. We can rewrite this to:

$$cov[x, y] = E[xy] - \mu E[y] \tag{3}$$

Now we take another definition of covariance: $cov[x, y] = E[(x - E[x])(y - E[y])]$. Since $E[x] = \mu$ we can write:

$\Rightarrow cov[x,y] = E[(x-\mu)(y-E[y])]$
$\Rightarrow cov[x,y] = E[xy - xE[y] - \mu y + \mu E[y]]$
$\Rightarrow cov[x,y] = E[xy] - E[xE[y]] - \mu E[y] + \mu E[E[y]]$. We can simplify terms using LIE:
$\Rightarrow cov[x,y] = E[xy] - E[xE[y]]$. Assuming that x and y are independent R.V., we simplify to this equation.

$$cov[x,y] = E[xy] - \mu E[y] \tag{4}$$

$\Rightarrow$ Now we see that equation (3) = (4). We substitute $y = E[E[y|x]] = E[g(x)]$.
$\therefore cov[x,y] = E[xy] - \mu E[y] = E[y(x-\mu)] = E[E[g(x)](x-\mu)] = E[g(x)(x-\mu)]$ ■.

0(ii). Question

Show that when $x \leq \mu$, $g(x)(x-\mu) > g(\mu)(x-\mu)$ AND $x > \mu$, $g(x)(x-\mu) > g(\mu)(x-\mu)$. Use to prove $E[g(x)(x-\mu)]$.

0(ii). Proof:

When $x \leq \mu$, we know that $g(x) \leq g(\mu)$ because $g(x)$ is an increasing function of x.
$\Rightarrow (x-\mu) \leq 0$ because $x \leq \mu$
$\Rightarrow g(x)(x-\mu) \geq g(\mu)(x-\mu)$ because multiplying an inequality by a negative value flips the sign.

When $x > \mu$, we know that $g(x) > g(\mu)$ because $g(x)$ is an increasing function of x.
$\Rightarrow (x-\mu) > 0$ because $x > \mu$
$\Rightarrow g(x)(x-\mu) > g(\mu)(x-\mu)$ because multiplying an inequality by a positive value preserves the original sign.

Given this equation:

$$E[g(x)(x-\mu)] = \int_0^{\mu} g(x)(x-\mu)f(x)dx + \int_{\mu}^{\infty} g(x)(x-\mu)f(x)dx \tag{5}$$

We know that $g(x)(x-\mu) > g(\mu)(x-\mu)$, so we can integrate both sides of the inequality found in the first part. As $g(x)$ is a positive function, $\mu \geq 0$ so we can write:
$\Rightarrow \int_0^{\mu} g(x)(x-\mu)f(x)dx > \int_0^{\mu} g(\mu)(x-\mu)d\mu \geq 0$

Looking at the second term in equation (5), we can write a similar expression since we know that $g(x)(x-\mu) > g(\mu)(x-\mu)$:
$\Rightarrow \int_{\mu}^{\infty} g(x)(x-\mu)f(x)dx > \int_{\mu}^{\infty} g(\mu)(x-\mu)d\mu > 0$
$\therefore E[g(x)(x-\mu)] = \int_0^{\mu} g(x)(x-\mu)f(x)dx + \int_{\mu}^{\infty} g(x)(x-\mu)f(x)dx > 0$ because the first term is ≥ 0 and the second is strictly > 0, so the sum of these terms must be strictly positive ■.

1(a). Question

Define the OLS residual for observation i as: $\hat{u}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i$. Show that $\bar{y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}$.

1(a). Proof:

We can take the expected value of the given expression:
$\Rightarrow E[\hat{u}_i] = E[y_i] - E[\hat{\beta}_0] - E[\hat{\beta}_1 x_i]$. From the sample FOC we know that $E[\hat{u}_i] = 0$ from $E[x_i \hat{u}_i] = 0$
$\Rightarrow 0 = E[y_i] - E[\hat{\beta}_0] - E[\hat{\beta}_1 x_i]$
$\Rightarrow \frac{1}{N}\sum_{i=1}^{N} y_i = \hat{\beta}_0 + \hat{\beta}_1 \frac{1}{N}\sum_{i=1}^{N} x_i$
$\therefore \bar{y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}$ $\blacksquare$.

1(b). Question

Show that $y_i - \bar{y} = \hat{\beta}_1(x_i - \bar{x}) + \hat{u}_i$

1(b). Proof:

From (a) we know that $\bar{y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}$
$\Rightarrow y_i - \bar{y} = y_i - \hat{\beta}_0 - \hat{\beta}_1 \bar{x}$
Given $\hat{u}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i$:
$\Rightarrow y_i - \bar{y} = \hat{\beta}_0 + \hat{\beta}_1 x_i + \hat{u}_i - \hat{\beta}_0 - \hat{\beta}_1 \bar{x}$
$\therefore y_i - \bar{y} = \hat{\beta}_1(x_i - \bar{x}) + \hat{u}_i$ $\blacksquare$.

1(c). Question

Show that $\frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})\hat{u}_i = \frac{1}{N}\sum_{i=1}^{N} \hat{u}_i$

1(c). Proof:

From (a) we know that $\bar{y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}$
$\Rightarrow y_i - \bar{y} = y_i - \hat{\beta}_0 - \hat{\beta}_1 \bar{x}$. We want to show that:

$$\frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y}) = \frac{1}{N}\sum_{i=1}^{N} \hat{u}_i \tag{6}$$

so that we can claim $\frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})\hat{u}_i = \frac{1}{N}\sum_{i=1}^{N} \hat{u}_i^2$. Given that $\hat{u}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i$
$\Rightarrow \frac{1}{N}\sum_{i=1}^{N} \hat{u}_i = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)$
$\Rightarrow \frac{1}{N}\sum_{i=1}^{N} \hat{u}_i = \frac{1}{N}\sum_{i=1}^{N} y_i - \hat{\beta}_0 - \hat{\beta}_1 \frac{1}{N}\sum_{i=1}^{N} x_i$ we obtain:
$\Rightarrow \frac{1}{N}\sum_{i=1}^{N} \hat{u}_i = \bar{y} - \hat{\beta}_0 - \hat{\beta}_1 \bar{x}$
From $\Rightarrow y_i - \bar{y} = y_i - \hat{\beta}_0 - \hat{\beta}_1 \bar{x}$ we can find $\frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})$:
$\Rightarrow \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y}) = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{\beta}_0 - \hat{\beta}_1 \bar{x})$
$\Rightarrow \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y}) = \bar{y} - \hat{\beta}_0 - \hat{\beta}_1 \bar{x}$
$\Rightarrow \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y}) = \bar{y} - \hat{\beta}_0 - \hat{\beta}_1 \bar{x} = \frac{1}{N}\sum_{i=1}^{N} \hat{u}_i$. Taking equation (6) and multiplying both sides by $\hat{u}_i$:
$\therefore \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})\hat{u}_i = (\bar{y} - \hat{\beta}_0 - \hat{\beta}_1 \bar{x})^2 = \frac{1}{N}\sum_{i=1}^{N} \hat{u}_i^2$ $\blacksquare$.

1(d). Question

Show that $\hat{\beta}_1 \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})(x_i - \bar{x}) = \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})^2 - \frac{1}{N}\sum_{i=1}^{N} \hat{u}_i^2$

1(d). Proof:

Take $\hat{\beta}_1$ inside the summation for the expression on the left side of the given equation
$\Rightarrow \frac{1}{N}\sum_{i=1}^{N}\hat{\beta}_1(y_i-\bar{y})(x_i-\bar{x})$. From (b) we know that $y_i-\bar{y}=\hat{\beta}_1(x_i-\bar{x})+\hat{u}_i$ so that $\hat{\beta}_1(x_i-\bar{x})=y_i-\bar{y}-\hat{u}_i$. We substitute:
$\Rightarrow \frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})((y_i-\bar{y})-\hat{u}_i)$
$\Rightarrow \frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2-(y_i-\bar{y})\hat{u}_i$. From (c) and equation (6) we know that $y_i-\bar{y}=\hat{u}_i$
$\Rightarrow \frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2-\hat{u}_i^2$
$\therefore \frac{1}{N}\sum_{i=1}^{N}\hat{\beta}_1(y_i-\bar{y})(x_i-\bar{x})=\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2-\frac{1}{N}\sum_{i=1}^{N}\hat{u}_i^2$ ■.

1(e). Question

Show that $\rho_{xy}^2=R^2$

1(e). Proof:

From lecture we know that: $R^2=1-(SSR/SS)=1-\frac{\frac{1}{N}\sum_{i=1}^{N}\hat{u}_i^2}{\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2}$
We are given $\rho_{xy}=\frac{\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})(x_i-\bar{x})}{(\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2*\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2)^{1/2}}$. We can square both sides to get the desired term:

$$\rho_{xy}^2=\frac{(\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})(x_i-\bar{x}))^2}{\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2*\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2)} \tag{7}$$

From (e) we are given $\hat{\beta}_1=\frac{\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})(x_i-\bar{x})}{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2}$. Substituting into equation (7) we obtain:

$$\rho_{xy}^2=\frac{(\hat{\beta}_1\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2)^2}{\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2*\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2)} \tag{8}$$

Distributing the square in the numerator and canceling like terms below, we get:

$$\rho_{xy}^2=\frac{\hat{\beta}_1^2\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2}{\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2} \tag{9}$$

Using $\hat{\beta}_1=\frac{\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})(x_i-\bar{x})}{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2}$, we rewrite the numerator of equation (9):
$\Rightarrow \hat{\beta}_1^2\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2=\hat{\beta}_1\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})(x_i-\bar{x})$

From (d) we know that $\frac{1}{N}\sum_{i=1}^{N}\hat{\beta}_1(y_i-\bar{y})(x_i-\bar{x})=\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2-\frac{1}{N}\sum_{i=1}^{N}\hat{u}_i^2$ so we can plug in the latter expression for the numerator from equation (9) and complete the proof:

$$\rho_{xy}^2=\frac{\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2-\frac{1}{N}\sum_{i=1}^{N}\hat{u}_i^2}{\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2}=1-\frac{\frac{1}{N}\sum_{i=1}^{N}\hat{u}_i^2}{\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2}=R^2\blacksquare \tag{10}$$

2. OVB Dataset

```
#load libraries
library(dplyr)
library(ggplot2)
library(magrittr)
library(reshape2)
library(stargazer)
library(lubridate)
library(lmtest)
library(ivpack)
library(kableExtra)
library(sandwich)
```

```
#read in dataset
ovb_raw <- read.csv("/Users/sofia/Box/Cal (sofiaguo@berkeley.edu)/2018-19/Spring 2019/Econ 142/PSETS/PSE
```

Find the coefficient ρ_{we} between log wages and education for females in the sample given equation:

$$\rho_{xy} = \frac{\frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})(x_i - \bar{x})}{(\frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})^2 * \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2)^{1/2}} \tag{11}$$

```
#restrict the sample to women only
ovb_fem <- filter(ovb_raw, female ==1)

#define terms
N <- length(ovb_fem$female)
x_i <- ovb_fem$educ
y_i <- ovb_fem$logwage
y_bar <- mean(ovb_fem$logwage)
x_bar <- mean(ovb_fem$educ)

#calculate corr. coeff
rho_fem <- (1/N)*(sum((y_i- y_bar)*(x_i - x_bar)))/((1/N)*(sum((y_i- y_bar)^2)*(1/N)*(sum((x_i- x_bar)^
rho_fem
```

```
## [1] 0.473167
```

```
#run the OLS regression and get R^2
reg_fem <- summary(lm(logwage ~ educ, data = ovb_fem))
reg_fem$r.squared
```

```
## [1] 0.223887
```

We see that the equality holds true:
$\rho_{we}^2 = 0.4731^2 = 0.2238 = R^2$.

```
#verify the same R^2 for switching x and y
reg_fem_inv <- summary(lm(educ ~ logwage, data = ovb_fem))
reg_fem_inv$r.squared
```

```
## [1] 0.223887
```

To show 1(e) with the reversed regressors we complete the exercise again:

From lecture we know that: $R^2 = 1 - (SSR/SS) = 1 - \frac{\frac{1}{N}\sum_{i=1}^{N}\hat{u}_i^2}{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$

We are given $\rho_{yx} = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y})}{(\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2 * \frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2)^{1/2}}$. We can square both sides to get the desired term:

$$\rho_{yx}^2 = \frac{(\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y}))^2}{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2 * \frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2)} \tag{12}$$

From (e) we are given $\hat{\beta}_1 = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y})}{\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2}$. Substituting into equation (12) we obtain:

$$\rho_{yx}^2 = \frac{(\hat{\beta}_1 \frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2)^2}{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2 * \frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2)} \tag{13}$$

Distributing the square in the numerator and canceling like terms below, we get:

$$\rho_{yx}^2 = \frac{\hat{\beta}_1^2 \frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2}{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2} \tag{14}$$

Using $\hat{\beta}_1 = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y})}{\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2}$, we rewrite the numerator of equation (12):
$\Rightarrow \hat{\beta}_1^2 \frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2 = \hat{\beta}_1 \frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y})$

From (d) we know that $\frac{1}{N}\sum_{i=1}^{N}\hat{\beta}_1(x_i-\bar{x})(y_i-\bar{y}) = \frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2 - \frac{1}{N}\sum_{i=1}^{N}\hat{u}_i^2$ so we can plug in the latter expression for the numerator from equation (12) and complete the proof:

$$\rho_{yx}^2 = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2 - \frac{1}{N}\sum_{i=1}^{N}\hat{u}_i^2}{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2} = 1 - \frac{\frac{1}{N}\sum_{i=1}^{N}\hat{u}_i^2}{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2} = R^2 \blacksquare \tag{15}$$

3(a). Constructing test stat to test if means are the same

```
#compute mean log wages and standard error for female non-immigrants
ovb_fem_nimm <- filter(ovb_fem, imm == 0)

nimm_mean_wage <- mean(ovb_fem_nimm$logwage)
nimm_mean_wage
```

```
## [1] 2.886378
```

```
std <- function(x) sd(x)/sqrt(length(x))
se_nimm <- std(ovb_fem_nimm$logwage)
se_nimm
```

```
## [1] 0.007019041
```

```
#compute mean log wages and standard error for female immigrants
ovb_fem_imm <- filter(ovb_fem, imm == 1)

imm_mean_wage <- mean(ovb_fem_imm$logwage)
imm_mean_wage
```

```
## [1] 2.706393
```

```r
std <- function(x) sd(x)/sqrt(length(x))
se_imm <- std(ovb_fem_imm$logwage)
se_imm
```

```
## [1] 0.01606555
```

```r
sqrt(se_imm^2 + se_nimm^2)
```

```
## [1] 0.01753194
```

To test if the means are equal, I set the $H_0 : \mu_{imm} - \mu_{non-imm} = 0$ and $\mu_{imm} - \mu_{non-imm} \neq 0$.

$$t = \frac{\mu_{imm} - \mu_{non-imm}}{\sqrt{SE^2_{imm} + SE^2_{non-imm}}} = \frac{2.706 - 2.886}{0.01753194} = -10.26617 \tag{16}$$

Testing at the 95% confidence level we find that $|-10.266| > 1.96 \Rightarrow$ We reject the H_0 that the means are equal at the $\alpha = 5\%$ level.

3(b). Another test for mean equality

```r
#run a regression of logwage on constant and immigrant statues
reg_wage_im <- summary(lm(logwage ~ imm, data = ovb_fem))
reg_wage_im$coefficients
```

```
##               Estimate  Std. Error   t value     Pr(>|t|)
## (Intercept)  2.8863783 0.007153705 403.48021 0.000000e+00
## imm         -0.1799858 0.016531954 -10.88714 1.854621e-27
```

```r
#is it equal to the diff. in means?
imm_mean_wage - nimm_mean_wage
```

```
## [1] -0.1799858
```

The coefficient on immigrants is equal to the difference in the means I found, but the standard error is off by approximately 0.001.

We calculate the test statistic using this regression's estimates by setting $H_0 : \hat{\beta}_1 = \mu_{imm} - \mu_{non-imm} = 0$ and $\hat{\beta}_1 \neq 0$.

$$t = \frac{\hat{\beta}_1}{\sqrt{SE(\hat{\beta}_1)}} = \frac{-0.179985}{0.0165319} = -10.8871 \tag{17}$$

Testing at the 95% confidence level we find that $|-10.8871| > 1.96 \Rightarrow$ We reject the H_0 that the means are equal at the $\alpha = 5\%$ level. This is not the same test statistic as the one in part (a) because there is likely non-constant variance among the residuals across immigrants and non immigrants.

3(c). Fitting heteroskedasticity robust standard errors

```r
ols <- lm(logwage ~ imm, data = ovb_fem)
ols$robse <- vcovHC(ols, type = 'HC1')
ols$robse
```

```
##              (Intercept)           imm
## (Intercept)  4.927052e-05 -4.927052e-05
## imm         -4.927052e-05  3.072910e-04
```

```r
sqrt(3.072910e-04)
```

```
## [1] 0.01752972
```

We find this variance-covariance matrix from the sandwich package:

$$VcoV = \begin{bmatrix} 4.927052 * 10^{-5} & -4.927052 * 10^{-5} \\ -4.927052e * 10^{-5} & 3.072910 * 10^{-4} \end{bmatrix}$$

From lecture we know that this matrix is generally:

$$VcoV = \begin{bmatrix} (SE(\hat{\beta}_0))^2 & Cov(\hat{\beta}_1, \hat{\beta}_0) \\ Cov(\hat{\beta}_0, \hat{\beta}_1) & (SE(\hat{\beta}_1))^2 \end{bmatrix}$$

So we can calculate the standard errors for $\hat{\beta}_1$ and $\hat{\beta}_0$ by taking the square root of the diagonal elements in the matrix:

$$SE(\hat{\beta}_0) = \sqrt{4.927052 * 10^{-5}} = 0.007019296 \tag{18}$$

$$SE(\hat{\beta}_1) = \sqrt{3.072910 * 10^{-4}} = 0.01752972 \tag{19}$$

We can see that these robust standard errors are slightly bigger than the ones we estimated in (b), because the robust calculations do not assume homoskedastic errors - thus the standard error calculated is larger (coefficients less accurate) to compensate for the non-constance variance of errors.