Sofia Guo PSET 5 Econ 136

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1. Calculate a spot rate for each date

```
#add spot rate column
table <- t_quotes %>%
  mutate(`Discount Factor` = Price/(100 + (Coupon/2)),
         `Spot Rate` = 2*(1/`Discount Factor`)^(1/i) -2)

kable(table, "latex", caption = "U.S. Treasury quotes", booktabs = T, digits = 4) %>%
  kable_styling(latex_options = "hold_position")
```

Table 1: U.S. Treasury quotes

Maturity	i	Coupon	Price	Discount Factor	Spot Rate
8/15/19	1	0.750	99.1641	0.9879	0.0244
2/15/20	2	1.375	98.8750	0.9820	0.0182
8/15/20	3	1.500	98.4609	0.9773	0.0154
2/15/21	4	2.250	99.4453	0.9834	0.0084
8/15/21	5	2.125	99.0703	0.9803	0.0080
2/15/22	6	2.000	98.5781	0.9760	0.0081
8/15/22	7	1.625	97.0938	0.9631	0.0108
2/15/23	8	2.000	98.0938	0.9712	0.0073
8/15/23	9	2.500	100.0000	0.9877	0.0028
2/15/24	10	2.750	101.1328	0.9976	0.0005

2. Repo Rates

From lecture we know that:

$$r_{repo} = (\frac{p_1 - p_0}{p_0})(\frac{365}{t_1 - t_0})$$

From the problem, we are given:

- $p_1 = 10,000,000 =$ repurchase price
- $t_1 = 1$
- $t_0 = 0$ because the trade is overnight
- $r_{repo} = 0.024 = 2.4\%$

We solve for $p_0 =$ the original price of the bonds:

$$
\begin{aligned}
0.024 &= (\frac{10,000,000 - p_0}{p_0})(\frac{365}{1}) \\
\Rightarrow 0.0000065 &= \frac{10,000,000 - p_0}{p_0} \\
\Rightarrow 0.0000065 &= \frac{10,000,000}{p_0} - 1 \\
\Rightarrow 1.0000065 &= \frac{10,000,000}{p_0} \\
\therefore p_0 &= 9,999,933
\end{aligned}
$$

3. Perpetuity closed form solution proof

From lecture we're given:

$$P_{perp} = \frac{C}{n}\sum_{i=1}^{\infty}\frac{1}{(1+\frac{Y}{n})^i}$$

We are also given the Taylor series expansion for $|x| > 1$:

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$$

We can rewrite the first equation as:

$$P_{perp} = \frac{C}{n}\sum_{i=1}^{\infty}\left(\frac{1}{1+\frac{Y}{n}}\right)^i$$

and set $x = \frac{1}{1+\frac{Y}{n}}$. We can then substitute x into the Taylor Series expansion and subtract 1 because the index of the sum for P_{perp} begins at 1 while the Taylor series index begins at 0 (and the first term is 1, so we can subtract that):

$$
\begin{aligned}
P_{perp} &= \frac{C}{N}\sum_{i=1}^{\infty}\left(\frac{1}{1+\frac{Y}{N}}\right)^{i} \\
&= \frac{C}{N}\sum_{i=1}^{\infty}x^{i} \\
&= \frac{C}{N}\left(\frac{1}{1-\frac{1}{1+\frac{Y}{N}}}-1\right) \\
&= \frac{C}{N}\left(\frac{1}{1-\left(\frac{N}{Y+N}\right)}-1\right) \\
&= \frac{C}{N}\left(\frac{1}{\frac{Y+N}{Y+N}-\frac{N}{Y+N}}-1\right) \\
&= \frac{C}{N}\left(\frac{1}{\frac{Y}{Y+N}}-1\right) \\
&= \frac{C}{N}\left(\frac{Y+N}{Y}-1\right) \\
&= \frac{C}{N}\left(\frac{Y}{Y}+\frac{N}{Y}-\frac{Y}{Y}\right) \\
&= \frac{C}{N}\left(\frac{N}{Y}\right)=\frac{C}{Y}\blacksquare.
\end{aligned}
$$

4. Okun's Law and the Taylor Rule

A decrease in the unemployment gap would increase $(Y_i - \tilde{Y})/\tilde{Y}$ as they are inversely related by $-c$ according to Okun's Law. Thus, according to Taylor's Rule, an increase in output $(Y_i - \tilde{Y})/\tilde{Y}$ would *increase* the real interest rate r_i by half the output gap; thus the central bank would increase its overnight real interest rate by $\frac{1}{2} * (Y_i - \tilde{Y})/\tilde{Y}$ for a given decrease in $U_i - \tilde{U}$.

5. Bank profit and losses

1. For the first term $r_{10y}^{customer} - r_{10y}^{CM}$, the bank makes profit from the difference between the long term loan interest they charge on customers and the equivalent lending rate in capital markets. They lose money if the capital markets rate grows higher than their customer lending rate (i.e. the right tail of the yield curve increases).

2. For the second term $r_{10y}^{CM} - r_{3m}^{CM}$, the bank makes profit if the long term capital markets borrowing rate is higher than the short term borrowing rate paid for deposits (when the yield curve is "normal"). The bank loses money when the yield curve inverts, or when the long term rates become smaller than the short term rates.

3. For the third term $r_{3m}^{CM} - r_{3m}^{customer}$, the bank makes profit if the captial market short term rate is higher than what they are paying customers for short term deposits. They lose money when the rate they pay customers for deposits is higher than the rate at which capital markets are paying for the same deposits.

6. TD Ameritrade Bonds

From class we're given the equation for annuities:

$$P_{ann} = \sum_{i=1}^{\infty} \frac{\frac{C}{N}}{(1+\frac{Y}{N})^i} - \sum_{i=NT+1}^{\infty} \frac{\frac{C}{N}}{(1+\frac{Y}{N})^i}$$

Since we are asked to derive the price of the bond at period $i = 0$, we can rewrite the formula based on the Taylor series in the previous questions:

$$P_{ann} = 1 + \frac{C}{Y} - \left(1 - \frac{1}{(1+\frac{Y}{N})^{NT}}\right)$$

Then, we can plug in the information we are given so that the formula becomes the closed-form solution:

$$P_{ann} = 1 + \frac{2.5}{Y} - \left(1 - \frac{1}{(1+\frac{Y}{N})^{NT}}\right)$$