Sofia Guo Econ 136 PSET 6

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1. Calculate DV01 of 2-yr annual pay par bond in 4% yield env.

We know that a par bond indicates that the coupon is equal to yield. Thus:

- $n = 1$
- $t = 2$
- $Y = 0.04$
- $C = 4$

We find the DV01 by taking the derivative of the price function w.r.t. yield and plugging in the relevant information:

$$\begin{aligned}
DV01 &= -\frac{1}{10000} * \frac{dP}{dY}(P_{coupon}(Y)) \\
&= -\frac{1}{10000} * \left(-\frac{C}{Y^2} + \frac{\left[\frac{C}{Y^2}\left(1+\frac{Y}{n}\right)^{nt} + t\left(1+\frac{Y}{n}\right)^{nt-1} * \left(100 - \frac{C}{Y}\right)\right]}{\left(1+\frac{Y}{n}\right)^{2nt}} \right) \\
&= -\frac{1}{10000} * \left(-\frac{4}{0.04^2} + \frac{\left[\frac{4}{0.04^2}\left(1+\frac{0.04}{1}\right)^{2} + 2\left(1+\frac{0.04}{1}\right)^{2-1} * \left(100 - \frac{4}{0.04}\right)\right]}{\left(1+\frac{0.04}{1}\right)^{4}} \right) \\
&= -\frac{1}{10000} * \left(-2500 + \frac{[2500\,(1.0816) + 2\,(1.04) * 0]}{(1.169858)} \right) \\
&= -\frac{1}{10000} * (-2500 + 2311.3916) \\
&= -\frac{1}{10000} * (-188.6083) \\
&= 0.01886
\end{aligned}$$

2. Calculate the DV01 of 15 yr annual pay floating rate annuity in 6% yield env.

We know that a par bond indicates that the coupon is equal to yield. Thus:

- $n = 1$
- $t = 25$
- $Y = 0.06$

We find the $DV01_{floatingannuity}$ by taking the derivative of the price function w.r.t. yield and plugging in the relevant information:

$$
\begin{aligned}
DV01_{floatingannuity} &= -\frac{1}{10000} * -\frac{100t}{\left(1+\frac{Y}{n}\right)^{nt+1}} \\
&= -\frac{1}{10000} * -\frac{100 * 25}{\left(1+\frac{0.06}{1}\right)^{25+1}} \\
&= -\frac{1}{10000} * -\frac{2500}{(1.06)^{26}} \\
&= 0.05495
\end{aligned}
$$

3. Calculate 4-year swap rate (par coupon) on Feb 15, 2019 given PSET 5 Q1 results

First, we know that the par coupon rate = swap rate. From lecture we're given how to calculate the par coupon rate.

$$C_{par} = n\frac{1 - DF_{nt}}{\sum_{i=1}^{nt} DF_i}$$

We are also given:

- $t = 4$
- $n = 2$ because the question was about Treasury bonds, which have semi-annual payments

Thus to get C_{par} we want:

$$C_{par} = n\frac{1 - DF_8}{\sum_{i=1}^{8} DF_i}$$

From PSET 5 the answers are:

Table 1: U.S. Treasury quotes

Maturity	Coupon	Price	Discount.Factor	Spot.Rate
8/15/19	0.750	99.1641	0.9879	2.44
2/15/20	1.375	98.8750	0.9752	2.52
8/15/20	1.500	98.4609	0.9627	2.55
2/15/21	2.250	99.4453	0.9508	2.54
8/15/21	2.125	99.0703	0.9395	2.51
2/15/22	2.000	98.5781	0.9283	2.49
8/15/22	1.625	97.0938	0.9168	2.50
2/15/23	2.000	98.0938	0.9053	2.50
8/15/23	2.500	100.0000	0.8942	2.50
2/15/24	2.750	101.1328	0.8828	2.51

So we plug in the discount factors for $i = 1, 2, \ldots, 8$:

$$
\begin{aligned}
C_{par} &= SwapRate \\
&= n\frac{1 - DF_8}{\sum_{i=1}^{8} DF_i} \\
&= 2 * \frac{1 - 0.89424}{0.97525 + 0.96267 + 0.95084 + 0.93953 + 0.92834 + 0.91681 + 0.90527 + 0.89424} \\
&= 0.028304
\end{aligned}
$$

4. Reproduce table in slide 16 and calculate change in swap price if forward rates increase 150 bp

Table 2: Swap Valuation

Year	Par	Spot	Forward	New.Forward	DF	DF.New	Fixed	Float	PV.Fixed	PV.Float
1	8.75	8.75	8.75	10.25	0.9195	0.0889	0.1	0.0875	0.0920	0.0805
2	9.00	9.01	9.27	10.77	0.8415	0.0850	0.1	0.0927	0.0842	0.0780
3	9.25	9.28	9.82	11.32	0.7662	0.0812	0.1	0.0982	0.0766	0.0752
4	9.50	9.56	10.41	11.91	0.6940	0.0775	0.1	0.1041	0.0694	0.0722
5	9.75	9.86	11.04	12.54	0.6250	0.0739	0.1	0.1104	0.0625	0.0690
6	10.00	10.16	11.72	13.22	0.5594	0.0703	0.1	0.1172	0.0559	0.0656

We plug in the new discount factors:

$$
\begin{aligned}
C_{par} &= SwapRate \\
&= 6\frac{1 - DF_6}{\sum_{i=1}^{6} DF_i} \\
&= 6 * \frac{1 - 0.0703}{0.0888 + 0.0849 + 0.0811 + 0.0774 + 0.0738 + 0.0703} \\
&= 11.711
\end{aligned}
$$

The swap price change increases because the forward rate increases.

5(a). Design a swap hedge

- The swap rate is is the Treasury bond coupon rate
- The swap is a payer because he is short the fixed rate bond
- The notional amount is 110 million USD
- The tenor of the swap is 30 years
- The payment frequency is semi-annual because it is a Treasury bond

5(b). Mechanics

The swap hedges the bond because the client now pays a fixed rate instead of a floating rate.