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Taxi pooling in New York City: a network-based approach to social sharing problems

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Taxi services are a vital part of urban transportation, and a major contributor to traffic congestion and air pollution causing substantial adverse effects on human health[1,2]. Sharing taxi trips is a possible way of reducing the negative impact of taxi services on cities[3], but this comes at the expense of passenger discomfort in terms of a longer travel time. Due to computational challenges, taxi sharing has traditionally been approached on small scales[4,5], such as within airport perimeters[6,7], or with dynamical ad-hoc heuristics[8–10]. However, a mathematical framework for the systematic understanding of the tradeoff between collective benefits of sharing and individual passenger discomfort is lacking. Here we introduce the notion of shareability network which allows us to model the collective benefits of sharing as a function of passenger inconvenience, and to efficiently compute optimal sharing strategies on massive datasets. We apply this framework to a dataset of millions of taxi trips taken in New York City, showing that the cumulative trip length can be cut by 40%, leading to similar reductions in service cost, traffic, and emissions. This benefit comes with split fares and minimal passenger discomfort quantifiable as an additional travel time of up to five minutes, hinting towards a wide passenger acceptance of such a shared service. Simulation of a realistic online dispatch system demonstrates the feasibility of a shareable taxi service in New York City. Shareability as a function of trip density saturates fast, suggesting effectiveness of the taxi sharing system also in cities with much sparser taxi fleets. We anticipate our methodology to be a starting point to the development and assessment of other ride sharing scenarios and to a wide class of social sharing problems where spatio-temporal conditions for sharing, the incurred discomfort for individual participants, and the collective benefits of sharing, can be formally defined.

Great hope is placed today in the rapid deployment of digital information and communication technologies that could help make cities "smarter"[11], and to manage vehicular traffic[12] – and the hazardous air pollution that results from it[1,2] – more efficiently. The extensive use of real-time information in today's apps make it possible to design, for instance, new, smarter taxi systems. However, municipal authorities, city residents, and other stakeholders may be reluctant to invest in it until its benefits have been quantified[3]. This is the goal of the present paper.

The concept of ride-sharing or carpooling at the basis of a shared taxi service is a long-standing proposition for decreasing road traffic, originating during the "oil crisis" in the 1970s[3]. Classical studies on carpooling typically rely on survey data[13] and discuss possible incentives or psychological attitudes[14–16]. From a theoretical perspective, trip-sharing is traditionally seen as an instance of "dynamic pickup and delivery" problems[4,5], in which a number of goods or customers must be picked up and delivered efficiently at specific locations within well-defined time windows. Such problems are typically solved by means of linear programming, in which a function of the system variables is optimised subject to a set of equations that describe the constraints, e.g. the pickup and delivery time windows of each customer. While linear programming tasks can be solved with standard approaches of Operations Research or with constraint programming[17], their computational feasibility heavily depends on the number of variables and equations used to describe the problem at hand. Most previous taxi studies have therefore focused on small-scale routing problems, such as within airport perimeters[6,7]. Large urban taxi systems, in contrast, involve thousands of vehicles performing hundreds of thousands of trips per day. Because of the immense computational challenges, the traditional approaches are unsuitable for assessing the benefit and the induced passenger discomfort of a taxi-sharing system at the city level.

Here we introduce the notion of shareability network and apply classical methods from graph theory to solve the taxi trip-sharing problem in a provably efficient way. The starting point of our analysis is a dataset composed of the records of over 150 million taxi trips originating or ending in Manhattan in the year 2011 by all 13,586 registered cabs. For each trip, the record reports the vehicle ID, the Global Positioning System (GPS) coordinates of the pickup and drop-off locations, and corresponding times. Pickup and drop-off locations have been associated with the closest street intersection in the road map of Manhattan (Supplementary Information). We impose a natural network structure on an otherwise unstructured, gigantic search space of the type explored in traditional linear programming. To this end we define two parameters: the *shareability parameter* k, standing for the maximum number of trips that can be shared, and the *quality of service parameter* Δ, which stands for the maximum delay a customer tolerates in a shared taxi service trip. Further, let $T_i = (o_i, d_i, t_i^o, t_i^d), i = 1 \dots k$ be k trips where o_i denotes the origin of the trip, d_i the destination, and t_i^o, t_i^d the starting and ending times, respectively. We say that multiple trips T_i are *shareable* if there exists a route connecting all the o_i and d_i in any order

where each o_i precedes the corresponding d_i (except for configurations where single trips are only concatenated, such as $o_1 \rightarrow d_1 \rightarrow o_2 \rightarrow d_2$), such that each customer is picked up and dropped at the respective origin and destination locations with delay at most Δ, with the delay computed as the time difference to the historic trip in the dataset. Imposing a bound of k on shareability implies that the k trips can be combined using a taxi of corresponding capacity (Fig. 1g). Deciding whether two or more trips can be shared necessitates knowledge of the travel time between arbitrary intersections in Manhattan, which we estimated using an ad-hoc heuristic (Extended Data Fig. 1, Extended Data Table 1). Note that our method only concerns the sharing of non-vacant trips, but these make up the majority of taxi traffic[18].

For the case $k = 2$, the *shareability network* associated with a set $\mathcal{T}$ of trips is obtained by assigning a node T for each trip in $\mathcal{T}$, and by placing a link between two nodes T_i and T_j if the two trips can be shared for the given value of Δ (Fig. 1a and b). The value of Δ has a profound impact on topological properties of the resulting shareability network. Increasing Δ capitalises on well-known effects of time-aggregated networks such as densification[19,20], capturing the intuitive notion that, the more patient the customers, the more opportunities for trip sharing arise (Fig. 2a and b). For values of $k > 2$, the shareability network has a hyper-graph structure in which up to k nodes can be connected by a link simultaneously. Because of computational reasons, the shareability parameter k has a substantial impact on the feasibility of solving the problem. A solution is tractable for $k = 2$, heuristically feasible for $k = 3$, while it becomes computationally intractable for $k \geq 4$ (Supplementary Information). This constraint implies that taxi sharing services, and social sharing applications in general, will likely be able to combine only a limited number of trips. However, as we show below, even the minimum possible number of trip combinations ($k = 2$) can provide immense benefits to a dense enough community like the City of New York.

With the shareability network, classical algorithms for solving maximum matching on graphs[21,22] can be used to determine the best trip sharing strategy according to two optimisation criteria: $a)$ maximising the number of shared trips, or $b)$ minimising the cumulative time needed to accommodate all trips. To find the best solution according to $a)$ or $b)$, it is sufficient to compute a maximum matching or a weighted maximum matching on the shareability network, respectively (Fig. 1c and e). Since a shared trip can be served by a single taxi instead of two, the number of shared trips can be used as a proxy for the reduction in number of circulating taxis. For instance, an 80% rate of shared trips translates into a 40% reduction of the taxi fleet. Other important objectives such as total system cost and emissions are reasonably approximated by criterion $b)$.

Using a maximum value of $\Delta = 10\,\mathrm{min}$ and all trips performed in New York City in the year 2011, the resulting shareability network has more than 150 million nodes and over 100 billion

links. We first consider trip sharing opportunities under a model in which the entire shareability network is known beforehand, and maximum matchings are computed on the entire graph. This omniscient *Oracle* approach models an artificial scenario in which trip sharing decisions can be taken considering not only the current taxi requests, but also all future ones, serving as a theoretical upper bound for sharing opportunities. In practice, the Oracle model is useful to assess the benefits of social sharing systems where bookings are placed well ahead of time (Fig. 3a). Therefore, even with the low and reasonable value of $\Delta = 2\,\mathrm{min}$, the average percentage of shareable trips is close to 100% (Fig. 3b).

In practical systems, only trip requests issued in a relatively short time window are known at decision time, corresponding to a small time-slice of the shareability network. In the following, we therefore focus on trip sharing opportunities in a realistic model in which the trip sharing decision for a trip T_1 considers only trips which start within a short interval around its starting time t_1^o. More formally, we retain in the shareability network only links connecting trips T_i and T_j such that $|t_i^o - t_j^o| \leq \delta$, where δ is a *time window* parameter. This *Online* model is representative of a scenario in which a customer using an "e-hailing" application issues a taxi request reporting pickup/drop-off locations, and after the small time window δ receives feedback from the taxi management system whether a shared ride is available. This parameter is fundamental in the Online model: the larger δ, the more trip sharing opportunities can be exploited, for the same reasons of network time-aggregation as with Δ (Extended Data Fig. 2). However, δ should be kept reasonably small to be acceptable by a potential customer. Therefore, in what follows, we set $\delta = 1\,\mathrm{min}$.

As expected, reducing the time horizon δ from practically infinite in the Oracle model to $1\,\mathrm{min}$ in the Online model considerably reduces trip sharing opportunities for low values of Δ. For instance, when $\Delta = 1\,\mathrm{min}$, the Oracle model allows sharing of 94.5% of the trips, but the Online model only less than 30%. However, the situation is much less penalising for the Online model when the delay parameter is increased within reasonable range. When $\Delta = 5\,\mathrm{min}$, the Online model can exploit virtually *all* available trip sharing opportunities (Fig. 3b). Concerning saved travel time, results are similarly promising (Fig. 3d). When $\Delta = 5\,\mathrm{min}$, we can save 32% of total travel time with the Online model, compared to 40% savings in the optimal Oracle model.

Is it possible to even further improve efficiency by increasing the number k of shareable trips? When $k = 3$, the shareability network becomes a shareability hyper-network, for which maximum matching is solvable only in approximation using a heuristic algorithm which is computationally feasible for relatively small networks only[23,24]. Because of this methodological issue and the combinatorial explosion of sharing options, we calculated the number of shared trips and the fraction of saved travel time for $k = 3$ only in the Online model – which by definition features much smaller shareability networks. Simulations show that increasing the number of shareable

trips k provides noticeable benefits only when the quality of service parameter Δ crosses a threshold around $\Delta_{\mathrm{crit}} \approx 150\,\mathrm{sec}$ (Fig. 3d and e). When $\Delta = 300\,\mathrm{sec}$, the number of saved taxi trips is increased from about 50% with $k = 2$ to about 60% with $k = 3$, which is however well below the 66.7% maximum theoretical percentage of shared trips. This suboptimal result suggests that the effort for implementing a service for sharing $k > 2$ trips may not be well justified. Further, to become widely accepted, a multi-shared taxi service might require vehicles of higher capacity and/or physically separated, private compartments, possibly inflating overhead for $k > 2$. Since the benefit of multi-sharing is not that high, it might not cover these additional expenses.

Our analysis shows that New York City offers ample opportunities for trip sharing with minimal passenger discomfort, without having to resort to a computationally demanding sharing strategy in which already started trips would be re-routed on the fly, and that these opportunities are realistic to be implemented in a new dispatch system. In order to assess to which extent our results could be generalised to cities with lower taxi densities than New York, we studied how the number of shareable trips in a given day changes as a function of the total number of trips (Fig. 3c). The average number of daily trips in New York is highly concentrated around 400,000. Hence, we have generated additional low density situations by subsampling the dataset, randomly removing increasing fractions of vehicles from the system. The resulting shareability values are excellently fit by saturation curves well-known from biochemical systems (Supplementary Information). At around 100,000 trips, or 25% of the daily average, we already reach saturation and near maximum shareability. This fast saturation suggests that taxi sharing systems would be effective even in cities with taxi fleet densities much lower than New York. More generally, the framework of shareability networks can be used to study other social sharing opportunities such as ride sharing of cars, bikes, etc. or the communal usage of equipment which is characterised by considerable unit cost and infrequent use, stimulating new forms of sharing and models of ownership.

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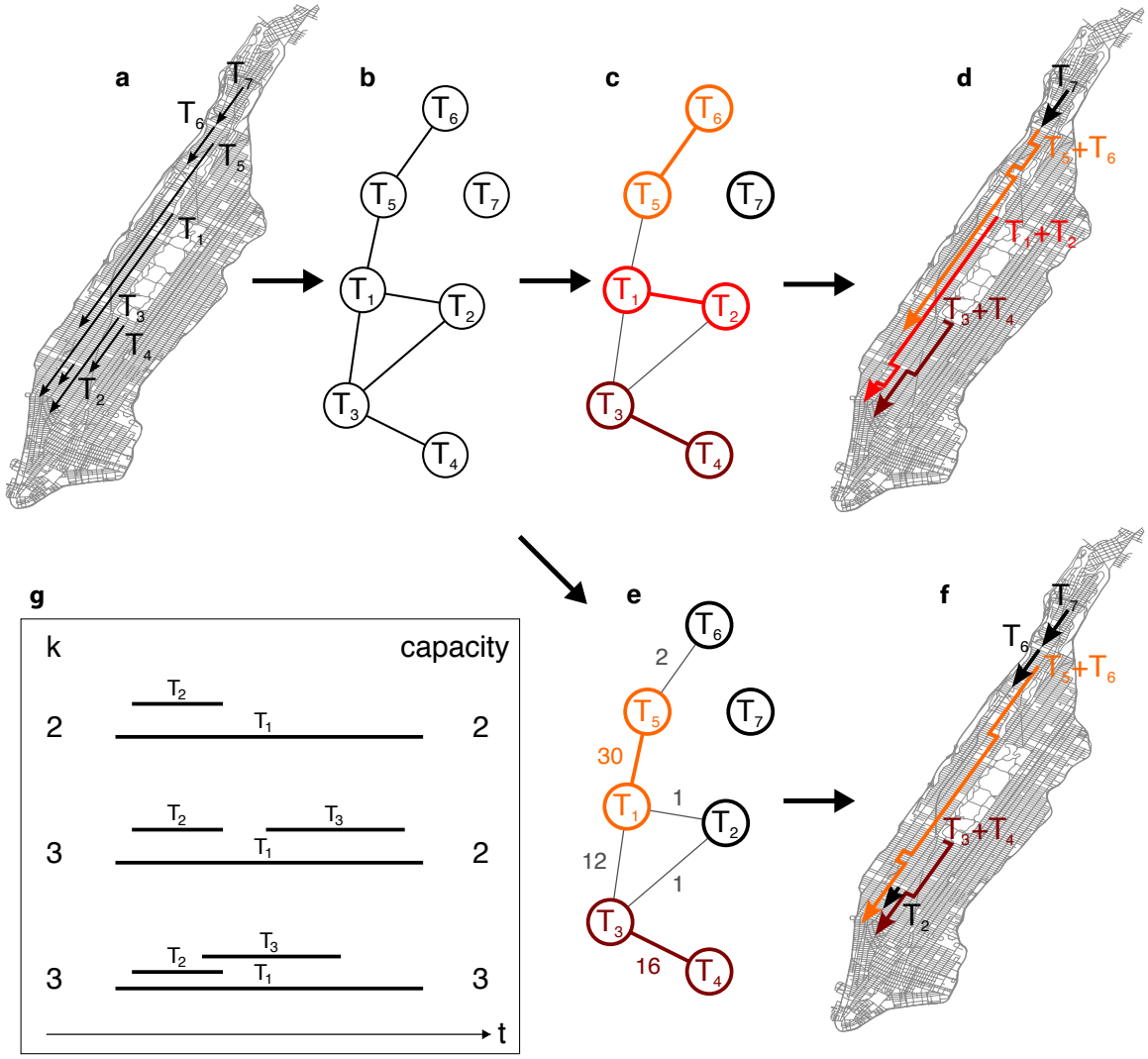
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Figure 1: Construction and application of shareability networks. **a**, Example of seven trips, $T_1, \ldots, T_7$, requested and to be shared in Manhattan, New York City. **b**, Construction of shareability network for $k = 2$. Trips that could potentially be shared are connected, given the necessary time constraints to hold which we assume here to be the case. Trips 1 and 4 cannot be shared because the total length of the best shared route would be longer than the sum of the single routes. Likewise, trip 7 is an isolated node because it cannot possibly be shared with other trips. **c**, Maximum matching of the shareability network gives the maximum number of trip pairs, i.e. the maximum number of shared trips. **d**, Implementation (routing) of the maximum matching solution. **e**, Alternatively, maximum weighted matching of the shareability network gives the solution with the minimal total travel time, which in this case leads to a different solution than unweighted maximum matching. Here, only two pairs of trips are shared, but the amount of travel time saved, given by the sum of link weights of the matching, $30+16$, is optimal. **f**, Implementation (routing) of the weighted maximum matching solution. **g**, k-sharing and taxi capacity. Each of the three cases involves a number of trips T_i to be shared, but ordered differently in time t. The top case corresponds to a feasible sharing according to our model with $k = 2$, and the trips can be accommodated in a taxi with capacity ≥ 2. The middle case corresponds to a model with $k = 3$ since three trips are combined, but the three trips can be combined in a taxi with capacity two since two of the trips are non-overlapping. The bottom case corresponds to $k = 3$, but here a taxi capacity ≥ 3 is needed to accommodate the combined trips.

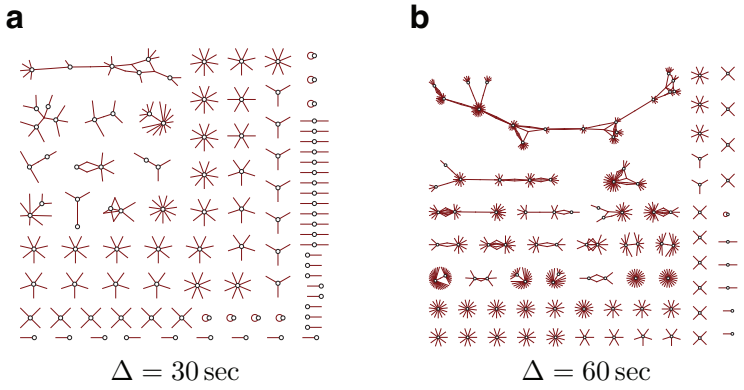

Figure 2: Influence of time constraints on shareability network. Exemplary subset of the shareability network corresponding to 100 consecutive trips for values of **a**, $\Delta = 30\,\text{sec}$ and **b**, $\Delta = 60\,\text{sec}$, showing network densification effects and thus an increase of sharing opportunities with longer time-aggregation. Open links point to trips outside the considered set of trips. Isolated nodes are represented as self-loops. Node positions are not preserved across the networks.

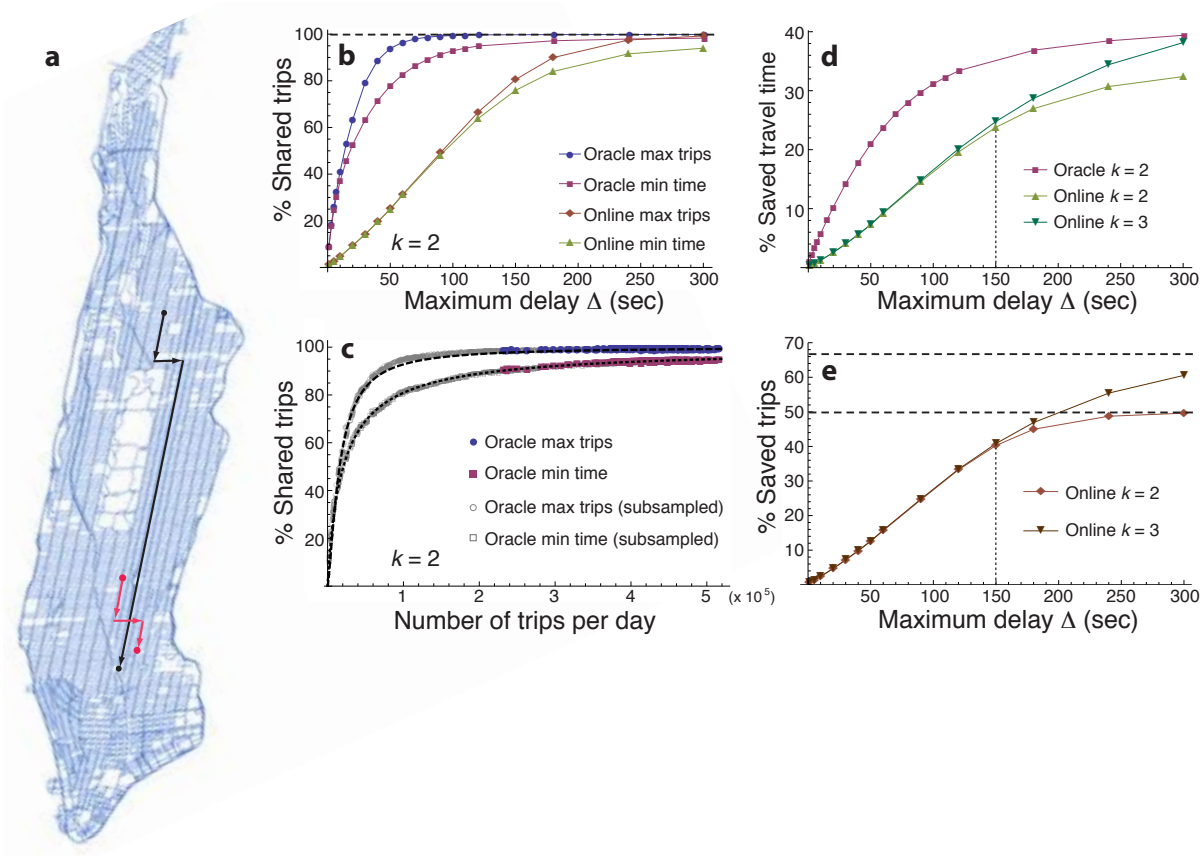

Figure 3: The benefits of trip sharing. **a**, Street network of Manhattan, and examples of trips that can be shared under the omniscient Oracle model, but not under the Online model. The starting time of the red trip is much later than that of the black trip, but in the Online model trip sharing decisions must be taken within a very short time window $\delta = 1\,\mathrm{min}$ to notify customers of trip sharing opportunities as soon as possible after their order. **b**, Percentage of shared trips as a function of the trip time delay Δ in the Oracle and in the Online model for the two considered optimisation criteria of maximising shared trips (max trips) and minimising total travel time (min time), when up to $k = 2$ trips can be shared. **c**, Shareability as a function of trips per day in the Oracle model. Typical days in New York City feature around 400,000 trips with near maximum shareability. Subsampling data by randomly removing vehicles reveals the underlying saturation curves, fit (dashed lines) by a simple function of type $f(x) = \frac{Kx^n}{1+Kx^n}$ with the two parameters K and n well-known in adsorption processes and biochemical systems (Supplementary Information). The fast, hyperbolic saturation implies that taxi sharing could be effective even in cities with vehicle densities much lower than New York, or when the willingness to share is low. **d**, Percentage of saved travel time as a function of Δ for $k = 2$ and $k = 3$. Although δ is reduced from practically infinite in the Oracle model to $\delta = 1\,\mathrm{min}$ in the Online model, saved travel time is well above 30% for $\Delta = 300\,\mathrm{sec}$, for $k = 3$ almost reaching the maximum possible value from the Oracle

model with $k = 2$. **e**, Percentage of saved travel time as a function of Δ with $k = 2$ and $k = 3$. The theoretically possible maximum (dashed lines) of 50% for $k = 2$ and 66.7% for $k = 3$ are closely approximated. For $\Delta < 150\,\mathrm{sec}$ (dotted line), the benefits of 3-sharing over 2-sharing are negligible.

Supplementary Information for

Taxi pooling in New York City: a network-based approach to social sharing problems

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In this supplementary information we present the detailed methods, including the handling of the data set, the formal derivation of the network-based approach used to quantify the benefits of a shared taxi system, and essential extended details of the analysis.

Data set and pre-processing

The data set contains origin-destination data of all 172 million trips with passengers of all 13,586 taxicabs in New York during the calendar year of 2011. Each vehicle is associated with a license, a so-called *medallion*, which is synonymously used as a name for the vehicles. These medallion taxis are the only vehicles in the city permitted to pick up passengers in response to a street hail. A medallion may be purchased from the City at infrequent auctions, or from another medallion owner. Because of their high prices medallions and most cabs are owned by investment companies and are leased to drivers. There are 39,437 unique driver IDs in the data set, which corresponds to 2.9 drivers per medallion on average. The data set contains a number of fields from which we use the following: medallion ID, origin time, destination time, origin longitude, origin latitude, destination longitude, destination latitude. Times are accurate to the second, positional information has been collected via Global Positioning System (GPS) technology by the data provider. Out of our control are possible biases due to urban canyons which might have slightly distorted the GPS locations during the collection process[25]. All IDs are given in anonymized form, origin and destination values refer to the origins and destinations of trips, respectively.

For creating the street network of Manhattan we used data from openstreetmap.org. We filtered the streets of Manhattan, selecting only the following road classes: primary, secondary, tertiary, residential, unclassified, road, living street. Several other classes were deliberately left out, such as footpaths, trunks, links or service roads, as they are unlikely to contain delivery or pickup locations. Next we extracted the street intersections to build a network in which nodes are intersections and directed links are roads connecting those intersections (we use directed

links because a non-negligible fraction of streets in Manhattan are one-way). The extracted network of street intersections was then manually cleaned for obvious inconsistencies or redundancies (such as duplicate intersection points at the same geographic positions), in the end containing 4091 nodes and 9452 directed links. This network was used to map-match the GPS locations from the trip data set. We only matched locations for which a closest node in the street intersection network exists with a distance less than than $100\,\mathrm{m}$. Finally, from the remaining 150 million trips we discarded about 2 million trips that had identical starting and end points, and trips that lasted less than one minute.

A network-based approach for sharing taxi rides

In contrast to typical approaches based on linear programs[5,26] and references therein, we show in this supplementary information in detail how our new approach allows polynomial-time, i.e., feasible, computation of the optimal ride sharing strategy when at most two trips can be combined, and polynomial-time computation of a constant-factor approximation of the optimal solution when $k > 2$ trips can be shared. Notice, though, that the degree of the involved polynomials increases with k. In practice, the approach turns out to be computationally feasible for $k = 3$, while it becomes impractical for larger values of k.

The goal of the trip sharing strategy can be either minimizing the number of trips performed or the total travel cost for a given set of trips, subject to a quality of service constraint (maximum allowed delay at delivery/destination). The former goal allows quantifying the actual number of taxis needed to satisfy the current taxi demand with a shared taxi service. By assuming that cost of a trip equals the travel time, the latter goal becomes a proxy of the carbon emissions generated by the shared taxi fleet to accommodate the total traffic demand; in fact, these emissions are roughly proportional to the total travel time. By comparing the total travel time of the shared taxi service with that of the traditional, non-shared taxi service, we can quantify the expected reduction in pollution achieved by a shared versus a traditional taxi service.

The high-level idea, elaborated rigorously in the following sections, is to cast the problem of identifying the best trip sharing strategy as a network problem, where nodes of the network represent taxi trips, and links connect trips that can be combined. The resulting network is called the *shareability network*. A maximum tolerated time delay Δ at both pickup and delivery location regulates the density of the shareability network – the higher Δ the more sharing opportunities arise but the lower the quality of service becomes due to the increased delays. We show that the problem of finding the optimal trip sharing strategy when at most two trips can be combined is equivalent to the problem of finding the maximum matching in the shareability network, which can be solved in time $O(m\sqrt{n})$, where n is the number of nodes and m the number of links in the network. Notice that the shareability network is likely sparse, i.e. the average node degree is a constant which does not depend on n, hence the above time complexity reduces to $O(n\sqrt{n})$. More specifically, the maximum matching in the shareability network corresponds to the trip combination strategy that minimizes the number of performed trips. If links in the trip graph are weighted with the travel cost reduction, i.e. the difference between the duration

of combined ride and the two single rides, then the problem of finding the trip combination that minimizes the total travel cost is equivalent to the problem of finding the maximum weighted matching in the shareability network, which is also solvable in polynomial time.

If we relax the assumption that at most two trips can be combined, the complexity of the problem increases. In fact, the problem(s) at hand becomes equivalent to the weighted matching problem on k-bounded hyper-networks, which is NP-complete when $k > 2$ in general hyper-networks. However, polynomial-time algorithms are known that compute a solution which is within a constant factor from optimal. In particular, when the number of combined trips is at most k, for any constant $k > 2$, simple greedy algorithms can be used to produce a solution within a factor k from optimal.

We first present the case in which at most two trips can be combined, and then proceed to present the more general (and complex) case of an arbitrary number of combined trips. To ease presentation, we assume single passenger trips.

The two-trips sharing case

In this section, we assume that at most two trips can be combined. Notice that this is a stricter condition than assuming that the maximum taxi capacity is two. The difference between the two assumptions is exemplified in Fig. 1g of the main text. We have three trips T_1, T_2, and T_3. Assuming that delay constraints on passenger delivery are satisfied, the three trips can be combined in a single trip even using a taxi with capacity two if the passenger of T_2 is loaded onboard taxi performing T_1 at time t_2, unloaded at time $t_2^{'} > t_2$, and the passenger of trip T_3 is loaded at time $t_3 > t_2^{'}$ – cfr. the middle case in Fig. 1g of the main text. This combination of trips is not allowed in our model, since only two trips at most can be combined. Notice, on the other hand, that any shared trip obtained by combining at most two single trips can be realized using a taxi with capacity two. More generally, any k combination of trips can be performed using a taxi with capacity k. Hence, the trip combination solutions presented in the following can be accomplished using a taxi fleet where each taxi has capacity k, where k is the upper bound on the number of trips combined in a single trip.

Let $S = (T, L)$ be the (undirected) *shareability network* defined as follows. The node set $T = \{T_1, \ldots, T_n\}$ corresponds to the set of all possible n trips. The link set $L = \{L_1, \ldots, L_m\}$ is built as follows: link $(T_i, T_j) \in L$ connects nodes T_i and T_j if and only if trips T_i and T_j can be combined. The trip obtained combining trips T_i and T_j is denoted $T_{i,j}$ in the following. Whether trips T_i, T_j can be combined depends on spatial/temporal properties of the two trips, and on an upper bound Δ on the maximum delivery delay customers can tolerate. How to derive the set of combinable trips (link set L) given a travel data set such as the one at hand is a problem we defer to the next section. In the remainder of this section, we assume that L is known and given as input to the problem.

Definition 1. *A set $\mathcal{T}$ of* possibly combined trips*, where combined trips are composed of two single trips, is defined as $\mathcal{T} = \mathcal{T}_1 \cup \mathcal{T}_2$, where $\mathcal{T}_1 \subseteq T$ is a set of single trips, and $\mathcal{T}_2$ is a set of combined trips $T_{i,j}$, for some $i, j \in \{1, \ldots, n\}$.*

Definition 2 (Feasible trip set)**.** *A set $\mathcal{T}$ of possibly combined trips is* feasible *if and only if all trips in T appear once in $\mathcal{T}$, formally:*

$$\forall T_i \in T, \quad (T_i \in \mathcal{T}_1) \vee (\exists_1 j \text{ s.t. } T_{i,j} \in \mathcal{T}_2) \; .$$

Notice that any trip $T_i \in T$ can appear only once in a feasible trip set, either as a single trip (if $T_i \in \mathcal{T}_1$), or combined with another trip (if $T_i \in \mathcal{T}_2$). The two travel optimization problems we solve in the following are formally defined as follows:

Definition 3 (MINIMUMNUMBERTRIP – MNT)**.** *Given the shareability network $S = (T, L)$, determine a feasible trip set of minimum cardinality.*

Definition 4 (MINIMIZETOTALTRAVELCOST – MTTC)**.** *Given the weighted shareability network $S = (T, L)$ where each link $(T_i, T_j) \in L$ is weighted with $w_{ij} = c(T_i) + c(T_j) - c(T_{i,j})$, where $c(T_x)$ denotes the cost of trip T_x, and $c(T_{i,j})$ the cost of the combined trip $T_{i,j}$; determine a feasible trip set such that the total travel cost is minimized.*

Regarding problem MTTC, we observe that we can assume w.l.o.g. that $c(T_{i,j}) < c(T_i) + c(T_j)$, since otherwise we can remove link (T_i, T_j) from L without impacting the optimal solution.

Definition 5. (MATCHINGS) *Let $S = (T, L)$ be a shareability network. A matching on S is a set of links $L' \subseteq L$ such that no two links in L' share an endpoint. A maximum matching on S is a matching on S of maximum cardinality. If links in S are weighted, a maximum weighted matching on S is a matching L' on S such that the sum of weights of links in L' is maximum.*

Lemma 1. *A set $\mathcal{T}$ of possibly combined trips is feasible if and only if its subset $\mathcal{T}_2$ of combined trips is a matching of S.*

Proof. Let $\mathcal{T}$ be any feasible set of combined trips. Since $\mathcal{T}$ is feasible, it contains either single trips, or trips obtained by combining two trips. Let $\mathcal{T}_2 \subseteq \mathcal{T}$ be the set of combined trips in $\mathcal{T}$. For any combined trip $T_{i,j} \in \mathcal{T}_2$, we consider the corresponding link (T_i, T_j) in the shareability network. Notice that, for any other link of the form (T_i, T_h) (or (T_h, T_j)) in the shareability network, the corresponding combined trip $T_{i,h}$ (or $T_{h,j}$) is not in $\mathcal{T}_2$, since otherwise conditions on maximal trip combination would be violated. It follows that no two links corresponding to the combined trips in $\mathcal{T}_2$ share a node, i.e., the links corresponding to trips in $\mathcal{T}_2$ are a matching on S.
The proof of the reverse implication is similar. Any matching M of S uniquely determines a set of combinable trips $\mathcal{T}_2$. A feasible set $\mathcal{T}$ of possibly combined trips is then obtained from $\mathcal{T}_2$ by adding as single trips all trips whose corresponding nodes in S are not part of the matching M. □

Theorem 1. *Let $M_{\max} \subseteq L$ be a maximum matching on S. Then, the minimum cardinality of a feasible set of possibly combined trips is $n - |M_{\max}|$.*

Proof. By Lemma 1, the cardinality of any feasible set $\mathcal{T}$ of possibly combined trips is $n-|M|$, where $|M|$ is the cardinality of the matching M corresponding to the subset $\mathcal{T}_2$ of combined trips in $\mathcal{T}$. The proof then follows by observing that the minimum cardinality of a feasible set is obtained when $|M|$ is maximum, i.e., when M is a maximum matching for S. □

The proof of Theorem 1 suggests a polynomial time algorithm for solving MNT, which is reported below. The feasible set $\mathcal{T}$ of trips to be performed is initialized to the entire set of single trips T. Given the shareability network S, a maximum matching $M_{\max}$ on S is computed using, e.g., Edmond's algorithm[27].For any edge $(T_i, T_j) \in M_{\max}$, the combined trip $T_{i,j}$ is included in $\mathcal{T}$, while the individual trips T_i and T_j are removed from $\mathcal{T}$. After all edges in $M_{\max}$ have been considered and processed as above, $\mathcal{T}$ contains a set of (possibly combined) trips of minimum size that satisfies all customers, i.e., it is a feasible trip set of minimum size. The time complexity of MAXMATCH is determined by the complexity of the matching algorithm. Considering that the shareability network is likely to be very sparse in practice, the Edmond's matching algorithm yields $O(n\sqrt{n})$ time complexity.

Algorithm MAXMATCH
Input: the shareability network $S = (T, L)$
Output: the set $\mathcal{T}$ of (possibly combined) trips to be performed
1. $\mathcal{T} = T$
2. Build a maximum matching $M_{\max}$ on S
3. **for each** $(T_i, T_j) \in M_{\max}$ **do**
4. $\quad \mathcal{T} = \mathcal{T} \cup \{T_{i,j}\}; \mathcal{T} = \mathcal{T} - \{T_i, T_j\}$
5 **return** $\mathcal{T}$

Algorithm MAXMATCH for optimally solving MNT.

Theorem 2. *Let $M_{\max_w} \subseteq L$ be a maximum weighted matching on S, where S is link weighted as described in Definition 4. Then, the feasible set of possibly combined trips of minimum total travel cost has cost*

$$c_{\min} = \sum_{i=1,\dots,n} c(T_i) - \sum_{(T_i,T_j)\in M_{\max_w}} c(T_i) + c(T_j) - c(T_{i,j}) \; .$$

Proof. By Lemma 1, the subset $\mathcal{T}_2$ of combined trips of any feasible trip set $\mathcal{T}$ corresponds to a matching M on S. For any edge $(T_i, T_j) \in M$, the travel cost reduction due to the combined trip $T_{i,j}$ with respect to the cost of the two single trips T_i, T_j is $c(T_i) + c(T_j) - c(T_{i,j})$. Thus, the total travel cost for any feasible trip set $\mathcal{T}$ is given by the total travel cost of the single trips ($\sum_{i=1,\dots,n} c(T_i)$), minus the sum of the cost savings achieved by the combined trips in M, i.e., $\sum_{(T_i,T_j)\in M} c(T_i) + c(T_j) - c(T_{i,j}) = \sum_{(T_i,T_j)\in M} w_{ij}$. The proof then follows by observing that, if $M_{\max_w}$ is a maximum weighted matching on S, then the sum of the cost savings is maximized, and the total travel cost of $\mathcal{T}$ is minimized. □

The algorithm WEIGHTEDMAXMATCH to find the feasible trip set of minimum travel cost is similar to MAXMATCH, the only difference being that the maximum matching algorithm in step 2 is replaced with a maximum weighted matching algorithm. For instance, we can use Edmond's algorithm for weighted matching[25], which on a sparse graph yields time complexity $O(n^2 \log n)$.

The k-trips sharing case

In this section, we generalize the results presented in the previous section to the more challenging scenario in which an arbitrary number $k > 2$ of trips can be combined. The only assumption we make about the value of k in this section is that k does not depend on the total number of trips n, i.e., $k = O(1)$ in asymptotic notation. Considering that k is also an upper bound on taxi capacity needed to accommodate the k combined trips, assuming k a small constant is reasonable in any practical case.

We first present how a combination of up to k trips can be represented in form of a k-bounded shareability hyper-network. Some definitions are in order before proceeding further.

Definition 6. *A set $\mathcal{T}$ of* possibly combined trips*, where combined trips are composed of at most $k \geq 2$ single trips, is defined as $\mathcal{T} = \mathcal{T}_1 \cup \cdots \cup \mathcal{T}_k$, where $\mathcal{T}_1 \subseteq T$ is a set of single trips, and $\mathcal{T}_h$, with $2 \leq h \leq k$, is a set of combined trips $T_{i_1,\ldots,i_h}$, for some $i_1, \ldots, i_h \in \{1, \ldots, n\}$.*

Definition 7 (Feasible trip set)**.** *A set $\mathcal{T}$ of possibly combined trips is* feasible *if and only if all trips in T appear once in $\mathcal{T}$, formally:*

$$\forall T_j \in T, \quad (T_j \in \mathcal{T}_1) \vee (\exists_1 h, \ell \text{ s.t. } (T_{i_1,\ldots,i_h} \in \mathcal{T}_h) \wedge (i_\ell = j)) \ .$$

Notice that also in this case, any trip $T_i \in T$ can appear only once in a feasible trip set, either as a single trip, or in a combined trip.

Definition 8. *A* hyper-network *H is a pair $H = (T, \mathcal{L})$ where $T = (T_1, \ldots, T_n)$ is a set of nodes (representing single trips in the context at hand), and $\mathcal{L}$ is a set of non-empty subsets of T called hyper-links. The* size *of a hyper-link is the number of nodes it connects. A hyper-network whose hyper-links have size $\leq k$, for some integer $k \geq 2$, is called a k-*bounded hyper-network*.*

Definition 9. *A (hyper-)matching M on the hyper-network $H = (T, \mathcal{L})$ is a subset of the hyper-links in $\mathcal{L}$ such that each node in T appears in at most one hyper-link.*

Similarly to the previous section, given a set of trips T, we can represent all possible combinations of up to k trips – defined according to some quality of service criterion – with a k-bounded hyper-network, which we call the *shareability hyper-network*. Formally, the trip hyper-newtork is defined as the k-bounded hyper-network $H = (T, \mathcal{L})$, where $L_i = (T_{i_1}, T_{i_2}, \ldots) \in \mathcal{L}$ if and only if trips $T_{i_1}, T_{i_2}, \ldots$ can be combined. Notice that, if hyper-link $L_i = (T_{i_1}, T_{i_2}, \ldots)$ belongs to the shareability hyper-network, so do all hyper-links formed of any subset of the nodes connected by L_i. This is due to the fact that, if, say, trip $T_{1,2,3,4}$ is feasible, so do trips

$T_{1,2}$, $T_{1,2,3}$, etc. We call a hyper-link in H *maximal* if its incident nodes are not a subset of any other hyper-link in H.

We are now ready to formally define the two considered optimization problems.

Definition 10 (k-MINIMUMNUMBERTRIP – kMNT)**.** *Given the shareability hyper-network $H = (T, \mathcal{L})$, determine a feasible trip set $\mathcal{T}$ of minimum cardinality.*

Definition 11 (k-MINIMIZETOTALTRAVELCOST – kMTTC)**.** *Given the weighted shareability hyper-network $H = (T, \mathcal{L})$ where each link $L_i = (T_{i_1}, T_{i_2}, \dots) \in \mathcal{L}$ is weighted with $w_i^c = \sum_{i_j \in L_i} c(T_{i_j}) - c(T_{i_1,i_2,\dots})$, where $c(T_{i_j})$ denotes the cost of trip T_{i_j}, and $c(T_{i_1,i_2,\dots})$ the cost of the combined trip $T_{i_1,i_2,\dots}$; determine a feasible trip set $\mathcal{T}$ such that the total travel cost is minimized.*

Lemma 2. *A set $\mathcal{T}$ of possibly combined trips is feasible if and only if its subset $\mathcal{T}_c = \mathcal{T} - \mathcal{T}_1$ of combined trips is a (hyper-)matching of H.*

Proof. The proof is along the same lines of the proof of Lemma 1. □

Theorem 3. *Let $H = (T, \mathcal{L})$ be a shareability hyper-network, and assign weight $w_i = |L_i| - 1$ to each hyper-link $L_i \in \mathcal{L}$. Then, the minimum cardinality of a feasible set of possibly combined trips is $n - \sum_{L_i \in M_{\max_w}} w_i$, where $M_{\max_w}$ is a maximum weighted matching of H.*

Proof. By Lemma 2, any feasible trip set $\mathcal{T}$ uniquely defines a matching M in the shareability hyper-network H. Consider any hyper-link L_i in the matching M. By definition, w_i represents the number of trips that are saved by performing the combined trip corresponding to hyper-link L_i instead of performing all single trips in L_i. For instance, if $L_i = \{T_{i_1}, \dots, T_{i_k}\}$, the combination of k trips allows reducing the number of performed trips from k to 1; i.e., the total number of trips is reduced of $w_i = k - 1$. Based on this observation, the total number of trips performed for feasible trip set $\mathcal{T}$ equals $n - \sum_{L_i \in M} w_i$. The proof then follows by observing that the total number of trips is minimized when $\sum_{L_i \in M} w_i$ is maximized, i.e., when M is a maximum weighted matching for H. □

Unfortunately, the maximum (weighted) matching problem on k-bounded hyper-networks is NP-complete for $k > 2$ on general hyper-networks, hence finding the optimal solution to kMNT is likely computationally hard. However, a simple greedy heuristic can be used to find a k-approximation of the optimal solution in time $O(m \log m)$, where m is the number of hyper-links in the hyper-network, which yields $O(n \log n)$ complexity under our working assumption of sparse shareability hyper-network. In the greedy heuristic, a hyper-link L_i of maximum weight is added to the current matching at each iteration, and hyper-links sharing at least one node with L_i are removed from the set of candidate hyper-links for matching before proceeding to the next iteration. Observe that better approximation ratios can be obtained at the price of increased (but still polynomial) time complexity using, for instance, the algorithm[23]which finds a $2(k+1)/3$ approximation of the optimal solution. Note that the weighted maximum matching problem on hyper-networks is equivalent to the weighted set packing problem. The greedy algorithm for finding a k-approximation to the optimal kMNT solution is reported below.

Algorithm GREEDYKMATCHING
Input: the shareability hyper-network $H = (T, \mathcal{L})$ with weights w_i on hyper-links
Output: the set $\mathcal{T}$ of (possibly combined) trips to be performed
1. $\mathcal{T} = T$
2. Build a weighted matching M_w of H using the greedy heuristic
3. **for each** $L_i = (T_{i_1}, \dots, T_{i_j}) \in M_w$ **do**
4. $\quad \mathcal{T} = \mathcal{T} \cup \{T_{i_1,\dots,i_j}\}; \mathcal{T} = \mathcal{T} - \{T_{i_1}\} - \dots - \{T_{i_j}\}$
5 **return** $\mathcal{T}$

Algorithm GREEDYKMATCHING for finding a k-approximation to kMNT.

Theorem 4. *Let $H = (T, \mathcal{L})$ be the shareability hyper-network, where each $L_i = (T_{i_1}, T_{i_2}, \dots) \in \mathcal{L}$ is weighted with the weight $w_i^c = \sum_{i_j \in L_i} c(T_{i_j}) - c(T_{i_1,i_2,\dots})$ representing the cost saving in performing the combined trip versus the collection of single trips. Then, the feasible set of possibly combined trips of minimum total travel cost has cost*

$$c_{\min} = \sum_{i=1,\dots,n} c(T_i) - \sum_{L_i \in M_{\max_w}} w_i^c \,,$$

where $M_{\max_w}$ is a maximum weighted matching of H.

Proof. The proof is along the same lines of the proof of Theorem 3. □

The greedy heuristic for computing a k approximation of the optimal solution to kMTTC can be straightforwardly obtained from Algorithm GREEDYKMATCHING by using weights w_i^c instead of w_i to label hyper-links in the shareability hyper-network.

Building the shareability network

In this section, we describe a method for producing the shareability (hyper-)network, given a set of single trips $T = \{T_1, \dots, T_n\}$ and a quality of service criterion Δ. We present in detail the method for $k = 2$, and shortly describe how the technique can be generalized to arbitrary values of k.

Each trip $T_i \in T$ is characterized by the following quantities: the trip origin o_i and destination d_i, that we can think of as pairs of (lat, lon) coordinates; the start time st_i; and the arrival time at_i. We start defining a notion of feasible trip combination based on a quality of service criterion Δ.

Definition 12. *The combined trip $T_{i,j}$ is* feasible *if and only if a trip route can be found such that the following conditions are satisfied:*

a) $st_i \leq pt_i \leq st_i + \Delta$*;*

b) $st_j \leq pt_j \leq st_j + \Delta$*;*

c) $dt_i \leq at_i + \Delta$*;*

d) $dt_j \leq at_j + \Delta$*;*

where pt_x *is the pickup time at* o_x *in the combined trip, and* dt_x *is the delivery time at* d_x *in the combined trip.*

The above definition is motivated by the fact that a customer might be willing to wait at most some extra time Δ at her pickup location (and in general she might not be able to show up at o_i before time st_i), as well as to arrive at destination with delay at most Δ (early arrivals are likely not to be a problem for customers).

Theorem 5. *Building the shareability network* $S = (T, L)$ *starting from the trip set* $T = \{T_1, \ldots, T_n\}$ *requires* $O(n^2)$ *time.*

Proof. In the worst-case, we have to consider all $O(n^2)$ possible pairs of trips T_i, T_j. For each pair, the feasibility condition for the combined trip $T_{i,j}$ can be verified in $O(1)$ as follows. Observe that only four routes are possible for trip $T_{i,j}$: $o_i \rightarrow o_j \rightarrow d_i \rightarrow d_j$, $o_i \rightarrow o_j \rightarrow d_j \rightarrow d_i$, $o_j \rightarrow o_i \rightarrow d_i \rightarrow d_j$, and $o_j \rightarrow o_i \rightarrow d_j \rightarrow d_i$. Let us consider a specific route, e.g., $o_i \rightarrow o_j \rightarrow d_i \rightarrow d_j$. Condition a) for feasibility is always satisfied by setting a pickup time at o_i in the desired time window. The pickup time at o_j can then be computed as follows: $pt_j = pt_i + tt(o_i, o_j)$, where $tt(x, y)$ denotes the travel time between x and y. The delivery time at d_i is defined as follows: $dt_i = pt_j + tt(o_j, d_i)$. Finally, the delivery time at d_j is defined as $dt_j = dt_i + tt(d_i, d_j)$. Thus, the feasibility condition for route $o_i \rightarrow o_j \rightarrow d_i \rightarrow d_j$ can be verified by checking whether a value of pt_i that simultaneously satisfies the four conditions below exists, which requires $O(1)$ time:

$$st_i \leq pt_i \leq st_i + \Delta \quad \text{(S1)}$$

$$st_j \leq pt_i + tt(o_i, o_j) \leq st_j + \Delta \quad \text{(S2)}$$

$$pt_i + tt(o_i, o_j) + tt(o_j, d_i) \leq at_i + \Delta \quad \text{(S3)}$$

$$pt_i + tt(o_i, o_j) + tt(o_j, d_i) + tt(d_i, d_j) \leq at_j + \Delta \quad \text{(S4)}$$

The feasibility conditions for the other routes can be verified similarly. If there exists at least one route which satisfies the feasibility condition, then trip $T_{i,j}$ is feasible, and link (T_i, T_j) is included in the trip graph. Otherwise, trips T_i and T_j cannot be combined.
Observe that the number of trip pairs to consider for combination can be reduced by considering only trip pairs T_i, T_j such that: i) $st_j \leq at_i + \Delta$; and ii) $st_i \leq at_j + \Delta$. Simultaneously satisfying conditions i) and ii) is a necessary (but not sufficient) condition for feasibility of trip $T_{i,j}$. In practice, this heuristic considerably reduces the running time of the shareability network construction algorithm, although the worst-case time complexity remains $O(n^2)$. □

The algorithm for building the trip graph reported in the proof of Theorem 5 can be extended in a straightforward way to the case of combinations of up to k trips, yielding a time complexity of $O(n^k)$; in particular, all possible routes connecting k origins with k destinations, subject to the condition that each origin must precede the respective destination in the route, must be considered. The number of possible such routes grows exponentially with k, which is however assumed to be a small constant in our model. For instance, 60 possible routes connecting origins with destinations must be considered when $k = 3$.

Computing travel times

Knowledge of estimated travel times between arbitrary origin/destination in the road map is a pre-requisite for checking the trip sharing conditions, and, hence, to build the shareability network. Since we do not have access to such detailed information for the city of New York, we designed a travel time estimation heuristic starting from the data set of taxi trips in New York City.

Given is a set of actually performed trips $\mathcal{T} = \{T_1, \ldots, T_k\}$, where each trip $T_i = (o_i, d_i, tt_i)$ is defined by an origin location o_i, a destination location d_i, and a travel time tt_i. While in the original data set origin and destination of a trip are defined as raw GPS (lat, lon) coordinates, in the following we assume origin and destination of a trip are taken from the set $\mathcal{I}$ of street intersections in the road map. To convert raw GPS coordinates into an intersection in $\mathcal{I}$, we associate o_i (or d_i) to the closest intersection based on geodesic distance, subject to the condition that the distance to the closest intersection is below a threshold such as twice the average GPS accuracy, set to $100\,\mathrm{m}$. Thus, in the following we assume o_i and d_i are indeed distinct elements of the set $\mathcal{I}$ of possible intersections in the road map, i.e., $\forall T_i \in \mathcal{T}, o_i, d_i \in \mathcal{I}$. We also define the set $\mathcal{S} = \{S_1, \ldots, S_h\}$ of *streets* as the set of all road segments connecting two adjacent intersections in the road map.

Given the trip set $\mathcal{T}$ as defined above, the problem to solve is estimating the travel time x_i for each street $S_i \in \mathcal{S}$, in such a way that the average relative error (computed across all trips) between the actual travel time tt_i and the estimated travel time et_i for trip T_i computed starting from the x_is (compound with a routing algorithm) is minimized. Once error minimizing travel times for each street in $\mathcal{S}$ are determined, the travel time between any two intersections $I_i, I_j \in \mathcal{I}$ can be computed starting from the x_is, using a routing algorithm that minimizes the travel time between any two intersections. Besides the trip set $\mathcal{T}$, we are also given the array $Le = (l_i)$ of the lengths of the streets in $\mathcal{S}$.

In the following, we define the problem at hand more formally. First, we partition the trip set in time sliced subsets $\mathcal{T}_1, \ldots, \mathcal{T}_{24}$, where subset $\mathcal{T}_i$ contains all trips whose starting time is in hour i of the day. Finer partitioning (e.g., per hour and weekday, per hour and weekday and month, etc.) is possible, if needed. In the following, to simplify notation, we re-define $\mathcal{T}$ as any of the time-sliced subsets $\mathcal{T}_i$. In fact, the travel time estimation process can be performed independently on each of the time-sliced trip subsets. When a time-sliced trip set $\mathcal{T}$ is considered, classes $\mathcal{T}^1, \ldots, \mathcal{T}^h$ of *equivalent trips* are formed, where two trips

T_u, T_v are said to be equivalent if and only if $(o_u = o_v) \wedge (d_u = d_v)$. Notice that, under the assumption that the routing algorithm is deterministic (i.e., it always computes the same route given the same starting and ending intersections I_i and I_j), the set of streets in the optimal route from origin to destination is the same for any two trips $T_u, T_v \in \mathcal{T}^{i,j}$, where $\mathcal{T}^{i,j}$ is the class of trips with origin I_i and destination I_j. Thus, all the trips in equivalence class $\mathcal{T}^{i,j}$ can be considered as multiple samples of the travel time on the same set of streets. All trips in $\mathcal{T}^{i,j}$ are then replaced by a single trip $T_{i,j}$ with corresponding origin and destination, and travel time $\bar{tt}_{i,j}$ equal to the average of the travel times of all trips in $\mathcal{T}^{i,j}$. After this step is performed for all equivalence classes, we are left with an aggregate set $\mathcal{T}_{\text{agg}}$ of singleton, non-equivalent trips $T_{i,j}$, and corresponding travel times $\bar{tt}_{i,j}$.

The travel time estimation heuristic is reported below. Initially, trips are filtered to remove "loop" trips (i.e., trips with the same origin and destination), as well as excessively "short" or "long" trips. After a step in which initial routes are computed using a pre-selected initial speed v_{init} (the same for all streets), a second trip filtering step is performed, in which excessively "fast" and "slow" trips are removed from the travel time estimation process. The rationale for this filtering is removing "noisy" data which could have been resulted from very specific conditions (say, a snowstorm could have caused many slow trips). Including "noisy" data in the travel time estimation process would bias the estimation process to partially compensate for "noisy" trips, increasing the error experienced in the remaining portion of trips.

An iterative process is started after the second trip filtering step. The iterative process is composed of two nested iterations. In the outer iteration, new routes for the trips are computed based on the updated travel time estimation of street segments. After routes are computed, new trip travel time estimations are determined, and the average relative error across all trips is computed. Furthermore, an offset value is computed for each street segment, indicating whether travel times of all trips in which a street segment is included are under- or over-estimated. Then, an inner loop is started, with the purpose of refining street travel time estimations based on the computed offset values: an increase/decrease step $\mathtt{k}$ is initialized, and used to tentatively change street travel time estimates based on the offset value (tentative updated trip travel times are accepted only if the resulting average speed v on the trip is such that $0.5\,\text{m/sec} < v < 30\,\text{m/sec}$). The tentative estimations are accepted if the newly computed average relative error is decreased with respect to the current value. Otherwise, another iteration of the inner loop is started with a smaller value of $\mathtt{k}$. This process is repeated until either the street travel time estimations are updated, or the value of $\mathtt{k}$ has reached a specified minimum value. The outer iterative process terminates when there is no updated street travel time estimation after the execution of the inner loop.

After the iterative process, the algorithm has produced a travel time estimation for each street included in at least one optimal route for trips (set $\mathcal{S}_{\text{trip}}$). The travel time for the remaining streets is then computed according to a simple heuristic: the travel time for each street having an intersection in common with at least one street in $\mathcal{S}_{\text{trip}}$ is estimated based on the average speed estimated in neighboring streets. This process is repeated until the travel time on all streets can be estimated. Finally, at step 7 the travel time between any two possible intersections I_i, I_j

in the street map is computed by first computing the optimal route between I_i and I_j using Dijkstra algorithm with the estimated trip travel times, and then computing the travel time by summing up the travel time of the streets in the optimal route. Notice that we use the Dijkstra algorithm[21](repeated $|\mathcal{I}|^2$ times) to compute all-to-all shortest paths instead of the classical Floyd-Warshall algorithm since the graph corresponding to the street network is very sparse. Thus, repeating $|\mathcal{I}|^2$ times the Dijkstra algorithm yields $O(|\mathcal{I}|^2 \log |\mathcal{I}|)$, which is lower than the $O(|\mathcal{I}|^3)$ complexity of Floyd-Warshall.

The travel time estimation algorithm has been executed on the set of about 150 millions trips performed in New York City during weekdays, in year 2011. The performance of the travel time estimation algorithms for the 24 trip classes (corresponding to time of day) is summarized in Extended Data Table 1. The table reports the average relative error computed on all trips retained after the filtering steps, the percentage of trips retained in the data set after filtering, and the number of streets included in at least one optimal route. As seen from the table, the algorithm provides travel time estimations incurring an average relative error of 15%. The vast majority of trips is retained in the data set after filtering (more than 97% on the average). Furthermore, the vast majority of street segments are included in at least one optimal route: considering that the total number of (directed) street segments in Manhattan is 9452, on average 91.7% of the streets are included in at least one optimal route. For the remaining streets, step 6 of the algorithm is used to estimate street travel time.

To study the travel speeds estimated by our algorithm we calculated travel speeds across different times of the day. The travel time estimations are reasonable, with a relatively lower average speed of around $5.5\,\mathrm{m/sec}$ estimated during rush hours (between 8am and 3pm), and peaks around $8.5\,\mathrm{m/sec}$ at midnight. Further evidence for a reasonable estimation is highlighted also by Extended Data Fig. 1, reporting the estimated travel speed on each street segment at four different times of day: 0am, 8am, 4pm, and 22pm. As expected, travel speeds tend to reduce during daytime. Also, the algorithm is able to faithfully model the higher speed on highways (on the left-hand side of Manhattan).

Algorithm for travel time estimation

Input: the (sub)set $\mathcal{T}$ of performed trips;the set $\mathcal{I}$ of intersections;
the set $\mathcal{S}$ of streets; the vector Le of lengths for streets in $\mathcal{S}$

Output: a travel time estimation matrix $ET(i,j)$, where $et_{i,j}$ is the estimated time for traveling from intersection I_i to intersection I_j

1. **Equivalent trip reduction**
   - group in class $\mathcal{T}^{i,j}$ all trips T_u such that $o_u = I_i$ and $d_u = I_j$
   - for each class $\mathcal{T}^{i,j}$, replace all trips in $\mathcal{T}^{i,j}$ with a single trip $T_{i,j}$ with $o_{i,j} = I_i, d_{i,j} = I_j$, and $tt_{i,j} = \bar{tt}_{i,j} = \frac{\sum_{T_u \in \mathcal{T}^{i,j}} tt_u}{|\mathcal{T}^{i,j}|}$
   - let $\mathcal{T}_{\mathrm{agg}}$ be the collection of trips $T_{i,j}$
2. **First trip filtering**
   - for each $T_{i,j} \in \mathcal{T}_{\mathrm{agg}}$, remove $T_{i,j}$ from $\mathcal{T}_{\mathrm{agg}}$ if $i = j$ *//remove "loop" trips*
   - for each $T_{i,j} \in \mathcal{T}_{\mathrm{agg}}$, remove $T_{i,j}$ from $\mathcal{T}_{\mathrm{agg}}$ if $(\bar{tt}_{i,j} < 2min)$ or $(\bar{tt}_{i,j} > 1h)$ *//remove "short" and "long" trips*
3. **Initial route computation**
   - for each $S \in \mathcal{S}$, set same initial speed $v_S = v_{\mathrm{init}}$; set travel time to $t_S = \frac{L(S)}{v_S}$
   - for each $T_{i,j} \in \mathcal{T}_{\mathrm{agg}}$, compute optimal route $I_i \rightarrow I_j$ using Dijkstra algorithm
   - for each $T_{i,j} \in \mathcal{T}_{\mathrm{agg}}$, let $\mathcal{S}^{i,j} = \{S_1^{i,j}, \ldots, S_k^{i,j}\}$ be the set of streets in the optimal route for $T_{i,j}$
4. **Second trip filtering**
   - for each $T_{i,j} \in \mathcal{T}_{\mathrm{agg}}$, compute the average speed $as_{i,j} = \frac{\sum_h L(S_h^{i,j})}{tt_{i,j}}$
   - for each $T_{i,j} \in \mathcal{T}_{\mathrm{agg}}$, remove $T_{i,j}$ from $\mathcal{T}_{\mathrm{agg}}$ if $(as_{i,j} < 0.5\,\mathrm{m/sec})$ or $(as_{i,j} > 30\,\mathrm{m/sec})$ *//remove "slow" and "fast" trips*
5. **Iterative steps**
   5.1 set `again`=**true**
   5.2 **while** `again` **do**
   - `again`=**false**
   - for each $T_{i,j} \in \mathcal{T}_{\mathrm{agg}}$, compute optimal route $I_i \rightarrow I_j$ using Dijkstra algorithm
   - for each $T_{i,j} \in \mathcal{T}_{\mathrm{agg}}$, let $\mathcal{S}^{i,j} = \{S_1^{i,j}, \ldots, S_k^{i,j}\}$ be the set of streets in the optimal route for $T_{i,j}$
   - for each $T_{i,j} \in \mathcal{T}_{\mathrm{agg}}$, compute $et_{i,j} = \sum_{S \in \mathcal{S}^{i,j}} t_S$ *//trip travel time estimation*
   - let $\mathcal{S}_{\mathrm{trip}} = \bigcup_{\mathcal{T}^{i,j} \in \mathcal{T}_{\mathrm{agg}}} \mathcal{S}^{i,j}$
   - `RelErr`=$\sum_{T_{i,j} \in \mathcal{T}_{\mathrm{agg}}} \frac{|et_{i,j} - \bar{tt}_{i,j}|}{\bar{tt}_{i,j}}$
   - for each $S \in \mathcal{S}_{\mathrm{trip}}$, let $T_S = \{T_{i,j} \in \mathcal{T}_{\mathrm{agg}} | S \in \mathcal{S}^{i,j}\}$ *//set of trips including S in the current route*
   - for each $S \in \mathcal{S}_{\mathrm{trip}}$, compute $O_S = \sum_{T_{i,j} \in T_S} (et_{i,j} - \bar{tt}_{i,j}) \cdot |\mathcal{T}_{i,j}|$ *//offset computation*
   - `k`=1.2
   5.3 **while true do**
   - for each $S \in \mathcal{S}_{\mathrm{trip}}$, do the following
   - if $O_S < 0$, then $t_S = k \cdot t_S$; else $t_S = \frac{t_S}{k}$ *//street travel time estimate is increased/reduced based on offset*
   - for each $T_{i,j} \in \mathcal{T}_{\mathrm{agg}}$, compute $et'_{i,j} = \sum_{S \in \mathcal{S}^{i,j}} t_S$ *//tentative updated trip travel time estimation*
   - `NewRelErr`=$\sum_{T_{i,j} \in \mathcal{T}_{\mathrm{agg}}} \frac{|et'_{i,j} - \bar{tt}_{i,j}|}{\bar{tt}_{i,j}}$ *//compute new relative error*
   - if `NewRelErr`<`RelErr` then do the following *//new estimates better than previous ones*
   - for each $T_{i,j} \in \mathcal{T}_{\mathrm{agg}}$, $et_{i,j} = et'_{i,j}$ *//update travel time estimates*
   - `RelErr`=`NewRelErr`
   - `again`=**true**; **goto** step 5.2 *//perform another iteration*
   - else *// new estimates worse than previous ones*
   `k`=1 + (`k`−1) · 0.75 *//reduce the street travel time increase/decrease step*
   if `k`< 1.0001 then exit from loop at step 5.3 *//if `k` is too small, exit from inner loop*
   else **goto** step 5.3 *//otherwise, perform another iteration with smaller `k`*
6. **Computation of estimated travel time for remaining streets**
   - $\mathcal{ES} = \mathcal{S}_{\mathrm{trip}}$; $\mathcal{NS} = \mathcal{S} - \mathcal{S}_{\mathrm{trip}}$
   - let $N(S)$ be the set of streets sharing an intersection with street S
   - for each $S_i \in \mathcal{NS}$ compute $n_{S_i} = |N(S_i) \cap \mathcal{ES}|$
   - order the streets in $\mathcal{NS}$ in decreasing order of n_{S_i}
   - for each $S_i \in \mathcal{NS}$ in the ordered sequence
   $v_{S_i} = \frac{\sum_{S_j \in N(S_i) \cap \mathcal{ES}} v_{S_j}}{|N(S_i) \cap \mathcal{ES}|}$; $t_{S_i} = \frac{L(S_i)}{v_{S_i}}$; $\mathcal{ES} = \mathcal{ES} \cup \{S_i\}$;$\mathcal{NS} = \mathcal{NS} - \{S_i\}$
   - repeat above step until $\mathcal{NS} = \emptyset$
6. **Travel time estimation**
   -for each possible pair of intersections (I_i, I_j), compute optimal route $I_i \rightarrow I_j$ using Dijkstra algorithm with estimated travel time t_S for each street S
   - let $\mathcal{S}^{i,j}$ be the set of streets in the optimal route for (I_i, I_j)
   - $ET(i,j) = \sum_{S_h \in \mathcal{S}^{i,j}} \frac{L(S)}{v_S}$
   - **return** ET

Travel time estimation algorithm.

Robustness of day of week (Oracle model)

To assess whether there is any noticeable difference in terms of trip sharing opportunities between weekend and week days, we have repeated the analysis above for the 104 weekend days. There is no major difference in terms of trip sharing opportunities in weekend versus weekdays. However, the average number of trips per day during weekend days is about 17% lower than that during week days ($\approx 350K$ versus $\approx 418K$ trips per day). Only minimal differences in total trip travel time savings between week and weekend days are observed, with slightly better savings achieved during weekdays. As shown next, this is due to the strong relation between trip sharing opportunities and the number of performed trips.

Shareable trips versus trips per day (Oracle model)

To better understand the relationship between trip sharing opportunities and number of trips performed in a day, we have ordered the days for increasing number of performed trips, and plotted the corresponding percentage of shared trips in the day. The resulting plot is reported in Fig. 3c in the main text. Typical days in New York City feature around 400,000 trips with almost near maximum shareability. Days with a small number of trips are rare and happen mostly during special events. For example, the most noticeable drop in trip sharing opportunities occurs on day 240, August 28th 2011, during which hurricane Irene hit New York City. On this (weekend) day, only about $26,500$ trips were performed, and trip sharing opportunities dropped to about 87% when $\Delta = 2\,\mathrm{min}$. As such, data points below 300,000 trips per day are too sparse to make statistically reasonable assessments. Hence we have generated additional low density situations by subsampling our dataset, randomly removing various fractions of vehicles from the system in the following way: For each day in the data set, we randomly selected a percentage c of the taxis in the trace, and deleted the corresponding trips from the data set. We varied c from 95% down to 1%, generating a number of trips per day as low as 1,962.

To the set of resulting shareability values we have fit a saturation curve of the form $f(x) = \frac{Kx^n}{1+Kx^n}$, where K and n are two (non-integer) parameters, Fig. 3c in the main text. Curves of this form appear in the well-known Hill equation in biochemistry, describing saturation effects in the binding of ligands to macromolecules and in similar processes[28].Fits to both the shared trip maximization and time minimization conditions match very well (for both, $R^2 > 0.99$), we used a standard Levenberg-Marquardt algorithm for obtaining least squares estimates. The best fit parameters read $K = 1.1 \times 10^{-4}$, $n = 0.92$ for time minimization, and $K = 1.5 \times 10^{-6}$, $n = 1.39$ for shared trip maximization. Since for time minimization we have $n \approx 1$, the fit works here almost as well (again $R^2 > 0.99$) with the functional form $f(x) = \frac{Kx}{1+Kx}$ that has only the one parameter K, known as the Langmuir equation[29], with $K = 4.4 \times 10^{-5}$. The Langmuir equation describes the relationship between the concentration of a gas (or compound) adsorbing to a solid surface (or binding site) and the fractional occupancy of the surface. Since an increasing density of taxis – the "particles" – implies that more trip pairs – the "surface" – can be covered, the Langmuir equation can thus be seen as an analogy to the saturation effects in

shareability if a homogeneous distribution of trips and taxis is assumed. The second parameter n that appears in the Hill equation is used as a measure for cooperative binding in enzyme kinetics: If $n > 1$, an enzyme which has already a bound ligand increases its affinity for other ligand molecules. It is unclear if the analogy can be stretched to understand why $n = 1.39$ works best for shared trip maximization. In any way, the fast, hyperbolic saturation implies that taxi sharing could be effective even in cities with taxi vehicle densities much lower than New York, and in case of low market penetration of the sharing system a high return on investment.

Increasing the number of shared trips

We next investigate what happens when we increase the number k of sharable trips from 2 to 3. We remark that the computational complexity of the trip matching task with $k = 3$ is orders of magnitude higher than the same task with $k = 2$, for the following reasons:

- The computation of the shareability hyper-network is challenging. In fact, we now have to compare triplets, instead of pairs, of candidate trips. For each triplet, we have 60 possible valid routes connecting the three sources/destinations of the trips, instead of 4 possible routes with $k = 2$. For each valid route, we then have to check whether the trips can actually be shared, meaning that the computational time for calculating the shareability network with $k = 3$ is at least 15 times higher than that needed to compute the trip sharing graph with $k = 2$.

- We now have to solve a matching problem on hyper-networks, instead of on simple networks. While matching on graphs can be solved in polynomial time, matching on general hyper-networks belongs to the class of NP-hard problems, i.e. problems that are "difficult to solve". To get around this computational challenge, we use a greedy, polynomial-time heuristic that first builds the maximum matching considering only triplets of trips, then applies standard matching on the remaining trips. This heuristic is known to build a solution which is, in the worst-case, within a constant factor from the optimal solution.

To tackle these computational challenges, we computed the number of shared trips and the fraction of saved travel time only in the Online model, and for selected days of the year. Notice that in the Online model trips can be shared only when their starting times are within a temporal window of δ, thus significantly reducing the number of candidate trips for sharing (and, hence, computational time needed to compute the shareability hyper-network) with respect to the Oracle model in which also trips with starting times in time windows larger than δ can be shared.

We first present the results referring to a day (day 300) in which about $450,000$ trips were performed, which is about the average number of trips per day recorded in our data set. Figures 2D and E in the main text report the percentage of saved taxi trips and of saved travel time as a function of the quality of service parameter Δ, when the time window parameter δ is set to $1\,\mathrm{min}$. As seen from the figure, increasing the number of sharable trips provides some

benefit only when the quality of service parameter Δ is large enough for such trips taxi sharing opportunities to become available. This value of Δ is approximately equal to $150\,\mathrm{sec}$. For larger values of Δ, the advantage of triple trip sharing versus double trip sharing becomes perceivable. When $\Delta = 300\,\mathrm{sec}$, the number of saved taxi trips is increased from about 50% with $k = 2$ to about 60% with $k = 3$. While with $k = 2$ nearly all trips can be shared, resulting in about halving the number of performed trips, relatively less trips can be combined in a triple trips when $k = 3$. In fact, the achieved percentage of saved trips with $k = 3$ is 60%, which is below (but not too much) the percentage of 66.6% that would result if all trips would be shared in a triple trip.

Similarly to the percentage of saved trips, also when the percentage of total traveled time is considered the difference between double and triple trip sharing becomes perceivable only for values of $\Delta \geq 150\,\mathrm{sec}$. Increasing the number of shared trips from 2 to 3 allows a further saving of about 10% in terms of total traveled time, which is achieved when $\Delta = 300\,\mathrm{sec}$.

Extended Data Fig. 2 reports the percentage of saved taxi trips and of saved total travel time as a function of the time window parameter δ, for two specific values of Δ. While increasing δ beyond $120\,\mathrm{sec}$ provides little benefits in terms of saved taxi trips when $k = 2$, we still can observe some benefit for $\delta > 120\,\mathrm{sec}$ with $k = 3$. This is due to the fact that with $\delta = 120\,\mathrm{sec}$ we already obtain near-ideal performance when $k = 2$, corresponding to halving the number of trips. When $k = 3$, there is more room for improvement, and a near-ideal performance is approached only when $\delta = 180\,\mathrm{sec}$ and $\Delta = 5\,\mathrm{min}$. This means that all triple trip sharing opportunities can be exploited only with relatively "patient" taxi customers. Concerning percentage of saved travel time, we observe that triple trip sharing achieves as far as 45% saving, which is significantly higher than that achieved by double trip sharing. However, similar to the percentage of taxi trips, such high savings can be obtained only with relatively "patient" taxi customers.

Finally, we compare the potential of triple versus double taxi trip sharing in a relatively less crowded day (day 250), when only $\approx 250,000$ trips were performed. Extended Data Fig. 2 reports the achieved reduction in number of taxi trips and in total travel time as a function of δ, when $\Delta = 5\,\mathrm{min}$. While with $k = 2$ near-ideal taxi sharing can be achieved also with low taxi traffic, with $k = 3$ a higher number of taxi trip requests is needed to fully exploit the potential of triple trip sharing. The situation is different in terms of saved total travel time, which is consistently benefiting from a higher number of taxi requests for both double and triple trip sharing.

Summarizing, based on the analysis above we can state that triple trip sharing does provide substantial benefits versus double trip sharing, but for this to occur we need a reasonable number of taxi requests, and relatively "patient" taxi customers, for which waiting for a few minutes at taxi request time and upon arrival at destination is acceptable. With "impatient" customers, double trip sharing is much more effective than triple trip sharing: it is computationally efficient, and provides nearly the same performance as triple trip sharing. Since the benefits to the community in terms of reduced number of taxis and reduced pollution with triple trip sharing versus double trip sharing are considerable, an interesting question raised by our analysis is

whether the New York City municipality can design a fare system that motivates customers to be “patient”.

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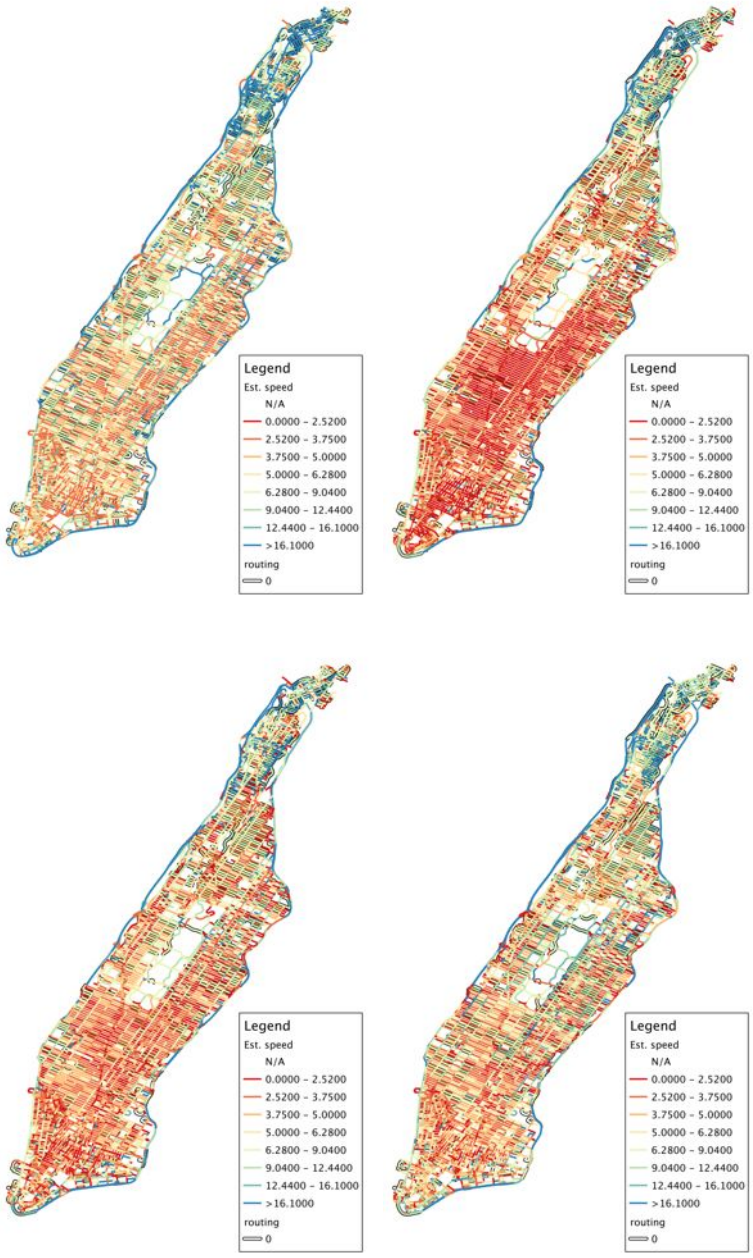
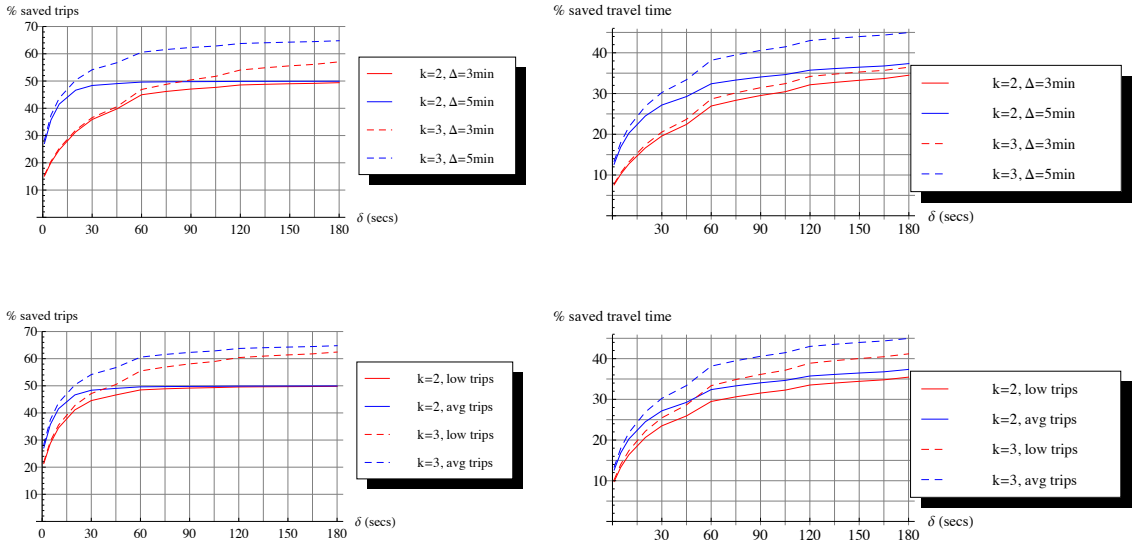


Extended Data Figure 1: Estimated speed map at 0am (top left) and at 8am (top right). Estimated speed map at 4pm (bottom left) and 10pm (bottom right). Travel time for streets in bold (routing) is estimated at step 6 of the algorithm.



Extended Data Figure 2: Percentage of saved taxi trips (top left) and percentage of saved travel time (top right) in New York City as a function of δ in the Online model. The quality of service parameter is set to $\Delta = 3\,\text{min}$ and $\Delta = 5\,\text{min}$. Percentage of saved taxi trips (bottom left) and percentage of saved travel time (bottom right) as a function of δ in the Online model, in a day with relatively low taxi traffic ("low"), and with average taxi traffic ("avg"). The quality of service parameter is set to $\Delta = 5\,\text{min}$. Each plot reports two curves: one referring to the case where at most two trips can be shared ($k = 2$), and one referring to the case where at most three trips can be shared ($k = 3$).

Hour	**Avg. Rel. Error**	**% trips after filtering**	$\lvert\mathcal{S}_{\text{trip}}\rvert$
0	0.1541	94.91	8582
1	0.1301	96.64	8664
2	0.1433	97.76	8781
3	0.1463	98.12	8673
4	0.1438	98.16	8707
5	0.1448	98.17	8443
6	0.1517	98.04	8485
7	0.1535	98.04	8554
8	0.1560	98.01	8600
9	0.1541	97.97	8596
10	0.1568	97.96	8629
11	0.1644	97.85	8650
12	0.1639	97.97	8833
13	0.1547	98.12	8820
14	0.1486	98.20	8622
15	0.1553	98.19	8866
16	0.1388	98.13	8687
17	0.1486	98.05	8853
18	0.1457	97.86	8845
19	0.1540	97.61	8718
20	0.1602	97.23	8698
21	0.1649	96.85	8600
22	0.1693	96.54	8641
23	0.1799	95.78	8578
avg	0.1534	97.59	8671.9

Extended Data Table 1: Summary of travel time estimation performance.