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Exploring the structure of the U.S. intercity passenger air transportation network: a weighted complex network approach

Zengwang Xu · Robert Harriss

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Abstract The U.S. airline network is one of the most advanced transportation infrastructures in the world. It is a complex geospatial structure that sustains a variety of dynamics including commercial, public, and military operations and services. We study the U.S. domestic intercity passenger air transportation network using a weighted complex network methodology, in which vertices represent cities and edges represent intercity airline connections weighted by average daily passenger traffic, non-stop distance, and average one-way fares. We find that U.S. intercity passenger air transportation network is a small-world network accompanied by dissortative mixing patterns and rich-club phenomenon, implying that large degree cities (or hub cities) tend to form high traffic volume interconnections among each other and large degree cities tend to link to a large number of small degree cities. The interhub air connections tend to form interconnected triplets with high traffic volumes, long non-stop distances, and low average one-way fares. The structure of the U.S. airline network reflects the dynamic integration of pre-existing urban and national transportation infrastructure with the competitive business strategies of commercial airlines. In this paper we apply an emerging methodology to representing, analyzing, and modeling the complex interactions associated with the physical and human elements of the important U.S. national air transport and services infrastructure.

Z. Xu (✉) · R. Harriss
Houston Advanced Research Center, The Woodlands, TX, USA
e-mail: zxu@harc.edu

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Introduction

Understanding the geospatial co-evolution of critical urban infrastructures is an important challenge for geography, civil engineering, urban planning, and other aspects of the new science of cities. As an influential transportation infrastructure, the United States air transportation network has played an important role in facilitating the mobility of people and goods. The demand for air transport services is projected to grow significantly in coming decades with consequences for the economy, environment, and lifestyles (Irwin and Kasarda 1991). For decades the air transportation system has evolved into a network with complex geospatial and topological structures and large heterogeneity in its connections and dynamics (Barrat et al. 2004a). Since air transportation networks are a geospatial network on which a variety of societal dynamics take place, it is increasingly becoming the focus of a great deal of research in complexity science, network science, and in geography (Vowles 2006).

The geospatial properties of the U.S. air transportation network evolve in space and time reflecting the influence of economic, demographic and public policy factors (Gillen and Morrison 2005; Swan 2002). Following the passage of the Airline Deregulation Act in 1978, a hub-and-spoke network structure became the dominant structure of the U.S. air transportation system, in which hubs served as intermediate vertices to consolidate traffic to and from spokes. In these structures, hubs are linked by high capacity interhub connections and spokes are linked to hubs through low capacity spoke-hub connections (Bryan and O'Kelly 1999; Goetz and Sutton 1997; Horner and O'Kelly 2001; Jaillet et al. 1996; O'Kelly 1998; O'Kelly and Bryan 1998; O'Kelly and Miller 1994; Shaw 1993). Recently, the emergence of a number of successful low cost carriers has resulted in another geospatial structure—point-to-point connections that currently coexist within the hub-and-spoke structure in the U.S. air transportation network. The point-to-point connections provide direct connection between cities saving time and resources in comparison to a similar trip on the hub-and-spoke structure (Alderighi et al. 2005; Gillen and Morrison 2005). The emergence of the point-to-point structure adds an element of geospatial and dynamic complexity to the properties of the U.S. airline system.

As in other complex geospatial networks, interactions between the structure and function of the system have feedback effects on the future evolution of the system (Strogatz 2001). For example, the structural properties of air transportation system have an influence on the scheduling, traffic, and choice of aircraft size (Brueckner 2004). Moreover, the structure of the air transportation network affects the operational business strategies such as the full services strategy adopted in the hub-and-spoke structure and the value-based strategy utilized in the point-to-point structure (Alderighi et al. 2005; Gillen and Morrison 2005). The interaction between structure and dynamics in U.S. air transportation system has driven its evolution for decades and it has made the U.S. air transportation system an increasingly complex networked system.

Network studies in geography can date back to 1960s (Haggett and Chorley 1969; Kansky 1963). At that time, networks were small and network studies relied on full knowledge of the networks, i.e., researchers knew the exact number of nodes in the network and how they are interconnected. At the present time, networks become increasingly complex and it is no longer possible to know precisely how many nodes and edges exist in complex networks such as the Internet and many social networks. A "new" science of networks (Barabasi 2002; Watts 2004) has emerged in last 10 years to study complex networks from "system" perspective based on their universal structural properties, the underlying mechanism of the network evolution, and the interrelation between the structure and dynamics occurring on networks (Newman 2003). The "new" science of networks will inform the representation, analysis, modelling, and simulation of dynamic processes occurring on socio-technical infrastructures and will expand the perspective of Geographic Information Science (GISci) (Sui 2006). Particularly, the current representational basis of GIS is dominated by a field or an object view of the world framed by Euclidean geometry in an absolute sense and ignores most of the interconnections among the components in a geospatial system. This lack of system thinking inherently poses a major barrier for GISci to embrace spatial relations, interactions, and connectivity, and therefore limits the introduction of geographic perspectives in modelling and simulating dynamic processes, such as propagation of infectious diseases and innovation diffusion (Batty 2003; Sui 2007). In this paper we apply an emerging methodology to representing, analyzing, and modelling the complex interactions within physical and human systems associated with the important U.S. national air transport and services infrastructure.

Current studies on the structure of air transportation networks have addressed the problems of optimal network design using a variety of operations research models and algorithms (Lederer and Nambimadom 1998; Magnanti and Wong 1984; Minouz 1989). We hypothesize that the emerging "new" science of networks offers GISci a "system" level analysis and "holistic" perspective on the study of large scale geospatial networks. In particular, recently proposed complex weighted network methodologies (Barrat et al. 2004a; Bathelemy et al. 2005; Park et al. 2004) provide a new set of approaches to pursue studies of systemic considerations in airline network geospatial design and economic performance. In this paper we integrate the static hub-and-spoke and point-to-point geospatial structures with an analysis of the dynamic air transportation services that evolve in response to travel demand. Our goal is to explore the potential for a universal methodological framework to characterize the co-evolution of critical socio-technical airline infrastructure systems.

In the remainder of the paper, second section describes the U.S. intercity air transportation network; third section introduces the methods and results of our analysis of the U.S. intercity passenger air transportation network based on the understanding of the architectural properties of the network, fourth section provides a weighted network model to simulate the evolution of the network; fifth section concludes with a summary and discussion of this study.

The U.S. intercity passenger air transportation networks

The U.S. intercity passenger air transportation networks we study will be characterized by cities, termed vertices, and intercity air connections, termed edges. Each air connection (edge) has three attributes: (1) the non-stop distance between cities in the contiguous U.S. (2) the average daily passengers travelling between cities in the contiguous U.S. and (3) the average one-way fare, which is the average price paid by all fare paying passengers. We acquired data from the Department of Transportation quarterly fare reports (Domestic Airline Fares Consumer Report) that report the average prices being paid by consumers in the 1,000 largest domestic city-pair markets within the 48 contiguous states from the third quarter of 1996 to the present. These markets account for about 75% of all 48-state passengers and 70% of total domestic passengers. Our dataset for network analysis consists of 16 networks for all quarters from 2002 to 2005, and each includes more than 1,000 largest domestic city-pair markets. However, we do not include very small cities that primarily provide non-commercial air services in our analysis.

Our dataset has a few limitations as indicated by (Grais et al. 2004), such as: passengers counted between a city pair are not only those non-stop airline passengers, but also those who stop at intermediate cities without leaving airports and passengers who depart and return to their origin city within 24 h are counted twice. Despite these minor limitations, the dataset is a comprehensive representation of one of the largest infrastructural networks in the contiguous U.S. Table 1 provides descriptive statistics for the entire dataset. Figure 1 maps the spatial distribution of the network in the fourth quarter of 2005, in which proportional circles represent the number of air connections of cities and the width of links represents the volume of air traffic. Air traffic is notably concentrated on trunk routes between the northeastern U.S., California, and Florida. We analyze all 16 networks, but will use the network in the fourth quarter of 2005 for a representation of the fundamental network properties of the U.S. intercity passenger air transportation system.

The analysis of the U.S. intercity passenger air transportation networks

Network study increasingly pervades all of science (Strogatz 2001), and it usually falls into three categories: (1) characterizing the structure of empirical networks; (2) modeling networks; and (3) investigating the interplay between network structure and dynamics taking place on networks (Costa et al. 2007; Newman 2003). However, most network studies, no matter which category they emphasize, rely on measures capable of characterizing relevant topological features to identify the unifying principles and statistical properties commonly existing among empirical networks (Costa et al. 2007). Based on a number of topological and weighted network measures, we investigate the network properties of the U.S. intercity passenger airline network using methodologies reported in recent advances of network science (Bagler 2008; Guimera and Amaral 2004).

Small-world properties

Many real-world networks, including airline networks (Bagler 2008; Guida and Maria 2007; Guimera and Amaral 2004), have been found to be small-world networks (Watts 1999; Watts and Strogatz 1998), in which their connectivity is as high as and their clustering is much higher than that of the same size random network. The connectivity of the network is measured by the average path length (APL) that is the average shortest path length between any pairs of vertices in networks (Watts and Strogatz 1998), and the clustering of the network is measured by the clustering coefficient (CC) that is

Table 1 Descriptive statistics of the 16 networks for all quarters of 2002–2005

Year	Qua.	Nodes (Cities)	Edges	Distance (miles)			Passengers			Fare (dollars)		
				Min	Max	Ave	Min	Max	Ave	Min	Max	Ave
2002	1	255	5,773	72	2,724	1,060	10	9,066	139	54	477	199
	2	262	6,395	72	2,783	1,061	10	8,067	142	53	475	196
	3	266	6,305	72	2,783	1,061	10	7,712	140	39	479	189
	4	253	6,159	72	2,783	1,064	10	8,950	143	50	598	188
2003	1	250	5,819	72	2,724	1,056	10	9,253	139	51	455	198
	2	260	6,192	72	2,783	1,063	10	9,069	148	50	456	195
	3	261	6,186	72	2,783	1,072	10	8,788	144	60	486	197
	4	261	6,319	72	2,783	1,068	10	9,969	146	66	465	195
2004	1	255	5,929	81	2,724	1,072	10	10,960	148	69	512	203
	2	263	6,478	81	2,783	1,073	10	10,375	157	66	485	198
	3	267	6,537	81	2,783	1,078	10	9,298	151	56	445	191
	4	260	6,557	81	2,783	1,064	10	11,637	153	54	441	188
2005	1	253	6,374	81	2,724	1,078	10	12,034	150	57	449	189
	2	263	6,771	67	2,783	1,080	10	11,532	162	59	470	198
	3	272	6,636	67	2,783	1,088	10	10,388	159	66	442	202
	4	272	6,566	67	2,783	1,076	10	10,761	158	57	428	207

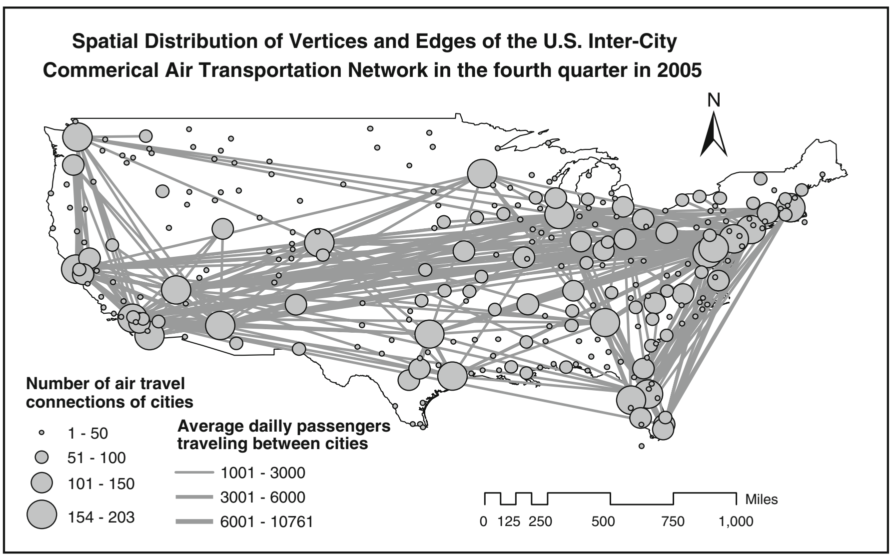


Fig. 1 The spatial distribution of the vertices and edges in the contiguous U.S. inter-city air transportation network for the fourth quarter of 2005

the average of interconnection rate of every vertex's neighbourhood (Watts and Strogatz 1998). We computed the APL and CC of 16 U.S. intercity passenger airline networks and compared them with those of the same size random networks (Tables 2, 3). The U.S. intercity passenger airline networks show the characteristics of a small-world network. Although the network includes hundreds of cities and thousands of connections, any two cities are typically connected in no more than two steps. The world-wide airport network (Guimera et al. 2005) the airport network of India (Bagler 2008), the Italian airport network (Guida and Maria 2007), the airport network of China (Li and Cai 2004) were also found to be small-world networks. However, the U.S. intercity passenger airline network has the smallest average path length and the largest clustering coefficient of any airline network studied to date. This finding reflects

Table 2 Comparison between airline networks with same-size random networks in terms of their average path lengths and clustering coefficients

Quarters	Average path length		Clustering coefficients	
	Airline networks	Random networks	Airline networks	Random networks
2002-1	1.91	1.81	0.74	0.18
2002-2	1.90	1.80	0.75	0.19
2002-3	1.90	1.81	0.75	0.18
2002-4	1.89	1.79	0.78	0.20
2003-1	1.93	1.80	0.75	0.19
2003-2	1.93	1.82	0.74	0.19
2003-3	1.92	1.83	0.76	0.19
2003-4	1.87	1.80	0.77	0.19
2004-1	1.92	1.82	0.78	0.19
2004-2	1.86	1.82	0.76	0.19
2004-3	1.85	1.80	0.77	0.19
2004-4	1.84	1.79	0.77	0.20
2005-1	1.86	1.79	0.78	0.21
2005-2	1.85	1.79	0.75	0.20
2005-3	1.87	1.83	0.75	0.19
2005-4	1.90	1.81	0.73	0.19

Table 3 The average path length and clustering coefficient of different airports networks

Airport networks	Average path length	Clustering coefficient
Indian	2.26	0.66
Italian		
June1, 2005–May 31, 2006	1.98	0.1
July 16, 2005–August 14, 2005	1.98	0.1
Nov. 2005	2.14	0.07
World wide	4.4	0.62
China	2.01	0.73

the fact that the U.S. currently has the most mature and efficient air transport infrastructure in the world.

City connectivity and weights based on intercity air connections

The topology of a network can be represented by an adjacency matrix a_{ij}, where a_{ij} is 1 if the vertex i and vertex j are connected and 0 otherwise. The degree of a vertex, defined as $k_i = \sum_j a_{ij}$, is the number of topological connections of the vertex to other vertices. The larger the degree, the more connected the vertex. However, the vertex degree cannot take into consideration the weights of the connections in a weighted network. Another centrality measure, vertex strength, is used in weighted networks to take into account not only the topological connections, but also the actual weights of the connections (Barrat et al. 2004a). The vertex strength can be defined as $s_i = \sum_{j=1}^{N} a_{ij} w_{ij}$, where w_{ij} is the weight of the connection between vertex i and vertex j, i.e., passengers, distance and fare. The strength of vertices measures the total weight of their connections handled by vertices. The degree of the vertex is the number of airline connections between the city and other cities, and the vertex strength is measured as the cumulative air traffic, distance, or fare handled through the city.

The centrality measures of air transportation networks can identify the most connected vertices (cities) through their degree and most travel intensive vertices (cities) through their strength. The distributions of centrality measures provide a holistic view of the structure and organization of the whole network. For a large number of complex networks, their degree distributions have been found following the scale-free behavior, which is characterized by a power-law $P(k) \sim k^{-\gamma}$. The scaling exponent γ is related to that of the cumulative degree distribution $P(>k) \sim k^{-\gamma_{cum}}$ by $\gamma = 1 + \gamma_{cum}$.

We calculate the cumulative degree distribution $P(>k)$ (Fig. 2) and cumulative strength distribution $P(>s)$ (Fig. 3), which are defined respectively as the

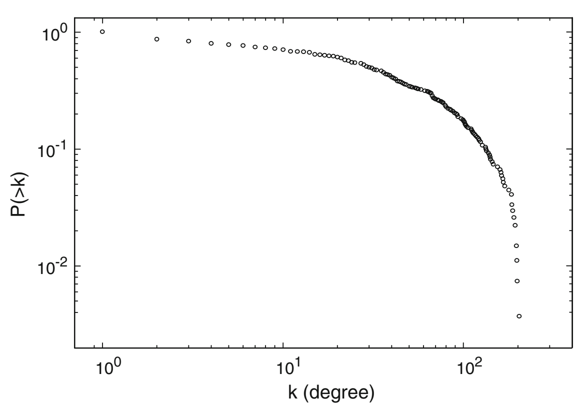


Fig. 2 The cumulative distribution of the degree of the U.S. passenger air transportation network in the fourth quarter of 2005

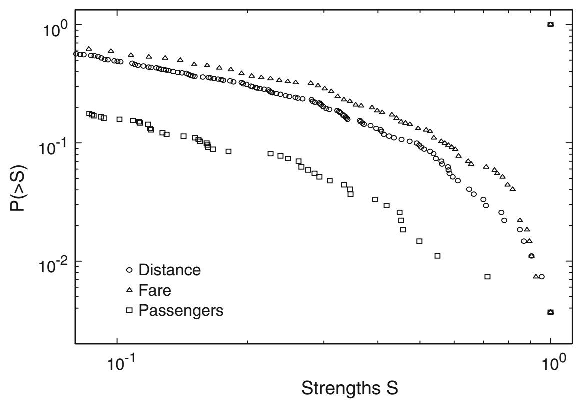
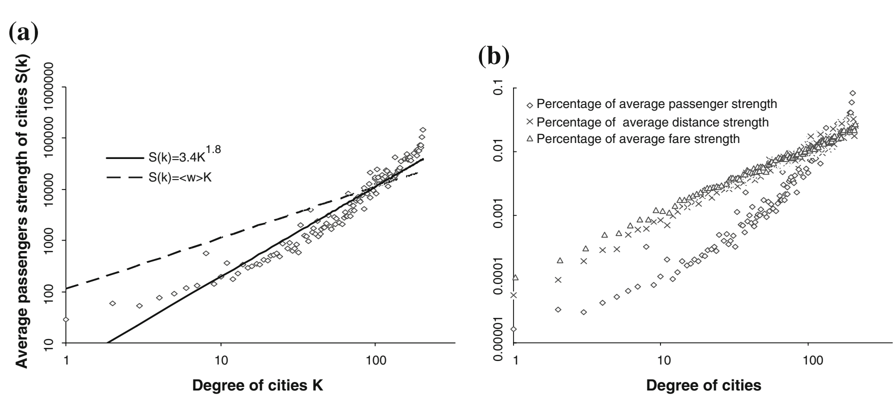


Fig. 3 The cumulative distribution of strengths (passengers, distance, and fare) of the U.S. intercity air transportation network in the fourth quarter of 2005. Passengers, distance, and fare are normalized according to their minimum and maximum values that can be seen in Table 1

probability that any given vertex has degree greater than k or strength greater than S. All distributions have truncated power-law tails, which have exponent $\gamma = 2.63 \pm 0.33$ for degree distribution, $\gamma = 2.59 \pm 0.25$ for fare strength distribution, $\gamma = 2.51 \pm 0.18$ for distance strength distribution, and $\gamma = 2.13 \pm 0.18$ for passenger strength distribution. These distributions, which termed "scale-free", have signalled the presence of high-level heterogeneity in airline connectivity and air traffic handled by cities in the air transportation networks, in which a majority of the vertices (cities) have low degree and strength, and a small number of vertices (cities) have high degree and strength. This scale-free structure has proved to be robust to random failure but vulnerable to targeted attack (Callaway et al. 2000).

The study of the relationship between strength and degree of vertices provides further characterization of the relationship between the topological structure and the weight distribution of the networks (Barrat et al. 2004a). Intuitively, vertices of large degree (more connected vertices) usually have more strength than those of small degree. If this is the case in the U.S. intercity air transportation network, the more connected cities would be expected to be associated with more air traffic, distance or fare. However, our analysis shows that the strength of vertices is not linearly proportional to their degree and that the average strength of passengers $S(k)$ at vertices increase with degree k as, $S(k) \sim k^{\gamma}$ with $\gamma = 1.7 \pm 0.1$. This relation between degree and strength is similar to that of India airport network ($\gamma = 1.43 \pm 0.06$) (Bagler 2008) and to that of worldwide airport network ($\gamma = 1.5 \pm 0.1$) (Guimera and Amaral 2004). Figure 4 shows this relation for the U.S. intercity air transportation network operations in the fourth quarter of 2005. As expected for a mature airline system, the 16 networks for 2002–2005 exhibit similar properties. The <w> in Fig. 4 denotes the average weight (the average daily passenger between cities) of the network. The dashed line represents a linear relationship between strength and degree of vertices on the assumption that the strength of vertices is linearly proportional to their degree. In Fig. 4, the dashed line ($S(k) = \langle w \rangle \times k$, where <w> = 112.2), and the solid line ($S(k) \sim k^{1.8}$) intersect when $k \approx 85$. It implies that for vertices whose degree is less than 85 ($k < 85$), their strength, i.e., the air traffic handled by the cities, is lower than the average level (<w>); for vertices whose degree is greater than 85 ($k > 85$), their strength is higher than the average level and increases exponentially. Highly connected cities handle more air traffic with the 85 more connected cities exhibiting especially high

Fig. 4 (**a**) Passenger strength $S(k)$ as a function of the vertex degree k in the network in the fourth quarter of 2005; (**b**) the variation of all three normalized strengths (passengers, non-stop distance, and fare) of cities with their degree



capacities. This phenomenon is rooted in the hub-and-spoke structure, or core-periphery structure (Goetz and Sutton 1997), in which a small number of hubs have a large number of airline connections and handle much more air traffic than spokes. The large hubs, especially those shared by several competing carriers, attract air traffic due to competitive forces generated by passenger demand (Bania et al. 1998). In the network of the fourth quarter of 2005, there are 58 out of 272 cities with degree greater than 85. These cities are consistent with core hubs cities (22 hub cores, 6 gateway core, 25 semi-core) identified by (Goetz and Sutton 1997).

We associate each city with three strengths, i.e., cumulative passenger demand, non-stop distance, and fare. Comparison among these three normalized strengths (Fig. 4b) shows that cities with degree greater than 85 tend to handle a higher percentage of passengers than is the case for corresponding distance and fares; cities with degree less than 85 handle a higher percentage of distance and fares. In other words, air carriers operating from cities with degree less than 85 cumulate more miles and more total fares, but they do not transport as many passengers. This shows the disadvantage of spoke cities, which may be caused by the historic evolution of the geospatial structure of the network that was initially designed for sea and land transport systems, the economy of the spoke cities, and the lack of competitive carriers. This inefficiency might provide the market space for point-to-point connection operated by low cost carriers. On the other hand, hub cities with degree greater than 85 are more efficient due to larger and highly competitive markets as a result of the consolidation of spoke cities to the hub cities.

We also did not find clear relationships between the passenger, fare, and distance attributes. Particularly, the relation between distance and fare is not significantly linear, i.e., long distance does not always mean high fare (Fig. 5).

We calculated the weights distribution of airline connections in the networks. The cumulative distributions of passengers in the U.S. intercity passenger air transportation networks for the 16 quarters spanning 2002 to 2005 can be approximated by a power function, $P(q) \sim q^{\gamma}$ with $\gamma = 2.54 \pm 0.05$. The distribution of fare weight has a power-law tail with exponent $\gamma = 2.72 \pm 0.33$, and the distribution of distance weight has a power-law tail with exponent

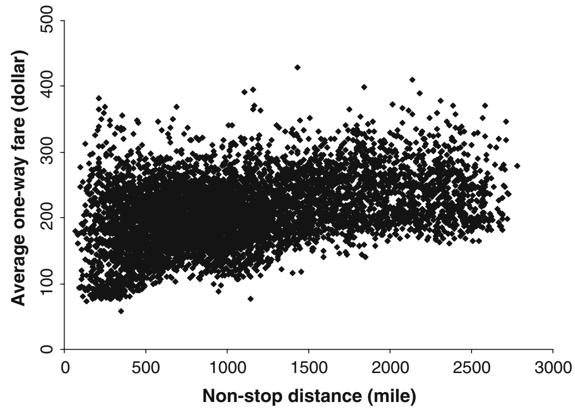


Fig. 5 Scatter plot of the average one-way fare versus non-stop distance on the air connections linking 6,566 city pairs in the fourth quarter of 2005

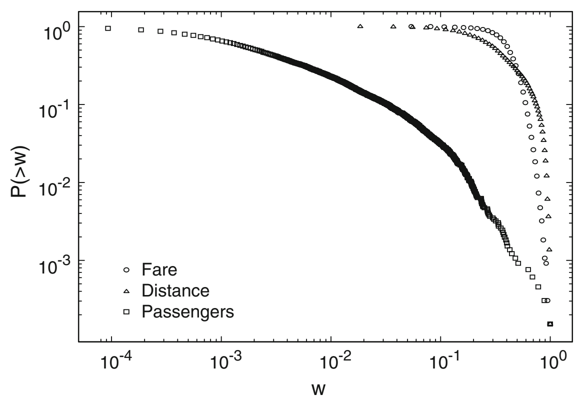


Fig. 6 The cumulative distribution of weights (fare, distance, and passengers) in the U.S. airline transportation network in the fourth quarter of 2005. The weights are all normalized according to their minimum and maximum values in Table 1

$\gamma = 2.20 \pm 0.19$. Figure 6 illustrates the weight distributions in the U.S. air transportation network in the fourth quarter of 2005, but all other quarters have the same behavior. This power-law behavior reflects the high level heterogeneity in air travel between cities in the U.S.

In contrast to passenger loads, the distributions of non-stop distances and average one-way fares have more pronounced cut-off at high magnitudes of fare or distance (Fig. 6). Their exponents were estimated according to the methodology recommended by Claust et al. (2007). These power-law behaviors and cut-off tails signal the trade-off between number of airline connections and the passengers, distance, and cost on the connections. The difference in the statistical distributions reflects different dynamics taking place on the U.S. air transportation system. In particular, power-law behavior in passenger distribution reflects a high heterogeneity in air traffic over the U.S. air transportation. It may be due to the regional character of dominant business and social networks, such as the tourism business in Florida and Los Angeles, along with other factors that influence the operational business strategies of airlines, such as the largest low-cost carrier, Southwest airline focusing south-western part of U.S. and operating with a value-based business strategy on point-to-point connections.

Topological and weighted clustering

The clustering property of networks is characterized by their clustering coefficients (Barabasi et al. 2003; Barrat et al. 2004a; Bathelemy et al. 2005; Guimera and Amaral 2004). The clustering coefficient of a vertex i is defined as the fraction of existing edges in its neighborhood, i.e., $C_i = N_i^{existingedges} \big/ N_i^{possibleedges}$, where $N_i^{existingedges}$ represents the number of edges actually existing in the neighborhood of vertex i, and $N_i^{possibleedges}$ represents the number of possible edges in the neighborhood of vertex i if all neighbors are fully connected. It actually measures how interconnected the neighborhood is by counting the number of interconnected triples existing in the neighborhood. If all neighboring vertices of vertex i are fully connected, C_i will be 1. If none of vertex i's neighboring vertices are connected each other, C_i will be 0. The clustering coefficient of a network C is the average of clustering coefficient over all vertices in the network. It represents the overall cohesiveness of the network. The clustering coefficient can also be averaged over vertices of the same degree $C(k)$. In fact, $C(k)$ as a function of degree k in many real-world networks exhibits a power-law decay in which high degree vertices increasingly have low clustering coefficients (Ravasz and Barabási 2003; Vázquez et al. 2002), which indicates a hierarchical structure of networks (Barrat et al. 2004a). In the case of airline networks, we found that the decaying behavior of clustering coefficient $C(k)$ follows a scaling law $C(k) \sim k^{-1}$in their long tails (Fig. 7). This behavior reflects a hierarchical structure in which the neighboring cities of low degree cities are very well interconnected and therefore the low degree cities have a high clustering coefficient, while high degree cities (or hubs) are connected to many vertices that are not well inter-connected to each other. Geographers have recognized the similar interurban hierarchical network structure from their study on hierarchical diffusion of innovation (Pred 1977).

The aforementioned clustering coefficient only considers the topological structure of networks. In weighted networks like the air transportation network, connections between cities have different magnitudes of air traffic, distance or fare. A weighted variant of the clustering coefficient takes into account the topological clustering (the number of closed triplets in the neighborhood) and their total relative weight with respect to the strength of the vertex (Barrat et al. 2004a). The weighted clustering coefficient of vertex i is defined as (Barrat et al. 2004a; Bathelemy et al. 2005).

$$C_i^w = \frac{1}{s_i(k_i - 1)} \sum_{j,h} \frac{(w_{ij} + w_{ih})}{2} a_{ij} a_{ih} a_{jh} \quad (1)$$

where, C_i^w is the weighted clustering coefficient of vertex i who has two neighboring vertices, i.e., vertex j and h; w_{ij} and w_{ih} are the weights between vertex i and vertex j, and between vertex i and vertex h, and the

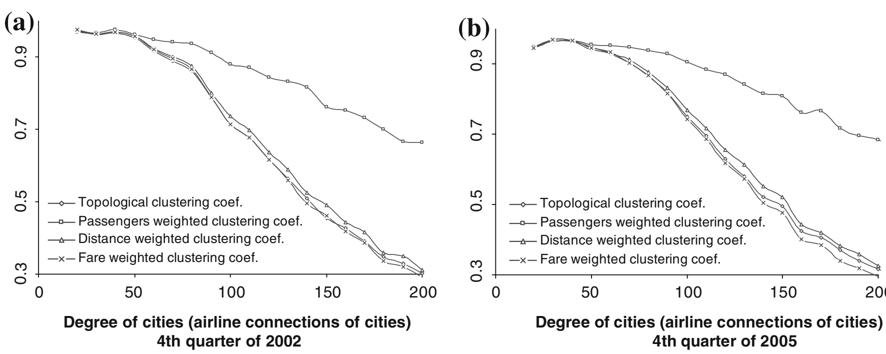


Fig. 7 The variation of topological and weighted clustering coefficients ($C(k)$ and $C^w(k)$) with city degree for the U.S. passenger air transportation networks in the fourth quarter of 2002 (**a**) and 2005 (**b**)

weight could be distance, passengers, or fare; S_i is the strength of vertex i; k_i the degree of vertex i; $a_{ij} = 1$, $a_{ih} = 1$, and $a_{jh} = 1$ when vertex i is connected to vertex j, vertex i is connected vertex h, and vertex j is connected to vertex h, respectively. They are equal to 0 otherwise. The weighted clustering coefficient of a network C^w is the average of weighted clustering coefficients over all vertices of the network. The weighted clustering coefficient can also be averaged over vertices of the same degree $C^w(k)$.

C^w and $C^w(k)$ provide global information on the correlation between weights and topology in network clustering by comparing them with their topological counterparts C and $C(k)$ (Barrat et al. 2004a). For a network with $C^w > C$, its interconnected triplets are more likely formed by edges with large weights. On the other hand, a network with $C^w < C$ will have many interconnected triplets generated by edges with low weights, and the high weight edges do not form the interconnected triplets (Barrat et al. 2004a; Bathelemy et al. 2005; Costa et al. 2007; Park et al. 2004).

The topological clustering coefficient $C(k)$, passenger weighted clustering coefficient $C^{Pass}(k)$, distance weighted clustering coefficient $C^{Dist}(k)$, and fare weighted clustering coefficient $C^{Fare}(k)$ were calculated for all the U.S. passenger air transportation networks from the first quarter of 2002 to the fourth quarter of 2005. All of these networks show a consistent decay behavior of $C(k)$ as shown in Fig. 6. The data for Fig. 6 are from the fourth quarter of 2005, but they are representative graph for all the networks in the time periods we studied. This decay behavior of $C(k)$ was also observed in other complex networks, such as the worldwide airport network, the airline network in India, and scientific collaboration networks. These results show that cities with large degree (i.e., a large number of air connections) tend to have small clustering coefficients. The role of large cities as hubs that serve provide long-distance, non-stop connections is consistent with a small clustering coefficient (Barrat et al. 2004a). In the U.S. airline network when a city degree is less than 60 ($k < 60$), both $C(k)$ and the weighted clustering coefficients ($C^{Pass}(k)$, $C^{Dist}(k)$, and $C^{Fare}(k)$) tend to remain high in spite of a few fluctuations (Fig. 7).

High clustering coefficients imply that cities with less than 60 airline connections have high interconnection among their neighboring cities. For example, Gainesville, FL connects to 21 other cities and those 21 cities are highly interconnected topologically and their interconnections have high weights (Fig. 8a).

When $k > 60$, both $C(k)$ and the weighted clustering coefficients ($C^{Pass}(k)$,$C^{Dist}(k)$, and $C^{Fare}(k)$) exhibit a pronounced decay behavior. A high volume of air traffic and reduced topological clustering contribute to the passenger weighted clustering coefficient, $C^{Pass}(k)$, decreasing much slower than topological clustering coefficient $C(k)$. In other words, $C(k)$ is increasingly offset by the air traffic. It indicates that the interconnected triplets in the neighborhood of large degree cities primarily serve large volume inter-hub traffic air connections. This "rich-club phenomenon" (Zhou and Mondragon 2004), which occurs when well-connected vertices tend to form interconnections among each other, was also observed in worldwide airport network (Barrat et al. 2004a) and in India's airport network (Bagler 2008). In fact, the "rich-club" concept is embedded in the aforementioned hub-and-spoke structure. Figure 8b analyzes the air connections among the neighboring cities of a large degree hub city, Chicago, IL, which connects to 198 other cities. The topological clustering coefficient of Chicago is 0.31 but its passenger weighted clustering coefficient is 0.70 due to the large traffic volume among interconnected hub cities. This is a specific example of a hub-and-spoke structure that connects the Chicago hub to a number of spoke cities with low traffic volume and forms interconnections among hubs (large degree cities) with high traffic volume.

Cities with degree greater than 60 ($k > 60$) exhibit distance and fare weighted clustering coefficients ($C^{Dist}(k)$, and $C^{Fare}(k)$) very similar to the decay behavior of their topological clustering coefficients ($C(k)$). However, the $C^{Dist}(k)$ is always larger than $C(k)$. But the $C^{Fare}(k)$ is equal to or greater than $C(k)$ when $k < 60$, and smaller than $C(k)$ when $k < 60$. The larger the degree k, the smaller the $C^{Fare}(k)$ when compared to $C(k)$. These results imply that interconnected triplets among large degree cities are more likely formed by the relative long distance air connections with relative low fares when $k > 60$. The relative long distance and low average one way fare between large degree cities can be considered a consequence of recent movement of low cost carriers on the medium/long-haul travel with no/less frills. This effect becomes increasingly apparent when the network of the fourth quarter of 2005 (Fig. 7a) is compared to 2002 (Fig. 7b).

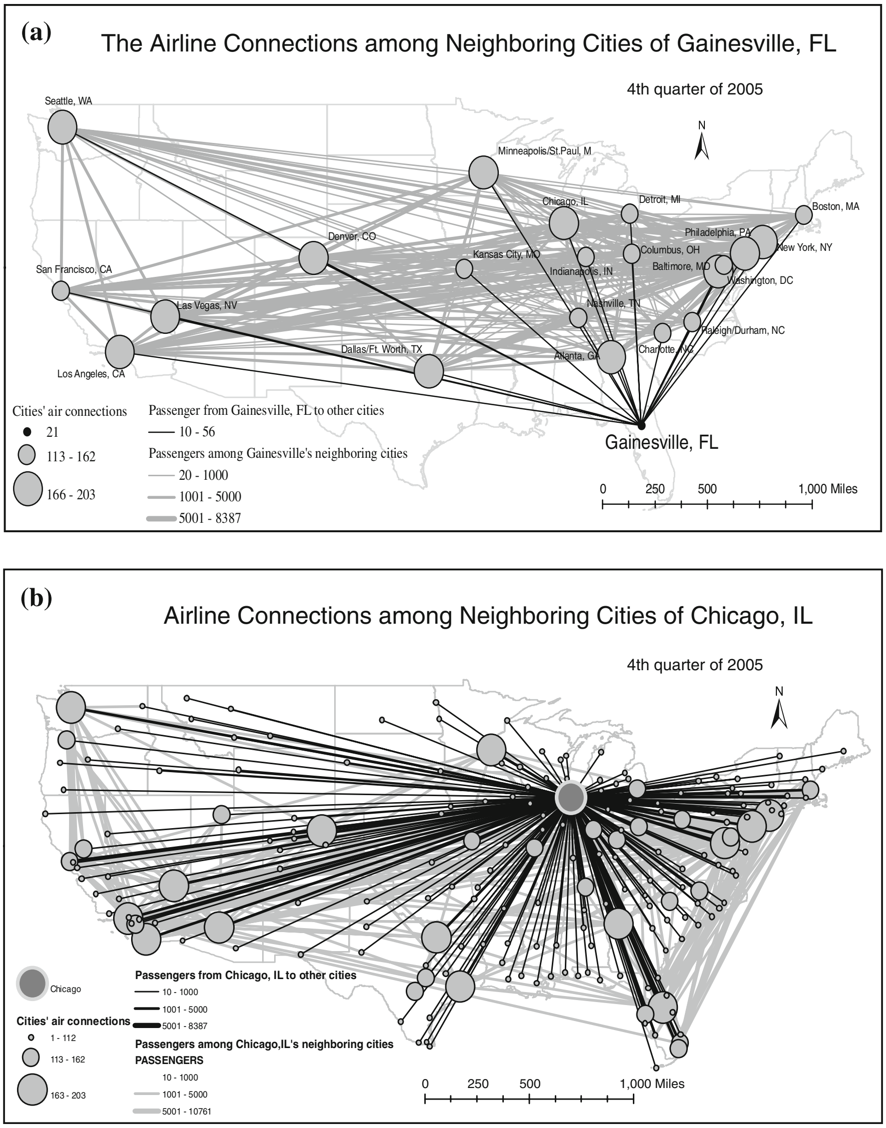


Fig. 8 The airline connections among the neighbourhood of a small-degree city, Gainesville, FL (**a**) and a large-degree city, Chicago, IL (**b**)

Topological and weighted degree correlations

A further property of complex networks investigates how vertices of different degrees connect to each other. The average degree of nearest neighbors, $k_{nn}(k)$ for vertices of degree of k, is used to investigate this degree correlation structure in complex networks (Barrat et al. 2004a). For a vertex i with degree k, the total degree of its nearest neighbors is $k_{nn,i} = \sum_{j=1}^{N} a_{ij}k_j$, where the k_j is the degree of vertex j that is one of N neighbors of vertex i, and a_{ij} is 1 if vertex i is connected to vertex j and 0 otherwise. For a network, the $k_{nn}(k)$ is the average of $k_{nn,i}$ over vertices of the same degree k; it is therefore a function of k. In a network without degree correlation, $k_{nn}(k)$ is independent of k. In a network with the presence of degree correlation, $k_{nn}(k)$ will change with degree k. When $k_{nn}(k)$increases with k, vertices with high degree tend to connect to high degree vertices. This property is also referred to in physics and social science as "assortative mixing" (Bagler 2008; Barrat et al. 2004a). In contrast, the high degree vertices have the tendency to connect to low degree vertices when $k_{nn}(k)$ has a decreasing behavior. This property is referred to as "dissortative mixing".

In order to take into account the relative weights on connections in a weighted network, the weighted average nearest neighbor degree (Barrat et al. 2004a) for vertex i is defined as

$$k^w_{nn,i} = \frac{1}{s_i}\sum_{j=1}^{N} a_{ij} w_{ij} k_j \qquad (2)$$

where $k^w_{nn,i}$ is the weighted nearest neighbor degree for vertex i; S_i is the strength of vertex i, and $s_i = \sum_j w_{ij}$; a_{ij} is 1 if vertex i is connected to vertex j and 0 otherwise; w_{ij} is the weight between vertex i and vertex j, and it could be distance, passengers, or fare; and k_j is the degree of vertex j that is one of the N neighbors of vertex i. This definition is a local weighted average of the nearest neighbor degree according to the normalized weight of the connecting edges, w_{ij}/s_i (Barrat et al. 2004a). For a network, $k^w_{nn,i}(k)$ is the average weighted nearest neighbor degree over vertices of degree k.

The comparison of $k^w_{nn,i}$ and $k_{nn,i}$ of a vertex or a network reveals the relationship between weights and topology in degree correlation structure of networks. For a vertex i, if $k^w_{nn,i} > k_{nn,i}$, its large weight edges tend to connect to high degree neighbors; if $k^w_{nn,i} < k_{nn,i}$, its small weight edges tend to link high degree neighbors or large weight edges tend to connect to low degree neighbors. The $k^w_{nn,i}$ thus measures the effective affinity to connect with high or low degree neighbors according to their weights (Barrat et al. 2004a). For a weighted network, the behavior of the function $k^w_{nn}(k)$, the same as its topological counterpart, can disclose the weighted assortative or dissortative properties.

We calculated the average nearest neighbor degree $k_{nn,i}$ and the passenger, distance, and fare weighted average nearest neighbor degrees, $k^{Pass}_{nn,i}$, $k^{Dist}_{nn,i}$, and $k^{Fare}_{nn,i}$, for U.S. passenger air transportation networks spanning the period from the first quarter of 2002 to the fourth quarter of 2005. The $k_{nn,i}$, $k^{Pass}_{nn,i}$, $k^{Dist}_{nn,i}$, and $k^{Fare}_{nn,i}$ have a consistent decaying behavior for all the networks. Figure 9 illustrates our result for the air transportation network of the fourth quarter of 2005. The results showed that $k_{nn,i}$ is independent on k when $k < 30$. However, $k_{nn,i}$ shows a decay behavior with the increase of k when $k \geq 30$ (Fig. 9); U.S. passenger air transportation network exhibit the dissortative mixing, i.e., large degree cities link to a large number of small degree neighboring cities. This result is consistent with that of India's airport network (Bagler 2008), but different from that of the worldwide airport network (Barrat et al. 2004a), in which the assortative behavior was observed. The $k^{Pass}_{nn,i}$, $k^{Dist}_{nn,i}$, and $k^{Fare}_{nn,i}$ also show an apparent decay behavior when $k \geq 30$ (Fig. 9). But the decay of $k^{Pass}_{nn,i}$ is much slower than $k_{nn,i}$. When the degree k increases, $k_{nn,i}$ drops quickly, but $k^{Pass}_{nn,i}$ remains very high and much larger than $k_{nn,i}$. This result implies that the high degree cities connect to a large number of low degree cities, but a few high volume traffic connections, which connect to a few high degree vertices, balance the calculation of $k^{Pass}_{nn,i}$ and results in $k^{Pass}_{nn,i}$ remaining relatively high. The comparison between the behaviors of $k^{Pass}_{nn,i}$ and $k_{nn,i}$ further reflects the hub-and-spoke structure of the U.S. passenger air transportation network, in which the hub cities connect to a large number of small degree spoke cities, and the high capacity interhub connections link to other high degree hubs identified by the $k^{Pass}_{nn,i} > k_{nn,i}$. This is the characteristic of the hub-and-spoke network structures (O'Kelly and Bryan 1998). The dissortative mixing concept is actually embedded in the hub-and-spoke structure of the airline network.

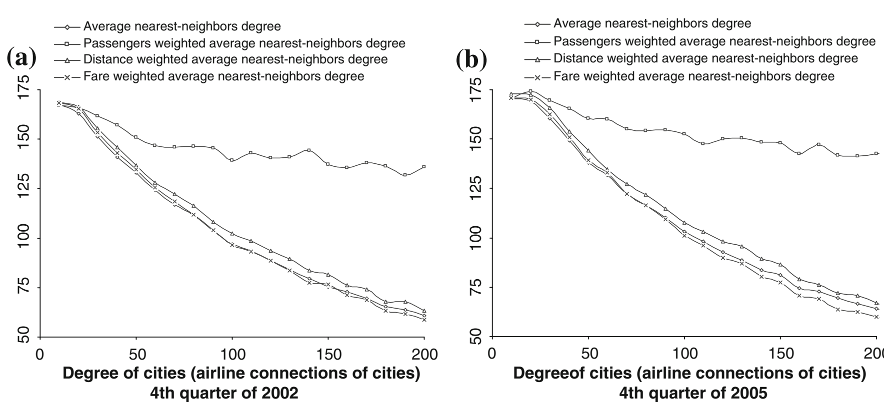


Fig. 9 The variation of topological and weighted average nearest-neighbors degrees with city degree for the U.S. passenger air transportation network in the fourth quarter of 2002 (**a**) and 2005 (**b**)

In addition, when $k \geq 30$, $k_{nn,i}^{Dist}$ and $k_{nn,i}^{Fare}$ have a very similar decaying behavior as $k_{nn,i}$. However, with the increase of k, $k_{nn,i}^{Dist}$ is greater than $k_{nn,i}$ but $k_{nn,i}^{Fare}$ tend to be increasingly smaller than $k_{nn,i}$. This behavior implies that high degree cities tend to connect to other high degree cities over long distances and with relative low fares and it is consistent with what we have observed from the decaying behavior of clustering coefficients. The effect is pronounced when comparing the fourth quarter of 2005 (Fig. 9b) to the fourth quarter of 2002 (Fig. 9a).

Betweenness centrality

A vertexs' degree is a measure of vertex centrality in terms of its local connectivity. A global measure of the centrality of vertices or edges characterizing their betweenness has been used to characterize the role of vertices or edges in control of the communication in networks (Brandes 2001, 2008; Freeman 1977; Newman 2001; Newman and Girvan 2004). In essence, the betweenness centrality for a vertex or edge is to count the number of shortest paths going through them. The betweenness centrality of a vertex v or an edge e is defined respectively as

$$cv_b(v) = \sum_{s \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}} \tag{3}$$

$$ce_b(e) = \sum_{s \neq t} \frac{\sigma_{st}(e)}{\sigma_{st}} \tag{4}$$

where σ_{st} represent is the number of shortest paths between vertex s to vertex t (s and t are any vertices in the network); $\sigma_{st}(v)$ or $\sigma_{st}(e)$ is the number of shortest paths between vertex s to vertex t and passing through vertex v or edge e. The high betweenness centrality means that there are more shortest paths going through the vertex or edge. It gives a theoretical estimation of optimal "traffic" on the vertices or edges according to the topological structure of networks. In general, high degree vertices have larger number of topological connections and they usually have high betweenness centrality. But it is not always the case. It should noted that high-degree vertices do not necessarily have high betweenness centrality. For example, in world-wide airport network, some gateway airports linking different countries have high betweenness centrality but small degree of connecticity and this anomaly usually signals the existence of community structure in the network (Guimera and Amaral 2004). In the U.S. network, the anomaly between betweenness centrality and degree (Fig. 10a) is not as severe as found in the world-wide airport network (Guimera et al. 2005). The cumulative probability distribution of edge betweenness (Fig. 10b) and vertex between (Fig. 10c) follow power law distribution.

A weighted network model

Based on our analysis of the structure of the network provided in previous sections and a general model for the evolution of weighted networks (Barrat et al. 2004b, 2004c; Guimera and Amaral 2004), we simulated the network evolution with a weighted network model that was able to reconstruct the major structural properties of the U.S. intercity airline network system. Although influenced by complex demographic and economic mechanisms, the evolution of U.S. air transportation network is subjected to certain regularities. In the early stage of the network

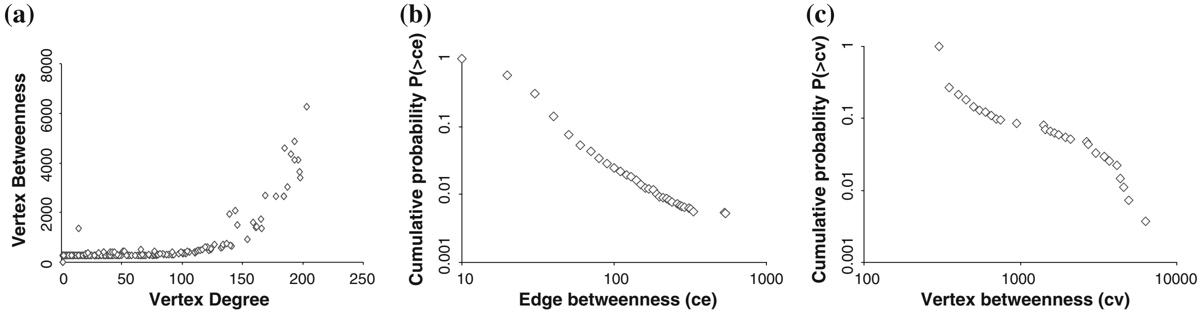


Fig. 10 (**a**) Scatter plot of vertex betweenness verse vertex degree; (**b**) cumulative probability of edge betweenness; (**c**) cumulative probability of vertex betweenness for the passenger air transportation network for the 4th quarter of 2005

evolution, the airports were only established at big cities that had large market potential. These airports have now become the major hub airports. Newly constructed airports are more likely to link to the large airports (with high market potential or handling high air traffic) and preferably as close as possible. When new airports are added to the network, they also attract more air traffic to the entire network. These rules were included mathematically in our modelling process.

We use the actual vertices in the network of the 4th quarter of 2005 to construct the network model. We sort the vertices according to their degrees, and assume that the vertices enter the model according to the order number (Fig. 11). The highest degree vertex (order number is 1) enters the model first. The degree of vertices is logarithmically related to their order number ($m = 295.54 - 53.526\ln(x)$ where x is the order number of vertex entering the model and m is the degree of this vertex).

The network model starts with a seed network of 10 fully connected vertices. These vertices are the 10 highest degree vertices in the network of the 4th quarter of 2005. Afterwards, the order number of the first vertex entering the model is 11. At each time step, a vertex will be added to the network model according to their order number in Fig. 11 and two evolutionary mechanisms will be simulated:

(1) A rule termed spatially constrained strength driven attachment (Barrat et al. 2005). A new vertex chosen from the rest of the vertices in the network of the 4th quarter of 2005 will be added to the model, and it will link to m ($m = 295.54 - 53.526\ln(x)$) existing vertices, which are chosen according to a probability in favor of nearest (in Euclidean distance) vertex with largest strength. The probability can be represented as follows,

$$\mathrm{Prob}_i = \frac{s_i \exp(-d_{ni}/r_c)}{\sum s_j \exp(-d_{nj}/r_c)}, \; j \in V(i) \tag{5}$$

where Prob_i is the probability for vertex i to be chosen to link to the new vertex; $V(i)$ represents all neighbors of vertex i and S_i and S_j represent respectively the strengths of vertex i and vertex j; r_c is a typical distance(we set r_c = 500 miles) and d_{bi} is the Euclidean distance between vertex n and vertex i. The consequences of this rule are that new vertices are more likely to link to the nearest vertices with largest strength. It implies that the new airline connections favor nearby cities handling large traffic volumes.

(2) A rule documenting the local spillover of stimulated traffic. Each newly established edge (n,i) is assigned a weight w_0 (we assign w_0 as 1) . The introduction of this new vertex and edges will attract additional traffic δ_i to the system through vertex i and it not only updates the strength of vertex i to $s_i + w_0 + \delta_i$ but also spillovers to other vertices and edges in the network (Fig. 12). Here we only assume that δ_i will only spillovers to the immediate neighbors of vertex i proportional to their weights w_{ij}, as follows,

$$w_{ij} = w_{ij} + \Delta w_{ij} \tag{6}$$

$$\Delta w_{ij} = \delta_i \frac{w_{ij}}{s_i} \tag{7}$$

After rule#2 is simulated, the network growth process goes back to add another new vertex and links until the final network size is reached.

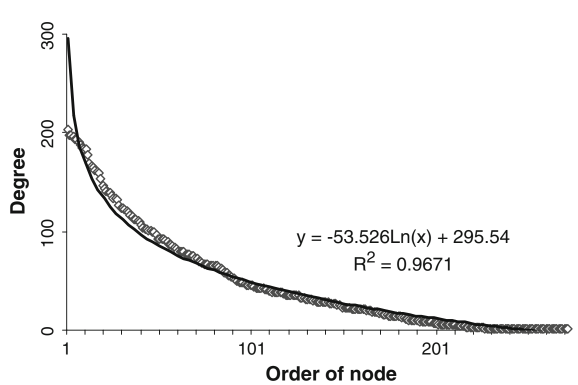


Fig. 11 Scatter plot of vertex degree versus order number entering the model and their estimated logarithmic relation

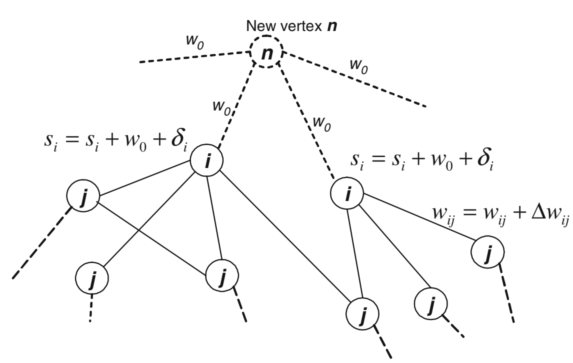


Fig. 12 Illustration of a new vertex n to be linked to the existing vertices (i_s) in the network model. Each vertex i has a number of neighboring vertices (j_s)

Reasonable heterogeneity in the value of δ_i, w_0, r_c could produce networks with different structural properties. For example, different values on r_c or different decay functions could produce network with different degree of assortaticity and clustering (Barrat et al. 2005; Guimera and Amaral 2004). Our study is focused on the evolution mechanism rather than how these parameters affect the network properties. Our model has been run for 100 times. The average results (Fig. 13) show that the model is able to reconstruct the major structural properties of the U.S. intercity passenger air transportation networks.

Summary and conclusions

This study has investigated the structural properties of the U.S. air transportation system, which is a complex socio-technical networked system that is the product of the co-evolution of dynamic operational airline business strategies with a relatively static geospatial infrastructure. Our primary findings have demonstrated that the U.S. intercity passenger airline network is the most developed and mature air transportation system studied to date. The complex evolution of this socio-technical system reflects the historical distribution of urban locations, adaptation to dynamic spatial and seasonal

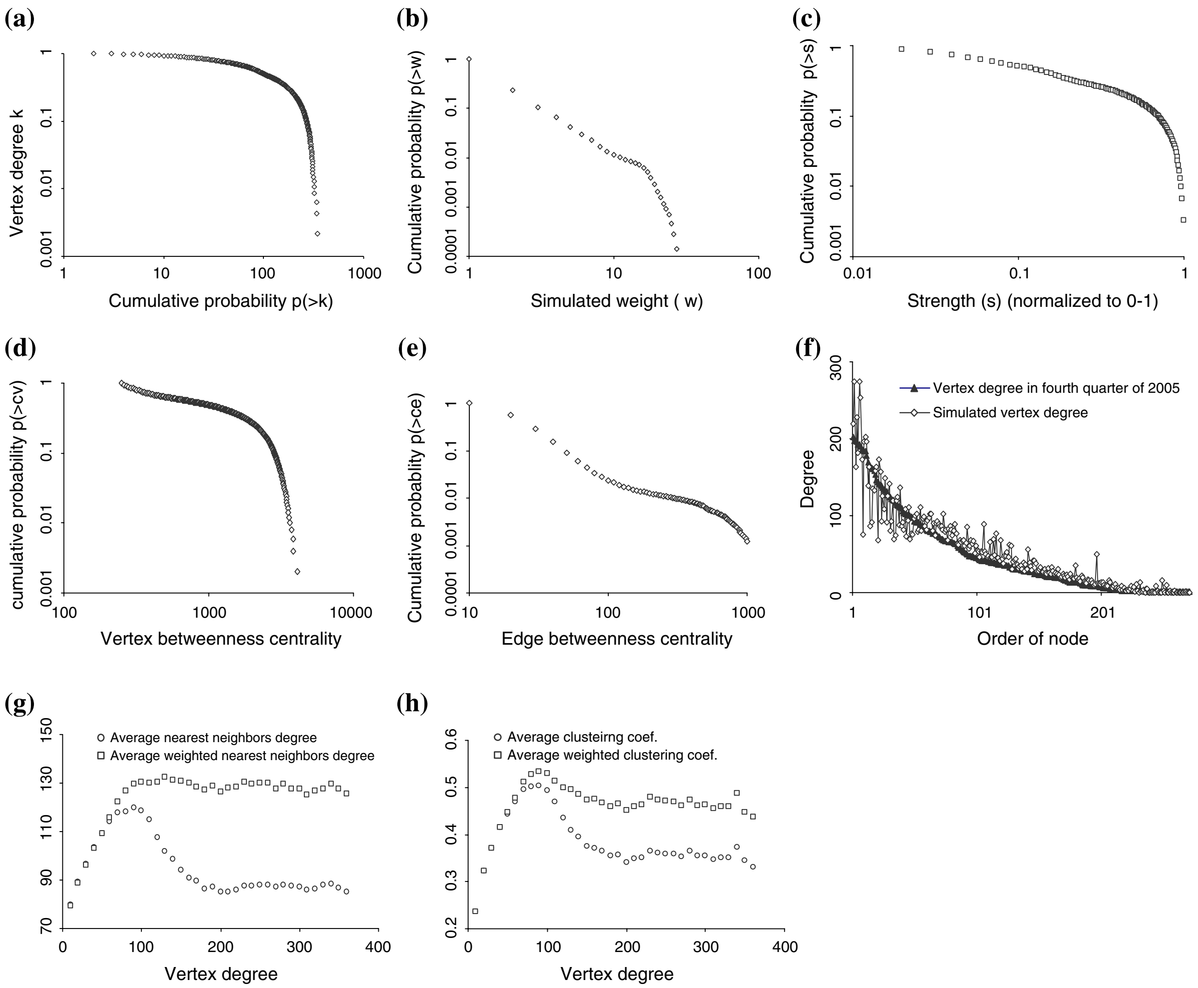


Fig. 13 The structural properties of simulated networks (average over 100 simulated networks): (**a**) cumulative degree distribution; (**b**) cumulative weight distribution; (**c**) cumulative strength distribution; (**d**) cumulative distribution of vertex betweenness; (**e**) cumulative distribution of edge betweenness; (**f**) comparison between the simulated vertex and the actual vertex degree; (**g**) average (weighted) nearest neighbors degree verse degree; (**h**) average (weighted) clustering coefficient verse degree

travel demand factors, and the highly competitive nature of the airline industry.

We documented that the U.S. intercity passenger air transportation network is a small-world, scale-free network. A rich-club phenomenon was observed in which large degree cities tend to form interconnections among each other to sustain high volumes of traffic. A dissortative mixing pattern reflects the tendency of large degree cities to link to a large number of small degree cities to enhance total passenger demand in a cost effective and infrastructure efficient design. We also demonstrated that a weighted network model, coupling the topological and weighting dynamics, is able to reconstruct the structural properties of the U.S. intercity air transportation network.

System interactions of social, geopolitical, and economic interactions and fluctuations enhance the complexity of the U.S. air transportation network. This complexity in the network topology and dynamics cannot be adequately characterized by using a hub-and-spoke analysis. Our results demonstrated that recent advances in network science offer an improved quantitative characterization of the large scale structural properties associated with U.S. airline geospatial networks. Although we were able to reconstruct the major structural properties of the U.S. intercity passenger air transportation network, our model had only limited capabilities for characterizing the dynamics of other weights like distance and fares. Further integration of social and economic rules governing airline operations remain as an important challenge for achieving a comprehensive understanding of the socio-technical dynamics associated with the U.S. airline system.

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