Filtration of ideal gas and C_5H_{12}

1 The Physics

We simulate 2d flow of ideal gas and C_5H_{12} through a porous medium with the help of Darcy's law:

$$\boldsymbol{v}_i = \frac{1}{\mu}\hat{K} \cdot f(s) \cdot \nabla P \tag{1}$$

where $\hat{K}$, the specific permeability. It depends only on the geometry of the medium. We assume isotropy of space, so K is a scalar. μ is the dynamic viscosity.

As an approximation, $f(s) = s^2$ for the first component, and $f(s) = (1-s)^2$ for the second.

The continuity equation for each component becomes:

$$\varphi\frac{\partial \rho_i}{\partial t} + div(\rho_i \boldsymbol{v}_i) = 0 \tag{2}$$

where $\rho_i = \frac{m_i}{V}$.

We use the Tait equation to relate liquid density to pressure:

$$\frac{\rho - \rho_0}{\rho} = C\log_{10}\frac{B+P}{B+P_0} \tag{3}$$

where $C = 0.2105$, $\rho_0 = \frac{1}{67.28\frac{m^3}{mol}}$, $P_0 = 0.1MPa$, $B = 35MPa$, in the case of C_5H_{12}.

Ideal gas equation of state:

$$P = \frac{RT}{\mu}\rho \tag{4}$$

2 Boundary Conditions

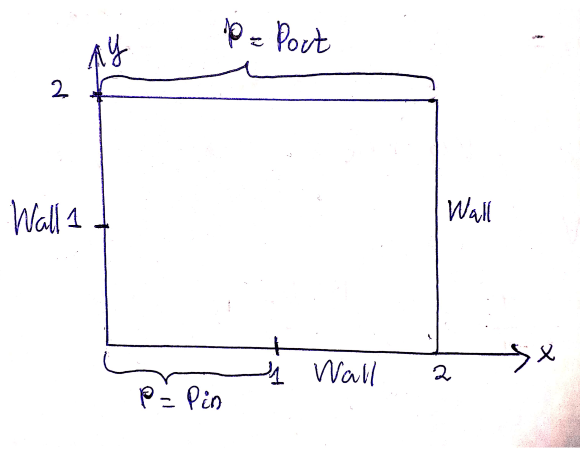

Figure 1: Boundary Conditions

On pressure:

$$\begin{cases} P = P_{in} & \text{at } y = 0 \text{ and } x \in [0, 1] \\ P = P_{out} & \text{at } y = 2 \\ \frac{\partial P}{\partial x} = 0 & \text{at } x = 0, 2 \\ \frac{\partial P}{\partial y} = 0 & \text{at } y = 0 \text{ and } x \in [1, 2] \end{cases}$$

On velocities:

$$\begin{cases} u = 0 & \text{at } x = 0, 2 \\ v = 0 & \text{at } y = 0 \text{ and } x \in [1, 2] \\ \frac{\partial v}{\partial y} = 0 & \text{at } y = 2 \\ \frac{\partial v}{\partial y} = 0 & \text{at } y = 0 \text{ and } x \in [0, 1] \end{cases}$$

On density - ?
On saturation - ?

3 Algorithm

Euler method for discretization with respect to time. (TODO: upgrade to predictor-corrector).

Second order scheme in space, with the use of ghost cells.

1. Calculate densities for each component using EOS.
2. Find the pressure and saturation with the help of binary search.
3. Use Darcy's law to calculate velocities.