Simulation of Position Control for X-Y Table with Nonlinear Friction and Backlash

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***Abstract*—This paper presents a position controller for X-Y table with nonlinearities. The nonlinear elements include coulomb friction, viscous friction and backlash. A design of fuzzy-PD position controller is presented. The validity and effectiveness of the proposed controller is verified by simulations.**

Keywords-X-Y table; friction; backlash; fuzzy control

I. INTRODUCTION

Motion system with X-Y table has been widely used in the fields of machining, plotting, sculpturing, and so on. The motion system includes two perpendicular mechanical shafts driven by servo motors through leading screws [3]. Each shaft can drive the table in one-dimensional movement and the synthesis of the both shafts can obtain a two-dimensional motion. In some applications, X-Y table can be considered as a linear system and traditional PID controller can perform well for the system. Nowadays, many applications need motion control systems with high precision and fast positioning. The conventional control strategy can not guarantee the high control performance due to nonlinear characteristics of friction in the mechanism. Several model-based friction compensation schemes have been studied so far. However, designing a controller using nonlinear friction model can be difficult due to its variable model parameters and also the nonlinear effect in practical application [4]. In this paper, a fuzzy-PD strategy for control of X-Y table with friction is investigated. The fuzzy-PD control is a non-model control method and can be implemented easily in practice. The validity of the proposed control algorithm is demonstrated by simulation based upon the Stribeck friction model.

II. MODELING OF X-Y TABLE

Both of the shafts in X-Y table have the similar motion characteristics, so we can take a single one to investigate for simplicity.

A. Modeling of servo system

Servo motor controlled by the current vector scheme with zero d-component of stator current is frequently used in industrial X-Y table applications [1]. The model of the servo motor can be expressed as

$$u = L\dot{i} + R_m i + \dot{E} \tag{1}$$

$$u = K_{cp}[K_{vp}(V_c - \dot{\theta}) - i] \tag{2}$$

$$L\dot{i} = K_{cp}[K_{vp}(V_c - \dot{\theta}) - i] - K_e\dot{\theta} - R_m i \tag{3}$$

where, L is the armature inductance, R_m is stator resistor, $\dot{E}$ is back EMF, K_{vp} is velocity loop proportional gain, K_{cp} is current loop proportional gain, K_e is back EMF coefficient, θ is angular displacement, $\dot{\theta}$ is angular velocity and V_c is input reference velocity.

The equation for torque equilibrium is

$$T_m = (J_r + J_{eL})\ddot{\theta} + \frac{kK_{bs}}{2\pi}(K_{bs}\theta - x) \tag{4}$$

In the above equation, the electromagnetic torque of the motor T_m is proportional to the stator current, that is $T_m = K_t i$, K_t is the motor torque coefficient. J_r is the moment of inertia of motor rotor and J_{eL} is the table inertia. $\ddot{\theta}$ is the angular acceleration. k is stiffness coefficient. K_{bs} is the screw-pitch and x is the table axial displacement.

Considering the kinetic characteristics, the kinetic balance equation is expressed by

$$k(K_{bs}\theta - x) = m\ddot{x} + B\dot{x} + F\,\mathrm{sgn}(\dot{x}) \tag{5}$$

where, B is the damping constant, F is the friction force of lead real, m is the quality of the table, $\dot{x}$ is the velocity and $\ddot{x}$ is the acceleration, $sgn(\dot{x})$ is sign function , which only depends on the direction of $\dot{x}$.

The relationship between voltage and current of motor stator can be expressed in a transfer function. Similarly, we can also get transfer function for the kinetic equation (5). The transfer functions are obtained as

$$G_1(s) = \frac{1}{\mathrm{Ls} + R_m} = \frac{I(s)}{U(s)} \tag{6}$$

$$G_2(s) = \frac{1}{\mathrm{ms} + B} = \frac{V(s)}{F(s)} \tag{7}$$

B. Friction Model

The Stribeck model is adopted to describe the nonlinear friction behavior in the X-Y table motion system. The friction force is a function of the velocity, which is described as

$$F(\dot{q}) = \{[F_c + (F_s - F_c)e^{-(\dot{q}/\dot{q}_s)^2}] + Q\dot{q}\}\,\mathrm{sgn}(\dot{q}) \tag{8}$$

where F_s is the maximum static friction, F_c is the coulomb friction force, $\dot{q}$ is the input velocity, Q is the viscous

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coefficient and $\dot{q}_s$ is the Stribeck critical velocity, which is a experiential parameter.

C. Backlash Model

Modern gear trains in X-Y table may be manufactured with smaller tolerances. However, friction and wear can reduce tolerances and then introduces appreciable backlash where originally it might have been negligible.

The backlash nonlinearity is shown in Figure 1. A mathematical model is given by

$$x_{out} = \begin{cases} x_{in} - b, & \dot{x}_{in} > 0 \, and \, \dot{x}_{out} > 0 \\ x_{out}(t-), & others \\ x_{in} + b, & \dot{x}_{in} < 0 \, and \, \dot{x}_{out} < 0 \end{cases} \tag{9}$$

where x_{in} denotes the feed motor displacement, x_{out} is the feed shaft displacement, b is the clearance on the deadband, $x_{out}(t-)$ is the previous x_{out}. The motion of shaft displacement x_{in} is delayed from motion of motor displacement x_{out} when the x_{out} changes its direction.

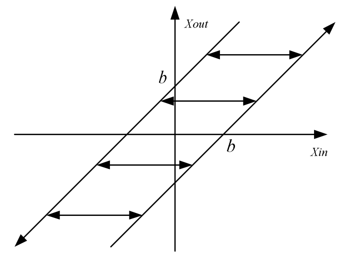

Figure 1. Basklash nonliearity

The controlled system model containing friction and backlash can be obtained from (2) to (9), as shown in Figure 2.

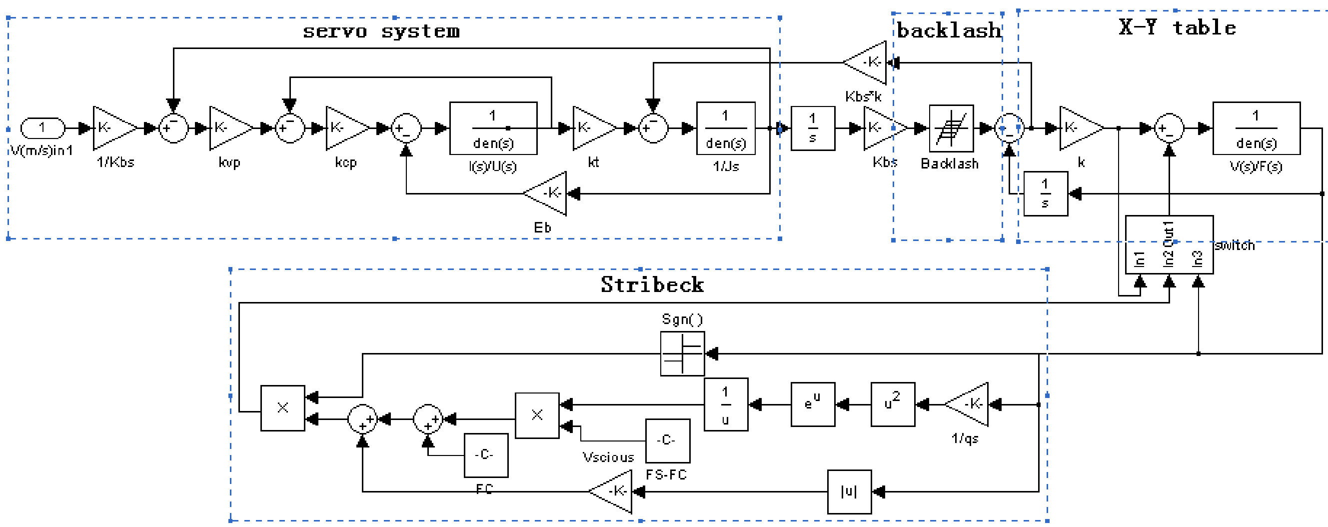

Figure 2. Controlled system model

III. FUZZY-PD CONTROLLER

The design of position loop is very important to the stability and high speed performance of the servo system, so it must be quick-run, high-precise and no over shoot [6]. Traditional way (PID) is difficult to satisfy the need of high performance, especially, when there is an external disturbance or a nonlinear element in system. In this part, a position controller called fuzzy-PD controller is suggested [2], based on the model referred above.

The structure of the control system is illustrated in Figure 3. The controlled X-Y table model contains the motor dynamics, friction and backlash nonlinearities described above. The fuzzy-PD controller is in the potion control loop and the current feedback control and velocity control loop are implemented within motor servo driver. Therefore, the velocity command is produced form the output of the fuzzy-PD controller. In fact, the velocity command signal is switched between a fuzzy controller and a PD controller depending on the position error and the change in error in the dynamic control process. Thus switching strategy is the key issue in such combined type controller because the proper switching can achieve satisfied dynamic behaviors.

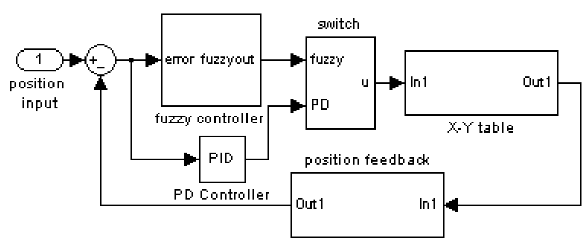

Figure 3. Block diagram of fuzzy-PD control System

In the fuzzy-PD control system shown in Figure 3, there is a "switch" in the controller, which is used to regulate the output weight of the fuzzy controller and the PD controller. Here, the PD controller is used to do a coarse tuning and the fuzzy controller is used to a fine tuning in the regulating process. So the point for switching the two controllers should be carefully designed. A weighted average method is adopted to make the switching according to the position error range, which is described as

$$u(e)=\begin{cases} u_{PD}(e) & |e|\geq e_2 \\ u_{FZ}(e,\Delta e) & |e|\leq e_1 \\ u_{PD}(e)\times a+u_{FZ}(e,\Delta e)\times(1-a) & e_1\leq|e|\leq e_2 \end{cases} \quad (10)$$

where e is the position error, Δe is the position error in change, $u_{PD}(e)$ is the output of the PD controller, $u_{FZ}(e)$ is the output of fuzzy controller, e_1 is lower error limit and e_2 is the higher error limit. a is a weight coefficient, which depends on the value of the position error, as shown in (11),

$$a=\frac{e^{k_e\times|e|}-e^{k_e\times e_1}}{e^{k_e\times e_2}-e^{k_e\times e_1}} \quad (11)$$

where k_e is a scale factor.

The switching function in M language is shown in Figure 4.

```
if abs(u(2))>=e2
sys=u(3);
elseif abs(u(2))>e1 && abs(u(2))<=e2
a=(exp(k*abs(u(2)))-exp(k*e1))
/(exp(k*e2)-exp(k*e1));
sys=u(1)*a+u(3)*(1-a);
else abs(u(2))<=e1
sys=u(1);
```

Figure 4. M code of switching

Detail of the fuzzy controller is shown in Figure 5, in which input is the position error and the change of position error. Here, K_e is the error scaling factor, K_{ec} is the scaling factor of error variance and K_u is the output scale factor of fuzzy controller. The selection of the three coefficients can influence the system control performances. When the scaling factor K_e increases, regulation speed of the system becomes fast, and the dead zone effect begins to decline, but an oversize of the scaling factor K_e can lead to a big overshoot and a long settling time, even course an oscillation. When the factor K_{ec} declines, regulation speed and change rate of steady state error increases, but if it is too small, system will become unstable. The larger K_u can decrease the rise time, but if K_u is too large, system will lose stability.

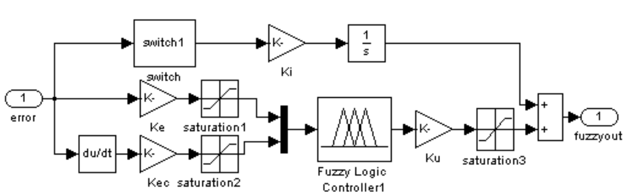

Figure 5 Schematic diagram of fuzzy controller

There is an integral element in the fuzzy controller that is used to reduce static error. Whether the integral element is introduced depends on the range of position error. If and only if $e(t)\leq e_1$, the integral element is added to the controller through a switch function, which is described as

$$s(e)=\begin{cases} 0 & e_1<|e|<e_2, |e|\geq e_2 \\ 1 & |e|\leq e_1 \end{cases} \quad (12)$$

The error e and change of the error Δe have to be quantized and make a fuzzification before fed into the fuzzy controller, the process of the fuzzification transforms e and Δe into the setting of linguistic variables. In this paper, the following linguistic variables as shown in (13) and associated membership functions are used.

$$\begin{aligned} L &= \{NB, NM, NS, ZO, PS, PM, PB\} \\ M &= \{M_{NB}, M_{NM}, M_{NS}, M_{ZO}, M_{PS}, M_{PB}\} \end{aligned} \quad (13)$$

For each linguistic value, $l\in \mathrm{L}$, we assign a pair of numbers $n_e(l)$ and $n_{\Delta e}(l)$ to the input e and Δe via the associated membership function M_l expressed as

$$n_e(l)=M_l(k_e e),\quad n_{\Delta e}(l)=M_l(k_{ce}\Delta e) \quad (14)$$

Fuzzy rules are written in the form: "if e is l_e and Δe is $l_{\Delta e}$, then u_i" l_e, $l_{\Delta e}\in L$. That is shown in (15) and the fuzzy rules are shown in TABLE I.

$$p(e,\Delta e)=\min(n_e(l_e), n_{\Delta e}(l_{\Delta e})) \quad (15)$$

For simplicity, we adopt the center of gravity defuzzification method to perform the defuzzification. The output of fuzzy controller is written as follows:

$$u_{FZ}(t)=\mu_t[e(t),\Delta e(t)]+s[e(t)]\times k_i\int e(t)\,dt \quad (16)$$

TABLE I. FUZZY RULES

		Δe						
	u	NB	NM	NS	ZO	PS	PM	PB
	NB	NB	NB	NB	NM	NM	NS	ZO
e	NM	NB	NB	NM	NM	NS	ZO	PS
	NS	NM	NM	NS	NS	ZO	ZO	ZO
	ZO	NM	NM	NS	ZO	PS	PM	PM
	PS	ZO	ZO	ZO	PS	PS	PM	PM
	PM	NS	ZO	PS	PM	PM	PB	PB
	PB	ZO	PS	PM	PM	PB	PB	PB

IV. SIMULATION OF THE SYSTEM

A simulation study for the X-Y table with nonlinear friction and backlash was performed using the models and controller described above. The parameters [5] of the models in the simulation are shown in TABLE II.

TABLE II. MODEL COEFFICIENTS

K_{vp}	K_{cp}	R_m	K_t
45A*s/rad	2V/A	1.04Ω	0.82N*m/A
E	K_{bs}	F_s	F_c
0.18V*s/rad	1.59mm/rad	26.97N	18.92N
q_s	L	J	B
0.017m/s	52.7mH	2.99kg*m^2	15KN*s/m
m	k	Q	b
48.8kg	0.4MN/m	56.6	0.002

The simulation result shown in Figure 6 is the velocity step response. Considering the friction effect, the Figure 6 a) indicates that the driving force and Stribeck force are increasing simultaneously at start and the table keeps in stationary state before the driving force reaches the maximum static friction force. Once the driving force surpasses the maximum static friction force, the table begins to move with a rising velocity and viscous friction comes into effect. When the velocity is approaching the reference value, the driving force begins to decline and finally approximately equals to the friction force. The lag velocity response caused by the nonlinear factors is evidently shown in Figure 6 b), including both friction and backlash in the velocity control system.

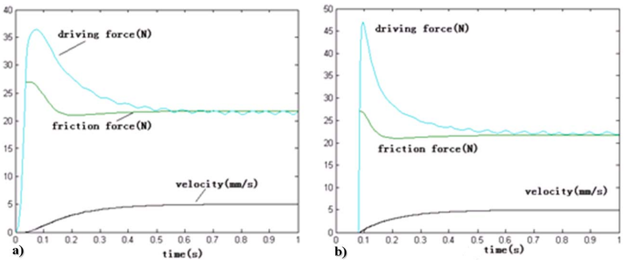

Figure 6 Velocity step responses

The positioning response of the table under the control of the proposed fuzzy-PD controller is shown in Figure 7. The driving force is lager enough to overcome the nonlinear friction and eliminate the standstill interval at the starting of the motion. Under the driving force, the table is moved with a rapid increasing velocity, and it quickly arrives at the set position without any overshoot.

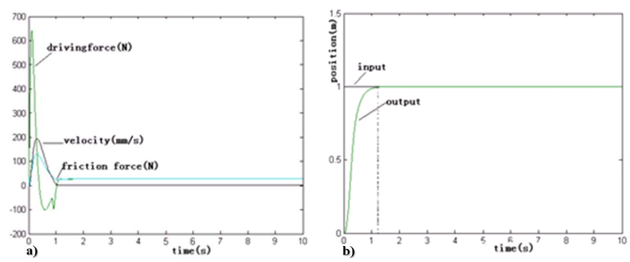

Figure 7 Position step response with the fuzzy-PD controller

The comparison of two control methods, PID position controller and the fuzzy-PD position controller in the presence of nonlinear friction and backlash generated in the X-Y table gear chain is shown in Figure 8. The position tracking responses of PID control and fuzzy-PD control is shown in Fig. 8 a) and b), respectively. The position reference is a sinusoidal function of time.

The friction and backlash influence in the position tracking control system is visible in the relatively large dead band in the point where the table changes its movement direction. The fuzzy-PD controller obviously decreases the dead band and tracking error in the position response comparing with the PID controller, as shown in Figure 8 a (1) and b (1). The delayed velocity response particularly caused by the backlash is also improved by using the fuzzy-PD controller (Figure 8 a (2) and b (2)).

The ripples on the driving force response for the two controllers are shown in Figure 8 a (3) and b (3). The fussy-PD controller produces a slightly lower force peak for the driving system due to the strategy of combination of fuzzy logical and linear control.

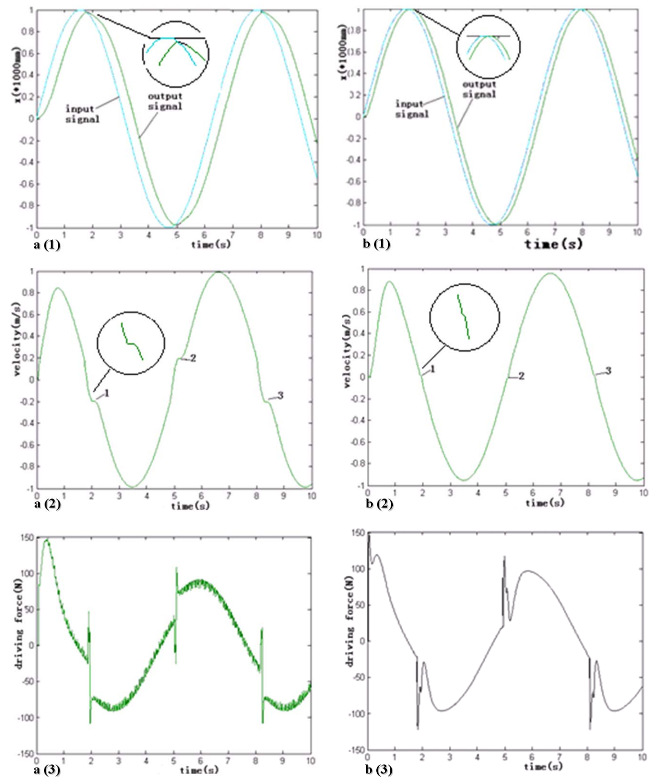

Figure 8. Position responses of PID control and fuzzy-PD control

V. CONCLUTION

A fuzzy-PD controller for position control of X-Y table with nonlinear friction and backlash was presented in this paper. The non-model based controller is designed to improve the position tracking performance in the presence of friction and backlash. The validity of the proposed control algorithm was examined by simulation.

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