Exploring portfolio returns

A four-factor model analysis of diverse risk sources

Introduction

The aim of this study is to understand how different sources of risk cause portfolios to generate different expected returns. The analysis is mainly based on the three-factor model proposed by Fama and French (1993), to which we add a fourth one, leading to a four-factor model. The four factors represent the different sources of risk. The model is applied in Section 2 to ten size-based portfolios built on UK stocks. As they are size-based, portfolios are ordered increasingly according to the size of the market capitalization of the included firms. This section allows us to identify which are the most significant sources of risk affecting the expected returns of the PTFs. Besides implementing the four-factor model, in Section 3 we also perform cross-section regressions to test if risk exposure explains how mean returns vary across PTFs. Before proceeding with any model, Section 1 reports a preliminary analysis based on the summary statistics and data visualization of the expected returns of the ten PTFs.

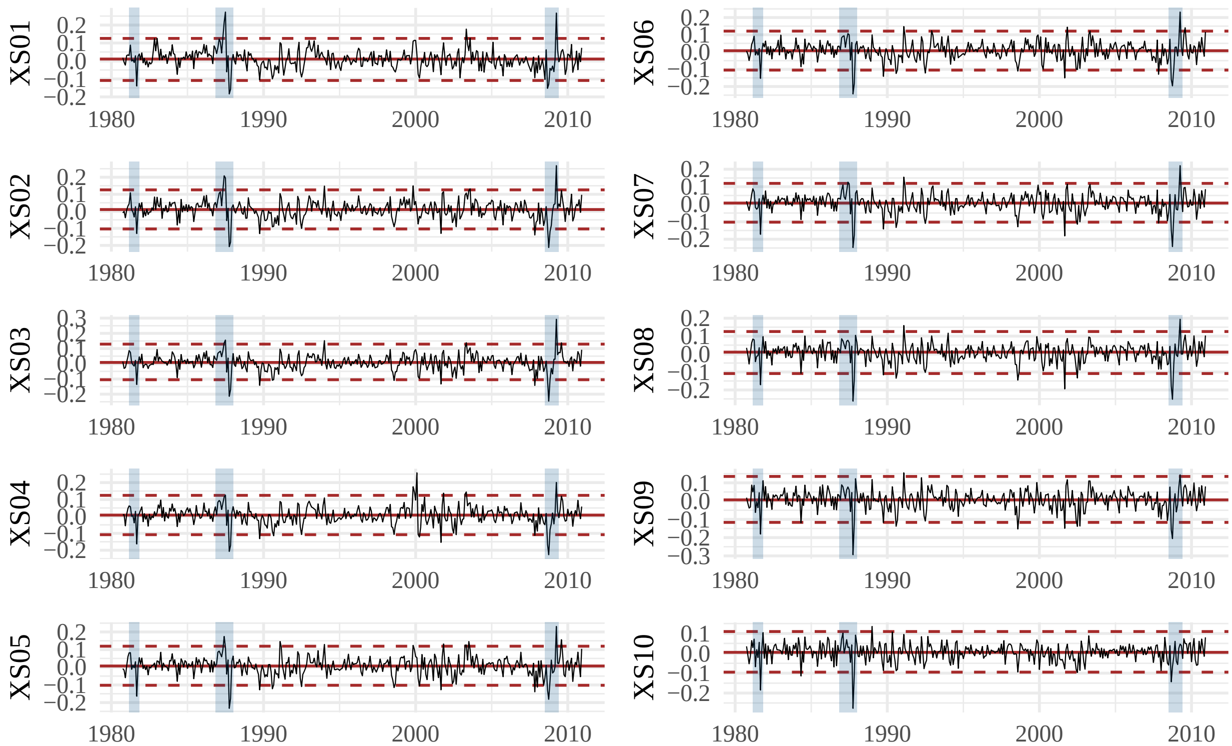
1. Data Visualization and Summary Statistics

Before starting with our analysis, we create the excess returns as the difference between the returns of the portfolios and the UK risk-free return. These will be the measures that we use throughout the whole project. As a first thing, we decide to plot the time series of the excess returns (Figure 1) to compare them between the portfolios, and more specifically to identify the outliers. From the plots, we notice that, despite having different compositions, the PTFs appear to have quite similar patterns, and in particular display prominent spikes in common time periods:

- In 1981 all PTFs drop, probably as a consequence of the 1979 Oil Crisis;
- In 1987 all PTFs exhibit a sharp decrease, anticipated by a positive spike only in the lower-cap PTFs. This could be associated with Black Monday (October 19th 1987); [1]
- In autumn 2008 excess returns plummeted in every PTF, likely due to the financial crisis, and then strongly recovered in Spring 2009 (except for the tenth PTF, which shows a smoother recovery).

[1] This makes sense because, in the UK, Black Monday was anticipated by a period of growth that has been perceived more strongly by lower-cap portfolios due to their more volatile nature.

Figure 1: Excess returns by portfolio



We move now to visualizing the summary statistics of the excess returns by portfolio, to better appreciate the differences and similarities.

Table 1 shows that as capitalization increases, the mean of excess returns decreases from PTF01 to PTF10. In particular, PTF01's mean excess return more than doubles PTF10's one. Interestingly, the standard deviation is essentially the same in all PTFs, meaning that they share a similar volatility, which is a similar risk. The only exception is the tenth PTF, whose standard deviation is 0.10 pp lower with respect to the other PTFs.

We also compute the Sharpe Ratio and we observe that it is decreasing as the market capitalization increases. Since the standard deviation (denominator of the Sharpe Ratio) is almost the same for all PFTs, the value of the ratio is determined only by the numerator (mean of PTF return minus mean of risk free rate). Indeed as the mean of returns across portfolio decreases, also the Sharpe Ratio decreases. From a financial point of view, PTF01 is the one that provides the highest return for each unit of overall risk.

Looking at the Skewness of the PTFs, we can notice that as the market capitalization increases, there is a general tendency for asymmetry to become more and more negative, implying that PTFs built on large-cap firms have many negative outliers.

Table 1: Summary statistics of the excess returns

name	mean	var	st_dev	min	q25	q50	q75	max	skewness	kurtosis	Sharpe ratio
XS01	0.011	0.003	0.054	-0.184	-0.021	0.010	0.038	0.273	0.484	6.435	0.201
XS02	0.010	0.003	0.055	-0.213	-0.017	0.011	0.040	0.268	-0.039	5.949	0.191
XS03	0.009	0.003	0.054	-0.246	-0.017	0.011	0.041	0.293	-0.269	7.071	0.163
XS04	0.008	0.003	0.055	-0.227	-0.021	0.009	0.037	0.258	-0.195	6.018	0.144
XS05	0.007	0.003	0.055	-0.233	-0.019	0.010	0.036	0.231	-0.293	5.358	0.132
XS06	0.008	0.003	0.055	-0.243	-0.020	0.011	0.036	0.232	-0.514	5.603	0.140
XS07	0.007	0.003	0.054	-0.247	-0.019	0.010	0.036	0.221	-0.771	6.213	0.124
XS08	0.007	0.003	0.054	-0.262	-0.019	0.014	0.039	0.194	-1.016	6.564	0.122
XS09	0.007	0.003	0.055	-0.294	-0.022	0.014	0.041	0.155	-0.933	6.053	0.122
XS10	0.005	0.002	0.045	-0.277	-0.018	0.010	0.033	0.136	-1.037	7.318	0.111

These last aspects can be appreciated even more in the boxplots in Figure 2. Then 1^{st}, 2^{nd} and 3^{rd} quartiles are quite aligned in between the portfolios. Still, the outliers are not: going from small to large-cap portfolios, the positive outliers are decreasing while the negative ones are increasing, in terms of both number and magnitude.

Figure 2: Distribution of excess returns by portfolio

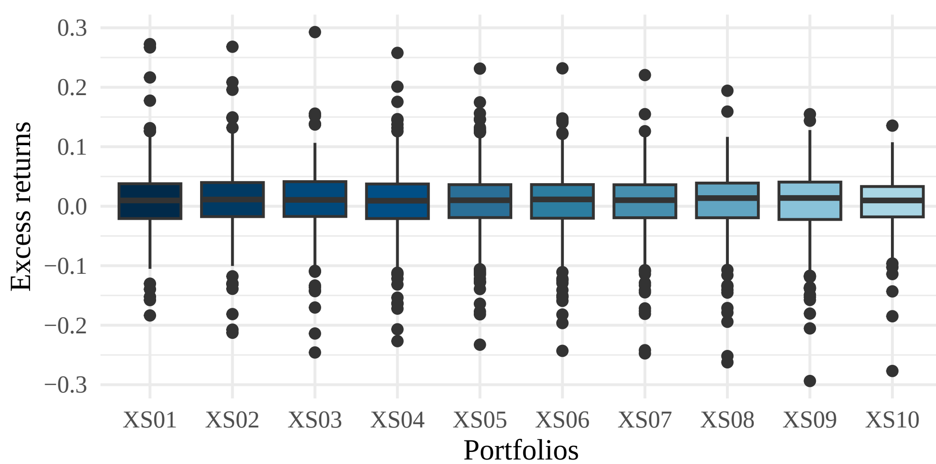
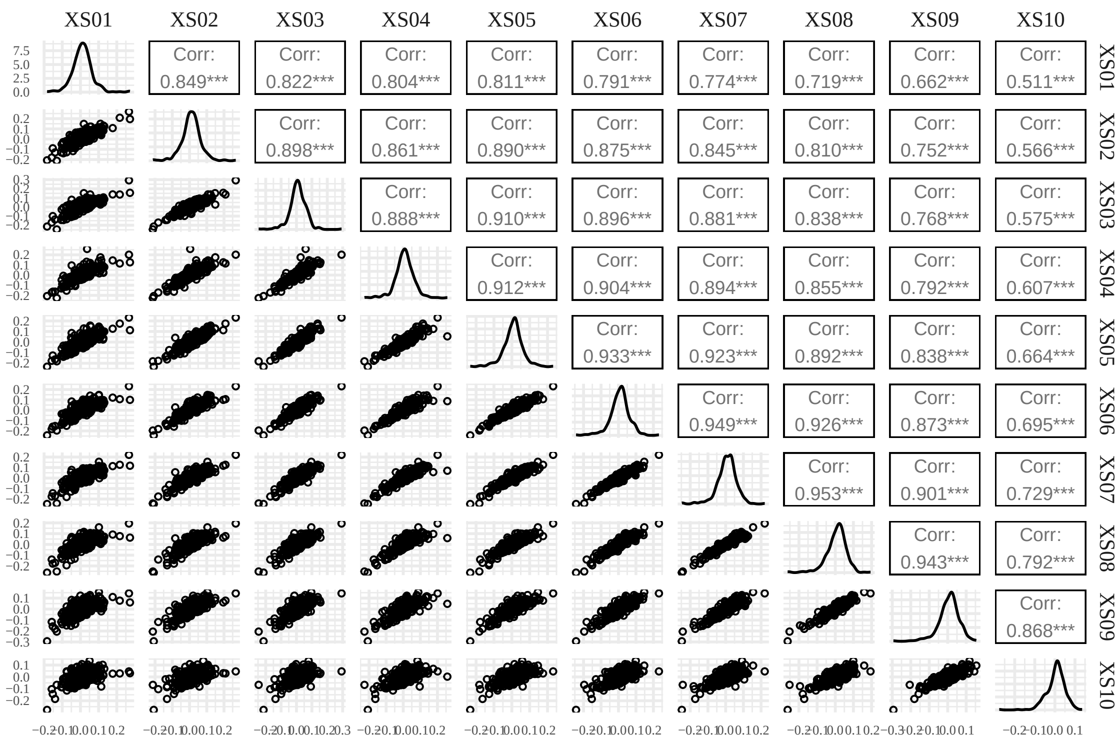


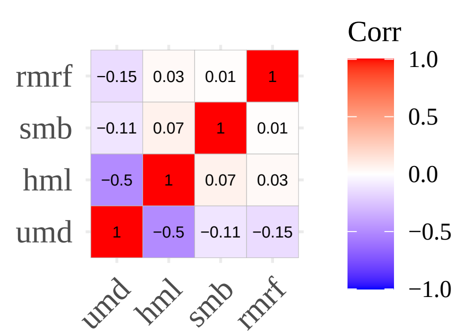
Figure 3 explains the correlation between PTFs' excess returns, both graphically and numerically, and plots their marginal distribution on the main diagonal. All PTFs are positively and significatively correlated and correlation is high in magnitude. The only exception is the tenth PTF, whose excess returns show a lower correlation with the other PTFs.

Figure 3: Correlations between portfolios' excess returns



Following Fama and French (1993)'s hints, in Figure 4 we have analysed the correlation between the explanatory variables, that are the risk factors. As found by the authors, *smb* and *hml* show a very small correlation. This suggests that the two factors have been built in such a way that the difference represented by *smb* can be considered free of the influence of BE/ME, and, similarly, *hml* can be considered free of the influence of the size factor[2]. As expected, *smb* and *hml* are also uncorrelated with *rmrf*. What is interesting to observe, instead, is that *umd* is negatively correlated with *hml* and the correlation is notable in terms of magnitude. Therefore this means that the two factors capture the same parts of variation.

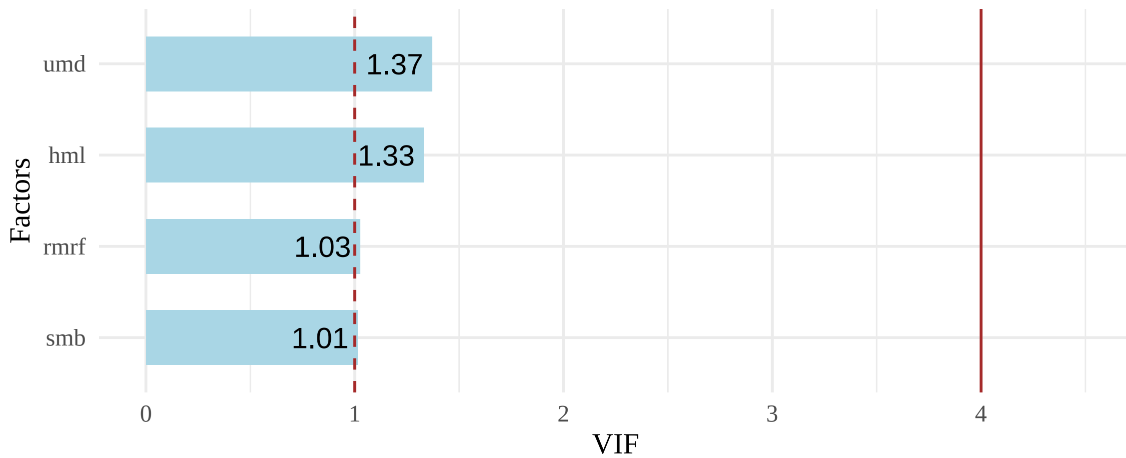
Figure 4: Correlations between factors



[2]Fama and French (1993) found a small negative correlation, while we found a small positive correlation, but results are extremely close in terms of magnitude, and this allows us to derive the same conclusions.

But as we can see in Figure 5 the variance inflation factors are all very low (close to 1, which is the minimum possible value) and don't even approach the threshold value of 4. Therefore we can conclude that there are no multicollinearity problems.

Figure 5: Variance inflation factor



For doubt's sake, we have also plotted the seasonality of the portfolios and discovered there are no such patterns, therefore we are not including the relative plots for brevity.

2. Multifactor Regressions

As anticipated, the following analysis is based on the three-factor model proposed by Fama and French (1993), which says that the expected return on a portfolio in excess of the risk-free rate is explained by the sensitivity of its return to three factors:[3]

1. The excess return on a broad market portfolio, that is the difference between the stock market return and the risk-free return (*rmrf*);
2. The returns on a size factor portfolio, that is long small-cap stocks and short large-cap stocks (*smb*);
3. The returns on a value portfolio, that is long stocks with high book-to-market value and short stocks with low book-to-market value (*hml*).

To which we add the returns on a momentum factor portfolio, that is long recent winners and short recent losers, in cumulative return terms (*umd*).

This leads to a four-factor model that can be represented as follows:

$$r_{it} = \alpha_i + \beta_i^{rmrf} RMRF_t + \beta_i^{smb} SMB_t + \beta_i^{hml} HML_t + \beta_i^{umd} UMD_t + \epsilon_{it}$$

[3] Fama and French (1996).

The current section presents the results obtained by applying the four-factor model to the ten portfolios using the time-series regression approach.[4]

As highlighted by Fama and French (1993), slopes and R-squared values give evidence of whether different risk factors capture common variations in stock returns.

Table 2 and 3 report the results of these regressions, displaying the estimated coefficients with the relative standard errors in parenthesis.

The *rmrf* coefficient represents the beta of the CAPM[5], that is the sensitivity of the PTF to market fluctuation. Table 2 and 3 show that *rmrf* coefficients are all statistically significant, suggesting that the excess return on a broad market PTF is a relevant risk component for the PTFs considered in the present analysis. Besides this, Table 2 and 3 also show that as the market capitalization increases (from PTF01 to PTF10), the *rmrf* coefficients increase towards 1 (and go beyond it in the eighth and ninth PTFs), meaning that small cap PTFs have lower volatility with respect to the market than large-cap PTFs. This makes sense because large-cap PTFs represent the biggest portion of the market and so they follow its behavior[6].

As concerns the size-related risk factor (*smb*), the coefficients are all statistically significant and generally decreasing. This makes sense: since PTFs are size-based, when *smb* increases (due to an increase in small-cap returns and/or a decrease in large-cap returns), the returns of smaller-cap PTFs are positively affected by such increase (and vice versa). An exception is provided by the first 4 PTFs, whose coefficients sightly increase. However, this increase is not significant since their confidence intervals overlap, as can be seen in Figure 6.

Moving to *hml*, coefficients are overall not significant, and this is coherent with the fact that PTFs are built according to the size, and not to the book-to-market values: this means that the PTFs are properly diversified including both stocks of firms in financial distress and not. A notable exception is observed for the fourth PTF, which shows a negative and significant coefficient at the 1% level, but this could be due to portfolio composition.

As regards *umd*, the momentum factor, it is evident that PTFs with positive coefficients have a lot of winners (or a few winners with high returns), while PTFs with negative coefficients have more losers. Among all the PTFs, a notable effect of the *umd* factor is observed only for PTF10, whose coefficient is positive and statistically significant at 1%. For all the others, the coefficients are either weakly or not at all statistically significant.

Lastly, we observe that the intercept is always zero. As explained by Fama and French (1996) this is a good signal about the quality of the model, because it leaves no unexplained returns: all the relevant risk components affecting these PTFs have been included in the model.

For what concerns the (adjusted) R-squared, we observe that the variation of excess returns captured by the risk-factors increases together with the market capitalization, indeed the goodness-of-fit measure go from about 0.60 in PTF01 to 0.96 in PTF10[7].

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[4] The coefficients are corrected for heteroskedasticity.

[5] The Capital Asset Pricing Model (CAPM) models the excess returns on a security (R_i) on the excess return on a broad market index (R_M). It can be represented by the equation $R_i = \beta_i R_M + \epsilon_i + \alpha_i$. β represents response of that particular stock's excess return to changes in the market index's excess return; $\beta > 1$ indicates cyclical stocks, while $\beta < 1$ indicates defensive stocks.

[6] Our results are different with respect to those obtained by Fama and French (1996): the PTFs analyzed in the paper show high risk for small-caps and low risk for large-caps and the regression exhibits similar betas, meaning that diversification in portfolios worked effectively.

[7] Actually, this result departs from the findings in Fama and French (1993) and Fama and French (1996), in which the R-squared values obtained when including all their risk-factors in the model were stably above 0.90 regardless of the PTF's characteristics.

Table 2: Multifactor Regressions Results (part b)

	Dependent variable:				
	XS01	XS02	XS03	XS04	XS05
rmrf	0.670^{***}	0.747^{***}	0.746^{***}	0.808^{***}	0.864^{***}
	(0.039)	(0.034)	(0.029)	(0.026)	(0.024)
smb	0.833^{***}	0.896^{***}	0.929^{***}	1.005^{***}	0.904^{***}
	(0.054)	(0.046)	(0.040)	(0.036)	(0.033)
hml	0.104^{*}	0.055	0.073^{*}	-0.106^{***}	0.041
	(0.056)	(0.048)	(0.041)	(0.037)	(0.035)
umd	0.095^{*}	-0.030	-0.002	0.087^{**}	0.054^{*}
	(0.051)	(0.043)	(0.038)	(0.034)	(0.032)
Constant	0.005^{***}	0.006^{***}	0.004^{**}	0.002^{*}	0.001
	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)
Observations	363	363	363	363	363
R^2	0.602	0.719	0.778	0.830	0.850
Adjusted R^2	0.598	0.716	0.776	0.828	0.848
Residual Std. Error (df = 358)	0.034	0.029	0.025	0.023	0.021
F Statistic (df = 4; 358)	135.549^{***}	229.535^{***}	314.474^{***}	436.290^{***}	505.626^{***}
Note:					*p<0.1; **p<0.05; ***p<0.01

Table 3: Multifactor Regressions Results (part b)

	Dependent variable:				
	XS06	XS07	XS08	XS09	XS10
rmrf	0.894^{***}	0.925^{***}	0.996^{***}	1.076^{***}	0.955^{***}
	(0.021)	(0.019)	(0.017)	(0.018)	(0.010)
smb	0.883^{***}	0.845^{***}	0.725^{***}	0.451^{***}	-0.150^{***}
	(0.029)	(0.026)	(0.023)	(0.024)	(0.014)
hml	0.021	-0.016	-0.034	-0.009	-0.008
	(0.030)	(0.027)	(0.024)	(0.025)	(0.014)
umd	0.014	0.009	0.046^{**}	-0.057^{**}	0.047^{***}
	(0.027)	(0.025)	(0.022)	(0.023)	(0.013)
Constant	0.002^{*}	0.001	0.0005	0.001	-0.00002
	(0.001)	(0.001)	(0.001)	(0.001)	(0.0005)
Observations	363	363	363	363	363
R^2	0.888	0.907	0.926	0.922	0.963
Adjusted R^2	0.886	0.906	0.925	0.921	0.962
Residual Std. Error (df = 358)	0.018	0.017	0.015	0.016	0.009
F Statistic (df = 4; 358)	706.798^{***}	876.378^{***}	$1,117.527^{***}$	$1,055.308^{***}$	$2,321.720^{***}$
Note:					*p<0.1; **p<0.05; ***p<0.01

Figure 6: Regression coefficients plot (XS01-XS05)

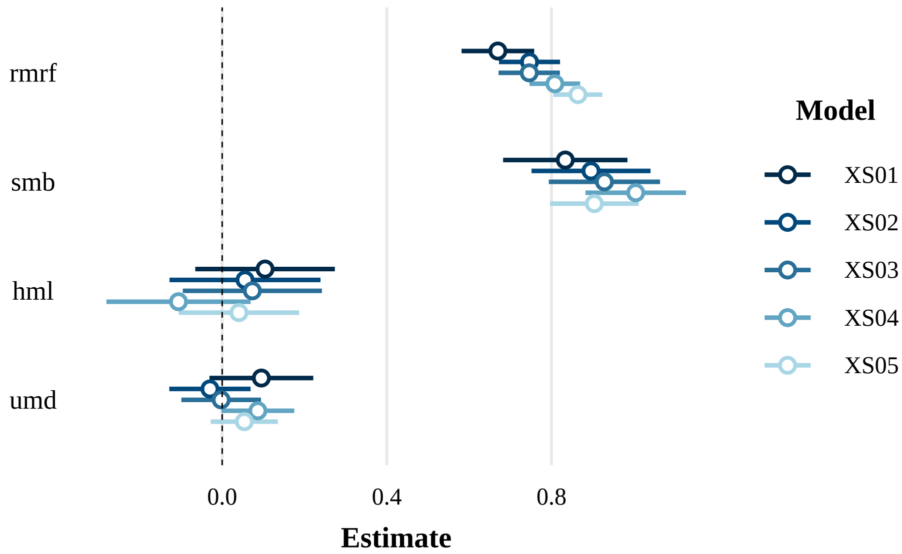
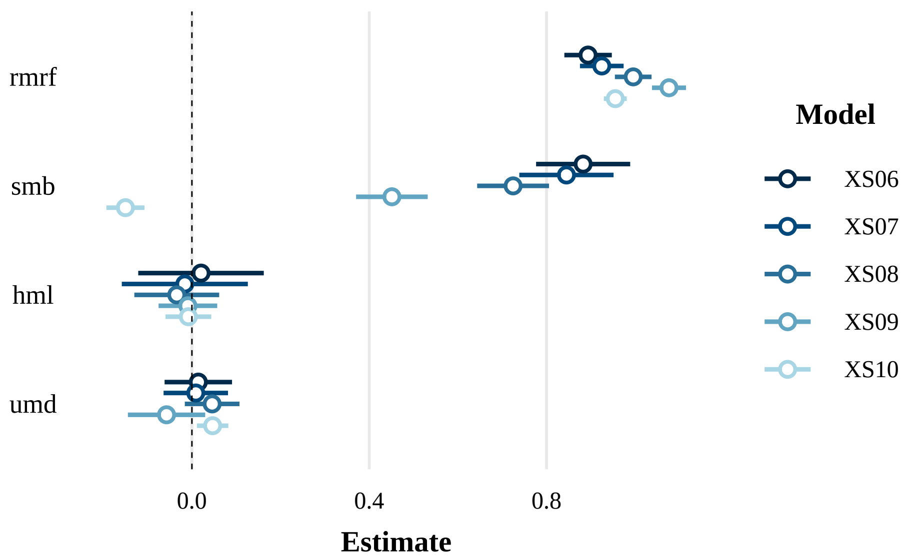


Figure 7: Regression coefficients plot (XS06-XS10)



We deepened the analysis by calculating the partial R-squared of the risk-factors in each portfolio[8]. Table 4 clearly shows that the most relevant factors in explaining the variation in excess returns are the market-related factor (*rmrf*) and the size factor (*smb*). As regards *rmrf*, it becomes increasingly more important as the capitalization in PTFs increases, indeed its partial R-squared goes from 0.45 in PTF01 to the impressive 0.96 in PTF10. *smb* presents very high values as well, even though not as much as *rmrf*. It is noticeable that it shows a similar increasing pattern, but somehow its relative importance drops in PTF09 and PTF10, in which its partial R-squared is respectively 0.49 and 0.25. Instead, the two remaining risk factors, *hml* and *umd*, seem not to be as important as theory would suggest in explaining the variance of excess returns.

Table 4: Partial R-squared

	rmrf	smb	hml	umd
XS01	0.450	0.404	0.010	0.010
XS02	0.581	0.516	0.004	0.001
XS03	0.649	0.605	0.009	0.000
XS04	0.726	0.687	0.022	0.018
XS05	0.778	0.672	0.004	0.008
XS06	0.835	0.725	0.001	0.001
XS07	0.869	0.747	0.001	0.000
XS08	0.906	0.731	0.006	0.012
XS09	0.912	0.492	0.000	0.017
XS10	0.962	0.250	0.001	0.035

3. Cross-Section Regressions

This section presents the results of the cross-section regressions. From Section 2 we saved the vector of coefficients $\boldsymbol{\beta}_i = [\beta_i^{rmrf}, \beta_i^{smb}, \beta_i^{hml}, \beta_i^{umd}]'$ for each portfolio i, to use them as regressors in a model that in turn includes β_i^{rmrf} and one of the remaining betas as explanatory variables and the time-mean excess return of each portfolio as dependent variable.

The general specification for our model is thus

$$\bar{r}_i = \lambda_0 + \lambda_1 \beta_i^{rmrf} + \lambda_2 \beta_i^k + v_i$$

where β_i^k is, in turn, β_i^{smb},β_i^{hml} or β_i^{umd}.

The cross-section regressions in Table 5 show that the mean returns are explained by only one risk component, that is the sensitivity of the PTFs to the market in relation to macroeconomic changes and other systematic shocks. Indeed, the coefficient associated with β_i^{rmrf} is the only one that is statistically significant across all models. Thus, at first glance, the only risk component that seems to be able to explain the mean returns of the PTFs is the market risk.

However, before drawing conclusions, two things must be observed. First, the sign of the coefficient corresponding to β_i^{rmrf} is always negative. Fama and French (1992) underline that, according to the SLB model, the relationship between average returns and β_i^{rmrf}s is expected to be positive even if they also found a negative relationship in the same paper. Moreover, Ang et al. (2006) also found negative coefficients in the cross-section regression and they explained it saying that "the low average returns to stocks with high idiosyncratic volatilities could arise because stocks with high idiosyncratic volatilities may have high exposure to aggregate volatility risk, which lowers their average returns". Note that Ang et al. (2006) examines idiosyncratic volatility at the firm level. Our result also makes sense from a mathematical point of view.

[8]Defining as "reduced model" a model that includes only three risk-factors, the partial R-squared tells us which percentage of the total variation not explained by the reduced model is then explained by the left-out risk-factor when it is included in the model, providing in turn a picture of the relative importance of the risk-factors

Indeed, in the previous sections we found that as market capitalization increases, PTFs' mean returns decrease while β_i^{rmrf}s increase, leading to a negative relationship between the two.

Second, it is worth noting that the non-significance of the coefficients associated with the other betas could be due to noise in our regressors, given the fact that the latters are random variables.

About the intercepts, we found them to be always positive and statistically different from zero. A possible interpretation is that they represent the expected return if the sensitivities to the risk components were zero, representing a sort of risk-free or (better) risk-independent returns in all the time periods considered.

All these considerations are to be made taking into account the small dimension of the sample, which only accounts for 10 observations.

Table 5: Cross-section Regressions Results

	Dependent variable:		
	mean_returns		
coeff rmrf	-0.010^{**}	-0.011^{**}	-0.014^{***}
	(0.003)	(0.003)	(0.003)
coeff smb	0.001		
	(0.001)		
coeff hml		0.006	
		(0.007)	
coeff umd			-0.011
			(0.007)
constant	0.022^{***}	0.023^{***}	0.026^{***}
	(0.003)	(0.003)	(0.002)
Observations	10	10	10
R^2	0.754	0.740	0.789
Adjusted R^2	0.684	0.665	0.729
Residual Std. Error (df = 7)	0.001	0.001	0.001
F Statistic (df = 2; 7)	10.728^{***}	9.944^{***}	13.081^{***}
Note:			*p<0.1; **p<0.05; ***p<0.01

Conclusions

We conclude that with the time-series approach, the relevant risk factors explaining the excess returns in our UK stocks PTFs are the broad market factor (*rmrf*), and, with lower relevance, the size factor (*smb*), as shown by the high significance of the coefficients and the partial R-squared values.

However, the cross-section regressions showed that only the sensitivity of the portfolios to the market is able to explain the mean excess returns. In particular, we find a negative relationship between average returns and β_i^{rmrf}, which is also found in the literature and it makes sense from a mathematical point of view.

Our results are different from the Fama and French (1992) ones.

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