Exploring portfolio returns

A four-factor model analysis of different risk sources

0. Data preparation

```
library(tidyverse)
library(ggfortify)
library(fpp2)
library(lmtest)
library(sandwich)
library(car)
library(readxl)
library(quantreg)
library(moments)
library(strucchange)
library(urca)
library(fable)
library(hts)
library(knitr)
library(extrafont)
library(ggplot2)
library(jtools)
library(huxtable)
library(officer)
library(flextable)
library(openxlsx)
ds <- read_excel("First_Assignment_dataset.xlsx")</pre>
ds$date = seq(as.Date(0, origin="1980-10-01"),length=363, by="1 month")
#converting returns in excess returns
ds = ds %>% mutate(XS01 = S1-rf,
                   XS02 = S2-rf,
                   XS03 = S3-rf,
                   XS04 = S4-rf,
                   XS05 = S5-rf,
                   XS06 = S6-rf,
                   XS07 = S7-rf,
                   XS08 = S8-rf,
                   XS09 = S9-rf,
                   XS10 = S10-rf)
#making dataset in long format
ds_long = ds %>% pivot_longer(XS01:XS10)
```

1. Data Visualization and Summary Statistics

Figure 1: Excess returns by portfolio

```
# excess return e outliers
theme_set(theme_minimal(base_family = "serif"))
xs01_plot = ds %>% ggplot()+
  geom hline(yintercept = mean(ds$XS01), col = 'brown')+
  geom hline(yintercept = quantile(ds\$XS01, 0.25)-1.5*IQR(ds\$XS01), linetype = 2, col = 'brown')+
  geom_hline(yintercept = quantile(ds$XS01, 0.75)+1.5*IQR(ds$XS01), linetype = 2, col = 'brown')+
  geom_line(aes(date, XS01),lwd = 0.2)+
  xlab(NULL)+
  annotate(geom = "rect",
         xmin = as.Date('1981-03-01'), xmax = as.Date('1981-11-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('1986-11-01'), xmax = as.Date('1988-01-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('2008-07-01'), xmax = as.Date('2009-06-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86",alpha=0.2)
xs02_plot = ds %>% ggplot()+
  geom_hline(yintercept = mean(ds$XSO2), col = 'brown')+
  geom_hline(yintercept = quantile(ds$XS02, 0.25)-1.5*IQR(ds$XS02), linetype = 2, col = 'brown')+
  geom_hline(yintercept = quantile(ds$XS02, 0.75)+1.5*IQR(ds$XS02), linetype = 2, col = 'brown')+
  geom_line(aes(date, XS02),lwd = 0.2)+
  xlab(NULL)+
  annotate(geom = "rect",
         xmin = as.Date('1981-03-01'), xmax = as.Date('1981-11-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('1986-11-01'), xmax = as.Date('1988-01-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('2008-07-01'), xmax = as.Date('2009-06-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86",alpha=0.2)
xs03_plot = ds \%% ggplot()+
  geom_hline(yintercept = mean(ds$XSO3), col = 'brown')+
  geom_hline(yintercept = quantile(ds$XS03, 0.25)-1.5*IQR(ds$XS03), linetype = 2, col = 'brown')+
  geom_hline(yintercept = quantile(ds\$XSO3, 0.75)+1.5*IQR(ds\$XSO3), linetype = 2, col = 'brown')+
  geom_line(aes(date, XS03),lwd = 0.2)+
  xlab(NULL)+
  annotate(geom = "rect",
```

```
xmin = as.Date('1981-03-01'), xmax = as.Date('1981-11-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('1986-11-01'), xmax = as.Date('1988-01-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('2008-07-01'), xmax = as.Date('2009-06-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86",alpha=0.2)
xs04_plot = ds %>% ggplot()+
  geom_hline(yintercept = mean(ds$XS04), col = 'brown')+
  geom_hline(yintercept = quantile(ds$XS04, 0.25)-1.5*IQR(ds$XS04), linetype = 2, col = 'brown')+
  geom_hline(yintercept = quantile(ds$XS04, 0.75)+1.5*IQR(ds$XS04), linetype = 2, col = 'brown')+
  geom_line(aes(date, XS04), lwd = 0.2)+
  xlab(NULL)+
  annotate(geom = "rect",
         xmin = as.Date('1981-03-01'), xmax = as.Date('1981-11-01'),
         ymin = -Inf, ymax = Inf,
        fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('1986-11-01'), xmax = as.Date('1988-01-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('2008-07-01'), xmax = as.Date('2009-06-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)
xs05_plot = ds %>% ggplot()+
  geom_hline(yintercept = mean(ds$XSO5), col = 'brown')+
  geom_hline(yintercept = quantile(ds\$XS05, 0.25)-1.5*IQR(ds\$XS05), linetype = 2, col = 'brown')+
  geom_hline(yintercept = quantile(ds\$XS05, 0.75)+1.5*IQR(ds\$XS05), linetype = 2, col = 'brown')+
  geom_line(aes(date, XS05),lwd = 0.2)+
  xlab(NULL)+
  annotate(geom = "rect",
         xmin = as.Date('1981-03-01'), xmax = as.Date('1981-11-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('1986-11-01'), xmax = as.Date('1988-01-01'),
         ymin = -Inf, ymax = Inf,
        fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('2008-07-01'), xmax = as.Date('2009-06-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86",alpha=0.2)
xs06_plot = ds %>% ggplot()+
  geom_hline(yintercept = mean(ds$XS06), col = 'brown')+
  geom_hline(yintercept = quantile(ds\$XS06, 0.25)-1.5*IQR(ds\$XS06), linetype = 2, col = 'brown')+
```

```
geom_hline(yintercept = quantile(ds$XS06, 0.75)+1.5*IQR(ds$XS06), linetype = 2, col = 'brown')+
  geom line(aes(date, XS06), lwd = 0.2)+
  xlab(NULL)+
  annotate(geom = "rect",
         xmin = as.Date('1981-03-01'), xmax = as.Date('1981-11-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('1986-11-01'), xmax = as.Date('1988-01-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('2008-07-01'), xmax = as.Date('2009-06-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)
xs07_plot = ds %>% ggplot()+
  geom_hline(yintercept = mean(ds$XSO7), col = 'brown')+
  geom_hline(yintercept = quantile(ds\$XSO7, 0.25)-1.5*IQR(ds\$XSO7), linetype = 2, col = 'brown')+
  geom_hline(yintercept = quantile(ds$XS07, 0.75)+1.5*IQR(ds$XS07), linetype = 2, col = 'brown')+
  geom_line(aes(date, XS07),lwd = 0.2)+
  xlab(NULL)+
  annotate(geom = "rect",
         xmin = as.Date('1981-03-01'), xmax = as.Date('1981-11-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('1986-11-01'), xmax = as.Date('1988-01-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('2008-07-01'), xmax = as.Date('2009-06-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86",alpha=0.2)
xs08_plot = ds %>% ggplot()+
  geom_hline(yintercept = mean(ds$XS01), col = 'brown')+
  geom_hline(yintercept = quantile(ds\$XS01, 0.25)-1.5*IQR(ds\$XS01), linetype = 2, col = 'brown')+
  geom_hline(yintercept = quantile(ds$XS01, 0.75)+1.5*IQR(ds$XS01), linetype = 2, col = 'brown')+
  geom_line(aes(date, XSO8), lwd = 0.2)+
  xlab(NULL)+
  annotate(geom = "rect",
         xmin = as.Date('1981-03-01'), xmax = as.Date('1981-11-01'),
         ymin = -Inf, ymax = Inf,
        fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('1986-11-01'), xmax = as.Date('1988-01-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('2008-07-01'), xmax = as.Date('2009-06-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86",alpha=0.2)
```

```
xs09_plot = ds %>% ggplot()+
  geom_hline(yintercept = mean(ds$XS09), col = 'brown')+
  geom_hline(yintercept = quantile(ds$XS09, 0.25)-1.5*IQR(ds$XS09), linetype = 2, col = 'brown')+
  geom hline(yintercept = quantile(ds$XS09, 0.75)+1.5*IQR(ds$XS09), linetype = 2, col = 'brown')+
  geom_line(aes(date, XS09),lwd = 0.2)+
  xlab(NULL)+
  annotate(geom = "rect",
         xmin = as.Date('1981-03-01'), xmax = as.Date('1981-11-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('1986-11-01'), xmax = as.Date('1988-01-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('2008-07-01'), xmax = as.Date('2009-06-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86",alpha=0.2)
xs10_plot = ds %>% ggplot()+
  geom_hline(yintercept = mean(ds$XS10), col = 'brown')+
  geom hline(vintercept = quantile(ds$XS10, 0.25)-1.5*IQR(ds$XS10), linetype = 2, col = 'brown')+
  geom_hline(yintercept = quantile(ds$XS10, 0.75)+1.5*IQR(ds$XS10), linetype = 2, col = 'brown')+
  geom_line(aes(date, XS10),lwd = 0.2)+
  xlab(NULL)+
  annotate(geom = "rect",
         xmin = as.Date('1981-03-01'), xmax = as.Date('1981-11-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('1986-11-01'), xmax = as.Date('1988-01-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86", alpha=0.2)+
  annotate(geom = "rect",
         xmin = as.Date('2008-07-01'), xmax = as.Date('2009-06-01'),
         ymin = -Inf, ymax = Inf,
         fill = "#014F86",alpha=0.2)
library(gridExtra)
library(grid)
grid.arrange(xs01_plot,
             xs06_plot,
             xs02_plot,
             xs07_plot,
             xs03_plot,
             xs08_plot,
             xs04_plot,
             xs09_plot,
             xs05_plot,
             xs10_plot,
             nrow = 5
```

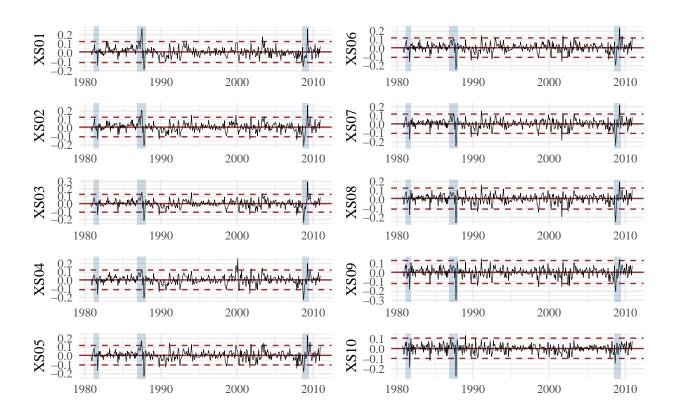


Table 1: Summary statistics of the excess returns

```
# summary
library(moments)
summary_tab = as.data.frame(ds_long %>%
                              group_by(name) %>%
                              summarise(mean=mean(value) %>% round(3),
                                         var=var(value)%>% round(3),
                                         st_dev=sd(value)%>% round(3),
                                         min=min(value)%>% round(3),
                                         q25=quantile(value, 0.25)%>% round(3),
                                         q50=quantile(value, 0.5)%>% round(3),
                                         q75=quantile(value, 0.75)%>% round(3),
                                         max=max(value)%>% round(3),
                                         skewness=skewness(value)%>% round(3),
                                         kurtosis=kurtosis(value)%>% round(3)) %>%
                              mutate("Sharpe ratio" = (c((mean(ds$S1)-mean(ds$rf))/sd(ds$S1),
                                                         (mean(ds$S2)-mean(ds$rf))/sd(ds$S2),
                                                         (mean(ds$S3)-mean(ds$rf))/sd(ds$S3),
                                                         (mean(ds$S4)-mean(ds$rf))/sd(ds$S4),
                                                         (mean(ds$S5)-mean(ds$rf))/sd(ds$S5),
                                                         (mean(ds$S6)-mean(ds$rf))/sd(ds$S6),
                                                         (mean(ds\$S7)-mean(ds\$rf))/sd(ds\$S7),
                                                         (mean(ds$S8)-mean(ds$rf))/sd(ds$S8),
                                                         (mean(ds$S9)-mean(ds$rf))/sd(ds$S9),
                                                         (mean(ds\$S10)-mean(ds\$rf))/sd(ds\$S10)))% roun
kable(summary_tab, caption = 'Summary statistics of the excess returns', align = 'l')
```

Table 1: Summary statistics of the excess returns

name	mean	var	st_dev	min	q25	q50	q75	max	skewness	kurtosis	Sharpe ratio
XS01	0.011	0.003	0.054	-0.184	-0.021	0.010	0.038	0.273	0.484	6.435	0.201
XS02	0.010	0.003	0.055	-0.213	-0.017	0.011	0.040	0.268	-0.039	5.949	0.191
XS03	0.009	0.003	0.054	-0.246	-0.017	0.011	0.041	0.293	-0.269	7.071	0.163
XS04	0.008	0.003	0.055	-0.227	-0.021	0.009	0.037	0.258	-0.195	6.018	0.144
XS05	0.007	0.003	0.055	-0.233	-0.019	0.010	0.036	0.231	-0.293	5.358	0.132
XS06	0.008	0.003	0.055	-0.243	-0.020	0.011	0.036	0.232	-0.514	5.603	0.140
XS07	0.007	0.003	0.054	-0.247	-0.019	0.010	0.036	0.221	-0.771	6.213	0.124
XS08	0.007	0.003	0.054	-0.262	-0.019	0.014	0.039	0.194	-1.016	6.564	0.122
XS09	0.007	0.003	0.055	-0.294	-0.022	0.014	0.041	0.155	-0.933	6.053	0.122
XS10	0.005	0.002	0.045	-0.277	-0.018	0.010	0.033	0.136	-1.037	7.318	0.111

Figure 2: Distribution of excess returns by portfolio

```
# boxplots
theme_set(theme_minimal(base_family = "serif"))
ds_long %>% ggplot()+
   geom_boxplot(aes(name, value, fill = name), show.legend = F) +
   scale_fill_manual(name="color", values=c("#012A4A", "#013A63", "#01497C", "#014F86", "#2A6F97", "#2C7
   theme(plot.title = element_text(hjust = 0.5, size = 12))+
   xlab("Portfolios")+
   ylab("Excess returns")
```

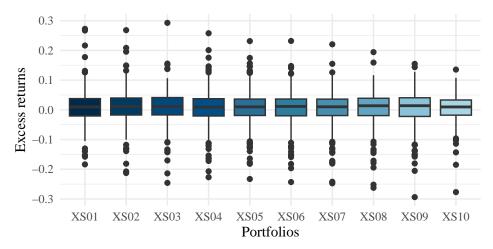


Figure 3: Correlations between portfolios' excess returns

```
text = element_text(size = 5),
strip.text = element_text(size = 8))
```

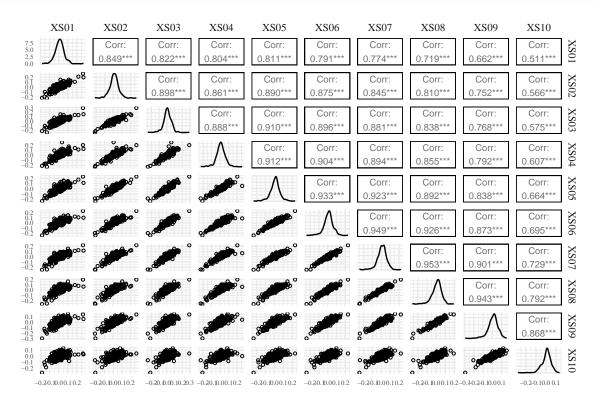


Figure 4: Correlations between factors

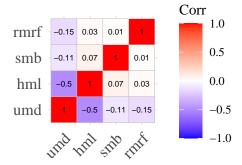
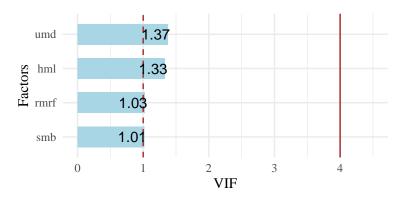


Figure 5: Variance inflation factor

```
vif_tab = vif(lm(XS01~umd+hml+smb+rmrf, data=ds)) %>%
  as.data.frame()
vif_tab = vif_tab %>%
  rename(vif = '.') %>%
  mutate(factors = row.names(vif_tab))
theme_set(theme_minimal(base_family = 'serif'))
vif tab %>%
  mutate(factors = reorder(factors, vif), decreasing = F) %>%
  ggplot()+
  geom_col(aes(y = factors, x = vif), fill = '#A9D6E5', width = 0.6)+
  theme(plot.title = element_text(hjust = 0.5, size = 12)) +
  xlim(0, 4.5)+
  ylab("Factors")+
  xlab("VIF")+
  geom_vline(xintercept = 4, color = "brown", linetype = "solid") +
  geom_vline(xintercept = 1, color = "brown", linetype = "dashed") +
  geom_text(aes(y = factors, x = vif, label = round(vif,2)), nudge_x=-0.18)
```



2. Multifactor Regressions

```
ptf.ts = ts(ds[,8:18], start = c(1980,10), frequency = 12)
ptf_lm = lm(cbind(XS01,XS02,XS03,XS04,XS05,XS06,XS07,XS08,XS09,XS10)~rmrf+smb+hml+umd, data = ds)
#interpreting coefficient, t-values, p-values
# storing slope coefficients
coefficients.ds = data.frame(ptf_lm$coefficients)
coefficients.ds = t(coefficients.ds)
#coefficients with sderr, t-values and p-values robust to heteroskedasticity
#this code is in order to obtain dataframe with everything we need:
#we have to export it and then we will save the estimates of the coefficients
#and use those as regressors on the 10 ptfs
coefficients_robust.ds = coeftest(ptf_lm, vcovHC = vcovHC(ptf_lm))
estimates = as.vector(coefficients_robust.ds[,1])
std_err = as.vector(coefficients_robust.ds[,2])
t_values = as.vector(coefficients_robust.ds[,3])
p values = as.vector(coefficients robust.ds[,4])
XS_{\text{vec}} = \text{paste0}("XS", c(rep(1,5),rep(2,5),rep(3,5),rep(4,5),rep(5,5),
               rep(6,5), rep(7,5), rep(8,5), rep(9,5), rep(10,5)))
xs_values = paste0(XS_vec, c(": Intercept",
                 ": rmrf",
                 ": smb",
                 ": hml",
                 ": umd"))
coefficients_robust.ds_final = data.frame(ptf_variables = xs_values,
                                           coefficient = estimates,
                                           std_error = std_err,
                                           t_value = t_values,
                                           p_value = p_values)
# getting r2
s = summary(ptf lm)
r2 XS1 = s$`Response XS1`$r.squared
adj_r2_XS1 = s$`Response XS1`$adj.r.squared
r2_XS2 = s$`Response XS2`$r.squared
adj_r2_XS2 = s$`Response XS2`$adj.r.squared
r2_XS3 = s$`Response XS3`$r.squared
adj_r2_XS3 = s$`Response XS3`$adj.r.squared
r2_XS4 = s$`Response XS4`$r.squared
adj_r2_XS4 = s$`Response XS4`$adj.r.squared
r2_XS5 = s$`Response XS5`$r.squared
adj_r2_XS5 = s$`Response XS5`$adj.r.squared
r2_XS6 = s$`Response XS6`$r.squared
adj_r2_XS6 = s$`Response XS6`$adj.r.squared
r2_XS7 = s$`Response XS7`$r.squared
adj_r2_XS7 = s$`Response XS7`$adj.r.squared
r2_XS8 = s$`Response XS8`$r.squared
adj_r2_XS8 = s$`Response XS8`$adj.r.squared
r2 XS9 = s$`Response XS9`$r.squared
```

```
adj_r2_XS9 = s$`Response XS9`$adj.r.squared
r2_XS10 = s$`Response XS10`$r.squared
adj_r2_XS10 = s$`Response XS10`$adj.r.squared
ptfs_r2 = c(rep(r2_XS1, 5), rep(r2_XS2, 5), rep(r2_XS3, 5), rep(r2_XS4, 5), rep(r2_XS5, 5),
    rep(r2_XS6, 5), rep(r2_XS7, 5), rep(r2_XS8, 5), rep(r2_XS9, 5), rep(r2_XS10, 5))
ptfs_adj_r2 = c(rep(adj_r2_XS1, 5), rep(adj_r2_XS2, 5), rep(adj_r2_XS3, 5), rep(adj_r2_XS4, 5),
                rep(adj_r2_XS5, 5), rep(adj_r2_XS6, 5), rep(adj_r2_XS7, 5), rep(adj_r2_XS8, 5),
                rep(adj_r2_XS9, 5), rep(adj_r2_XS10, 5))
coefficients_robust.ds_final$r2 = ptfs_r2
coefficients_robust.ds_final$adj_r2 = ptfs_adj_r2
# separated regression
ptf_lm1 = lm(XS01~rmrf+smb+hml+umd, data = ds)
ptf_lm2 = lm(XSO2~rmrf+smb+hml+umd, data = ds)
ptf_lm3 = lm(XSO3~rmrf+smb+hml+umd, data = ds)
ptf_lm4 = lm(XSO4~rmrf+smb+hml+umd, data = ds)
ptf_lm5 = lm(XS05~rmrf+smb+hml+umd, data = ds)
ptf_lm6 = lm(XS06~rmrf+smb+hml+umd, data = ds)
ptf_lm7 = lm(XSO7~rmrf+smb+hml+umd, data = ds)
ptf_lm8 = lm(XS08~rmrf+smb+hml+umd, data = ds)
ptf_lm9 = lm(XS09~rmrf+smb+hml+umd, data = ds)
ptf_lm10 = lm(XS10~rmrf+smb+hml+umd, data = ds)
library(stargazer)
```

Table 2: Multifactor Regressions Results (part b)

	$Dependent\ variable:$				
	XS01	XS02	XS03	XS04	XS05
rmrf	0.670***	0.747***	0.746***	0.808***	0.864***
	(0.039)	(0.034)	(0.029)	(0.026)	(0.024)
smb	0.833***	0.896***	0.929***	1.005***	0.904***
	(0.054)	(0.046)	(0.040)	(0.036)	(0.033)
hml	0.104*	0.055	0.073^{*}	-0.106****	0.041
	(0.056)	(0.048)	(0.041)	(0.037)	(0.035)
umd	0.095^{*}	-0.030	-0.002	0.087^{**}	0.054^{*}
	(0.051)	(0.043)	(0.038)	(0.034)	(0.032)
Constant	0.005^{***}	0.006^{***}	0.004**	0.002*	0.001
	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)
Observations	363	363	363	363	363
R^2	0.602	0.719	0.778	0.830	0.850
Adjusted R ²	0.598	0.716	0.776	0.828	0.848
Residual Std. Error ($df = 358$)	0.034	0.029	0.025	0.023	0.021
F Statistic (df = $4; 358$)	135.549***	229.535***	314.474***	436.290***	505.626**

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3: Multifactor Regressions Results (part b)

	$Dependent\ variable:$				
	XS06	XS07	XS08	XS09	XS10
rmrf	0.894***	0.925***	0.996***	1.076***	0.955***
	(0.021)	(0.019)	(0.017)	(0.018)	(0.010)
smb	0.883***	0.845***	0.725***	0.451***	-0.150****
	(0.029)	(0.026)	(0.023)	(0.024)	(0.014)
hml	0.021	-0.016	-0.034	-0.009	-0.008
	(0.030)	(0.027)	(0.024)	(0.025)	(0.014)
umd	0.014	0.009	0.046**	-0.057^{**}	0.047***
	(0.027)	(0.025)	(0.022)	(0.023)	(0.013)
Constant	0.002^{*}	0.001	0.0005	0.001	-0.00002
	(0.001)	(0.001)	(0.001)	(0.001)	(0.0005)
Observations	363	363	363	363	363
\mathbb{R}^2	0.888	0.907	0.926	0.922	0.963
Adjusted R ²	0.886	0.906	0.925	0.921	0.962
Residual Std. Error ($df = 358$)	0.018	0.017	0.015	0.016	0.009
F Statistic ($df = 4; 358$)	706.798***	876.378***	1,117.527***	1,055.308***	2,321.720***

Note:

*p<0.1; **p<0.05; ***p<0.01

Figure 6: Regression coefficients plot (XS01-XS05)

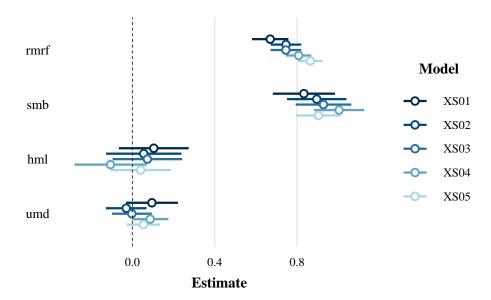


Figure 7: Regression coefficients plot (XS06-XS10)

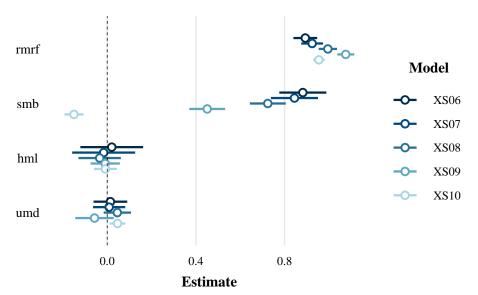


Table 4: Partial R-squared

```
XSO1 lm = lm(XSO1~rmrf+smb+hml+umd, data = ds)
XS02 lm = lm(XS02 rmrf + smb + hml + umd, data = ds)
XS03_lm = lm(XS03~rmrf+smb+hml+umd, data = ds)
XSO4 lm = lm(XSO4~rmrf+smb+hml+umd, data = ds)
XS05_{lm} = lm(XS05~rmrf+smb+hml+umd, data = ds)
XS06_{lm} = lm(XS06~rmrf+smb+hml+umd, data = ds)
XS07_{lm} = lm(XS07_{rmrf+smb+hml+umd}, data = ds)
XS08 lm = lm(XS08 \sim rmrf + smb + hml + umd, data = ds)
XS09_{lm} = lm(XS09~rmrf+smb+hml+umd, data = ds)
XS10_lm = lm(XS10~rmrf+smb+hml+umd, data = ds)
XS01_lm_noRMRF = lm(XS01~smb+hml+umd, data = ds)
XSO2_{lm}_{noRMRF} = lm(XSO2_{smb+hml+umd}, data = ds)
XSO3_lm_noRMRF = lm(XSO3~smb+hml+umd, data = ds)
XS04_lm_noRMRF = lm(XS04~smb+hml+umd, data = ds)
XSO5_lm_noRMRF = lm(XSO5~smb+hml+umd, data = ds)
XS06_lm_noRMRF = lm(XS06~smb+hml+umd, data = ds)
XS07_lm_noRMRF = lm(XS07_smb+hml+umd, data = ds)
XSO8 lm noRMRF = lm(XSO8~smb+hml+umd, data = ds)
XS09_{m_noRMRF} = lm(XS09_{mh}+hml+umd, data = ds)
XS10 lm noRMRF = lm(XS10~smb+hml+umd, data = ds)
XS01_lm_noSMB = lm(XS01~rmrf+hml+umd, data = ds)
XSO2_lm_noSMB = lm(XSO2~rmrf+hml+umd, data = ds)
XS03_lm_noSMB = lm(XS03~rmrf+hml+umd, data = ds)
XS04_lm_noSMB = lm(XS04~rmrf+hml+umd, data = ds)
XSO5_lm_noSMB = lm(XSO5~rmrf+hml+umd, data = ds)
XS06_lm_noSMB = lm(XS06~rmrf+hml+umd, data = ds)
XS07 lm noSMB = lm(XS07~rmrf+hml+umd, data = ds)
XS08_lm_noSMB = lm(XS08~rmrf+hml+umd, data = ds)
XS09_lm_noSMB = lm(XS09~rmrf+hml+umd, data = ds)
XS10_lm_noSMB = lm(XS10~rmrf+hml+umd, data = ds)
XSO1_lm_noHML = lm(XSO1~smb+rmrf+umd, data = ds)
XS02_lm_noHML = lm(XS02~smb+rmrf+umd, data = ds)
XSO3 lm noHML = lm(XSO3~smb+rmrf+umd, data = ds)
XS04_lm_noHML = lm(XS04~smb+rmrf+umd, data = ds)
XS05_lm_noHML = lm(XS05~smb+rmrf+umd, data = ds)
XS06_lm_noHML = lm(XS06~smb+rmrf+umd, data = ds)
XS07_lm_noHML = lm(XS07_smb+rmrf+umd, data = ds)
XS08_lm_noHML = lm(XS08~smb+rmrf+umd, data = ds)
XS09_{m_noHML} = lm(XS09_{m_mrf+umd}, data = ds)
XS10_{m_noHML} = lm(XS10_{m_noHML} = ds)
XSO1_lm_noUMD = lm(XSO1~smb+hml+rmrf, data = ds)
XSO2_{lm} = lm(XSO2 = lm) + lm + lm = lm(XSO2 = lm)
XSO3_lm_noUMD = lm(XSO3~smb+hml+rmrf, data = ds)
XS04_lm_noUMD = lm(XS04~smb+hml+rmrf, data = ds)
XSO5_lm_noUMD = lm(XSO5~smb+hml+rmrf, data = ds)
XS06_{lm}_{noUMD} = lm(XS06_{smb+hml+rmrf}, data = ds)
XS07 lm noUMD = lm(XS07~smb+hml+rmrf, data = ds)
```

```
XS08_lm_noUMD = lm(XS08~smb+hml+rmrf, data = ds)
XS09_lm_noUMD = lm(XS09~smb+hml+rmrf, data = ds)
XS10_lm_noUMD = lm(XS10~smb+hml+rmrf, data = ds)
partial_R2 = matrix(data = NA, nrow = 10, ncol = 4)
rownames(partial_R2) = c("XS01", "XS02", "XS03", "XS04", "XS05",
                                               "XS06", "XS07", "XS08", "XS09", "XS10")
colnames(partial_R2) = c("rmrf", "smb", "hml", "umd")
library("asbio")
XS01_lm = lm(XS01~rmrf+smb+hml+umd, data = ds)
XSO2_{lm} = lm(XSO2~rmrf+smb+hml+umd, data = ds)
XSO3_{lm} = lm(XSO3~rmrf+smb+hml+umd, data = ds)
XS04_{lm} = lm(XS04_{rmrf+smb+hml+umd}, data = ds)
XSO5_{lm} = lm(XSO5~rmrf+smb+hml+umd, data = ds)
XS06_{lm} = lm(XS06~rmrf+smb+hml+umd, data = ds)
XS07_{lm} = lm(XS07~rmrf+smb+hml+umd, data = ds)
XS08_{lm} = lm(XS08_{rmrf+smb+hml+umd}, data = ds)
XS09_lm = lm(XS09~rmrf+smb+hml+umd, data = ds)
XS10_{lm} = lm(XS10~rmrf+smb+hml+umd, data = ds)
XS01 lm noRMRF = lm(XS01~smb+hml+umd, data = ds)
XS02_{m_noRMRF} = lm(XS02_{m_noRMRF}, data = ds)
XSO3 lm noRMRF = lm(XSO3~smb+hml+umd, data = ds)
XSO4 lm noRMRF = lm(XSO4~smb+hml+umd, data = ds)
XSO5 lm noRMRF = lm(XSO5~smb+hml+umd, data = ds)
XS06_{m_noRMRF} = lm(XS06~smb+hml+umd, data = ds)
XS07_lm_noRMRF = lm(XS07_smb+hml+umd, data = ds)
XS08_{m_noRMRF} = lm(XS08_{m_noRMRF} = ds)
XS09_{m_noRMRF} = lm(XS09_{m_noRMRF} + lm(XS09_{m_noRMRF} + lm(XS09_{m_noRMRF} + lm(XS09_{m_noRMR} + lm(
XS10_lm_noRMRF = lm(XS10~smb+hml+umd, data = ds)
parial.R2_XS01_noRMRF = partial.R2(XS01_lm_noRMRF, XS01_lm)
parial.R2_XS02_noRMRF = partial.R2(XS02_lm_noRMRF, XS02_lm)
parial.R2_XS03_noRMRF = partial.R2(XS03_lm_noRMRF, XS03_lm)
parial.R2_XS04_noRMRF = partial.R2(XS04_lm_noRMRF, XS04_lm)
parial.R2_XS05_noRMRF = partial.R2(XS05_lm_noRMRF, XS05_lm)
parial.R2_XS06_noRMRF = partial.R2(XS06_lm_noRMRF, XS06_lm)
parial.R2_XS07_noRMRF = partial.R2(XS07_lm_noRMRF, XS07_lm)
parial.R2_XS08_noRMRF = partial.R2(XS08_lm_noRMRF, XS08_lm)
parial.R2 XS09 noRMRF = partial.R2(XS09 lm noRMRF, XS09 lm)
parial.R2_XS10_noRMRF = partial.R2(XS10_lm_noRMRF, XS10_lm)
XS01_lm_noSMB = lm(XS01~rmrf+hml+umd, data = ds)
XS02_lm_noSMB = lm(XS02~rmrf+hml+umd, data = ds)
XS03_lm_noSMB = lm(XS03~rmrf+hml+umd, data = ds)
XS04_lm_noSMB = lm(XS04~rmrf+hml+umd, data = ds)
XSO5_lm_noSMB = lm(XSO5~rmrf+hml+umd, data = ds)
XS06_lm_noSMB = lm(XS06~rmrf+hml+umd, data = ds)
XS07_lm_noSMB = lm(XS07~rmrf+hml+umd, data = ds)
XS08_lm_noSMB = lm(XS08~rmrf+hml+umd, data = ds)
```

```
XS09_lm_noSMB = lm(XS09~rmrf+hml+umd, data = ds)
XS10_lm_noSMB = lm(XS10~rmrf+hml+umd, data = ds)
parial.R2_XS01_noSMB = partial.R2(XS01_lm_noSMB, XS01_lm)
parial.R2_XS02_noSMB = partial.R2(XS02_lm_noSMB, XS02_lm)
parial.R2_XS03_noSMB = partial.R2(XS03_lm_noSMB, XS03_lm)
parial.R2 XS04 noSMB = partial.R2(XS04 lm noSMB, XS04 lm)
parial.R2 XS05 noSMB = partial.R2(XS05 lm noSMB, XS05 lm)
parial.R2_XS06_noSMB = partial.R2(XS06_lm_noSMB, XS06_lm)
parial.R2_XS07_noSMB = partial.R2(XS07_lm_noSMB, XS07_lm)
parial.R2_XS08_noSMB = partial.R2(XS08_lm_noSMB, XS08_lm)
parial.R2_XS09_noSMB = partial.R2(XS09_lm_noSMB, XS09_lm)
parial.R2_XS10_noSMB = partial.R2(XS10_lm_noSMB, XS10_lm)
XSO1_lm_noHML = lm(XSO1~smb+rmrf+umd, data = ds)
XSO2_lm_noHML = lm(XSO2~smb+rmrf+umd, data = ds)
XSO3_lm_noHML = lm(XSO3~smb+rmrf+umd, data = ds)
XSO4_lm_noHML = lm(XSO4~smb+rmrf+umd, data = ds)
XS05_lm_noHML = lm(XS05~smb+rmrf+umd, data = ds)
XS06_{m_noHML} = lm(XS06_{m_mrf+umd}, data = ds)
XS07_lm_noHML = lm(XS07_smb+rmrf+umd, data = ds)
XSO8 lm noHML = lm(XSO8~smb+rmrf+umd, data = ds)
XS09_lm_noHML = lm(XS09~smb+rmrf+umd, data = ds)
XS10 lm noHML = lm(XS10 \sim smb + rmrf + umd, data = ds)
parial.R2 XS01 noHML = partial.R2(XS01 lm noHML, XS01 lm)
parial.R2_XS02_noHML = partial.R2(XS02_lm_noHML, XS02_lm)
parial.R2_XS03_noHML = partial.R2(XS03_lm_noHML, XS03_lm)
parial.R2_XS04_noHML = partial.R2(XS04_lm_noHML, XS04_lm)
parial.R2_XS05_noHML = partial.R2(XS05_lm_noHML, XS05_lm)
parial.R2_XS06_noHML = partial.R2(XS06_lm_noHML, XS06_lm)
parial.R2_XS07_noHML = partial.R2(XS07_lm_noHML, XS07_lm)
parial.R2_XS08_noHML = partial.R2(XS08_lm_noHML, XS08_lm)
parial.R2_XS09_noHML = partial.R2(XS09_lm_noHML, XS09_lm)
parial.R2_XS10_noHML = partial.R2(XS10_lm_noHML, XS10_lm)
XS01 lm noUMD = lm(XS01~smb+hml+rmrf, data = ds)
XSO2_lm_noUMD = lm(XSO2~smb+hml+rmrf, data = ds)
XS03_lm_noUMD = lm(XS03~smb+hml+rmrf, data = ds)
XS04_lm_noUMD = lm(XS04~smb+hml+rmrf, data = ds)
XSO5_lm_noUMD = lm(XSO5~smb+hml+rmrf, data = ds)
XS06 lm noUMD = lm(XS06~smb+hml+rmrf, data = ds)
XS07_lm_noUMD = lm(XS07~smb+hml+rmrf, data = ds)
XSO8 lm noUMD = lm(XSO8~smb+hml+rmrf, data = ds)
XS09_lm_noUMD = lm(XS09~smb+hml+rmrf, data = ds)
XS10_lm_noUMD = lm(XS10~smb+hml+rmrf, data = ds)
parial.R2_XS01_noUMD = partial.R2(XS01_lm_noUMD, XS01_lm)
parial.R2_XS02_noUMD = partial.R2(XS02_lm_noUMD, XS02_lm)
parial.R2_XS03_noUMD = partial.R2(XS03_lm_noUMD, XS03_lm)
parial.R2_XS04_noUMD = partial.R2(XS04_lm_noUMD, XS04_lm)
parial.R2_XS05_noUMD = partial.R2(XS05_lm_noUMD, XS05_lm)
```

```
parial.R2_XS06_noUMD = partial.R2(XS06_lm_noUMD, XS06_lm)
parial.R2_XS07_noUMD = partial.R2(XS07_lm_noUMD, XS07_lm)
parial.R2_XS08_noUMD = partial.R2(XS08_lm_noUMD, XS08_lm)
parial.R2_XS09_noUMD = partial.R2(XS09_lm_noUMD, XS09_lm)
parial.R2_XS10_noUMD = partial.R2(XS10_lm_noUMD, XS10_lm)
partial_R2 = matrix(data = NA, nrow = 10, ncol = 4)
rownames(partial_R2) = c("XS01", "XS02", "XS03", "XS04", "XS05",
                          "XS06", "XS07", "XS08", "XS09", "XS10")
colnames(partial_R2) = c("rmrf", "smb", "hml", "umd")
partial_R2[,1] = c(parial.R2_XS01_noRMRF,
                   parial.R2_XSO2_noRMRF,
                   parial.R2_XSO3_noRMRF,
                   parial.R2_XSO4_noRMRF,
                   parial.R2_XS05_noRMRF,
                   parial.R2_XS06_noRMRF,
                   parial.R2_XS07_noRMRF,
                   parial.R2_XSO8_noRMRF,
                   parial.R2_XS09_noRMRF,
                   parial.R2_XS10_noRMRF)
partial_R2[,2] = c(parial.R2_XS01_noSMB,
                   parial.R2 XSO2 noSMB,
                   parial.R2_XS03_noSMB,
                   parial.R2 XS04 noSMB,
                   parial.R2_XS05_noSMB,
                   parial.R2_XS06_noSMB,
                   parial.R2_XS07_noSMB,
                   parial.R2_XSO8_noSMB,
                   parial.R2_XS09_noSMB,
                   parial.R2_XS10_noSMB)
partial_R2[,3] = c(parial.R2_XS01_noHML,
                   parial.R2_XSO2_noHML,
                   parial.R2_XS03_noHML,
                   parial.R2_XSO4_noHML,
                   parial.R2_XS05_noHML,
                   parial.R2_XS06_noHML,
                   parial.R2_XS07_noHML,
                   parial.R2_XSO8_noHML,
                   parial.R2_XS09_noHML,
                   parial.R2_XS10_noHML)
partial_R2[,4] = c(parial.R2_XS01_noUMD,
                   parial.R2_XSO2_noUMD,
                   parial.R2_XSO3_noUMD,
                   parial.R2_XSO4_noUMD,
                   parial.R2_XSO5_noUMD,
                   parial.R2_XS06_noUMD,
                   parial.R2_XS07_noUMD,
                   parial.R2_XSO8_noUMD,
```

Table 4: Partial R-squared

	rmrf	smb	hml	umd
XS01	0.450	0.404	0.010	0.010
XS02	0.581	0.516	0.004	0.001
XS03	0.649	0.605	0.009	0.000
XS04	0.726	0.687	0.022	0.018
XS05	0.778	0.672	0.004	0.008
XS06	0.835	0.725	0.001	0.001
XS07	0.869	0.747	0.001	0.000
XS08	0.906	0.731	0.006	0.012
XS09	0.912	0.492	0.000	0.017
XS10	0.962	0.250	0.001	0.035

3. Cross-Section Regressions

```
#cross section
x_sect_reg.df = t(ptf_lm$coefficients) %>% as.data.frame()
#creating 10 mean returns
ds_with_means = ds %>% mutate(across(S1:S10, ~mean(.)))
s_means = ds_with_means %>% dplyr::select(S1:S10) %>% slice_head() %>% t()
x_sect_reg.df = x_sect_reg.df %>% mutate(mean_returns = s_means)
# first model
model <- lm(mean_returns ~ rmrf + smb, data = x_sect_reg.df)</pre>
model_summary <- summary(model)</pre>
# coefficients and p-values
coefficients <- model_summary$coefficients</pre>
p_values <- coefficients[, 4]</pre>
# table
results_table <- data.frame(</pre>
  Coefficient = rownames(coefficients),
  Estimate = coefficients[, 1],
  Std_Error = coefficients[, 2],
 t_value = coefficients[, 3],
  p_value = p_values,
 r2=model_summary$r.squared,
  adj_r2=model_summary$adj.r.squared
# mod 2
model2 = lm(mean_returns~rmrf+hml, data = x_sect_reg.df)
model2_summary <- summary(model2)</pre>
```

```
# coefficients and p-values
coefficients <- model2_summary$coefficients</pre>
p_values <- coefficients[, 4]</pre>
# table
results_table2 <- data.frame(</pre>
  Coefficient = rownames(coefficients),
  Estimate = coefficients[, 1],
  Std Error = coefficients[, 2],
  t_value = coefficients[, 3],
 p_value = p_values,
 r2=model2_summary$r.squared,
  adj_r2=model2_summary$adj.r.squared
# mod 3
model3 = lm(mean_returns~rmrf+umd, data = x_sect_reg.df)
model3_summary <- summary(model3)</pre>
# coefficients and p-values
coefficients <- model3_summary$coefficients</pre>
p_values <- coefficients[, 4]</pre>
# table
results_table3 <- data.frame(</pre>
  Coefficient = rownames(coefficients),
  Estimate = coefficients[, 1],
  Std_Error = coefficients[, 2],
  t_value = coefficients[, 3],
  p_value = p_values,
 r2=model3_summary$r.squared,
  adj_r2=model3_summary$adj.r.squared
final_result=rbind(results_table, results_table2, results_table3) %>% select(Estimate:adj_r2)
```

Table 5: Cross-section regression results

Table 5: Cross-section Regressions Results

	$D\epsilon$	ependent vario	able:			
	mean_returns					
coeff rmrf	-0.010**	-0.011**	-0.014***			
	(0.003)	(0.003)	(0.003)			
coeff smb	0.001					
	(0.001)					
coeff hml	,	0.006				
		(0.007)				
coeff umd		, ,	-0.011			
			(0.007)			
constant	0.022^{***}	0.023***	0.026***			
	(0.003)	(0.003)	(0.002)			
Observations	10	10	10			
\mathbb{R}^2	0.754	0.740	0.789			
Adjusted R ²	0.684	0.665	0.729			
Residual Std. Error $(df = 7)$	0.001	0.001	0.001			
F Statistic (df = 2 ; 7)	10.728***	9.944***	13.081***			

Note:

*p<0.1; **p<0.05; ***p<0.01