TALLER 2 SEÑALES y SISTEMAS.

a) $e^{-a|t|}$, $a \in \mathbb{R}^+$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \qquad a \in \mathbb{R}^+$$

$$x(t) = e^{-a|t|} = \begin{cases} e^{at} & t<0 \\ e^{-at} & t\geq 0 \end{cases}$$

$$= \int_{-\infty}^{0} e^{at}\, \underset{\substack{\downarrow \\ \text{fodo com}}}{e^{-j\omega t}}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left[\frac{e^{(a-j\omega)t}}{a-j\omega}\right]_{-\infty}^{0} + \left[\frac{e^{-(a+j\omega)t}}{a+j\omega}\right]_{0}^{+\infty}$$

$e^{-\infty} = 0\ldots$

$$= \frac{1}{a-jn\omega} - \frac{1}{a+jn\omega}$$

$$= \frac{(a+jn\omega)-(a-jn\omega)}{(a-jn\omega)(a+jn\omega)}$$

$a^2-(jn\omega)^2$

$(a-b)\cdot(a+b) = a^2-b^2$ $\quad j^2 = -1$

$$\frac{}{a^2+n^2\omega^2}$$

$j = \sqrt{-1}$
$j^2 = -1$

$\cancel{a}+jn\omega - \cancel{a}+jn\omega = 2jn\omega$

$$= \frac{2jn\omega}{a^2+n^2\omega^2}$$

b) $\cos(\omega_c t)$, $\omega_c \in \mathbb{R}$.

$$\boxed{\int_{-\infty}^{\infty} e^{-j\omega t}\, dt = 2\pi\,\delta(\omega)}$$

$$\cos(\omega_c t) = \frac{e^{j\omega_c t}+e^{-j\omega_c t}}{2}$$

$$F\{\cos(\omega_c t)\} = \frac{1}{2}\int_{-\infty}^{\infty}(e^{j\omega_c t}+e^{-j\omega_c t})\, e^{-j\omega t}\, dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} e^{j(\omega_c-\omega)t}\, dt + \int_{-\infty}^{\infty} e^{-j(\omega_c+\omega)t}\, dt$$

$\delta(\alpha) = \delta(-\alpha) \rightarrow$ par

$$= \frac{1}{2}\left[2\pi\,\delta(\omega-\omega_c) + \underbrace{2\pi}_{\text{factor común}}\,\delta(\omega+\omega_c)\right]$$

$\delta(-\omega_c-\omega) =$
$\delta(-(\omega_c-\omega))$

$$= \frac{1}{\cancel{2}}\left\{\cancel{2}\pi\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right]\right\}$$

$$= \pi\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right]$$

c) $\sin(\omega_s t)$, $\omega_s \in \mathbb{R}$

$$\text{Sen}(\omega_s t) = \frac{e^{j\omega_s t}-e^{-j\omega_s t}}{2}$$

$$F\{\text{sen}(\omega_s t)\} = \frac{1}{2}\int_{-\infty}^{\infty}(e^{j\omega_s t}-e^{-j\omega_s t})\, e^{-j\omega t}\, dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} e^{j(\omega_s-\omega)t}\, dt - \int_{-\infty}^{\infty} e^{-j(\omega_s+\omega)t}\, dt$$

$$= \frac{1}{2}\left[\underline{2\pi}\, \delta(\omega-\omega_S) - \underline{2\pi}\, \delta(\omega+\omega_S)\right]$$

$$= \frac{1}{2}\left\{2\pi\left[\delta(\omega-\omega_S) - \delta(\omega+\omega_S)\right]\right\}$$

$$= \pi\left[\delta(\omega-\omega_S) - \delta(\omega+\omega_S)\right]$$

d) $f(t)\cos(\omega_c t)$, $\omega_c \in \mathbb{R}$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt \qquad x(t) = f(t)\cos(\omega_c t)$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} f(t)\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)e^{-j\omega t}\,dt$$

$$= \frac{1}{2}\left[F(\omega-\omega_c) + F(\omega+\omega_c)\right]$$

e) $e^{-a|t|^2}$, $a \in \mathbb{R}^+$.

$$x(t) = e^{-a|t|^2} = \left\{e^{-at^2} \quad t \geq 0\right.$$

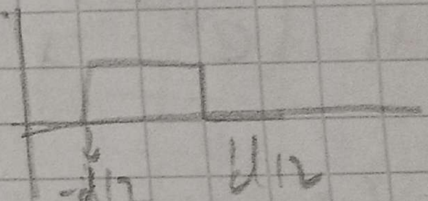
$$= \int_0^{+\infty} e^{-at}\, e^{-j\omega t}\,dt = \int e^{-(a+j\omega)t}\,dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{a+j\omega}\right]_0^{+\infty} = \sqrt{\frac{\pi}{a}}\cdot e^{\frac{-(\pi\omega)^2}{a}}$$

f) $A\,\mathrm{rect}\,d(t)$, $A, d \in \mathbb{R}$.

Pulso rectangular ancho d, centrado $t=0$

$$\text{rect}_d(t) = \begin{cases} 1, & -d/2 < t < d/2 \\ 0, & \text{en otro caso} \end{cases}$$



$x(t) = A \cdot \text{rect}_d(t)$

$$X(\omega) = \int_{-d/2}^{d/2} A \cdot e^{-j\omega t}\, dt = A \int_{-d/2}^{d/2} e^{-j\omega t}\, dt$$

Sustitución:

$u = -j\omega t$

$du = -j\omega\, dt$

$$= \left. \frac{e^{-j\omega t}}{-j\omega} \right]_{-d/2}^{d/2} = \left[\frac{e^{-j\omega(d/2)}}{-j\omega} - \frac{e^{-j\omega(-d/2)}}{-j\omega} \right]$$

$$\left[\frac{e^{-j\omega(d/2)}}{-j\omega} - \frac{e^{j\omega(d/2)}}{-j\omega} \right] = \frac{A}{-j\omega}\left[e^{-j\omega(d/2)} - e^{j\omega(d/2)} \right]$$

$$\frac{du}{-j\omega}$$

$$\frac{A}{-j\omega} \int e^{u}\, du$$

a) $\mathcal{F}\{e^{-jw_1 t}\cos(w_c t)\}$ → convolución → $f(t) = e^{-jw_1 t}\cos(w_c t)$ → tabla, pag 2

$w_1, w_c \in \mathbb{R}$;

Identidad Euler como:

$\cos(w_c t) = \frac{e^{jw_ct} + e^{-jw_ct}}{2}$ → tabla

$= e^{-jw_1t} \cdot \frac{e^{jw_ct} + e^{-jw_ct}}{2} = \frac{e^{-jw_1t}e^{jw_ct} + e^{-jw_1t}e^{-jw_ct}}{2}$ → factor común

$= \frac{e^{j(w_c-w_1)t} + e^{-j(w_c+w_1)t}}{2}$ → $\frac{1}{2}$

$\mathcal{F}\{e^{jw_0t}\} = 2\pi\delta(w - w_0)$ → tabla

Aplicando esto a cada término de $f(t)$

$\mathcal{F}\{f(t)\} = \frac{1}{2}\left[\mathcal{F}\{e^{j\underbrace{(w_c-w_1)}_{w_0}t}\} + \mathcal{F}\{e^{-j\underbrace{(w_c+w_1)}_{w_0}t}\}\right]$

$= \frac{1}{2}\left[2\pi\delta(w-(w_c-w_1)) + 2\pi\delta(w+(w_c+w_1))\right]$

factor común;

$= \frac{1}{2}\{2\pi[\delta(w-(w_c-w_1)) + \delta(w+(w_c+w_1))]\}$

$= \pi[\delta(w-(w_c-w_1)) + \delta(w+(w_c+w_1))]$ //

b) $\mathcal{F}\{u(t)\cos^2(w_ct)\}$, $w_c \in \mathbb{R}$.

$u(t) = \pi\delta(w) + \frac{1}{jw}$ → tabla → escalón unitario

$\cos^2(w_ct) = \frac{1+\cos(2w_ct)}{2}$ → tabla identidades trigonométricas

$= u(t)\cdot\left(\frac{1+\cos(2w_ct)}{2}\right) = \frac{u(t) + u(t)\cos(2w_ct)}{2}$ → $\frac{1}{2}$

Aplicando escalón unitario a cada uno

$\mathcal{F}\{f(t)\} = \frac{1}{2}\left[\underbrace{\mathcal{F}\{u(t)\}}_{\text{tabla}} + \underbrace{\mathcal{F}\{u(t)\cos(2w_ct)\}}_{\text{tabla, } w_0}\right]$

$= \frac{1}{2}\left\{\left(\pi\delta(w) + \frac{1}{w}\right) + \left(\frac{\pi}{2}[\delta(w-2w_c) + \delta(w+2w_c)] + \frac{jw}{2w_c^2 - w^2}\right)\right\} =$

c) $\mathcal{F}^{-1}\left\{\underbrace{\frac{7}{w^2+6w+45}}_{①} \times \underbrace{\frac{10}{(8+jw/3)^2}}_{②}\right\}$

$A + B \% C$

$+A \% B C$

①
$$\frac{7}{w^2+6w+45}$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow \frac{-6 \pm \sqrt{6^2-4(1)(45)}}{2(1)}$$

$a=1$
$b=6$
$c=45$

$$= \frac{-6 \pm \sqrt{36-180}}{2} = \frac{-6 \pm \sqrt{-144}}{2}$$

$$= \frac{-6}{2} \pm \frac{\sqrt{144}\sqrt{-1}}{2} = \frac{\cancel{-6}^{3}}{\cancel{2}_{1}} \pm \frac{\cancel{12}^{6} i}{\cancel{2}_{1}} = -3 \pm 6i$$

completar cuadrados formula:

$$x^2+bx+c = \left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

$$w^2+6w+45$$

$$\left(w+\frac{\cancel{6}^{3}}{\cancel{2}_{1}}\right)^2 - \left(\frac{\cancel{6}^{3}}{\cancel{2}_{1}}\right)^2 + 45$$

$$(w+3)^2-(3)^2+45$$

$$(w+3)^2-9+45$$

$$(w+3)^2+36$$

$$\frac{7}{(w+3)^2+6^2} \begin{matrix}\times 6 \\ \div 6\end{matrix} = \frac{1}{(w+3)^2+6^2} \qquad 6^2$$

propiedad:

$$F^{-1}\left\{\frac{a}{(w+c)^2+a^2}\right\} = e^{-ct}\sin(at)\cdot u(t) \rightarrow \text{tabla}$$

$$\frac{7}{6}\left\{\frac{6}{(w+3)^2+6^2}\right\} = \frac{7}{6}e^{-3t}\sin(6t)\cdot u(t)$$

②
$$\frac{10}{\left(8+\frac{jw}{3}\right)^2} \rightarrow \quad 8+\frac{jw}{3} = \left(\frac{24+jw}{3}\right)^2$$

$$\left(8+\frac{jw}{3}\right)^2 = \frac{(24+jw)^2}{9}$$

$$\frac{\frac{10}{1}}{\frac{(24+jw)^2}{9}} = \frac{10\cdot 9}{(24+jw)^2} = \frac{90}{(24+jw)^2}$$

$$\mathcal{F}^{-1}\left\{\frac{1}{(a+j\omega)^2}\right\} \to t e^{-at} \cdot u(t) \quad \text{— tabla}$$

$$\mathcal{F}^{-1}\left\{\frac{90}{(21+j\omega)^2}\right\} = 90\left\{\frac{1}{(21+j\omega)^2}\right\} = 90\, t e^{-21t} \cdot u(t)$$

Convolución: $f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau) g(t-\tau)\, d\tau$

$$= \left(\underbrace{\frac{7}{6} e^{-3t} \sin(6t)\, u(t)}_{f(\tau)}\right) * \left(\underbrace{90\, t e^{-21t} \cdot u(t)}_{g(t-\tau)}\right) \to \tau \le t$$

$\tau \geq 0 \to u(\tau)$

$u(t-\tau)$

$$= \frac{7}{6} \cdot 90 \int_0^t e^{-3\tau} \sin(6\tau) \cdot (t-\tau) e^{-21(t-\tau)}\, d\tau$$

$$= 105\, e^{-21t} \int_0^t (t-\tau) e^{21\tau} \sin(6\tau)\, d\tau \qquad e^{-3\tau} \sin(6\tau)(t-\tau) e^{-21t+21\tau}$$

$$\int_0^t (t-\tau) e^{21\tau} \sin(6\tau)\, d\tau \to \text{partes} \qquad e^{\tau(-3+21)} \sin(6\tau)(t-\tau) e^{-21t}$$

$$\int e^{ax} \sin(bx)\, dx = \frac{e^{ax}(a \sin(bx) - b \cos(bx))}{a^2+b^2} \to \text{tabla integrales}$$

$$\int_0^t (t-\tau) e^{21\tau} \sin(6\tau)\, d\tau = \left[\frac{(t-\tau) e^{21\tau}(21\sin(6\tau) - 6\cos(6\tau))}{477}\right]_0^t$$

$$- \int_0^t -\frac{e^{21\tau}(21\sin(6\tau) - 6\cos(6\tau))}{477}\, d\tau$$

$$= 105\, e^{-21t} \left[\frac{21 e^{21t} \sin(6t) - 6 e^{21t} \cos(6t) + 6}{477}\right]$$

$$= \frac{35}{159}\left(21 e^{-3t} \sin(6t) - 6 e^{-3t} \cos(6t) + 6 e^{-21t}\right) u(t)$$

$$90(t-\tau) e^{-21(t-\tau)} \cdot u(t-\tau) \to g(t-\tau)$$

$$(t-\tau) e^{21\tau}$$

$$t e^{21} - e^{21\tau}\,\tau$$

$u(t) \to$ se incluye fuera de la integral

$u(t)$ apaga el resultado para $t < 0$

porque la exponencial en 0 si t es negativo

d) $\mathcal{F}\{3t^3\}$

$3\mathcal{F}\{t^3\}$, derivada de una frecuencia → Tabla prop - Transformada

$\mathcal{F}[t^n x(t)] = j^n \frac{d^n}{d\omega^n} X(j\omega)$

$g^{-1}(1) = 2\pi f(\omega)$ → sin h encontrar

Aplicando propiedad

$$F\{1^3\} = j^3 \frac{d^3}{d\omega^3} F\{1\} = -j \frac{d^3}{d\omega^3}[2\pi\delta(\omega)]$$

$j = \sqrt{-1}$ 3 derivada delta de Dirac:

$j^2 = \sqrt{-1}^2 = -1$ $\int_{-\infty}^{\infty} \delta'''(\omega)\phi(\omega)\, d\omega = (-1)^3 \phi'''(0)$

$j^3 = -1 \cdot j = -j$

$F\{1^3\} = -2\pi\,\delta'''(\omega)$; luego $= 3(-2\pi\,\delta'''(\omega)) = -6j\pi\delta'''(\omega)$

e) $\frac{B}{T}\sum_{n=-\infty}^{\infty}\left(\frac{1}{a^2+(\omega-n\omega_0)^2} + \frac{1}{a+j(\omega-n\omega_0)}\right)$, $n \in \{0, \pm 1, \pm 2, \ldots\}$

$\omega_0 = 2\pi/T$ y $B, T \in \mathbb{R}^+$

① $\frac{1}{a^2+(\omega-n\omega_0)^2}$ → de par $F\{e^{-\alpha|t|}\} \to \frac{2\alpha}{a^2+\omega^2}$ $\times 2a$ $\div 2a$

② $\frac{1}{a+j(\omega-n\omega_0)}$ → de par $F\{u(t)e^{-\alpha t}\} \to \frac{1}{\alpha+j\omega}$

① $\frac{1}{2a}\cdot\frac{2a}{a^2+(\underbrace{\omega-n\omega_0}_{\omega})^2}$ → su inversa es: $\frac{1}{2a}e^{-a|t|}e^{jn\omega_0 t}$

② $\frac{1}{a+j(\omega-n\omega_0)}$ → su inversa es: $e^{-at}u(t)e^{jn\omega_0 t}$

Prop desplazamiento frecuencia: sin h encontrar $T = \frac{2\pi}{\omega_0}$

$$\sum_{n=-\infty}^{\infty} F(\omega-n\omega_0) \xrightarrow{F^{-1}} f(t)\cdot\sum_{k=-\infty}^{\infty}\delta(t-kT)$$

$$f(t) = \frac{B}{T}\left[\sum_{n=-\infty}^{\infty}\left(\frac{1}{2a}e^{-a|t|} + e^{-at}u(t)\right)e^{jn\omega_0 t}\right]$$

$$= \frac{B}{2aT}e^{-a|t|}\sum_{n=-\infty}^{\infty}e^{jn\omega_0 t} + \frac{B}{T}e^{-at}u(t)\sum_{n=-\infty}^{\infty}e^{jn\omega_0 t}$$

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Ser. de Fourier: [illegible]

$$\sum_{n=-\infty}^{\infty} e^{jn\omega t} = 2\pi \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

$$f(t) = \frac{2\pi B}{\cancel{2}aT} e^{-a|t|} \sum_{k=-\infty}^{\infty} \delta(t-kT) + \frac{2\pi B}{T} e^{-at} u(t) \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

$$= \frac{\pi B}{aT} \sum_{k=-\infty}^{\infty} e^{-a|kT|} \delta(t-kT) + \frac{2\pi B}{T} \sum_{k=0}^{\infty} e^{-akT} \delta(t-kT)$$

u(t) elimina los impulsos en $k<0$

$y(t)$ elimina las ... en $t<0$

- Aplicación en comunicaciones - modulación AM

$c(t) = A_c \cos(2\pi F_c t)$, con $A_c, F_c \in \mathbb{R}$

$$y(t) = \left(1 + \frac{m(t)}{A_c}\right) c(t).$$

$$y(t) = \left(1 + \frac{m(t)}{A_c}\right) A_c \cos(2\pi F_c t) = \underbrace{A_c \cos(2\pi F_c t)}_{\text{portadora pura}} + \underbrace{\frac{m(t) A_c \cos(2\pi F_c t)}{A_c}}_{\text{modulación en amplitud}}$$

Transformada de Fourier:

$$F\{A_c \cos(2\pi F_c t)\} = \frac{A_c}{2}[\delta(F - F_c) + \delta(F + F_c)]$$

$\omega = 2\pi F$

$$F\{\cos(\omega_0 t)\} = \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] \rightarrow \text{tabla}$$

Propiedad modulación:

$$F\{m(t)\cos(2\pi F_c t)\} = \frac{1}{2}[M(F - F_c) + M(F + F_c)] \qquad M(f) = F\{m(t)\}$$

Espectro completo:

$$Y(F) = \frac{A_c}{2}[\delta(F - F_c) + \delta(F + F_c)] + \frac{1}{2}[M(F - F_c) + M(F + F_c)]$$

DE LA FIGURA:

Entrada (señal DSB-CS recibida)

$x(t) = A_1 m(t) \cos(2\pi f_0 t + \theta_0)$ $\qquad \theta_0 = 0$

$x(t) = A_1 m(t) \cos(2\pi f_0 t)$ $\qquad F\{m(t)\cos(2\pi f_0 t)\}$

Espectro $X(f)$

$$X(f) = \frac{A_1}{2} M(f - f_0) + \frac{A_1}{2} M(f + f_0)$$

Salida del Mixer (Multiplicación portadora)

$$\frac{A_1 m(t)}{2} + \frac{A_1 m(t)}{2}\cos(4\pi f_0 t)$$

Espectro $= A_1/2\, M(f) + A_1/4\, M(f - 2f_0) + A_1/4\, M(f + 2f_0)$

Filtro para bajas (LPF)
Elimina las bandas altas $\pm 2f_0$.

$\frac{A_1}{2} m(t) \Rightarrow$ Espectro $\frac{A_1}{2} M(f)$

Escalamiento de Amplitud

$\frac{A_1}{2} m(t) \cdot \frac{2}{A_1} = m(t) \Rightarrow$ Espectro final: $M(f) \rightarrow$ se recupera señal original