exercise_09a

Proof the idempotence of the 'closing–opening' alternated filter.

Some operators have the special property that applying them more than once to the same image produces no further change after the first application. Such operators are said to be **idempotent**. Examples include the morphological operators opening and closing.

Starting with opening it is an erosion followed by a dilation. Also these two operation are idempotent :

$$\gamma_B(I) = \delta_{-B}(\varepsilon_{-B}(I))$$

δ dilation
ε erosion

For the closing it is the opposite:

$$\varphi_B(I) = \varepsilon_{-B}(\delta_{-B}(I))$$

We know that erosion and dilation are both idempotent. The application of them alterated to generate the opening and the closing generated an idempotent filter too, so a combination of it can be seen as a new idempotent operator:

$$\varepsilon_{-B}(\varepsilon_{-B}(I)) = \varepsilon_{-B}(I)$$

$$\delta_{-B}(\delta_{-B}(I)) = \delta_{-B}(I)$$

$$\underbrace{\varepsilon_{-B}(\delta_{-B}}_{\varphi_{-B}}(\underbrace{\varepsilon_{-B}(\delta_{-B}(I)}_{\varphi_{-B}(I)}))) = \underbrace{\varepsilon_{-B}(\delta_{-B}(I))}_{\varphi_{-B}(I)}$$

$$\underbrace{\delta_{-B}(\varepsilon_{-B}}_{\gamma_{-B}}(\underbrace{\delta_{-B}(\varepsilon_{-B}(I)}_{\gamma_{-B}(I)}))) = \underbrace{\delta_{-B}(\varepsilon_{-B}(I))}_{\gamma_{-B}(I)}$$

So I can do the same alternating opening and closing (thet is exactly the same as alternating erosion and dilation

$$\underbrace{\varphi_{-B}(\gamma_{-B}}_{opClo_{-B}}(\underbrace{\varphi_{-B}(\gamma_{-B}(I)}_{opClo_{-B}(I)}))) = \underbrace{\varphi_{-B}(\gamma_{-B}(I))}_{opClo_{-B}(I)}$$

$$\varepsilon_{-B}(\delta_{-B}(\delta_{-B}(\varepsilon_{-B}(\varepsilon_{-B}(\delta_{-B}(\delta_{-B}(\varepsilon_{-B}(I)))))))) =$$

$$\varepsilon_{-B}(\delta_{-B}(\underbrace{\delta_{-B}(\varepsilon_{-B}(\delta_{-B}(\varepsilon_{-B}(I))))}_{\gamma_{-B}(\gamma_{-B})})) =$$

$$\varepsilon_{-B}(\delta_{-B}(\delta_{-B}(\varepsilon_{-B}(I)))) = \varphi_{-B}(\gamma_{-B}(I)) \checkmark$$

The same for the closing-opening operation:

$$\underbrace{\gamma_{-B}\Big(\varphi_{-B}}_{clop_{-B}}\Big(\underbrace{\gamma_{-B}\big(\varphi_{-B}(I)\big)}_{clop_{-B}(I)}\Big)\Big) = \underbrace{\gamma_{-B}\big(\varphi_{-B}(I)\big)}_{clop_{-B}(I)}$$

$$\delta_{-B}(\varepsilon_{-B}(\varepsilon_{-B}(\delta_{-B}(\delta_{-B}(\varepsilon_{-B}(\varepsilon_{-B}(\delta_{-B}(I)))))))) =$$

$$\delta_{-B}(\varepsilon_{-B}(\underbrace{\varepsilon_{-B}(\delta_{-B}(\varepsilon_{-B}(\delta_{-B}(I))))}_{\varphi_{-B}(\varphi_{-B})})) =$$

$$\delta_{-B}(\varepsilon_{-B}(\varepsilon_{-B}(\delta_{-B}(I)))) = \gamma_{-B}(\varphi_{-B}(I))\ \checkmark$$