R05 Sampling and Estimation

Version 1.0

1. Introduction

A sample is a subset of a population. We can study a sample to infer conclusions about the population itself. For example, if all the stocks trading in the US are considered a population, then indices such as the S&P 500 are samples. We can look at the performance of the S&P 500 and draw conclusions about how all stocks in the US are performing. This process is known as sampling and estimation.

2. Sampling Methods

There are various methods for obtaining information on a population through samples. The information we obtain usually concerns a **parameter**, a quantity used to describe a population. To estimate a parameter, we use sample statistics. A **statistic** is a quantity used to describe a sample.

There are two reasons why sampling is used:

- Time saving: In many cases it will be very time consuming to examine every member of the population.
- Monetary saving: In some cases, examining every member of the population becomes economically inefficient.

There are two types of sampling methods:

- Probability sampling: Every member of the population has an equal chance of being selected. Therefore, the sample created is representative of the population.
- Non-probability sampling: Every member of the population may not have an equal chance of being selected. This is because sampling depends on factors such as the sampler's judgement or the convenience to access data. Therefore, the sample created may not be representative of the population.

All else equal, the probability sampling method is more accurate and reliable as compared to the non-probability sampling method.

In the subsequent sections, we will discuss the following sampling methods:

- Probability sampling
  - Simple random sampling
  - Systematic sampling
  - Stratified random sampling
  - Cluster sampling
- Non-probability sampling
  - Convenience sampling
  - Judgement sampling

2.1. Simple Random Sampling

Simple random sampling is the process of selecting a sample from a larger population in such a way that each member of the population has the same probability of being included in the sample.

Sampling distribution

If we draw samples of the same size several times and calculate the sample statistic, the sample statistic will be different each time. The distribution of values of the sample statistic is called a sampling distribution.

For example, say you select 100 stocks from a universe of 10,000 stocks and calculate the average annual returns of these 100 stocks. Let's say you get an average return of 15%. You repeat this process with a second sample of 100 stocks. This time, you get an average return of 14%. You keep repeating this process and each time you get a different average return. The distribution of these sample average returns is called a sampling distribution.

Sampling error

Sampling error is the difference between a sample statistic and the corresponding population parameter.

The sampling error of the mean is given by:

$$\text{Sampling error of the mean} = \bar{x} - \mu$$

For example, let's say you want to estimate the average returns of 10,000 stocks. You draw a sample of 100 stocks and calculate the average return of these 100 stocks as 15%. However, the actual average of the 10,000 stocks was 12%. Then the sampling error = 15% - 12% = 3%.

Systematic sampling: In this technique, we select every kth member of the population until we have a sample of the desired size. Samples created using this technique should be approximately random.

Instructor's Note: Researchers calculate the sampling interval 'k' by dividing the entire population size by the desired sample size.

2.2. Stratified Random Sampling

In stratified random sampling, the population is divided into subgroups based on one or more distinguishing characteristics. Samples are then drawn from each subgroup, with sample size proportional to the size of the subgroup relative to the population. Finally, samples from each subgroup are pooled together to form a stratified random sample.

The advantage of stratified random sampling is that the sample will have the same distribution of key characteristics as the overall population. This can help reduce the sampling error. Stratified random sampling therefore produces more precise parameter estimates than simple random sampling

For example, you divide the universe of 10,000 stocks as per their market capitalization such that you have 5,000 large cap stocks, 3,000 mid cap stocks, and 2,000 small cap stocks. In stratified random sampling, to select a total sample of 100 stocks, you will randomly select 50 large cap stocks, 30 mid cap stocks, and 20 small cap stocks and pool all these samples together to form a stratified random sample.

Example

Paul wants to categorize publicly listed stocks for his research project. He first divides the stocks into 15 industries. Then from each industry, he categorizes companies into three groups: small, medium, large. Finally, he divides these into value versus growth stocks. How many cells or strata does the sampling plan entail?

A. 20
B. 45
C. 90

Solution:

C is correct. This is an application of the multiplication rule of counting. The total number of cells is the product of 15, 3, and 2. Thus the answer is 90.

2.3 Cluster Sampling

Cluster sampling is similar to stratified random sampling as it also requires the population to be divided into subpopulation groups, called clusters. Each cluster is essentially a mini-representation of the entire population. Then some random clusters are chosen as a whole for sampling.

Instructor's Note: Clusters are generally based on natural groups separating the population. For example, you might be able to divide your data into natural groupings like city blocks, voting districts, or school districts.

The main difference between cluster sampling and stratified random sampling is that in cluster sampling, the whole cluster is selected; and not all clusters are included in the sample. In stratified random sampling, however, only a few members from each stratum are selected; but all strata are included in the sample.

The difference between simple random sampling, stratified random sampling, and cluster sampling is illustrated in the figure below:

As compared to SRS and stratified sampling, cluster sampling is less accurate because the chosen sample may be less representative of the entire population. However, this method is the most time-efficient and cost-efficient amongst the three.

2.4 Non-Probability Sampling

The two major types of non-probability sampling methods are:

- Convenience sampling: In this method, the researcher selects members from a population based on how easy it is to access the member i.e., data is collected from a conveniently available pool of respondents. The disadvantage of this method is that the sample selected may not be representative of the entire population. The advantage is that data can be collected quickly and at a low cost. Hence this method is particularly suitable for small-scale pilot studies.
- Judgmental sampling: In this method, the researcher uses his knowledge and professional judgment to selectively handpick members from the population. The disadvantage of this method is that the sampling may be impacted by the researcher's bias and the results may be skewed. The advantage of this method is that it allows the researcher to directly go to the target population of interest. For example, when auditing financial statements, experienced auditors can use their professional judgment to select important accounts or transactions that can provide sufficient audit coverage.

2.5 Sampling from Different Distributions

Instructor's Note: This section does not contain testable concepts. The core point is presented below.

In addition to selecting an appropriate sampling method, researchers also need to be careful when sampling from a population that is not under one single distribution. In such cases, the larger population should be divided into smaller parts, and samples should be drawn from the smaller parts separately.

3. Distribution of the Sample Mean and the Central Limit Theorem

The sample mean is a random variable with a probability distribution known as the statistic's sampling distribution. To understand this concept, consider the following population: last year's returns on every stock traded in the United States. We are interested in the mean return of all stocks but do not have time to calculate the population mean. Hence, we draw a sample of 50 stocks and compute the sample mean. We then draw another sample of 50 stocks and compute the sample mean. This exercise can be repeated several times giving us a distribution of sample means. This distribution is called the statistic's sampling distribution. The central limit theorem, explained below, helps us understand the sampling distribution of the mean.

3.1. The Central Limit Theorem

According to the central limit theorem, if we draw a sample from a population with a mean μ and a variance σ^2, then the sampling distribution of the sample mean:

- will be normally distributed (irrespective of the type of distribution of the original population).
- will have a mean of μ.
- will have a variance of σ^2/n.

For example, suppose the average return of the universe of 10,000 stocks is 12% and its standard deviation is 10%. Through central limit theorem we can conclude that if we keep drawing samples of 100 stocks and plot their average returns, we will get a sampling distribution that will be normally distributed with mean = 12% and variance of $10^2/100$ = 1%.

3.2 Standard Error of the Sample Mean

The standard deviation of the distribution of the sample means is known as the standard error of the sample mean.

When we know the population standard deviation, the standard error of the sample mean can be calculated as:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

When we do not know the population standard deviation (σ) we can use the sample standard deviation (s) to estimate the standard error of the sample mean:

$$s_{\bar{X}} = \frac{s}{\sqrt{n}}$$

Example

The mean of a population is 12 and the standard deviation is 3. Given that the population comprises of 64 observations, what is the standard error of the sample mean?
A. 0.375
B. 0.378
C. 0.667

Solution:

A is correct. Standard Error = $\sigma/\sqrt{n} = 3/\sqrt{64} = 0.375$

4. Point Estimates of the Population Mean

Statistical inference consists of two branches: 1) hypothesis testing and 2) estimation. Hypothesis testing addresses the question 'Is the value of this parameter equal to some specific value?' This is discussed in detail in the next reading. In this section, we will discuss estimation. Estimation seeks to answer the question 'What is this parameter's value?' In

estimation, we make the best use of the information in a sample to form one of several types of estimates of the parameter's value.

A **point estimate** is a single number that estimates the unknown population parameter.

Interval estimates (or confidence intervals) are a range of values that bracket the unknown population parameter with some specified level of probability.

4.1. Point Estimators

The formulas that we use to compute the sample mean and all other sample statistics are examples of estimation formulas or **estimators**. The particular value that we calculate from sample observations using an estimator is called an **estimate**. For example, the calculated value of the sample mean in a given sample is called a **point estimate** of the population mean.

The three desirable properties of an estimator are:

- Unbiasedness**:** Its expected value is equal to the parameter being estimated.
- Efficiency**:** It has the lowest variance as compared to other unbiased estimators of the same parameter.
- Consistency**:** As sample size increases, the sampling error decreases and the estimates get closer to the actual value.

5. Confidence Intervals for the Population Mean and Selection of Sample Size

A confidence interval is a range of values, within which the actual value of the parameter will lie with a given probability. Confidence interval is calculated as:

Confidence interval = point estimate ± (reliability factor x standard error)

where:
point estimate = a point estimate of the parameter
reliability factor = a number based on the assumed distribution of the point estimate and the degree of confidence (1 - α) for the confidence interval
standard error = standard error of the sample statistic providing the point estimate

The quantity 'reliability factor x standard error' is also referred to as the precision of the estimator. Larger values indicate lower precision and vice versa.

Calculating confidence intervals

To calculate a confidence interval for a population mean, refer to the table below and select t statistic or z statistic as per the scenario.

Sampling from		Small sample size(n<30)	Large sample size(n≥30)
Normal distribution	Variance known	z	z
	Variance unknown	t	t (or z)

Non –normal distribution	Variance known	NA	z
	Variance unknown	NA	t (or z)

Use the following formulae to calculate the confidence interval:

$$\text{Confidence interval} = \bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$\text{Confidence interval} = \bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

Sampling from a normal distribution with known variance

The most basic confidence interval for the population mean arises when we are sampling from a normal distribution with known variance.

The confidence interval will be calculated as:

$$\text{Confidence Interval} = \bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

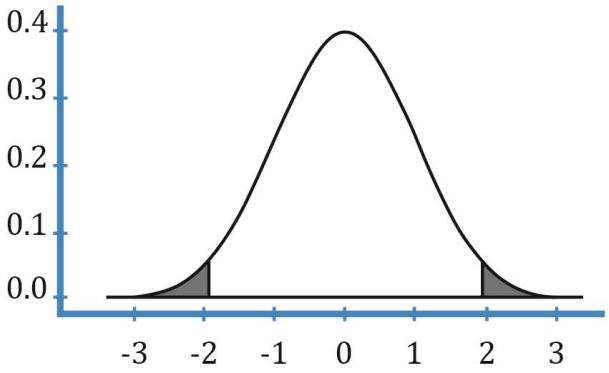
where:

$\bar{X}$ = sample mean

$z_{\alpha/2}$ = reliability factor

$\sigma/\sqrt{n}$ = standard error

The reliability factor ($z_{\alpha/2}$) depends purely on the degree of confidence. If the degree of confidence (1 – α) is 95% or 0.95, the level of significance (α) is 5% or 0.05. α/2 = 0.025. α/2 is the probability in each tail of the standard normal distribution. This is shown in blue in the figure below:



The reliability factors that are most frequently used when we construct confidence intervals based on the standard normal distribution are:

- 90% confidence intervals: α = 0.1, α/2 = 0.05. Reliability factor = $z_{0.05}$ = 1.65
- 95% confidence intervals: α = 0.05, α/2 = 0.025. Reliability factor = $z_{0.025}$ = 1.96
- 99% confidence intervals: α = 0.01, α/2 = 0.005. Reliability factor = $z_{0.005}$ = 2.58

Memorize these confidence intervals and the corresponding reliability factors.

Example

You take a random sample of stocks on the National Stock Exchange (NSE). The sample size is 100 and the average Sharpe ratio is 0.50. Assume that the Sharpe ratios of all stocks on the NSE follow a normal distribution with a standard deviation of 0.30. What is the 90% confidence interval for the mean Sharpe ratio of all stocks on the NSE?

Solution:

$$\text{Confidence Interval} = \bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 0.50 \pm 1.65 \frac{0.30}{\sqrt{100}} = 0.50 \pm 0.0495$$

Therefore, the 90% confidence interval for the mean Sharpe ratio of all stocks on the NSE is: 0.4505 to 0.5495.

As we increase the degree of confidence, the confidence interval becomes wider. If we use a 95% confidence interval, the reliability factor is 1.96. And the confidence interval ranges from 0.50 – 1.96 x 0.03 to 0.50 + 1.96 x 0.03 which is: 0.4412 to 0.5588. This range is wider than what we had with a 90% confidence interval.

Sampling from a normal distribution with unknown variance

If the distribution of the population is normal with unknown variance, we can use the t-distribution to construct a confidence interval.

The confidence interval can be calculated using the following formula:

$$\text{Confidence Interval} = \bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

This relationship is similar to what we've discussed before except here we use the t-distribution. Since the population standard deviation is not known, we have to use the sample standard deviation which is denoted by the symbol 's'. We will now see how to read the t-distribution table using an example.

Example

Given a sample size of 20, what is the reliability factor for a 90% confidence level?

Solution:

In order to answer this question, we need to refer to the t-table. A snapshot of the table is given below. This table shows the level of significance for one-tailed probabilities.

df	P = 0.10	P = 0.05	P = 0.025	P = 0.01	P = 0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032

6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845

The degrees of freedom in this case are n – 1 = 20 – 1 = 19. The level of significance is calculated as: 100(1 - α) % = 90% and α = 0.10. In order to calculate the reliability factor $t_{\alpha/2} = t_{0.10/2} = t_{0.05}$, we look at the row with df = 19. We then look at the column with p = 0.05. The value that satisfies these two criteria is 1.729. This is the reliability factor.

Example

An analyst wants to estimate the return on the Hang Seng Index for the current year using the following data and assumptions:

- Sample size = 64 stocks from the index
- Mean return for the stocks in the sample for the previous year = 0.12
- Variance = 0.081

It is given that the reliability factor for a 95% confidence interval with unknown population variance and sample size greater than 64 is 1.96. Assuming that the index return this year will be the same as it was last year, what is the estimate of the 95% confidence interval for the index's return this year?

Solution:

Confidence Interval $= \overline{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 0.12 \pm 1.96 \left[\frac{\sqrt{0.081}}{\sqrt{64}}\right] = 0.12 \pm 0.07 = 0.05$ to 0.19

Instructor's Note:
A conservative approach to confidence intervals relies on the t-distribution rather than the normal distribution, and use of the t-distribution will increase the reliability of the confidence interval.

5.1 Selection of Sample Size

The choice of sample size affects the width of a confidence interval. A larger sample size decreases the width of a confidence interval and improves the precision with which we can estimate the population parameter. This is obvious when we consider the confidence interval relationship, shown below:

$$\text{Confidence Interval} = \bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

All else being equal, when the sample size (n) is increased, the standard error ($s/\sqrt{n}$) decreases and we have a more precise (narrower) confidence interval. Based on this discussion it might appear that we should use as large a sample size as possible. However, we must consider the following issues:

- Increasing the sample size may result in sampling from more than one population.
- Increasing the sample size may involve additional expenses that outweigh the value of additional precision.

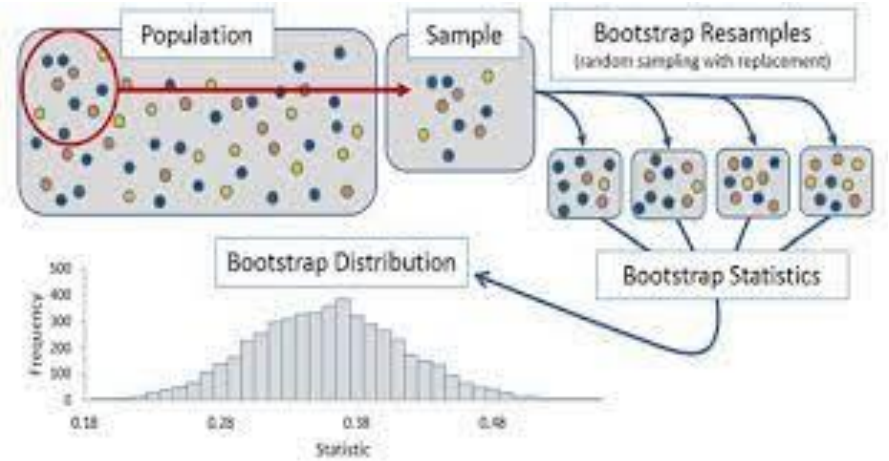
6. Resampling

Resampling is a computational tool in which we repeatedly draw samples from the original observed data sample for the statistical inference of population parameters. Two popular resampling methods are:

- Bootstrap
- Jackknife

Bootstrap

The bootstrap technique is illustrated in the figure below:



The technique is used when we do not know what the actual population looks like. We simply have a sample of size n drawn from the population. Since the random sample is a good representation of the population, we can simulate sampling from the population by

sampling from the observed sample i.e., we treat the randomly drawn sample as if it were the actual population.

Under this technique, samples are constructed by drawing observations from the large sample (of size n) one at a time and returning them to the data sample after they have been chosen. This allows a given observation to be included in a given small sample more than once. This sampling approach is called sampling with replacement.

If we want to calculate the standard error of the sample mean, we take many resamples and calculate the mean of each resample. We then construct a sampling distribution with these resamples. The bootstrap sampling distribution will approximate the true sampling distribution and can be used to estimate the standard error of the sample mean. Similarly, the bootstrap technique can also be used to construct confidence intervals for the statistic or to find other population parameters, such as the median.

Bootstrap is a simple but powerful technique that is particularly useful when no analytical formula is available to estimate the distribution of estimators.

Jackknife

In the Jackknife technique we start with the original observed data sample. Subsequent samples are then created by leaving out one observation at a time from the set (and not replacing it). Thus, for a sample of size n, jackknife usually requires n repetitions. The Jackknife method is frequently used to reduce the bias of an estimator.

Note that bootstrap differs from jackknife in two ways:

- Bootstrap repeatedly draws samples with full replacement.
- The researcher has to decide how many repetitions are appropriate.

7. Data Snooping Bias, Sample Selection Bias, Look-Ahead Bias, and Time Period Bias

There are many issues in sampling. Here we discuss four main issues.

7.1 Data-Snooping Bias

Data-snooping is the practice of analyzing the same data again and again, till a pattern that works is identified.

There are two signs that can warn analysts about the potential existence of data snooping:

- Too much digging/too little confidence: Many different variables were tested, until significant ones were found. Unfortunately, many researchers do not disclose the number of variables examined in developing a model.
- No story/no future: Without a reasonable economic rationale or story for why a variable should work, the variable is unlikely to have predictive power.

The best way to avoid this bias is to test the pattern on out-of-sample data to check if it holds.

7.2 Sample Selection Bias

When data availability leads to certain assets being excluded from the analysis, the resulting problem is known as sample selection bias.

Two types of sample selection biases are:

- Survivorship bias: For example, many mutual fund databases provide historical information about only those funds that currently exist. Since mutual funds usually shut down after a period of poor performance, this may inflate the average performance numbers of the database.
- Backfill bias: For example, if a new hedge fund is added to a given index, the fund's past performance is also backfilled into the index. Since new funds are usually added after a period of good performance, this may inflate the index performance.

7.3 Look-Ahead Bias

A test design is subject to look-ahead bias if it uses information that was not available to market participants at the time the market participants act in the model. For example, an analyst wants to use the company's book value per share to construct the P/B variable for that particular company. Although the market price of a stock is available for all market participants at the same point in time, fiscal year-end book equity per share might not become publicly available until sometime in the following quarter.

One way to avoid this bias is to use point-in-time (PIT) data. PIT data is stamped with the date when it was released. In the above example, PIT data of P/B would be stamped with the company's filing or press release date rather than the fiscal quarter end date.

7.4 Time-Period Bias

A test design is subject to time-period bias if it is based on a time period that may make the results time-period specific. A short time series is likely to give period-specific results that may not reflect a longer period. If a time series is too long, fundamental structural changes may have occurred in that time period.

Summary

LO.a: Compare and contrast probability samples with non-probability samples and discuss applications of each to an investment problem.

Probability sampling: Every member of the population has an equal chance of being selected. Therefore, the sample created is representative of the population.

Non-probability sampling: Every member of the population may not have an equal chance of being selected. This is because sampling depends on factors such as the sampler's judgement or the convenience to access data. Therefore, the sample created may not be representative of the population.

All else equal, the probability sampling method is more accurate and reliable as compared to the non-probability sampling method.

LO.b: Explain sampling error.

Sampling error is the difference between a sample statistic and the corresponding population parameter.

Sampling error of the mean $= \bar{x} - \mu$

LO.c: Compare and contrast simple random, stratified random, cluster, convenience, and judgmental sampling.

Simple random sampling is the process of selecting a sample from a larger population in such a way that each element of the population has the same probability of being included in the sample.

In stratified random sampling, the population is divided into subgroups based on one or more distinguishing characteristics. Samples are then drawn from each subgroup, with sample size proportional to the size of the subgroup relative to the population. Finally, samples from each subgroup are pooled together to form a stratified random sample.

Cluster sampling is similar to stratified random sampling as it also requires the population to be divided into subpopulation groups, called clusters. Each cluster is essentially a mini-representation of the entire population. Then some random clusters are chosen as a whole for sampling.

In convenience sampling, the researcher selects members from a population based on how easy it is to access the member i.e., data is collected from a conveniently available pool of respondents

In judgmental sampling, the researcher uses his knowledge and professional judgment to selectively handpick members from the population.

LO.d: Explain the central limit theorem and its importance.

According to the central limit theorem, if we draw a sample from a population with a mean μ

and a variance σ^2, then the sampling distribution of the sample mean:

- will be normally distributed (irrespective of the type of distribution of the original population).
- will have a mean of μ.
- will have a variance of σ^2/n.

LO.e: Calculate and interpret the standard error of the sample mean.

The standard deviation of the distribution of the sample means is known as the standard error of the sample mean.

When we know the population standard deviation, the standard error of the sample mean can be calculated as: $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

When we do not know the population standard deviation (σ) we can use the sample standard deviation (s) to estimate the standard error of the sample mean $s_{\bar{X}} = \frac{s}{\sqrt{n}}$

LO.f: Identify and describe desirable properties of an estimator.

The three desirable properties of an estimator are:

- Unbiasedness**:** Its expected value is equal to the parameter being estimated.
- Efficiency**:** It has the lowest variance as compared to other unbiased estimators of the same parameter.
- Consistency**:** As sample size increases, the sampling error decreases and the estimates get closer to the actual value.

LO.g: Contrast a point estimate and a confidence interval estimate of a population parameter.

A point estimate is a single value estimate that serves as an approximation for the actual value of the parameter.

A confidence interval is a range of values, within which the actual value of the parameter will lie with a given probability.

Confidence interval = Point estimate ± (reliability factor x standard error of point estimate)

LO.h: Calculate and interpret a confidence interval for a population mean, given a normal distribution with 1) a known population variance, 2) an unknown population variance, or 3) an unknown population variance and a large sample size.

Refer to the table below and select t statistic or z statistic as per the scenario.

Sampling from		**Small sample size(n<30)**	**Large sample size(n≥30)**
Normal distribution	Variance known	z	z
	Variance unknown	t	t (or z)
Non –normal distribution	Variance known	NA	z
	Variance unknown	NA	t (or z)

Use the following formulae to calculate the confidence interval:

$$\text{Confidence interval} = \bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$\text{Confidence interval} = \bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

LO.i: Describe the use of resampling (bootstrap, jackknife) to estimate the sampling distribution of a statistic.

Resampling is a computational tool in which we repeatedly draw samples from the original observed data sample for the statistical inference of population parameters. Two popular resampling methods are:

- Bootstrap: Constructs the sampling distribution of an estimator by repeatedly drawing samples from the original sample with replacement.
- Jackknife: Draws repeated samples while leaving out one observation at a time from the set, without replacing it.

LO.j: Describe the issues regarding selection of the appropriate sample size, data snooping bias, sample selection bias, survivorship bias, look-ahead bias, and time-period bias.

Increasing the sample size reduces the standard error and gives us narrower confidence intervals. However, while increasing sample size we must consider two things:

- Cost involved**:** Compare the cost of getting more data to the potential benefits of increasing precision.
- Risk of sampling from a different population**:** In the process of increasing sample size if we get data from a different population, then the accuracy will not improve.

Biases observed in sampling methods are:

- Data-snooping bias: Analyzing the same data again and again, till a pattern that works is identified. The best way to avoid this bias is to test the pattern on out-of-sample data to check if it holds.
- Sample selection bias: Excluding certain assets from the analysis due to unavailability of data. A type of sample selection bias is the *survivorship bias*, in which companies are excluded from analysis because they have gone out of business and data for them was not easily available.
- Look-ahead bias: Using information that became available at later periods in time in the analysis.
- Time-period bias**:** If the selected time period is too short, the results may only hold for that time period. If the time period is too long, then the results might not consider major structural changes in the economy.

