R41 Introduction to Fixed-Income Valuation

Version 1.0

1. Introduction

In this reading, we will see how to value different types of fixed-income securities such as fixed-rate bonds, floating-rate bonds, and money market securities. This reading will cover:

- How to price a bond.
- How bond prices and yields are calculated.
- The term structure of interest rates.
- Yield spreads over benchmark interest rates.

2. Bond Prices and the Time Value of Money

2.1 Bond Pricing with a Market Discount Rate

A bond's price is the present value of all future cash flows at the market discount rate. The discount rate is the rate of return required by investors given the risk of investment in the bond. It is also known as the **required yield**, or **required rate of return**.

$$\text{PV of bond} = \frac{\text{PMT}}{(1+r)^1} + \frac{\text{PMT}}{(1+r)^2} + \cdots + \frac{\text{PMT+FV}}{(1+r)^N}$$

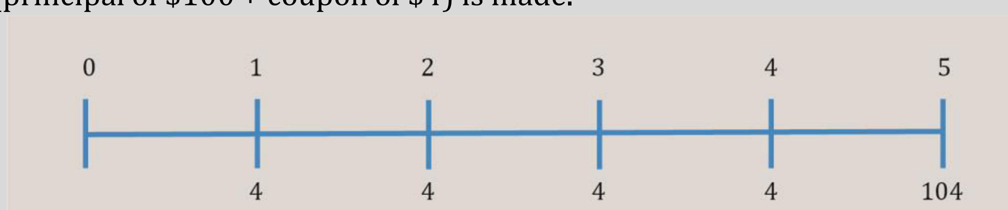
where:
PMT = coupon payment per period
FV= par value of the bond paid at maturity
r = market discount rate
N = number of periods until maturity

Example

The coupon rate on a bond is 4% and the payment is made once a year. The time to maturity is five years and the market discount rate is 6%. What is the bond price per 100 of par value?

Solution:

Start by drawing a timeline for the cash flows. The par value of the bond or principal is $100. A coupon payment of $4 is made every year. At maturity (at the end of five years), a payment of $104 (principal of $100 + coupon of $4) is made.



We are required to calculate the present value of bond at time t = 0. For that, we discount all the future cash flows at the market discount rate of 6%.

End of year	Type of cash flow	Amount	Present value
1	Coupon	4	3.77
2	Coupon	4	3.55

3	Coupon	4	3.36
4	Coupon	4	3.17
5	Coupon + principal	104	77.71
			91.56

Using the formula, PV = $\frac{4}{1.06} + \frac{4}{1.06^2} + \frac{4}{1.06^3} + \frac{4}{1.06^4} + \frac{104}{1.06^5}$ = 91.575

Note: The table and formula above were just for your understanding.

On the exam, solve this quickly using a financial calculator:

N = 5; I/Y = 6; FV = 100; PMT = 4; PV = ? PV = -91.575

Notice the present value of the bond (91.575) is lower than the par value of the bond (100).

Premium, Par, and Discount Bonds

Example

Assume we have another bond with a coupon rate of 8%, paid annually. If the market discount rate is again 6%, what is the price of the bond?

Solution:

Note that the coupon rate here is higher than the market rate. Calculate PV using these input values: N = 5; I/Y = 6; PMT = 8; FV = 100; CPT PV

PV = -108.425. In this case, the present value of the bond is higher than its par value. Such a bond is called a premium bond. Since the coupon rate (8%) is higher than the required return of 6%, investors pay more than the par value for the bond.

<u>What if the coupon rate is 6%?</u>
N = 5; I/Y = 6; PMT = 6; FV = 100; CPT PV

PV = -100. The present value is equal to the par value of the bond.

<u>What if the coupon rate is 2%?</u>
N = 5; I/Y = 6; PMT = 2; FV = 100; CPT PV

PV = -83.151. This type of bond, which sells at a price below its par value, is called a discount bond. Investors require a return of 6% but it pays a coupon of only 2%, so investors pay less.

Par, premium, or discount	Relationship between coupon rate & market discount rate
Par bond	Coupon rate = market discount rate
Discount bond	Coupon rate < market discount rate
Premium bond	Coupon rate > market discount rate

2.2 Yield to maturity

The yield to maturity (YTM) is the internal rate of return on the cash flows – the uniform interest rate that will make the sum of the present values of future cash flows equal to the price of the bond. It is the implied market discount rate. In simpler terms, it is a bond's internal rate of return - the rate of return on a bond including interest payments and capital gain if the bond is held until maturity. Yield to maturity is based on three important assumptions:

- The investor holds the bond to maturity.
- The issuer does not default on payments and pays coupon and principal as they come due.
- The investor is able to reinvest all proceeds (coupons) at the YTM. (This is an unrealistic assumption as the interest rates may increase or decrease after the bond is purchased. So the coupon may be reinvested at a higher or lower rate.)

Example

A $100 face value bond with a coupon rate of 10% has a maturity of 4 years. The price of the bond is $80. What is its yield to maturity?

Solution:

Based on YTM's definition, we come up with the equation below:

$$80 = \frac{10}{(1+r)^1} + \frac{10}{(1+r)^2} + \frac{10}{(1+r)^3} + \frac{110}{(1+r)^4}$$

This equation can be solved using trial and error but it is much easier to use a financial calculator: N = 4; PMT = 10; FV = 100; PV= -80; CPT I/Y = 17.34%. The bond is trading at a discount. So the coupon rate (10%) must be lower than market rate (17.34%).

Example

Calculate the yields to maturity for the following bonds:

Bond	Coupon Payment per Period	Number of Periods to Maturity	Price
A	2.50	3	102.80
B	2.00	5	97.76
C	0.00	48	23.425

Solution:

Bond A: $102.80 = \frac{2.5}{(1+r)} + \frac{2.5}{(1+r)^2} + \frac{102.5}{(1+r)^3}$, r = 1.54%

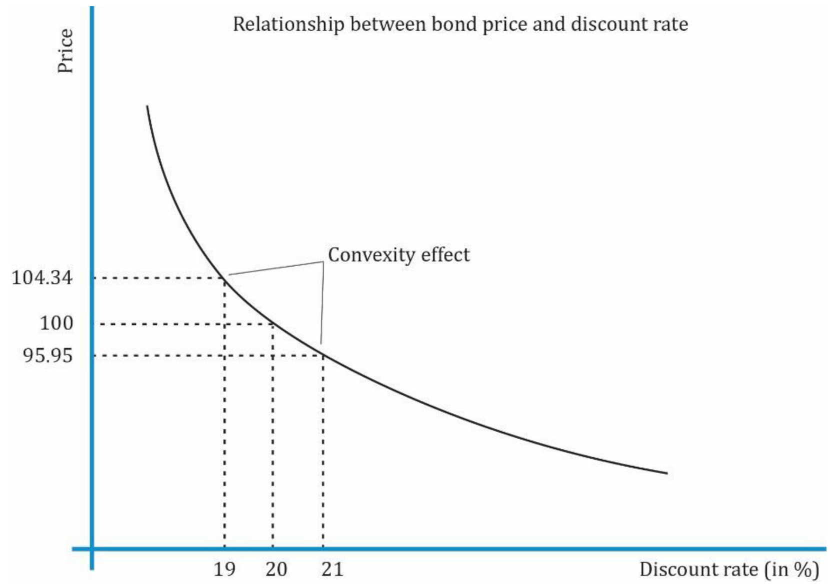
Bond B: $97.76 = \frac{2}{(1+r)} + \frac{2}{(1+r)^2} + \frac{2}{(1+r)^3} + \frac{2}{(1+r)^4} + \frac{102}{(1+r)^5}$, r = 2.48%

Bond C: $23.425 = \frac{100}{(1+r)^{48}}$, r = 3.07%

2.3 Relationships between the Bond Price and Bond Characteristics

In this section, we will study the relationship between the bond price and market discount rate. The underlying principle is that a bond's price changes whenever there is a change in market discount rate.

Inverse Effect and Convexity Effect



The bond price is inversely related to the market discount rate. As you can see from the exhibit above, when the discount rate goes down, the price of the bond increases. This is called the <u>inverse effect</u>. Remember that the price-yield relationship of a bond is not a straight line. Also note that price changes are not linear for the same amount of change in discount rate. At an interest rate of 20%, the price of the bond is 100. If the interest rate decreases from 20% to 19%, the bond price increases by 4.34%. However, if the interest rate increases from 20% to 21%, the bond price decreases by only 4.05%. This difference in price change is called the convexity effect.

Coupon Effect

Consider three bonds (A, B, C) which have the same time to maturity but different coupon rates.

Bond	Coupon Rate	Price At 20%	Price At 19%	% Change	Price At 21%	% Change
A	10.00%	58.075	60.950	4.95%	55.405	–4.60%
B	20.00%	100.000	104.339	4.34%	95.946	–4.05%
C	30.00%	141.925	147.728	4.09%	136.487	–3.83%

Note that the bond with lowest coupon rate has the highest interest rate sensitivity. In other words, a 1% change in interest rates causes a greater % change in the price of bond A (10% coupon) relative to bonds B (20% coupon) and C (30% coupon). This is called the coupon effect.

Maturity Effect

For the same coupon rate, a longer-term bond has a greater percentage price change than a shorter-term bond when their market discount rates change by the same amount. This is called the maturity effect.

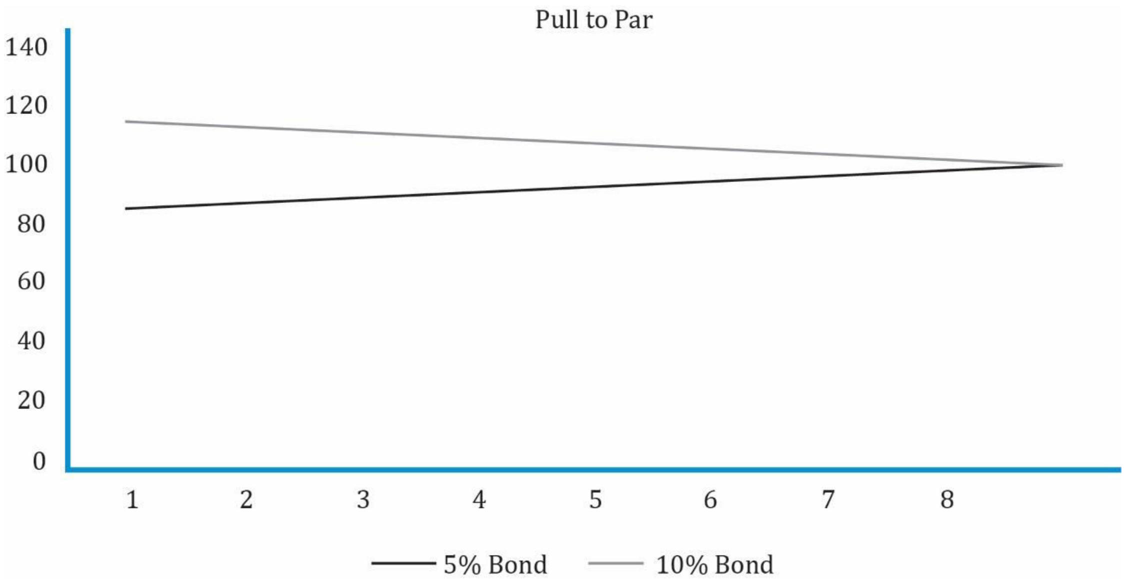
Summary of relationship between bond prices and discount rates	
Inverse effect	Bond price is inversely related to the discount rate.
Convexity effect	The percentage price change is more when discount rate goes down than when it goes up.
Coupon effect	For the same time to maturity and same change in discount rate the price of a low-coupon bond changes more than a high-coupon bond.
Maturity effect	For the same coupon rate and same change in discount rate, the price of a long-term bond changes more than a short-term bond.
Note: Low-coupon and Long-term bonds are more sensitive to changes in discount rates.	

Relationship between Bond's Price and Maturity

Even if the yield stays constant, bond prices change and come closer to par as time passes and as they near maturity date. The 'constant-yield price trajectory' illustrates the change in a bond's price over time if yields remain constant. This trajectory shows the "pull to par" effect. For a premium bond, the price decreases over time to par and for a discount bond, the price increases over time to par.

The exhibit below shows the constant-yield price trajectories for 5% and 10% annual coupon payment, 8-year bonds. Both bonds have a market discount rate of 7.5%. The 5% bond's initial price is 85.3567 and the 10% bond's initial price is 114.6433.

Year	**5% Bond**	**10% Bond**
1	85.3567	114.6433
2	86.7585	113.2415
3	88.2654	111.7346
4	89.8853	110.1147
5	91.6267	108.3733
6	93.4987	106.5013
7	95.5111	104.4889
8	97.6744	102.3256
	100.0000	100.0000



Instructor's Note

If the yield stays constant:

- A premium bond's price decreases to par value as its time to maturity approaches zero.
- A discount bond's price increases to par value as its time to maturity approaches zero.
- A par bond's value remains unchanged as it approaches maturity.

2.4 Pricing Bonds Using Spot Rates

Notice that we have used the same market discount rate for each of the future cash flows. Ideally, the basic approach to price a bond is to discount each cash flow at the corresponding market discount rate; hence, a sequence of discount rates must be used.

Spot rates are yields to maturity on zero-coupon bonds maturing at the date of each cash flow. Since zero-coupon bonds have no intermediate cash flows, the actual yield on a zero-coupon bond is used as the discount rate for a cash flow occurring at the same maturity date. Bond price (or value) determined using spot rates is sometimes referred to as the bond's '**no-arbitrage**' value. If a bond's price differs from its no-arbitrage value, an arbitrage opportunity exists in the absence of transaction costs.

$$PV = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \cdots + \frac{PMT+FV}{(1+Z_N)^N}$$

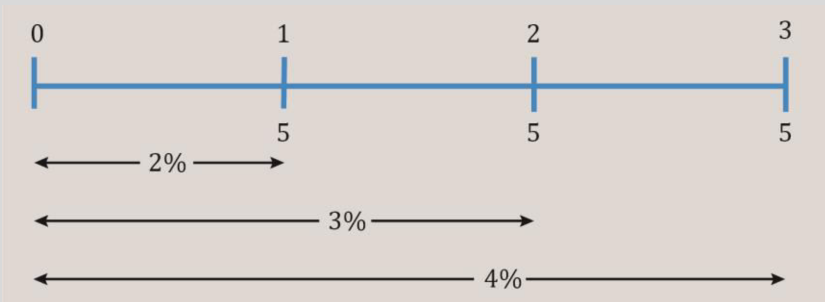
where:
PMT = coupon payment
FV = par value of the bond
Z_1 = spot rate or yield of zero-coupon bond for period 1
Z_2 = spot rate or yield of zero-coupon bond for period 2
Z_N = spot rate or yield of zero-coupon bond for period N

Example

The one-year spot rate is 2%, the two-year spot rate is 3%, and the three-year spot rate is 4%. What is the price of a three-year bond that makes a 5% annual coupon payment?

Solution:

Think of the bond as a portfolio of three zero-coupon bonds with one, two and three-year maturities with yields of 2%, 3%, and 4% respectively. Draw a timeline for the cash flows.



Bond's no-arbitrage value = $\frac{5}{1.02} + \frac{5}{1.03^2} + \frac{105}{1.04^3}$ = 102.96

Is there a single rate (yield) for the bond that equals the present value of cash flows to its purchase price? The YTM can be calculated as:

$$102.96 = \frac{5}{(1+r)^1} + \frac{5}{(1+r)^2} + \frac{105}{(1+r)^3}$$

Computing for r, we get 3.93%, which is the bond's yield to maturity.

3. Prices and Yields: Conventions For Quotes and Calculations

3.1 Flat Price, Accrued Interest, and the Full Price

When a bond is between coupon dates, its price has two parts: flat price and accrued interest. The sum of the two parts is called the full price or dirty price.

Full price or dirty price = Flat price + Accrued interest
$PV^{full} = PV^{flat} + AI$

Flat price = Full price – Accrued interest

Accrued interest = $\frac{t}{T}$ x PMT

where:
t = number of days since the last coupon payment
T = number of days between coupon payments
PMT = interest payment per period

As indicated in the above equation, the accrued interest is based on the number of days since the last coupon payment and the number of days between coupon payments. Here are two conventions for calculating the number of days:

- Actual convention: Uses the actual number of days, including weekends, holidays and leap days as in our example above.
- 30/360 convention: Assumes 30 days in a month and 360 days in a year.

Illustration of Flat Price and Dirty Price through an Example

Assume you own a two-year bond issued on 10 January 2017 with a par value of $100. The bond pays a coupon of 8% half-yearly. The first coupon is due on 10 July 2017, the next on 10 January 2018, the third on 10 July 2018, and the final one is paid along with principal at maturity date on 10 January 2019. The market discount rate is equal to the coupon rate and so the flat price is 100. An investor wants to purchase the bond from you on 31 May 2017. What is the full price he must pay?

You have held the bond for 140 days (since the bond was issued) and earned an interest for this period even though it has not been paid. The next coupon payment is $4. Assuming each period is 180 days the accrued interest is 4 x (140/180) = 3.11. The full price is 100.00 + 3.11 = 103.11. This is the amount you will receive on selling the bond.

Note that it is the flat (or clean) price which is quoted. The accrued interest increases every day until the coupon payment. So, if the accrued interest was included there would be a variation in bond quotes on a daily basis and a sudden drop after the coupon payment is made, which is misleading.

The full price of a bond can also be calculated using the equation shown below.

$$PV_{full} = PV \text{ x } (1 + r)^{\frac{t}{T}}$$

where:
t = number of days since the last coupon payment
T = number of days between coupon payments
PV = present value of the bond (not the flat price)

Example

A 5% French corporate bond is priced for settlement on 14 July 2015. The bond makes semi-annual coupon payments on 1 March and 1 September of each year and matures on 1 September 2020. Assuming a 30/360 day count convention for accrued interest, calculate the full price, the accrued interest, and the flat price per EUR 100 of par value for the following three yields to maturity: (I) 4.75%, (II) 5.00% and (III) 5.10%.

Solution:

Given the 30/360 day-count convention, there are 134 days between the last coupon on 1 March 2015 and the settlement date on 14 July 2015 (120 days for full months of March, April, May, and June, plus 14 days in July). Therefore, the fraction of the coupon period that has gone by is assumed to be 134/180. At the beginning of the period, there are 11 periods to maturity.

I) Stated annual YTM of 4.75%

The price at the beginning of the period is 101.198 per 100 of par value.

$$PV = \frac{2.5}{1.02375} + \frac{2.5}{1.02375^2} + \cdots + \frac{102.5}{1.02375^{11}} = 101.198$$

The full price on 14 July is PV(full) = 101.198 $\times \left(1.02375^{\frac{134}{180}}\right)$ = 102.979

The accrued interest is AI = 2.5 $\times \left(\frac{134}{180}\right)$ = 1.861

The flat price is PV(flat) = 102.979 - 1.861 = 101.118

II) Stated annual YTM of 5.00%

The price at the beginning of the period is 100.00 per 100 of par value.

$$PV = \frac{2.5}{1.025} + \frac{2.5}{1.025^2} + \cdots + \frac{102.5}{1.025^{11}} = 100.00$$

The full price on 14 July is PV(full) = 100.00 $\times \left(1.025^{\frac{134}{180}}\right)$ = 101.855

The accrued interest is AI = 2.5 $\times \left(\frac{134}{180}\right)$ = 1.861

The flat price is PV(flat) = 101.855 - 1.861 = 99.9942

III) Stated annual YTM of 5.10%

The price at the beginning of the period is 99.5256 per 100 of par value.

$$PV = \frac{2.5}{1.0255} + \frac{2.5}{1.0255^2} + \cdots + \frac{102.5}{1.0255^{11}} = 99.5256$$

The full price on 14 July is PV(full) = 99.5256 $\times \left(1.0255^{\frac{134}{180}}\right)$ = 101.4088

The accrued interest is AI = 2.5 $\times \left(\frac{134}{180}\right)$ = 1.861

The flat price is PV(flat) = 101.4088 - 1.861 = 99.5478

The flat price of the bond is a little below par value, even though the coupon rate and the yield to maturity are equal, because the accrued interest does not take into account the time value of money. The accrued interest is the interest earned by the owner of the bond for the time between the last coupon payment and the settlement date. However, that interest income is not received until the next coupon date. The calculation of accrued interest in practice neglects the time value of money. Therefore, compared to theory, the reported accrued interest is a little "too high" and the flat price is a little "too low."

3.2 Matrix Pricing

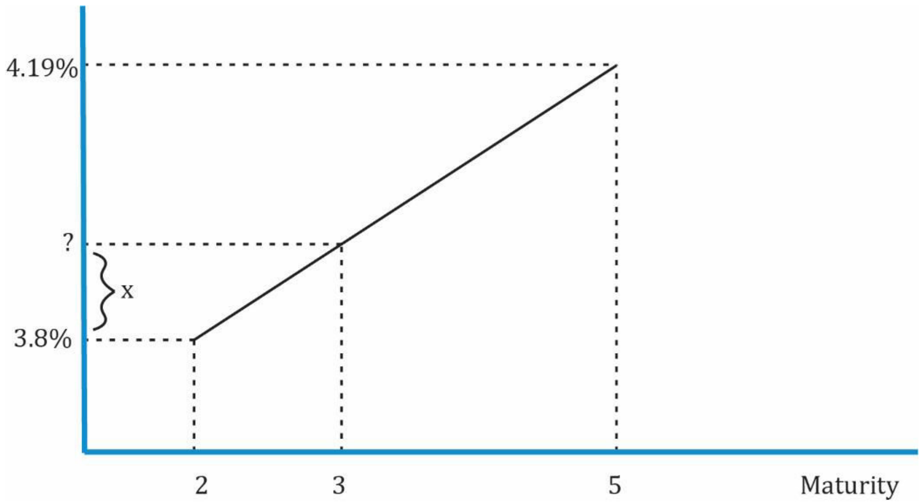
One can easily get information at what price a stock is trading. But, it is not easy to determine the price of most bonds as they do not trade as often as equity. Matrix pricing is an estimation process to find the market price of a not-so-frequently traded bond based on the prices of comparable bonds with similar times to maturity, type of issuer, coupon rates, and credit quality. This is also applicable for bonds that have not been issued yet.

For example, consider a three-year, 4% semi-annual bond X that you are trying to value. Comparable bonds whose prices are known are shown in the matrix below. Based on the four bonds, we can determine the price of bond X.

Matrix Pricing				
	2% coupon	**3% coupon**	**4% coupon**	**5% coupon**
Two years		98.5		102.25
		3.786%		3.821%
Three years			Bond X	
Four years				
Five years	90.25		99.125	
	4.181%		4.196%	

There are five steps to determine the value of a bond using the matrix pricing method:

1. Determine the YTM of comparable bonds, which is given to us already. If it were not given, you can calculate the YTM using the price and coupon. For example, YTM of a two-year, 3% coupon is: N = 4; PMT = 1.5; FV = 100; PV = -98.5; CPT I/Y. I/Y = 3.786%.
2. Determine the average yield for two-year bonds:
$$\frac{0.03786 + 0.03821}{2} = 0.038035$$
3. Determine the average yield for five-year bonds:
$$\frac{0.04181 + 0.4196}{2} = 0.041885$$
4. Calculate the three-market discount rate using linear interpolation. We have the data (yield) for two-year and five-year bonds, but not a three-year bond. From the graph you can see that the YTM of a three-year bond = 0.038 + *x*.



x can be calculated as $\frac{3-2}{5-2}$ x (0.041885 - 0.038035) = 0.001283. Discount rate for the three-year bond = 0.038035 + 0.001283 = 0.039318 = 3.39%.

5. Using 0.039318 as the discount rate, compute the price for the three-year bond.
   N = 6; I/Y = 1.966; PMT = 2; FV = 100; CPT PV. PV = -100.1906.

A few points to be noted on matrix pricing:

- Matrix pricing is used when underwriting new bonds to estimate the required yield spread over the benchmark rate.
- Benchmark rate is Libor, or a government bond with the same maturity as the bond being priced.
- Assume the YTM for a new bond is calculated using the matrix pricing method as 2.2% and a comparable government bond has a yield of 2.0%. The difference between the yield on a new bond over its benchmark rate is called the **required yield spread** or spread over the benchmark. In this case, it is 0.2%.
- Yield spreads are always specified in basis points where 1 basis point is one-hundredth of a percentage point. In this case, it is 20 basis points.

Example

An analyst decides to value an illiquid 3-year, 3.75% annual coupon payment corporate bond. Given the following information and using matrix pricing, what is the estimated price of the illiquid bond? Additional information: 2-year, 4.75% annual coupon payment bond priced at 105.60 and 4-year, 3.5% annual coupon payment bond priced at 103.28.

Solution:

The required yield on the 2-year, 4.75% bond priced at 105.60 is 1.871%.
The required yield on the 4-year, 3.50% bond priced at 103.28 is 2.626%.
The estimated market discount rate for a 3-year bond having the same credit quality is the average of two required yields: $\frac{1.871\% + 2.626\%}{2}$ = 2.249%.

The estimated price of the illiquid 3-year, 3.75% annual coupon payment corporate bond is 104.3078 per 100 of par value.

$$\frac{3.75}{1.02249} + \frac{3.75}{1.02249^2} + \frac{103.75}{1.02249^3} = 104.3078$$

3.3 Annual Yields for Varying Compounding Periods in the Year

Before discussing yield measures for money market instruments, it is important to understand the concept of periodicity. **Periodicity** is the number of compounding periods in a year, or number of coupon payments made in a year.

The **stated annual rate** for a bond will depend on the periodicity we are assuming. The stated annual rate is also called the annual percentage rate or APR.

Instructor's Note

A quarterly coupon paying bond has a periodicity of four, while a semi-annual bond has a periodicity of two, and a monthly-pay bond with a given annual yield would have a periodicity of twelve.

"Compounding more frequently within the year results in a lower (more negative) yield-to-maturity."

Consider a 5-year, zero-coupon bond priced at 80 per 100 par value. What is the stated annual rate for periodicity = 4, periodicity = 2, and periodicity = 1?

<u>When periodicity = 4</u>: compounding happens four times a year. N = 20; (5 years x 4 = 20). PMT = 0 as it is a zero-coupon bond. PV = -80; FV = 100; CPT I/Y = 1.12. This is the rate for each quarter. The stated annual rate is 1.12 x 4 = 4.487%.

<u>When periodicity = 2</u>: N = 10; PV = -80; PMT = 0; FV= 100; CPT I/Y = 2.2565. The stated annual rate is 2.25 x 2 = 4.51%.

<u>When periodicity = 1</u>: N = 5; PV = -80; PMT = 0; FV= 100; CPT I/Y = 4.56%. With a periodicity of 1, the stated annual rate is the same as the effective annual rate.

The formula for conversion based on periodicity is

$$\left(1 + \frac{\mathrm{APR_m}}{\mathrm{m}}\right)^{\mathrm{m}} = \left(1 + \frac{\mathrm{APR_n}}{\mathrm{n}}\right)^{\mathrm{n}}$$

Example

A 4-year, 3.75% semi-annual coupon payment government bond is priced at 97.5. Calculate the annual yield to maturity stated on a semi-annual bond basis and convert the annual yield to:

1. An annual rate comparable to bonds that make quarterly coupon payments.

2. An annual rate comparable to bonds that make annual coupon payments.

Solution to 1:

The stated annual yield to maturity on a semiannual bond basis can be calculated using a

financial calculator: N = 8; PMT = 1.875; FV = 100; PV = -97.5; CPT I/Y. I/Y = 2.2195%.
Hence, the stated annual yield to maturity = 2.2195% x 2 = 4.439%.

$$\left(1 + \frac{0.04439}{2}\right)^2 = \left(1 + \frac{APR_4}{4}\right)^4$$

APR_4 = 4.415%

The annual rate of 4.439% for compounding semiannually compares with 4.415% for compounding quarterly.

Solution to 2:

$$\left(1 + \frac{0.04439}{2}\right)^2 = (1 + APR_1)$$

APR_1 = 4.488%

The annual percentage rate of 4.439% for compounding semiannually compares with an effective annual rate of 4.488%.

The **effective annual rate** (EAR) is the yield on an investment in one year taking into account the effects of compounding. This rate has a periodicity of one as there is only one compounding period per year. EAR is used to compare the rate of return on investments with different frequency of compounding (periodicities).

Semiannual bond equivalent yield: Yield per semi-annual period times two. If the yield per semi-annual period is 2%, then the semi-annual bond equivalent yield is 4%.

3.4 Yield Measures for Fixed-Rate Bonds

- **Street convention:** It is the yield to maturity using a 30/360 day convention assuming payments are made on scheduled dates, even if the payment date fell on a weekend or a holiday.
- **True yield:** Yield to maturity calculated using an actual calendar of weekends and holidays. For instance, assume the coupon date falls on 15 March 2015, which is a Sunday. Street convention assumes the payment is made on that date, whereas true yield assumes the payment is made on 16 March if it is a business day. The coupon payment is discounted back from 16 March instead of 15 March.
- **Government equivalent yield:** Yield to maturity calculated using the actual day/count convention used for U.S. Treasuries.
- **Current yield:** Sum of the coupon payments received over the year divided by the flat price. It is also called the income or interest yield. Example: A 5-year, 8% semiannual coupon payment bond is priced at $960. Its current yield is 80/960 = 0.0833 = 8.33%. Current yield is not an accurate measure of the rate of return as it ignores the frequency of coupon payments, reinvestment income, and capital gain/loss on a bond.

$$\text{Current yield} = \frac{\text{Annual cash coupon payment}}{\text{Bond price}}$$

- **Yield-to-call:** Calculates the rate of return on a callable bond if it is bought at market price and held until the call date. The difference between YTM and the yield-to-call is that YTM assumes the bond is held to maturity. Calculation of yield-to-call is the same as YTM where N = number of periods to call date and FV= call price.
- **Yield-to-first call (YTFC):** It is the internal rate of return if the bond was bought at market price and held until the first call date.
- **Yield-to-second call:** Similarly, the yield on a callable bond if it was bought at market price and held to the second call date is called yield-to-second call.
- **Yield-to-worst:** Yield is calculated for every scenario. The lowest yield is called the yield-to-worst.
- **Option adjusted yield:** The option-adjusted yield is the required market discount rate whereby the price is adjusted for the value of the embedded option. For example, investors pay a lower price for the callable bond than if it were option-free. If the bond were non-callable, its price would be higher. The option-adjusted price is used to calculate the option-adjusted yield.

Example

An analyst observes the following statistics for two bonds:

	Bond A	Bond B
Annual Coupon Rate	6.00%	10.00%
Coupon Payment Frequency	Semi-annually	Quarterly
Years to Maturity	4 years	4 years
Price (per 100 par value)	95	110
Current Yield	?	?
Yield to Maturity	?	?

1. Calculate both yield measures for the two bonds.
2. How much additional compensation, in terms of yield to maturity, does a buyer of Bond A receive for bearing additional risk compared with Bond B

Solution to 1:

The current yield for Bond A is 6/95 = 6.316% and the yield to maturity for Bond A is 7.469%.

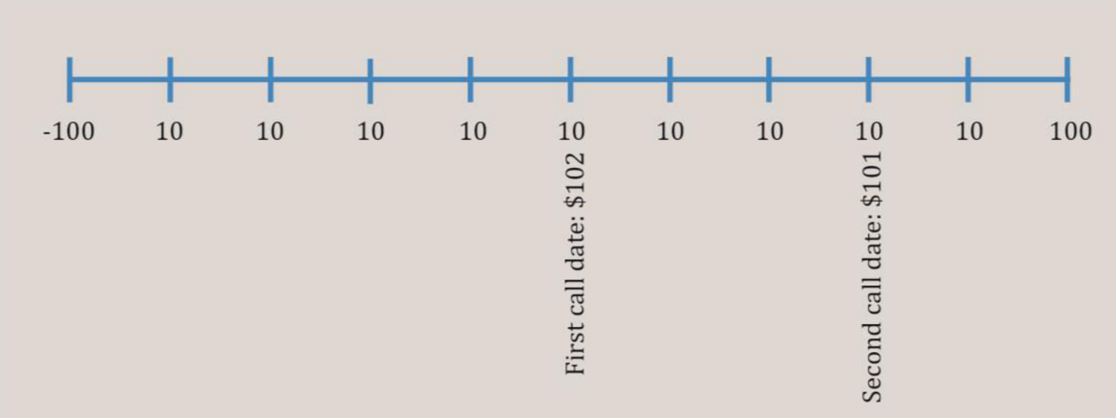
The current yield for Bond B is 10/110 = 9.091% and the yield to maturity for Bond B is 7.106%.

Solution to 2:

Compare the yields for the same periodicity to answer this question. 7.106% for a periodicity of four converts to 7.169% for a periodicity of two. The additional compensation for the greater risk in Bond A is 30 bps (0.07469 - 0.07169).

Example

Consider a bond which is selling for $100 and has the following cash flows:



Calculate the yield-to-first call and yield-to-second call.

Solution:

The yield-to-first call can be calculated as follows: PV= -100; N = 5; PMT = 10; FV = 102; CPT I/Y; I/Y = 10.325%. YTFC = 10.325%.

The yield-to-second call in our example will be 10.09%: PV= -100; N = 8; PMT = 10; FV = 101; CPT I/Y; I/Y = 10.09%.

Example

A bond with 4 years remaining until maturity is currently trading for 101.75 per 100 of par value. The bond offers a 5% coupon rate with interest paid semiannually. The bond is first callable in 2 years and is callable after that date on coupon dates according to the following schedule:

End of Year	Call Price
2	102.50
3	101.50
4	100.00

1. What is the bond's annual yield-to-first-call?

2. What is the bond's yield-to-worst?

Solution to 1:

The yield-to-first-call can be calculated with the following key strokes:

PV = -101.75, FV = 102.5, N = 4, PMT = 2.5, CPT I/Y = 2.6342.

To arrive at the annualized yield-to-first-call, the semiannual rate must be multiplied by two.

(2.6342 × 2 = 5.2684)

Solution to 2:

The yield-to-worst is 4.52%. The bond's yield to worst is the lowest of the sequence of yields-to-call and the yield to maturity. Yield-to-first-call = 5.27%, Yield-to-second-call = 4.84%, and Yield to maturity = 4.52%.

3.5 Yield Measures for Floating-Rate Notes

Floating-rate notes (FRN) are instruments where coupon/interest payments change from period to period based on a reference interest rate. Some important points to note about floating-rate notes:

- The objective is to protect the investor from volatile interest rates.
- The reference rate, usually a money market instrument such as a T-bill or an interbank offered rate like Libor, is used to calculate the interest payments. This rate is determined at the beginning of each period, but the interest is actually paid at the end of the period.
- Often, the coupon rate of an FRN is not just the reference rate, but a certain number of basis points, called the spread, is added to the reference rate.
- The specified yield spread over the reference rate is called the quoted margin.
- The spread remains constant throughout the life of the bond. The amount of spread depends on the credit quality of the issuer.

Example of a Floating Rate Note

Moody’s assigned a long-term credit rating of A2 to Nationwide, U.K.’s largest building society. Nationwide issued a perpetual floating-rate bond with a coupon rate of 6 month Libor + 240 basis points. The 2.4% quoted margin is a reflection of its credit quality. On the other hand, AAA-rated Apple sold a three-year bond at 0.05% over three-month Libor in 2013 as its credit risk was very low.

Coupon rate of a FRN = reference rate + quoted margin

The **required margin** is the spread demanded by the market. We saw that the quoted margin, or the spread over the reference rate, is fixed at the time of issuance. But what happens if the floater’s credit risk changes and investors demand an additional spread for bearing this risk? The required margin is the additional spread over the reference rate such that the FRN is priced at par on a rate reset date. If the required margin increases (decreases) because of a credit downgrade (upgrade), the FRN price will decrease (increase).

For example, assume a floater has a coupon rate of 3-month Libor plus 50 basis points. Six months after issuance, the issuer’s credit rating is downgraded and the market demands a required spread of 75 basis points. The coupon paid by the floater is lower than what the market demands. As a result, the floater would be priced at a discount to par as the cash flow is now discounted at a higher rate. The amount of the discount will be the present value of differential cash flows, i.e., the difference between the required and quoted margins. Conversely, if the credit rating of the issuer improves, the required margin would be below the quoted margin, and the market will demand a lower spread.

How required margin affects a floater's price at reset date	
Relationship between quoted and required margin	**Floater's price at reset date**
Required margin = quoted margin	Par
Required margin > quoted margin	Discount (below par)
Required margin < quoted margin	Premium (above par)

The required margin is also called the **discount margin.** FRNs can be valued using the model shown below.

$$PV = \frac{(\text{Index} + \text{QM}) * \frac{\text{FV}}{\text{m}}}{\left(1 + \frac{\text{Index+DM}}{\text{m}}\right)^1} + \frac{(\text{Index} + \text{QM}) * \frac{\text{FV}}{\text{m}}}{\left(1 + \frac{\text{Index+DM}}{\text{m}}\right)^2} + \cdots + \frac{(\text{Index} + \text{QM}) * \frac{\text{FV}}{\text{m}} + \text{FV}}{\left(1 + \frac{\text{Index+DM}}{\text{m}}\right)^N}$$

where:
PV = present value of the FRN
Index = reference rate, stated as an annual percentage rate
QM = quoted margin, stated as an annual percentage rate
FV = future value paid at maturity, or the par value of the bond
m = periodicity of the floating-rate note, or the number of payment periods per year
DM = discount margin; the required margin stated as an annual percentage rate
N = number of evenly spaced periods to maturity

Equation 1 for reference:

$$\text{PV of bond} = \frac{\text{PMT}}{(1+r)^1} + \frac{\text{PMT}}{(1+r)^2} + \cdots + \frac{\text{PMT+FV}}{(1+r)^N}$$

How to interpret the floating-rate note equation:

- Think of it as an extension of Equation 1, we will draw similarities between the two equations.
- $(\text{Index} + \text{QM}) * \frac{\text{FV}}{\text{m}}$ is the first interest payment similar to PMT of Equation 1, which is the coupon payment per period.
- (Index + QM) is the annual rate.
- Since it is divided by periodicity we get the interest payment for that period.
- In Equation 1, cash flows are discounted at 1 + r. For FRN, the cash flow for the first period is discounted at $1+\frac{\text{Index+DM}}{\text{m}}$, for the second period at $\left(1 + \frac{\text{Index+DM}}{\text{m}}\right)$, and so on.

This is considered a simple model because of the following assumptions:

- The value is calculated only on reset dates. There is no accrued interest, so the flat

price is the full price.
- The model uses a 30/360 day-count convention, which means periodicity is always an integer.
- The same reference rate is used in the numerator and denominator for all payment periods.

Example

A 3-year Italian floating-rate note pays 3-month Euribor plus 0.75%. Assuming that the floater is priced at 99, calculate the discount margin for the floater if the 3-month Euribor is constant at 1% (assume 30/360 day-count convention).

Solution:

The interest payment for each period is (1.00% + 0.75%) / 4 = 0.4375%.

The keystrokes to calculate the market discount rate are: PV = -99, PMT = 0.4375, FV = 100, N = 12, CPT I/Y = 0.5237 * 4, I/Y = 2.09%

The discount margin for the floater is 2.09% - 1% = 1.09% or 109 bps.

3.6 Yield Measures for Money Market Instruments

Money market instruments are short-term debt securities. They have maturities of one year or less, ranging from overnight repos to one-year certificates of deposit.

Differences in Bond Market and Money Market Yields	
Bond Market Yields	**Money Market Yields**
YTM is annualized and compounded.	Rate of return is annualized but not compounded; stated on a simple interest basis.
YTM calculated using the standard time value of money approach using a financial calculator.	Non-standard calculation using discount rates and add-on rates.
YTM stated for a common periodicity for all times to maturity.	Instruments with different times to maturity have different periodicities for the annual rate.

The calculation of interest of a money market instrument is different from calculating accrued interest on a bond. Money market instruments can be classified into two categories based on how the rates are quoted:

- **Discount rates**: T-bills, commercial paper (CP), and banker's acceptances are discount instruments. It means they are issued at a discounted price, and pay par value at maturity. They do not make any payments before maturity. The difference between the purchase price and par value at redemption is the interest earned. *Note: Do not confuse this discount rate with the rate used in TVM calculations.*

Price of a money market instrument quoted on a discount basis

$$PV = FV \times \left(1 - \frac{\text{days to maturity}}{\text{year}} \times DR\right)$$

where:
PV = present value of the money market instrument
FV = face value of the money market instrument
days to maturity = actual number of days between settlement and maturity
year = number of days in the year. Most markets use a 360-day year.
DR = discount rate, stated as an annual percentage rate (APR)

Money market discount rate

$$\text{Money market discount rate DR} = \left(\frac{\text{Year}}{\text{days to maturity}}\right) * \frac{FV-PV}{FV}$$

FV-PV = interest earned on the instrument (this is the discount)
$\frac{\text{year}}{\text{days}}$ = periodicity of the instrument

- **Add-on rates:** Bank term deposits, repos, certificates of deposit, and indices such as Libor/Euribor are quoted on an add-on basis. For a money market instrument quoted using an add-on rate, interest is added to the principal to calculate the redemption amount at maturity. In simple terms, if PV is the initial principal amount, days is the days to maturity, and year is the number of days in a year, then the amount to be paid at maturity is: $FV = PV + PV \times AOR \times \frac{\text{days to maturity}}{\text{year}}$ where AOR is the add-on rate stated on an annualized basis.

  Present value or price of a money market instrument quoted on an add-on basis

  $$PV = \frac{FV}{1 + \frac{\text{Days to maturity}}{\text{Year}} \times AOR}$$

  where:
  PV = present value of the money market instrument
  FV = amount paid at maturity including interest
  days = number of days between settlement and maturity
  year = number of days in a year
  AOR = add-on rate stated as an annual percentage rate

  Add-on rate

  $$AOR = \left(\frac{\text{Year}}{\text{Days}}\right) * \frac{FV-PV}{PV}$$

Instructor's Note

The primary difference between a discount rate (DR) and an add-on rate (AOR) is that the interest is included on the face value of the instrument for DR whereas it is added to the principal in case of AOR.

Example

Suppose that a banker's acceptance will be paid in 91 days. It has a face value of $1,000,000. It is quoted at a discount rate of 5%. What is the price of the banker's acceptance?

Solution:

$PV = FV * (1 - \frac{\text{Days to maturity}}{\text{Year}} * DR)$

FV = 1,000,000

Days = 91

Year = 360 days

DR = 5%

$PV = 1{,}000{,}000 * \left(1 - \frac{91}{360} * 0.05\right) = \$987{,}361$

Suppose that a Canadian pension fund buys a 180-day banker's acceptance (BA) with a quoted add-on rate of 4.38% for a 365-day year. If the initial principal amount is CAD 10 million, what is the redemption amount due at maturity?

Solution:

$$AOR = \left(\frac{\text{Year}}{\text{Days}}\right) * \frac{FV - PV}{PV}$$

$$0.0438 = \frac{365}{180} * \frac{FV - 10{,}000{,}000}{10{,}000{,}000}$$

FV = $10,216,000

Comparing Discount Basis with Add-On Yield

There are two approaches to compare the return of two money market instruments if one is quoted on a discount basis and the other on an add-on basis.

First approach: If you don't want to memorize one more formula, follow this approach:

1. Determine the present value of the instrument quoted on a discount basis.
2. Use the present value to determine the AOR.
3. Compare the two AORs to see which instrument offers a better return.

Second approach: Use the following relationship between AOR and DR:

Relationship between AOR and DR

$$AOR = \frac{DR}{1 - \frac{\text{Days to maturity}}{\text{Year}} * DR}$$

Example

A T-bill with a maturity of 90 days is quoted at a discount rate of 5.25%. Its par value is $100. Calculate the add-on rate.

Solution:

Using the first approach:

FV = 100; Days = 90; Year = 360 days; DR = 5.25%

$\text{PV} = 100 * \left(1 - \frac{90}{360} * 0.0525\right) = \$98.687$

$\text{AOR} = \frac{360}{90} * \frac{100-98.687}{98.687} = 5.32\%.$

Using the second approach:

$\text{AOR} = \frac{0.0525}{1 - \frac{90}{360} * 0.0525} = 5.3198\%$

Example

Money market Instrument	Quotation Basis	Number of Days in the Year	Quoted Rate
A	Discount Rate	360	3.23%
B	Discount Rate	365	3.46%
C	Add-on Rate	360	3.25%
D	Add-on Rate	365	3.35%

Given the four 90-day money market instruments, calculate the bond equivalent yield for each of them. Which instrument offers the highest rate of return if the credit risk is the same?

Solution:

A. The price of instrument A is $100 - \left(\frac{90}{360} \times 3.23\right) = 99.1925$ per 100 of par value. The bond equivalent yield is $\left(\frac{365}{90}\right) \times \frac{100 - 99.1925}{99.1925} = 3.30\%$.

B. The price of instrument B is $100 - \left(\frac{90}{365} \times 3.46\right) = 99.1468$ per 100 of par value. The bond equivalent yield is $\left(\frac{365}{90}\right) \times \frac{100 - 99.1468}{99.1468} = 3.49\%$.

C. The redemption amount per 100 of principal is $100 + \left(\frac{90}{360} \times 3.25\right) = 100.8125$. The bond equivalent yield is $\frac{365}{90} \times (100.8125 - 100) = 3.295\%$.

D. The quoted rate for instrument D of 3.35% is the bond equivalent yield. Instrument B offers the highest rate of return on a bond equivalent yield basis.

Periodicity of the Annual Rate

Another difference between yield measures in the money market and the bond market is the periodicity of the annual rate. Because bond yields to maturity are computed using interest rate compounding, there is a well-defined periodicity. For instance, bond yields to maturity for semi-annual compounding are annualized for a periodicity of two. Money market rates are computed using simple interest without compounding. In the money market, the periodicity is the number of days in the year divided by the number of days to maturity. Therefore, money market rates for different times to maturity have different periodicities.

$$\text{Periodicity of a money market instrument} = \frac{\text{Number of days in the year}}{\text{Number of days to maturity}}$$

Example

A 90-day T-bill has a BEY of 11%. Calculate its semiannual bond yield.

Solution:

The 11% BEY of the T-bill is based on a periodicity of 365/90. The periodicity of a semiannual bond is 2. Give this information, we can create the equation shown below and solve for r. We will get r = 0.1115 = 11.15%.

$$\left(1+\frac{0.11}{\frac{365}{90}}\right)^{\left(\frac{365}{90}\right)} = \left(1+\frac{r}{2}\right)^2$$

4. The Maturity Structure of Interest Rates

Maturity or Term Structure of Interest Rates

The term structure of interest rates is the relationship between interest rates and bonds with different times to maturity. The basic premise is that interest rates change when inflation rates are expected to change over a period of time. For example, a one-year zero-coupon bond may have an interest rate of 7.50%, while a two-year zero-coupon bond may have an interest rate of 9.75%, assuming all the other factors are the same (currency, credit rating, periodicity, etc.). The three common ways of representing the term structure of interest rates are given below:

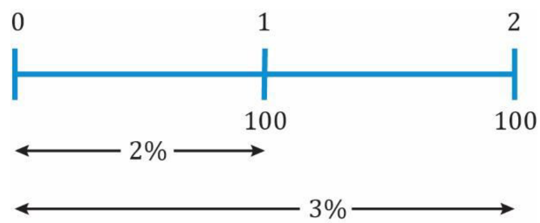
Spot rate curve: In our example above, the interest rates of zero-coupon bonds, 7.50% and 9.75% are called spot rates. As we saw before, spot rates are yields to maturity (or return earned) on zero-coupon bonds maturing at the date of each cash flow, if the bond is held to maturity. So, the spot rate curve is also called the zero or strip curve. The spot rate curve plots different maturities on the x-axis and corresponding spot rates on the y-axis.

Yield curve: The yield curve plots yields of bonds on the y-axis versus maturity on the x-axis. The main difference between a yield curve and a spot rate curve is that the yield curve considers the coupon payments as well. The assumption in a spot rate curve is that there are no coupon payments.

Par curve: The par curve plots yields to maturity for different maturities, but the bonds are assumed to be priced at par.

Forward Rates

A forward rate is an interest rate in the future. The one-year interest rate after one year is an example of a forward rate. The **implied forward rate** can be calculated using spot rates. As always, this is best understood with the help of an example. Consider a scenario where the one-year spot rate is 2%, and the two-year spot rate is 3%. This is illustrated below:



The forward rate f_2 is the rate over the second year. This can be calculated if we assume the following: $1 invested for two years at the 2-year spot rate of 3% should give same result as $1 invested for 1 year at the one-year spot rate of 2% and then again at the forward rate, f_2. Mathematically, this can be expressed as: $(1 + 0.03)^2 = (1 + 0.02)(1 + f_2)$

Solving for f_2, we get: $f_2 = 4.0098\%$.

The notation for a forward rate is expressed like this: 1y1y, 2y5y etc.; the first number refers to the length of the forward period in years from today and the second number refers to the tenor of the underlying bond. Thus, 1y1y is the rate for a 1-year loan one year from now; 2y5y is the rate for a 5-year loan, two years from now.

One of the applications of forward rates is that implied spot rates can be calculated as geometric averages of forward rates. Bonds can then be priced using implied spot rates. It gives the 1-year forward rate for zero-coupon bonds with various maturities. For example, 1y1y is the 1-year forward rate for a two-year bond.

Time Period	**Forward Rate**
0y1y	1.88%
1y1y	2.77%
2y1y	3.54%
3y1y	4.12%

Source: CFA Program Curriculum, Introduction to Fixed Income Valuation

Using the forward rates 0y1y and 1y1y, we can calculate the two-year spot rate as:

$(1.0188)(1.0277) = (1 + z_2)^2$

Calculating for z_2, we get 2.32%.

A **forward curve** plots the forward rates, which is an estimation of what investors expect the short-term interest rates to be. Each rate on the curve has the same time frame. Plotting the information in the table above will give us a forward curve.

Example

Maturity	Price	Yield to Maturity
1 year	97.25	2.476%
2 years	94.50	2.906%
3 years	91.30	2.819%

Compute the "1y1y" and "2y1y" implied forward rates stated on a semi-annual bond basis.

Solution:

$\left(1 + \frac{0.02476}{2}\right)^2 \times (1 + \text{IFR}_{2,2})^2 = \left(1 + \frac{0.02906}{2}\right)^4.$

$\text{IFR}_{2,2}$ = 0.0167 × 2 = 3.34%.

The 1y1y implied forward rate is 3.34%.

$\left(1 + \frac{0.02906}{2}\right)^4 \times (1 + \text{IFR}_{4,2})^2 = \left(1 + \frac{0.02819}{2}\right)^6.$

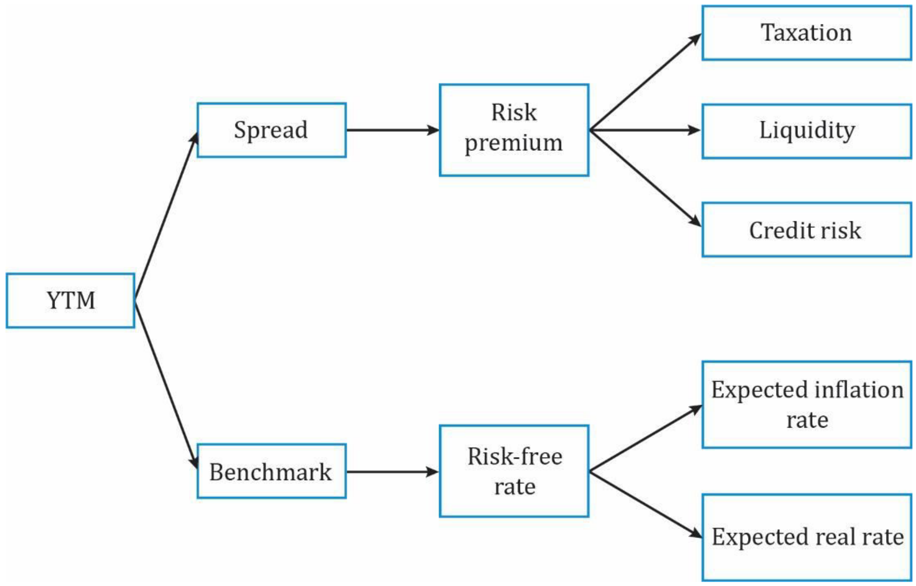
$\text{IFR}_{4,2}$= 0.0132 × 2 = 2.65%.

The 2y1y implied forward rate is 2.65%.

5. Yield Spreads

The yield spread is the difference in yield between a fixed-income security and a benchmark. Say the YTM of a 3-year corporate bond is 7.00%. The benchmark rate is 3-year Libor, which is 5.00%. The yield spread of the corporate bond relative to the benchmark is 2.00%. Generally, the benchmark reflects macroeconomic factors. A spread reflects microeconomic factors and aspects specific to the issuer, such as credit quality of the issuer and bond, tax status, etc.

The YTM of a bond can be broken down into the following components:



Two related concepts are the G-spread and the I-spread.

- The **G-spread** is the yield spread in basis points over an interpolated government bond. The spread is higher for bearing higher credit, liquidity, and other risks relative to the government bond.
- The **I-spread** is the yield spread of a specific bond over the standard swap rate in that currency of the same tenor.

The **Z-spread** (**zero-volatility spread**) is based on the entire benchmark spot curve. It is the constant spread that is added to each spot rate such that the present value of the cash flows matches the price of the bond. The Z-spread is also called the static spread as it is constant for all periods.

The **option-adjusted spread (OAS)** is the Z-spread adjusted for the value of an embedded option. Consider a callable bond with a Z-spread of 90 basis points, of which 10 basis points are due to the embedded call option. In this case, the OAS is 90 – 10 = 80 basis points. If a putable bond has a Z-spread of 180 basis points and the value of the put option is 10 basis points, the OAS is 180 + 10 = 190 basis points. Based on these simple scenarios, it should be clear that for a callable bond, the OAS is lower than the Z-spread, and for a putable bond, the OAS is higher than the Z-spread.

Example

A 5% annual coupon corporate bond with 3 years remaining to maturity is trading at 100.175. The 3-year, 3% annual payment government benchmark bond is trading at 100.50. The 1-year and 2-year government spot rates are 2.05% and 3.425% respectively. Calculate the G-spread, the spread between the yields to maturity on the corporate bond and the government bond having the same maturity.

Solution:

Solve for the time value of money approach on the corporate bond using the following keystrokes:

PV = -100.175, PMT = 5, FV = 100, N = 3, CPT I/Y = 4.936

The yield to maturity for the corporate bond is 4.936%.

The yield to maturity of the government bond can be calculated with the following keystrokes:

PV = -100.50, PMT = 3, FV = 100, N = 3, CPT I/Y = 2.824

The G-spread is 4.936% - 2.824% = 2.11% or 211 bps.

Summary

LO.a: Calculate a bond's price, given a market discount rate.

A bond's price is the present value of all future cash flows at the market discount rate, which is the rate of return required by investors given the risk of investment in the bond.

$$\text{PV of bond} = \frac{\text{PMT}}{(1+r)^1} + \frac{\text{PMT}}{(1+r)^2} + \cdots + \frac{\text{PMT+FV}}{(1+r)^N}$$

LO.b: Identify the relationships among a bond's price, coupon rate, maturity, and market discount rate (yield to maturity).

Inverse effect	Bond price is inversely related to discount rate.
Convexity effect	For the same coupon/maturity bond: the percentage price change is more when discount rate goes down than when it goes up.
Coupon effect	For the same time to maturity and same change in discount rate: prices of low-coupon bonds change more than a high-coupon bond.
Maturity effect	For the same coupon rate and same change in discount rate: prices of long-term bonds change more than short-term bonds.
Note: Low-coupon and Long-term bonds are more sensitive to discount rates.	

LO.c: Define spot rates and calculate the price of a bond using spot rates.

Spot rates are yields to maturity on zero-coupon bonds maturing at the date of each cash flow. Since zero-coupon bonds have no intermediate cash flows, the actual yield on a zero-coupon bond is used as the discount rate for a cash flow occurring at the same maturity date.

$$\text{PV} = \frac{\text{PMT}}{(1+Z_1)^1} + \frac{\text{PMT}}{(1+Z_2)^2} + \cdots + \frac{\text{PMT} + \text{FV}}{(1+Z_N)^N}$$

where: Z_N are the spot rates.

LO.d: Describe and calculate the flat price, accrued interest, and the full price of a bond.

When a bond is between coupon dates, its price has two parts: flat price and accrued interest. The sum of the two parts is called the full price or dirty price.

Full price or dirty price = flat price + accrued interest

$$\text{Accrued interest} = \frac{t}{T} * \text{PMT}$$

Full price of a fixed-rate bond between coupon payments:

$$\text{PV}_{\text{full}} = \text{PV} * (1+r)^{\frac{t}{T}}$$

LO.e: Describe matrix pricing.

Matrix pricing is an estimation process to find the market price of a not-so-frequently traded bond based on the prices of comparable bonds with similar times to maturity, type of issuer, coupon rates, and credit quality.

The steps involved in matrix pricing are:

- Determine the YTM of comparable bonds (which is usually given in the question).
- Determine the average yields of different comparable bonds.
- Calculate the market discount rate using linear interpolation.
- Using the discount rate, compute the price of the bond.

LO.f: Calculate annual yield on a bond for varying compounding periods in a year.

- The stated annual rate for a bond will depend on the periodicity we are assuming. The stated annual rate is also called the annual percentage rate or APR.
- The effective annual rate (EAR) is the yield on an investment in one year taking into account the effects of compounding. EAR is used to compare the rate of return on investments with different frequency of compounding (periodicities).
- The semiannual bond equivalent yield is the yield per semi-annual period times two.

LO.g: Calculate and interpret yield measures for fixed-rate bonds and floating-rate notes.

- Street convention is the yield to maturity using a 30/360-day convention assuming payments are made on scheduled dates.
- True yield is the yield to maturity calculated using an actual calendar with weekends and holidays.
- Current yield is the sum of the coupon payments received over the year divided by the flat price.
- Yield-to-call calculates the rate of return on a callable bond if it is bought at market price and held until the call date.

Floating-rate notes (FRN) are instruments where coupon/interest payments change from period to period based on a reference interest rate. The coupon rate of a FRN is equal to the sum of reference rate and quoted margin.

$$\text{PV} = \frac{(\text{Index} + \text{QM}) * \frac{\text{FV}}{\text{m}}}{\left(1 + \frac{\text{Index}+\text{DM}}{\text{m}}\right)^1} + \frac{(\text{Index} + \text{QM}) * \frac{\text{FV}}{\text{m}}}{\left(1 + \frac{\text{Index}+\text{DM}}{\text{m}}\right)^2} + \cdots + \frac{(\text{Index} + \text{QM}) * \frac{\text{FV}}{\text{m}} + \text{FV}}{\left(1 + \frac{\text{Index}+\text{DM}}{\text{m}}\right)^N}$$

LO. h. Calculate and interpret yield measures for money market instruments.

Money market instruments are short-term debt securities. They have maturities of one year or less, ranging from overnight repos to one-year certificates of deposit. Money market instruments can be classified into two categories based on how the rates are quoted; discount rate and add on rate.

$$PV = FV * (1 - \frac{\text{days to maturity}}{\text{year}} * DR)$$

$$PV = \frac{FV}{1 + \frac{\text{days to maturity}}{\text{year}} * AOR}$$

$$AOR = \frac{DR}{1 - \frac{\text{days to maturity}}{\text{year}} * DR}$$

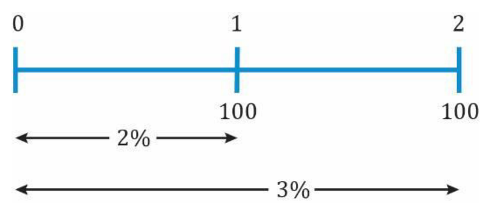
LO.i: Define and compare the spot curve, yield curve on coupon bonds, par curve, and forward curve.

- The spot curve is also called the zero or strip curve. The spot curve plots different maturities on the x-axis and corresponding spot rates on the y-axis.
- The yield curve plots yields of bonds on the y-axis versus maturity on the x-axis. The main difference between a yield curve and a spot curve is that the yield curve considers the coupon payments as well.
- The par curve plots yields to maturity for different maturities, but the bonds are assumed to be priced at par.
- Forward curve plots the forward rates, which is an estimation of what investors expect the short-term interest rates to be.

LO.j: Define forward rates and calculate spot rates from forward rates, forward rates from spot rates, and the price of a bond using forward rates.

A forward rate is an interest rate in the future. The one-year interest rate after one year is an example of a forward rate. The implied forward rate can be calculated using spot rates.

Consider a scenario where the one-year spot rate is 2%, and the two-year spot rate is 3%. This is illustrated below:



The forward rate f_2 is the rate over the second year. This can be calculated if we assume the following: $1 invested for two years at the 2-year spot rate of 3% should give the same result as $1 invested for 1 year at the one-year spot rate of 2% and then again at the forward rate, f_2.

Mathematically, this can be expressed as:

$(1 + 0.03)^2 = (1 + 0.02)(1 + f_2)$

Solving for f_2, we get: $f_2 = 4.0098\%$.

LO.k: Compare, calculate, and interpret yield spread measures.

The yield spread is the difference in yield between a fixed-income security and a benchmark.

- G-spread is the yield spread in basis points over an interpolated government bond. The spread is higher for bearing higher credit, liquidity, and other risks relative to the government bond.
- I-spread is the yield spread of a specific bond over the standard swap rate in that currency of the same tenor.
- Z-spread (zero-volatility spread) is based on the entire benchmark spot curve. It is the constant spread that is added to each spot rate such that the present value of the cash flows matches the price of the bond.
- Option-adjusted spread (OAS) is the Z-spread adjusted for the value of an embedded option.

Practice Questions

1. A 10-year, 8% annual-pay bond has a par value of $100. If it has a yield to maturity of 9%, the price of the bond is *closest* to:
   A. $93.58
   B. $95.64
   C. $101.25

2. A 5-year, 8% semiannual-pay bond has a par value of $100. What is the price of the bond if it has a yield to maturity of 10%?
   A. $91.54
   B. $92.28
   C. $94.15

3. A zero-coupon bond with a face value of $800 matures in 12 years. At a market discount rate of 8.5% and assuming annual compounding, the price of the bond is *closest* to:
   A. $300.561.
   B. $353.828.
   C. $485.487.

4. Suppose a bond's price is expected to decrease by 3% if its market discount rate increases by 100 basis points. If the bond's market discount rate decreases by 100 basis points, the bond price is *most likely* to change by:
   A. 3%.
   B. less than 3%.
   C. more than 3%.

5. The following information is provided about three bonds that are currently trading at par.

Bond	Coupon Rate	Maturity (years)
A	4%	20
B	4%	10
C	6%	10

   If the market discount rate for all three bonds increases by 100 basis points, which bond will *most likely* experience the greatest percentage change in price?
   A. Bond A.
   B. Bond B.
   C. Bond C.

6. If spot rates are 4% for one year, 4.25% for two years, and 4.5% for three years, the price of a $1,000 face value, 3-year, annual-pay bond with a coupon rate of 5% is *closest* to:

A. $998.74
B. $1,005.89
C. $1,014.19

7. An investor is considering selling a $1,000 face value, semi-annual coupon bond with a quoted price of 103 and accrued interest since the last coupon of $25. Ignoring transaction costs, how much will he receive at the settlement date?
   A. $1,025.
   B. $1,030.
   C. $1,055.

8. Matrix pricing allows investors to estimate the market discount rates and prices for bonds:
   A. with different coupon rates.
   B. that are infrequently traded.
   C. with different maturities.

9. Which of the following is *most likely* to be equal to the quoted price?
   A. accrued price.
   B. clean price.
   C. full price.

10. A bond with 10 years remaining until maturity is currently trading for 110 per 100 par value. The bond offers a 6% coupon rate with interest paid semiannually. The bond's annual yield to maturity is *closest* to:
    A. 4.74%
    B. 5.26%
    C. 6.28%

11. A bond with 5 years remaining until maturity is currently trading for 102 per 100 of par value. The bond offers 8% coupon rate with interest paid semiannually. The bond is callable at 103 in two years. What is the bond's yield-to-call?
    A. 8.1%
    B. 8.3%
    C. 8.7%

12. The following information is available for a banker's acceptance.
    PV = $1000
    FV = $1,100
    Number of days between settlement and maturity = 150
    Total number of days in the year = 365.

Which of the following is *most likely* to be the add-on-rate stated as an annual percentage rate?
A. 4.11%.
B. 24.33%.
C. 75.67%.

13. A floating rate note has a quoted margin of +50 basis points and a required margin of +25 basis points. The price of the note, on its next reset date, will be:
A. equal to par value.
B. less than par value.
C. more than par value.

14. A 365-day year bank certificate of deposit has an initial principal amount of $95 million and a redemption amount at maturity of $100 million. The number of days between settlement and maturity is 300. The bond equivalent yield is *closest* to:
A. 6.4%
B. 6.8%
C. 7.2%

15. Loop Inc. has issued semiannual $1,000 par value Floating Rate Note with 5 years to maturity, the reference rate is 180-day LIBOR and the quoted margin is 50 basis points. 180-day LIBOR is currently quoted at 4% and the margin for discount is 65 basis points. What is the *most likely* value of this FRN?
A. 996.73
B. 993.82
C. 973.77.

16. A yield curve created from a sequence of yields to maturity on zero-coupon bonds is the:
A. par curve.
B. spot curve.
C. forward curve.

17. The 3-year spot rate is 9.25%, and the 2-year spot rate is 9%. What is the 1-year forward rate two years from today?
A. 8.75%
B. 9.5%
C. 9.75%

18. An analyst wants to value a 3 year, 6% annual pay bond. The bond has par value of $1,000. The current spot rate is 4%, 1-year forward rate 1 year from now is 5% and 1-year forward rate 2 years from now is 6%. The value of this annual coupon pay bond is

closest to:

A. $1022.07.
B. $1024.35.
C. $1028.39.

19. A yield spread of a bond over an interest rate swap in the same currency and with the same tenor as the bond is *best* described as:
    A. I-spread.
    B. Z-spread.
    C. G-spread.

20. In case of putable bonds, the:
    A. OAS < Z-spread.
    B. OAS = Z-spread.
    C. OAS > Z-spread.

21. Which of the following is *most likely* correct about callable bonds
    A. Z-spread > OAS.
    B. Z-spread = OAS.
    C. Z-spread < OAS.

Solutions

1. A is correct. N = 10, I/Y = 9, FV = 100, PMT = 8; CPT → PV = -$93.58

2. B is correct. N = 10, I/Y = 5, FV = 100, PMT = 4; CPT → PV = $92.28

3. A is correct. Value of zerocoupon bond $= \text{Face value}/(1 + \text{coupon rate})^{N}$
   = $800/(1.085)^{12} = \$300.561$

4. C is correct. The relationship between bond prices and the market discount rate is not linear. The percentage price change is greater in absolute value when the market discount rate goes down than when it goes up by the same amount (the convexity effect). If a 100 basis point increase in the market discount rate will cause the price of the bond to decrease by 3%, then a 100 basis point decrease in the market discount rate will cause the price of the bond to increase by an amount more than 3%.

5. A is correct. All else equal, the longer the term to maturity, the greater the price volatility. All else equal, the lower the coupon rate, the greater the price volatility. Bond A has the longest maturity and the lowest coupon rate. Therefore, it will experience the greatest percentage change in price.

6. C is correct.
   $$\text{Bond value} = \frac{50}{1.04} + \frac{50}{1.0425^2} + \frac{1{,}050}{1.045^3} = \$1{,}014.19$$

7. C is correct. The seller will receive the full price, which is equal to the flat price plus the interest accrued from the last coupon date.
   Flat price = 1,000 x 103% = $1,030
   Full price = $1,030 + $25 = $1,055

8. B is correct. For bonds that are infrequently traded, matrix pricing is a price estimation process that uses market discount rates based on the quoted prices of similar bonds (similar times to maturity, coupon rates, and credit quality).

9. B is correct. The quoted price of a bond is equal to the clean price.

10. A is correct.
    N = 20, FV = 100, PMT = 3, PV = -110; CPT → I/Y = 2.37
    YTM = 2 x 2.37 = 4.74%

11. B is correct.

N = 4, FV = 103, PMT = 4, PV = -102; CPT → I/Y = 4.15%
YTM = 2 x 4.15% = 8.3%

12. B is correct.

$$PV = \frac{FV}{\left(1 + \frac{Days}{Year} * AOR\right)}$$

$$1{,}000 = \frac{1{,}500}{\left(1 + \frac{150}{365} * AOR\right)}$$

<u>Add-on rate</u>

$$AOR = \left(\frac{Year}{Days}\right) * \frac{FV - PV}{PV} = \left(\frac{365}{150}\right) * \frac{1100 - 1000}{1000} = 24.33\%.$$

13. C is correct. If the required margin is less than the quoted margin, the credit quality of the note has increased. Therefore, the price of the note on the reset date will be more than par value.

14. A is correct. For money market instruments, the BEY is equal to the add-on yield based on a 365-day year.
AOR = (365/ Days) x (FV-PV)/PV
= 365/300 x 5/95 = 6.40%

15. B is correct. Coupon rate for this FRN = $\frac{180\text{-day LIBOR} + \text{quoted margin}}{2} = \frac{4 + 0.5}{2} = \frac{4.5}{2} = 2.25\%$
Discount rate = $\frac{180\text{-day LIBOR} + \text{margin for discount}}{2} = \frac{4 + 0.65}{2} = \frac{4.65}{2} = 2.32\%$
Value of Fixed Rate Note
N = 10, PMT = 22.5, I/Y = 2.32%, FV = $1,000, CPT PV = 993.82

16. B is correct. The spot curve, also known as the strip or zero curve, is the yield curve constructed from a sequence of yields to maturity on zero-coupon bonds. The par curve is a sequence of yields to maturity such that each bond is priced at par value. The forward curve is constructed using a series of forward rates, each having the same timeframe.

17. C is correct. $(1.0925)^3 = (1.09)^2 \times (1 + 2y1y)$
$2y1y = (1.0925)^3 / (1.09)^2 - 1 = 9.75\%$

$2y1y \approx 3(9.25\%) - 2(9\%) = 9.75\%$

18. C is correct.

$$\text{Value of the bond} = \frac{60}{1.04} + \frac{60}{(1.04)(1.05)} + \frac{1,060}{(1.04)(1.05)(1.06)} = \$1.028.39$$

19. A is correct. The I-spread, or interpolated spread, is the yield spread of a specific bond over the standard swap rate in that currency of the same tenor. The yield spread in basis points over an actual or interpolated government bond is known as the G-spread. The Z-spread (zero-volatility spread) is the constant spread such that is added to each spot rate such that the present value of the cash flows matches the price of the bond.

20. C is correct.
OAS is the Z-spread after removing the impact of the embedded option.
OAS = Z-spread – Option value. The option value is negative since the options are a benefit to bondholders.

21. A is correct. OAS = Z-spread – Option value. The option value is positive since the options are a detriment to bondholders.

