R50 Portfolio Risk and Return: Part II

Version 1.0

1. Introduction

In this reading, we will discuss:

- The capital market line (CML).
- The two components of total risk: systematic and nonsystematic risk.
- The capital asset pricing model (CAPM) and the security market line (SML). The primary objective of this reading is to identify the optimal risky portfolio using CAPM.

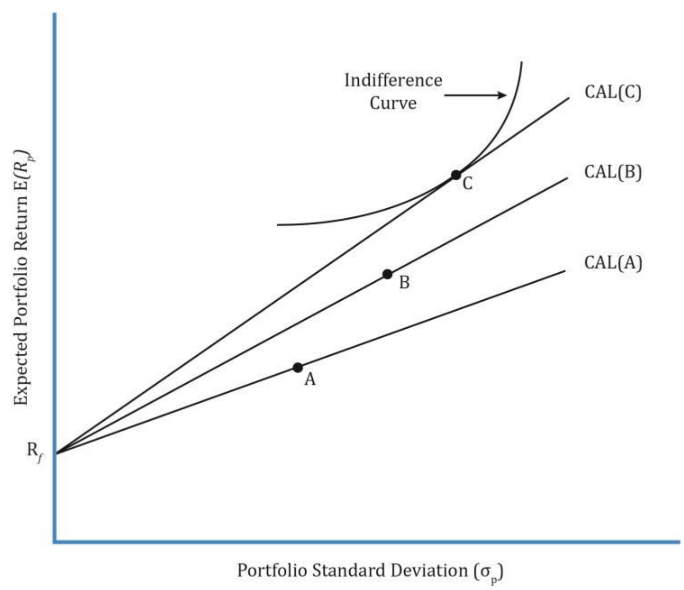
2. Capital Market Theory: Risk-Free and Risky Assets

2.1 Portfolio of Risk-Free and Risky Assets

Investors have different views of the market → Different optimal risky porfolios → Different capital allocation lines

This is a repetition of what we have seen in the previous reading. When a risk-free asset is combined with a risky asset, it results in higher risk-adjusted returns because the risk-free asset has zero correlation with the risky asset. This leads to the capital allocation line.

Investors have different views of the market (and different levels of risk aversion) which means the individual risky assets (e.g. securities) they choose to form their portfolio are different. Combining the capital allocation line with an investor's indifference curve leads to different optimal risky portfolios, as illustrated in the exhibit below.



We now explore whether a unique optimal risky portfolio exists for all investors. The answer is yes, if there is homogeneity of expectations.

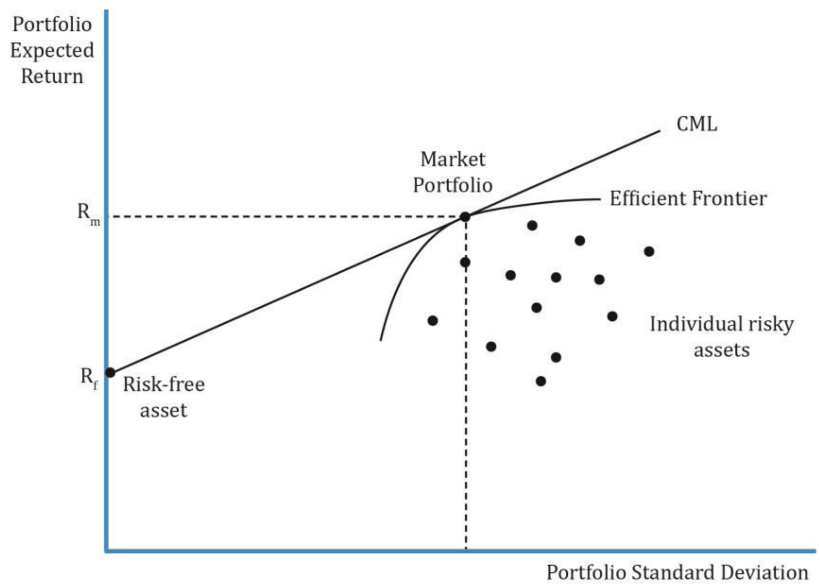
What is homogeneity of expectations?

- It means that all investors have the same economic expectations for an asset.
- For example, assume there are two securities. With homogeneity of expectations, all

investors have the same views on risk, return, price, cash flows, and the correlation between the two assets. This means the calculation to arrive at optimal risky portfolio for all investors would be the same.

3. Capital Market Theory: The Capital Market Line

Since all investors have the same expectations, they will construct only one efficient frontier. If there is one efficient frontier, there will be only one capital allocation line. The point where this capital allocation line is tangential to the efficient frontier is called the market portfolio. This is the optimal risky portfolio when all investors have the same expectations. The CML is a special case of the CAL where the efficient portfolio is the market portfolio.



Now, let us derive the equation for CML. We will use a basic equation of the form: Y = C + mX

where:

Y= R_p (portfolio return)
C = R_f (risk-free rate)
m = slope = $(R_m - R_f) / \sigma_m$
X = portfolio standard deviation = σ_p

The equation for CML can be written as

$$R_p = R_f + \left(\frac{R_m - R_f}{\sigma_m}\right) * \sigma_p$$

Any point along the CML is a combination of the risk-free asset and the market portfolio. At the point where the CML intersects y-axis, 100% is invested in the risk-free asset and its weight decreases as we go up along the CML.

What is the market?

Theoretically, the market includes all risky assets or anything that has value. Examples include stocks, bonds, real estate, etc.

The market portfolio should contain as many assets as possible, but it is not practical to include all assets in a single risky portfolio because not all assets are tradable and not all tradable assets are investable. Examples of non-tradable assets include the Eiffel Tower and human capital. Examples of tradable assets that are not investable include Class A shares on the Shanghai Stock Exchange that are not available for foreign investors.

Practically, we use major stock market indexes as a proxy for the market. This reading uses the S&P 500 index as the market's proxy as it represents approximately 80% of the U.S stock market capitalization and U.S. markets account for nearly 32 per cent of the world markets.

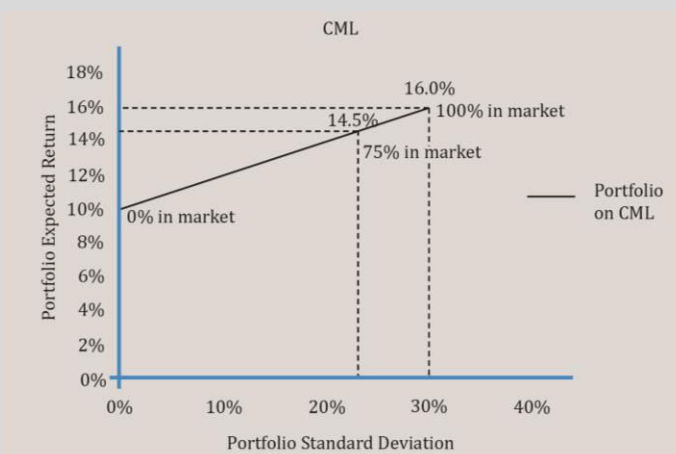
Example

Majid Khan wants to build a portfolio using T-bills and an index fund that tracks the KSE 100 Index. The T-bills have a return of 10%. The index fund has an expected return of 16% and a standard deviation of 30%. Draw the CML. Show the point where the investment in the market is 75%. What is the risk and return at this point?

Solution:

Risk and return when the investment in the market is 75%:
$\sigma_p = w_m * \sigma_m$ = 0.75 * 30 = 22.5% (*Note: Risk of T-bills is zero*)
$R_p = w_{r_f} \times R_f + w_m \times R_m$ = 0.25 x 10 + 0.75 x 16 = 14.5%

Let us now plot these values on the CML.

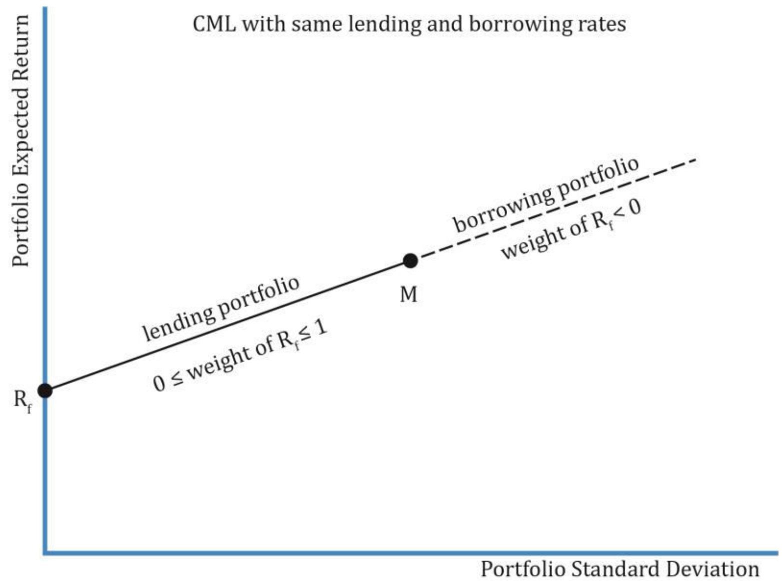


When the return on the portfolio is 10%, Majid is holding 100% of the T-bills and 0% of the market portfolio. As he increases his weight in the market portfolio to 75%, we see that the return has changed to 14.5% but his risk has also gone down to 22.5%. When he has invested 100% in the market portfolio, the risk and return are the same as the market portfolio.

4. Capital Market Theory: CML - Leveraged Portfolios

Our focus so far has been on the portfolios between the points R_f, the risk-free asset, and M, the market portfolio on the CML, where an investor is holding some combination of the two assets. The portfolios on this stretch between R_f and M are called lending portfolios because the investment in the assets comes from the investor's own wealth. He is lending (investing) his wealth at the risk-free rate.

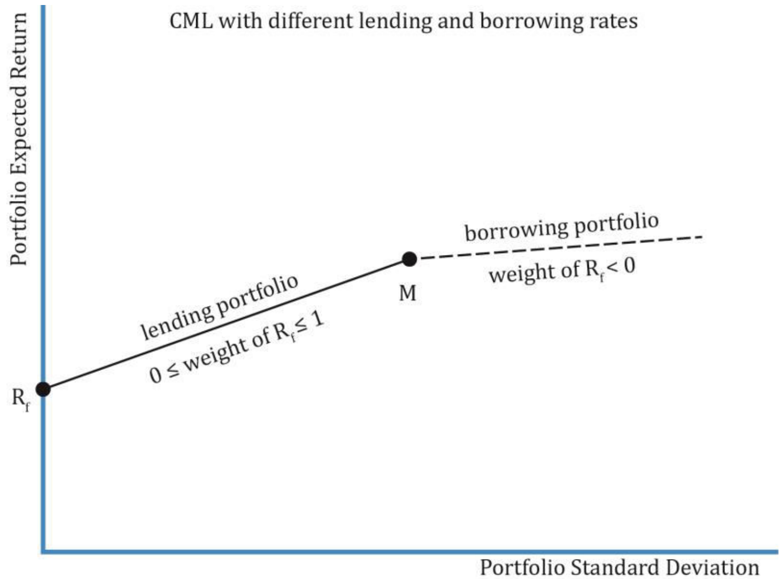
The dotted line in the exhibit below stretches beyond M on the CML represents the borrowing portfolios. This means that the investor is able to *borrow* money at the risk-free rate to invest more in the market portfolio.



- A portfolio beyond point M is a combination of the investor's own wealth and borrowed money at the risk-free rate.
- As we go up north-east beyond M on the CML, the investment in the market portfolio increases and so does the amount of borrowed money.
- Beyond point M, there is a negative investment in the risk-free asset, and the risky portfolio is called the leveraged portfolio. More risk leads to higher expected return.
- How much money the investor borrows to invest in a portfolio beyond M depends on his risk-return preferences, in other words, his utility function.
- The slope of the line between R_f and M is given as $\frac{R_m - R_f}{\sigma_m}$.

Different lending and borrowing rates

Often, investors are not able to borrow money at the risk-free rate. If the borrowing rate is higher than the risk-free rate, the CML is no longer a straight line. The slope of the line to the right of M is given by $\frac{R_m - R_b}{\sigma_m}$. Hence, there is a 'kink' in the CML. This is illustrated in the exhibit below. Using the data from the previous example, the slope of CML when the borrowing rate is the same as lending rate, we get slope as $\frac{16-10}{30} = 0.2$. Now, assume the borrowing rate is higher at 12%. The slope in this case will be $\frac{16-12}{30} = 0.133$, which is less than 0.2.



When the borrowing rate is higher than the lending rate, the return per unit of risk is less, relative to when the borrowing rate is equal to the lending rate.

Example

After a successful initial foray into the stock market, Majid Khan gets a little greedy and decides to build a leveraged portfolio. He invests 100,000 of his own money and 50,000 of borrowed money in the index fund. He expects to pay 10% on the borrowed money. The index fund has an expected return of 16% and a standard deviation of 30%. Calculate his expected risk and return.

Solution:

w_1 = Weight of the risk-free asset = Weight of borrowed money = -50,000/100,000 = -0.5

w_2 = Weight in risky asset = 150,000/100,000 = 1.5

Total weight = $w_1 + w_2$ = 1.5 - 0.5 = 1

Expected return with -50% invested in T-bills = $w_1R_b + w_2R_m$ = -0.5 (10) + 1.5 (16) = 19%.

Standard deviation with -50% invested in T-bills is $w_2 \times \sigma_m$ = 1.5 x 30 = 45%.

This example shows us that leverage amplifies both expected return and risk.

Example

Calculate the expected return and risk if Majid actually needs to pay 12% on the borrowed money. All other numbers are the same.

Solution:

The standard deviation or risk of the portfolio remains the same. But the expected return must be lower as the borrowing rate has increased. Let us calculate the slope of the borrowing part of the CML as $\frac{R_m - R_b}{\sigma_m} = \frac{16 - 12}{30} = 4/30$.

Expected return = 12% + (16 - 12)/30 * 45 = 18%.

Or, expected return with -50% invested in T-bills = $w_1R_b + w_2R_m$ = -0.5 (12) + 1.5 (16) = 18%.

Standard deviation with -50% invested in T-bills $\sigma_p = w_2 * \sigma_M$ =1.5 * 30 = 45%.

5. Systematic and Nonsystematic Risk

We have seen repeatedly that high returns come with high risk. But does all high risk lead to high returns? No. Total risk can be decomposed into systematic and nonsystematic risk.

Total variance = Systematic variance + Nonsystematic variance

5.1 Systematic Risk and Nonsystematic Risk

Systematic risk is non-diversifiable or market risk that affects the entire economy and cannot be diversified away. Investors get a return for systematic risk.

Examples of systematic risk are interest rates, inflation, natural disasters, unrest/coup attempts such as events in Middle East/Africa since 2011 and political uncertainty.

It is tough to avoid systematic risk. However, the effects can be minimized by selecting securities with low correlation to the rest of the portfolio.

Nonsystematic risk is a local risk that affects only a particular asset or industry. There is no compensation for nonsystematic risk as it can be diversified away.

Examples of nonsystematic risk are oil discoveries, non-approval for a drug, new regulations for telecom industry.

Nonsystematic risk may be avoided by diversifying a portfolio to include assets across countries, industries, and asset classes. Nonsystematic risk does not earn a return.

Pricing of risk**:** Pricing an asset is equivalent to estimating its expected rate of return; price and return are often used interchangeably. For example, if we know the cash flows for a bond, its price can be computed using the discount rate. Pricing of risk is this rate of return which reflects the systematic risk.

Instructor's Note

Systematic risk or market risk is the only risk for which investors get compensated. It cannot be eliminated with diversification.

A risk-free asset has zero systematic risk and zero nonsystematic risk.

Example

Describe the systematic and nonsystematic risk components of the following assets:

- A six-month Treasury bill.
- An index fund based on the S&P 500, with a total risk of 18%.

Solution:

- A six-month Treasury bill: For a T-bill, we know how much an investor would get paid at the end of six months. The systematic risk and nonsystematic risk are both zero; the total variance is therefore zero.
- An index fund based on the S&P 500, with a total risk of 18%: S&P 500 represents the market. There is no nonsystematic risk in a market portfolio. So, the total risk of 18% is systematic risk.

Example

Consider two assets X and Y. X has a total risk of 25% of which 15% is systematic risk. Asset Y is the S&P index fund mentioned above with a total risk of 18%, all of which is systematic risk. Which asset will have a higher expected rate of return?

Solution:

Based on total risk, asset X seems an obvious choice because higher risk results in higher return. However, it is more appropriate to compare the systematic risk of the two assets as only systematic risk earns a return. Asset Y will earn a higher expected return as its systematic risk of 18% is higher than asset X's 15%. An investor will not be compensated for assuming nonsystematic risk in asset X as it can be eliminated.

6. Return-Generating Models

Return-generating models provide an estimate of the expected return of a security given certain input parameters called factors. If the model uses many factors, it is called a multi-factor model. If it uses one factor, it is a single-factor model.

<u>Multi-factor Models</u>

The three commonly used multi-factor models are: macroeconomic, fundamental, and statistical models. These models use factors that are correlated with security returns.

- Macroeconomic models use economic factors such as economic growth, interest rate, unemployment rate, productivity, etc.
- Fundamental models use input parameters such as earnings, earnings growth, cash flow, market capitalization, industry-specific inputs, financial ratios like P/E, P/B, value/growth, etc.
- In the statistical models, there is no observable economic or fundamental connection between the input and security returns. Unlike a macroeconomic or fundamental model, there are also no pre-determined set of factors. Instead, historical or cross-sectional security returns are analyzed to identify factors that explain variance or covariance in returns.

Single-factor Models

In a single-factor model, only one factor is considered. A classic single factor model is the market model which is given by this equation:

$$R_i = \alpha_i + \beta R_m + e_i$$

where R_i is the dependent variable and R_m is the independent variable.

According to the market model, a stock's return can be decomposed into:

- α_i – Excess returns of the stock (difference between expected and realized returns).
- β – Systematic risk of a security or the stock's sensitivity to the market. For instance, how the stock's return varies when the market return moves up or down by 1%.
- R_m – Return on the market.
- e_i – Error term that affects stock returns due to firm-specific factors. There is an error term because not all of a stock's returns can be explained by market returns.

7. Calculation and Interpretation of Beta

Beta is a measure of systematic (or market) risk. Beta is a number and has no unit of measure. It tells us how sensitive an asset's return is to the market as a whole. Beta determines the amount by which a stock's return is expected to move for every 1 per cent increase or decrease in the market return. Beta is the ratio of the covariance of returns of stock i with returns on the market to the variance of market return.

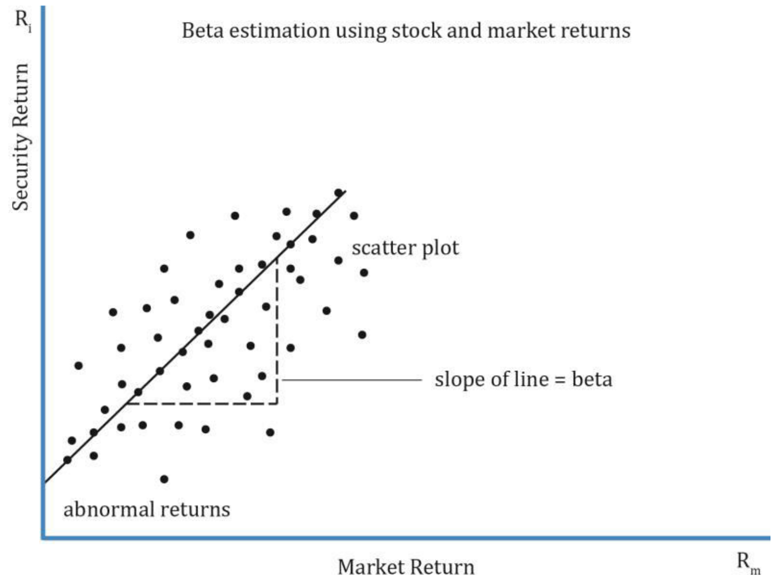
$$\beta = \frac{\text{Cov}(i,M)}{\sigma_M^2} = \frac{\rho_{iM} * \sigma_i}{\sigma_M}$$

Interpretation of beta values:

- **β > 0:** Return of the asset follows the market trend.
- **β < 0:** Return of the asset moves in an opposite direction to the market trend (negatively correlated with the market).
- **β = 0:** An asset's return has no correlation with the market. For example, a risk-free asset has a beta of zero.
- **β = 1:** Beta of market is equal to 1. If a stock has a beta of 1, it means it has the same volatility as that of the market.

How do we estimate beta?

Beta is estimated using a statistical method called linear regression analysis where stock returns are regressed against market returns. Historical security returns and historical market returns are used as inputs in this method. Let us say we consider a stock's returns for the past 100 months. The pairs (security, market return) are plotted to get a scatter plot. Each point on the graph represents a month. A regression line is drawn though the scatter plot that best represents the data points. The point where the line intercepts the y-axis is α (or excess returns). The slope of the line is equal to its beta.



A portfolio beta is calculated as the weighted average beta of the individual securities in the portfolio.

Example

Assuming the standard deviation of market returns is 20%, calculate the beta for the following assets:

- 6-month T-bill.
- A commodity with σ = 15%, and zero correlation with the market.
- A stock with σ = 25% and correlation with the market = 0.6.

Solution:

- 6-month T-bill: beta for a T-bill is zero as there is no correlation between a risk-free asset and the market.
- A commodity with σ = 15%, and zero correlation with the market: using equation 7, we can conclude that beta is zero if correlation with the market is zero.
- A stock with σ = 25% and correlation with the market = 0.6: $\beta = \frac{0.6 * 0.25}{0.2} = 0.75$

8. Capital Asset Pricing Model: Assumptions and the Security Market Line

The capital asset pricing model is a model used **to calculate the expected rate of return of a risky asset**, and gives the relationship between a security's return and risk. CAPM states that the expected return of assets reflects only their systematic risk, which is measured by beta. Systematic or market risk cannot be diversified. It means that two assets with same beta will have the same expected return irrespective of their individual characteristics.

$$r_e = R_f + \beta\,[E\,(R_{mkt}) - R_f]$$

where:

r_e = expected return

$R_f = \text{risk} - \text{free rate}$
$\beta = \text{systematic risk of security}$
$E\,(R_{mkt}) = \text{expected return of market}$
$E\,(R_{mkt}) - R_f\ = \text{market risk premium; it is the compensation for assuming market risk.}$

8.1 Assumptions of the CAPM

We will now take a look at the assumptions for arriving at CAPM:

1. Investors are risk-averse, utility-maximizing, rational individuals.
   Risk aversion means investors expect a higher return to accept a higher level of risk. Different investors have different levels of risk aversion or risk tolerance. Utility maximizing means investors prefer higher returns to lower returns. Individuals are rational means that all investors have the ability to gather and process information to make logical decisions. In reality, however, this is not true for most investors as personal biases and experiences shroud their judgment.
2. Markets are frictionless. There are no transaction costs and no taxes.
   Investors can borrow and lend as much as they want at the risk-free rate. One important assumption here is that there are no costs or restrictions on short-selling, which is not true in reality. This limits the CAPM.
3. All investors plan for the same, single holding period.
   CAPM is based on a single period instead of multiple periods because it is easy to calculate.
4. Investors have homogenous expectations.
   All investors analyze securities using the same probability distribution to arrive at identical valuations. This means all investors use the same estimates for expected returns, variance, and correlation between assets. Since the expected returns and standard deviation for all investors is the same, they will arrive at the same optimal risky portfolio, or the market portfolio in the CML.
5. Investments are infinitely divisible.
   Investors can hold a fraction of any asset. It means that they may invest as much as or little in any asset.
6. Investors are price takers.
   CAPM assumes that there are many small investors who cannot influence the security prices. They are price takers.

Example

FFC's standard deviation of returns is 25% and its correlation with the market is 0.6. The standard deviation of returns for the market is 20%. The expected market return is 10% and the risk free rate is 3%. What is FFC's expected return?

Solution:

$\beta = \frac{\rho_{iM} * \sigma_i}{\sigma_M} = \frac{0.6 * 0.25}{0.2} = 0.75$

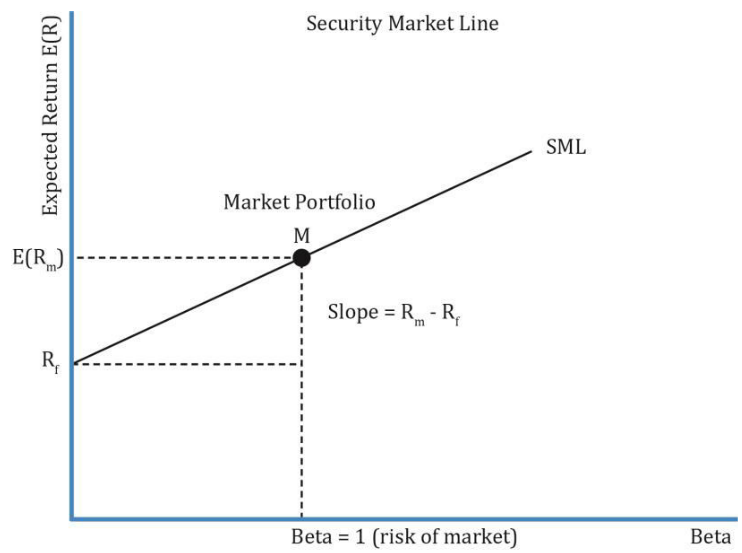
$r_e = R_f + \beta[E(R_{mkt}) - R_f]$

$r_e = 0.03 + 0.75[0.1 - 0.03] = 0.0825$

$r_e = 8.25\%$

8.2 The Security Market Line

The security market line (SML) is a graphical representation of the capital asset pricing model and applies to all securities, whether they are efficient or not. The graph has beta on the x-axis and expected return on the y-axis.



SML intersects y-axis at the risk-free rate.

Slope of the line = $\frac{R_m - R_f}{\beta} = R_m - R_f$ as β = 1 for the market. So, slope of the line is the market risk premium.

What is the difference between CML and SML?

Differences between CML and SML	
CML	**SML**
Does not apply to all securities; applies to only efficient portfolios. Only systematic risk is priced. So, CAL/CML can only be applied to those portfolios whose total risk is equal to systematic risk.	Applies to any security. It may include inefficient portfolios as well.

X-axis has the standard deviation of the portfolio.	X-axis has beta of the asset or portfolio.
$R_p = R_f + (\frac{R_m - R_f}{\sigma_m}) * \sigma_p$ where σ_p is the total risk of the portfolio.	$r_e = R_f + \beta[E(R_{mkt}) - R_f]$ where β is the systematic risk. SML is just a graphical representation of CAPM.
Slope of the CML is the Sharpe ratio: $\frac{R_m - R_f}{\sigma_m}$	Slope of the SML is the market risk premium: $E(R_{mkt}) - R_f$
Similarities between CML and SML	
Both CML and SML have expected portfolio return on the y-axis.	
The line stretches from the risk-free asset to the market portfolio in both the cases.	

Example

Ricky Ponting invests 10% in the risk-free asset, 40% in a mutual fund which tracks the market and 50% in a high-risk stock with a beta of 2.5. The risk-free rate is 5% and the expected market return is 10%. What is the portfolio beta and expected return?

Solution:

Risk-free asset: w = 0.1 r = 5% β = 0
Mutual fund: w = 0.4 r = 10% β = 1
High-risk stock: w = 0.5 r =? β = 2.5

Portfolio beta = weighted average beta of all assets = 0.1 x 0 + 0.4 x 1 + 0.5 x 2.5 = 1.65

Portfolio return = $r_f + \beta (r_m - r_f)$ = 0.05 + 1.65 [0.1 - 0.05] = 0.1325 = 13.25%

Alternative method:

First determine the return of the high-risk stock using its beta.

Expected return of high-risk stock = 0.05 + 2.5[0.1 - 0.05] = 0.175 = 17.5%

Portfolio return= weighted average return of all assets = 0.1 x 5 + 0.4 x 10 + 0.5 x 17.5 = 13.25%

9. Capital Asset Pricing Model: Applications

The CAPM is important both from a theoretical and practical perspective. In this section, we will look at some of the practical applications of CAPM in capital budgeting, performance appraisal, and security selection.

Estimate of Expected Return

Given an asset's systematic risk, the expected return can be calculated using CAPM.

- To estimate the current price of an asset, we discount future cash flows at the required rate of return calculated using CAPM.

- The required rate of return from the CAPM rate is also used in the capital budgeting process and to determine if a project is economically feasible.

10. Beyond CAPM: Limitations and Extensions of CAPM

Note: there is no explicit learning outcome associated with this section.

In this reading, we saw one return-generating model, the CAPM. But there are more models to estimate the return of an asset.

CAPM is popular because of its simplicity to estimate the expected return. However, there are several theoretical and practical limitations because of its unrealistic assumptions.

10.1 Limitations of the CAPM

Theoretical limitations of the CAPM are as follows:

- Single–factor model: Only systematic or market risk is priced. This assumption is restrictive as no other investment characteristic is considered.
- Single-period model: One of the assumptions of the model is that all investors hold assets for a single period, which is practically not true.

Practical limitations of the CAPM are as follows:

- **Market Portfolio:** A market portfolio must comprise all assets, including non-investable assets like Eiffel Tower, human capital, etc.
- **Proxy for a Market Portfolio:** When a true market portfolio comprising all assets cannot be created, a proxy such as S&P500 is used. But different analysts may use different proxies for the same asset based in the country. For example, one analyst may use S&P 500 as the proxy for equity while another may use DAX.
- **Estimation of beta risk:** Beta is an important input for the CAPM model. If not estimated correctly, the expected return will not be accurate as well. Beta is estimated using a long history of returns, which may vary according to the period used. For example, beta calculated using daily returns will be different than a one-year or five-year beta. Similarly, the risk of a company in the past may not represent its current or future risk.
- **CAPM does not predict returns accurately:** Studies have shown that actual returns do not reflect predicted returns.
- **Homogeneity in investor expectations:** CAPM assumes that all investors have the same expectations for securities that result in one optimal risky portfolio, the market portfolio. If investors have different views, then it will result in multiple optimal risky portfolios and SMLs.

10.2 Extensions to the CAPM

Other models are considered because of the limitations of CAPM. Of course, these models too come with their own limitations. The models can be broadly categorized into theoretical and practical models.

Theoretical models

In principle, theoretical models are similar to the CAPM but with additional risk factors. One example is the arbitrage pricing theory (APT) which takes the following form:

$\mathbf{E(R_p) = R_F + \lambda_1\beta_{p,1} + \ldots + \lambda_K\beta_{p,k}}$ where k is the number of risk factors, $\boldsymbol{\lambda_1}$ is the risk premium and $\boldsymbol{\beta_{p,k}}$ is the sensitivity of the portfolio to factor k.

The drawback of this model is that it is difficult to identify risk factors and estimate sensitivity to each factor.

Practical Models

The Fama-French three-factor model and four-factor model have been found to predict asset returns better than the CAPM, which considers only beta risk. The three factors included in the Fama-French model are relative size, relative book-to-market value, and beta of the asset. The four-factor model adds one more momentum factor to the three-factor model. These models have limitations too. They cannot be applied to all assets and there is no certainty that these will work in the future.

11. Portfolio Performance Appraisal Measures

Before selecting an investment manager, it is important for investors to understand the performance of a manager and the cost structure involved. In this section, we look at four measures that are commonly used in performance evaluation. These are:

Sharpe Ratio	$\frac{\text{Portfolio risk premium}}{\text{Portolio total risk}} = \frac{R_p - R_f}{\sigma_p}$
M-Squared	$M^2 = \frac{(R_p - R_f)\sigma_m}{\sigma_p} - (R_m - R_f)$
Treynor Ratio	$\frac{\text{Portfolio risk premium}}{\text{beta risk}} = \frac{R_p - R_f}{\beta_p}$
Jensen's Alpha	Actual portfolio return – expected return $= R_p - [R_f + \beta(R_m - R_f)]$

11.1 The Sharpe Ratio

Sharpe ratio is the excess return of the portfolio over the risk-free rate divided by the portfolio risk. It is the excess return per unit of risk. The higher the Sharpe ratio the better, all else equal. Sharpe ratio is the slope of the capital allocation line and represents the reward-to-variability ratio. The ex-ante and ex-post Sharpe ratio are given by:

Ex ante SR $= \frac{E(R_p) - R_F}{\sigma_p} \rightarrow$ evaluate the expected risk-adjusted return of the portfolio

Ex post $\widehat{SR} = \frac{\bar{R}_p - \bar{R}_F}{\hat{\sigma}_p} \rightarrow$ to evaluate historical risk-adjusted returns.

Advantages of Sharpe ratio

- Sharpe ratio can be easily calculated by using readily available market data.
- The Sharpe ratio is easy to interpret.
- The Sharpe ratio is the most widely recognized and used appraisal measure.

Limitations of Sharpe ratio

- The Sharpe ratio uses total risk; not systematic risk.
- The ratio itself is not informative. The Sharpe ratio of one portfolio must be compared with another to see which one is better. For instance, if a portfolio has a Sharpe ratio of 0.7, the number does not convey anything. But if there is another portfolio with a Sharpe ratio of 0.9, then we know it is superior to the one with 0.7.

11.2 The Treynor Ratio

Treynor ratio is the excess return of the portfolio over the risk-free rate divided by the systematic risk of the portfolio. The numerator must be positive for meaningful results. It does not work for negative beta assets.

The *ex-ante* and *ex-post* Treynor ratios are provided below.

$$TR = \frac{E(R_p) - R_f}{\beta_p}$$

$$\widehat{TR} = \frac{\bar{R}_p - \bar{R}_f}{\hat{\beta}_p}$$

Limitations of Treynor ratio

- The ratio itself is not informative. When two portfolios are compared, we know which one is superior but we do not know if its performance is better than the market portfolio.
- There is no information about the amount of underperformance or over performance. For instance, assume there are two portfolios A and B with Treynor ratios of 0.6 and 0.7 respectively. Though B is better than A, we have no information as to how much B's performance is better than A.

11.3 M-Squared

If M-Squared return (also known as risk adjusted performance measure or RAP) is greater than zero, the manager (portfolio) has positive risk-adjusted return. One way to get a positive M-Squared is when the risk is same as the market but R_P is greater than R_M. Another way is when the return is same as the market but at a lower risk. If M-Squared is zero, then the manager (portfolio) has the same risk-adjusted return as the market. If M-Squared return is negative, the manager (portfolio) has a lower risk-adjusted return than the market.

The *ex-ante* and *ex-post* M^2 are given by:

$$M^2 = \left[E(R_p) - R_f\right]\frac{\sigma_m}{\sigma_p} + R_f = \text{SR} \times \sigma_m + R_f \text{ (ex ante)}$$

$$\widehat{M^2} = \left(\bar{R}_p - \bar{R}_f\right)\frac{\hat{\sigma}_m}{\hat{\sigma}_p} + R_f = \widehat{\text{SR}} \times \hat{\sigma}_m + R_f \text{ (ex post)}$$

where: σ_m is the standard deviation of the market portfolio and σ_m/σ_p is the portfolio-specific leverage ratio.

The difference between the risk-adjusted performance of the portfolio and the performance of the market is called **M^2 alpha**.

Advantage: M-Squared is in units of the percent return which makes it more intuitive for the interpretation by the user.

Limitation: A limitation of this measure is that it uses total risk and not systematic risk.

11.4 Jensen's Alpha

Jensen's alpha is the difference between the actual return on a portfolio and the CAPM calculated, expected or required return. In other words, it is the plot of the excess return of the security on the excess return of the market. The intercept is Jensen's alpha and beta is the slope. Jensen's alpha can be calculated on both ex ante and ex post basis.

Like the Treynor ratio, it is based on systematic risk. Alpha is used to rank different managers and their portfolios. Since Jensen's alpha uses systematic risk, it is theoretically superior to M-Squared.

Interpreting Jensen's Alpha

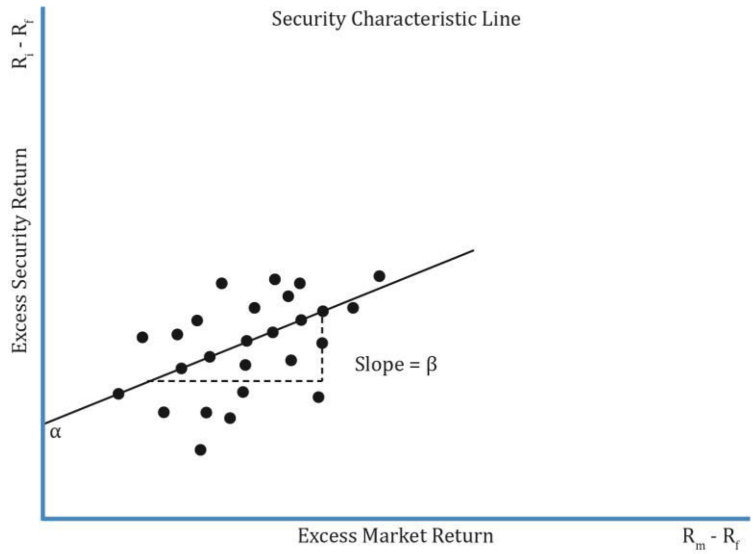
- If Jensen's alpha > 0, the manager (portfolio) has positive risk-adjusted return.
- If Jensen's alpha = 0, then the manager (portfolio) has the same risk-adjusted return as the market.
- If Jensen's alpha < 0, the manager (portfolio) has a lower risk-adjusted return than the market.

Instructor's Note:
Sharpe ratio and M-squared are total risk measures. Use these measures when a portfolio is not fully diversified.

Treynor ratio and Jensen's alpha are based on beta risk and should be used when a portfolio is well diversified.

12. Applications of the CAPM in Portfolio Construction

12.1 Security Characteristic Line



The SCL is a plot of the excess return of a security over the risk-free rate on the y-axis, against the excess return of the market on the x-axis. We saw earlier that the SML's intercept on the y-axis is the alpha and the line's slope is its beta. Similarly, the SCL's slope is the security's beta.

The SCL is obtained by regressing excess security return on the excess market return.

SCL: $R_i - R_f = \alpha_i + \beta_i * (R_m - R_f)$

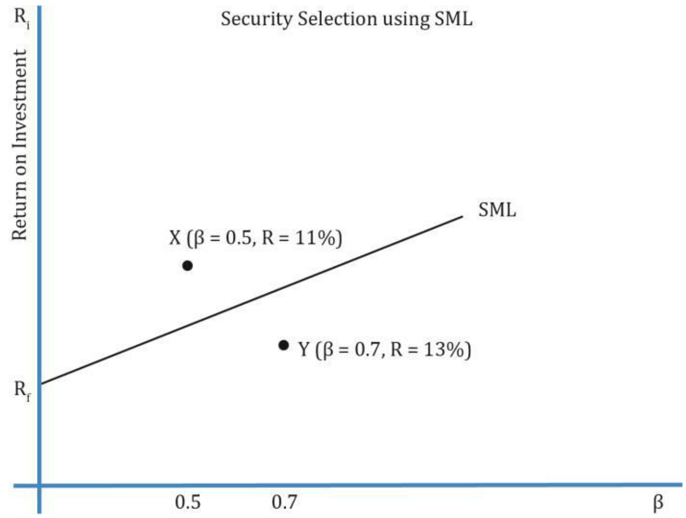
where:
$R_i - R_f$ = excess security return
$R_m - R_f$ = excess market return
α_i = Jensen's alpha or excess return

12.2 Security Selection

In CAPM, we assumed that investors have homogeneous expectations and assign the same value to all assets. So, all the investors arrive at the same optimal risky portfolio: the market portfolio. But, in reality, it does not actually happen.

The SML can also be used for security selection. Investors can plot a security's expected return and beta against the SML. The security is undervalued if it plots above the SML. The security is overvalued if it plots below the SML. The security is fairly priced if it plots on the SML.

As you can see in the exhibit below, security X is undervalued as it plots above the SML. At a risk level of $\beta = 0.5$, the return of X is greater than the security that plots on the SML. Similarly, Y must not be bought because the security on SML at a risk level of $\beta = 0.7$ has a higher return than Y.

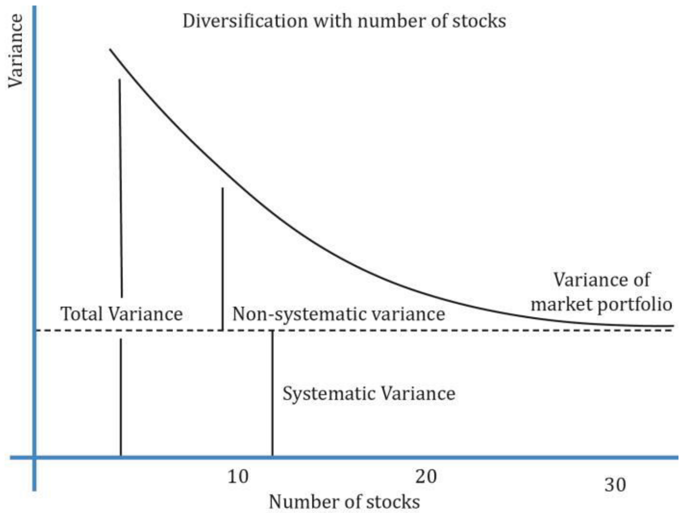


12.3 Implications of the CAPM for Portfolio Construction

Theoretically, investors should hold a combination of the risk-free asset and the market portfolio but it is impractical to own the market portfolio as it has a large number of securities. For example, S&P 500 is a good representation of the market as it has 500 stocks. But it can be shown that holding as few as 30 stocks can diversify away the non-systematic risk.

Interpretation of the exhibit below:

- It shows how variance decreases – nonsystematic risk, in particular – as stocks are added to the portfolio.
- Much of the non-systematic risk is diversified away with 30 stocks. As more stocks are added, the non-systematic risk progressively decreases approaching the systematic risk for 30 stocks.
- It is important that these stocks are not correlated with each other and must be randomly selected from different asset classes.


Diversification with number of stocks
Variance
Total Variance
Non-systematic variance
Variance of
market portfolio
Systematic Variance
10
20
30
Number of stocks

Summary

LO.a: Describe the implications of combining a risk-free asset with a portfolio of risky assets.

A combination of the risk-free asset and a risky asset can result in a better risk–return trade-off than an investment in only one type of asset because the risk-free asset has zero correlation with the risky asset. The risk of a portfolio is calculated using the following formula:

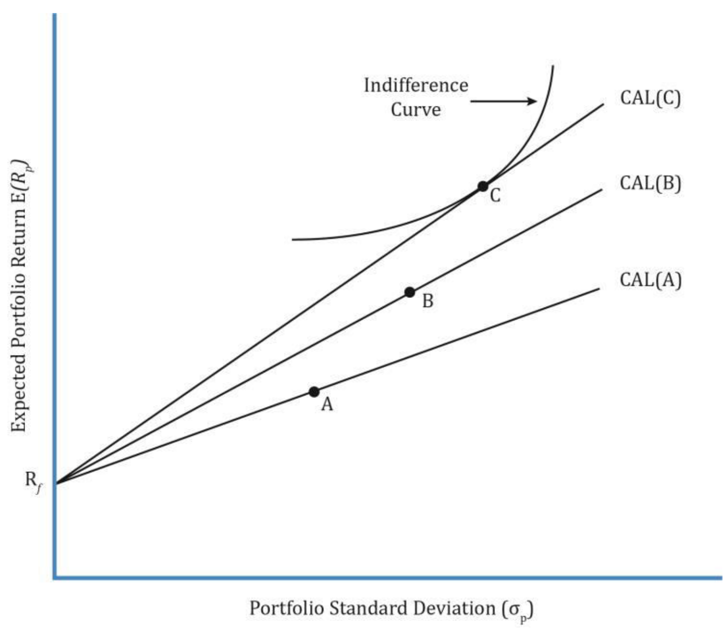
$$\sigma_p = \sqrt{w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * Cov_{1,2}}$$

As a risk-free asset has zero standard deviation and zero correlation of returns with a risky portfolio, it results in the following reduced equation:

$$\sigma_p = w_1 * \sigma_1$$

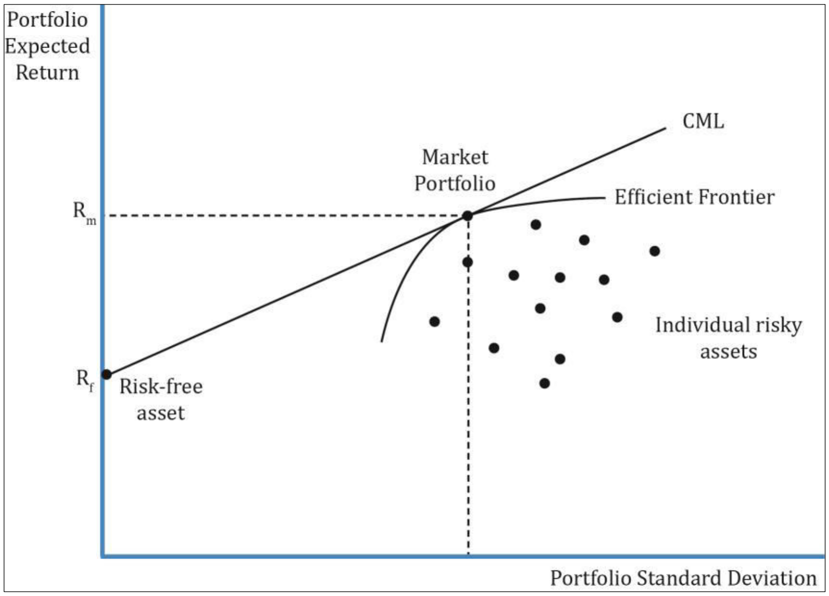
LO.b: Explain the capital allocation line (CAL) and the capital market line (CML).

Investors have different views of the market, which means the individual risky assets (e.g., securities) they choose to form their portfolio are different. This leads to different optimal risky portfolios, as illustrated in the figure below:



A unique optimal risky portfolio exists for all investors if there is homogeneity of expectations.

If there is one efficient frontier, there will be only one capital allocation line. The point where this capital allocation line is tangential to the efficient frontier is called the market portfolio. This is the optimal risky portfolio when all investors have the same expectations. The CML is a special case of the CAL where the efficient portfolio is the market portfolio.



$$R_p = R_f + \left(\frac{R_m - R_f}{\sigma_m}\right) * \sigma_p$$

LO.c: Explain systematic and nonsystematic risk, including why an investor should not expect to receive an additional return for bearing nonsystematic risk.

Systematic Risk: It is non-diversifiable or market risk that affects the entire economy and cannot be diversified away. Investors get a return for systematic risk. (Interest rates, inflation rates, natural disaster, political situation, etc.)

Nonsystematic Risk: It is a local risk that affects only a particular asset or industry. There is no compensation for nonsystematic risk as it can be diversified away. (Oil discoveries, non-approval for a drug, new regulations for telecom industry, etc.)

LO.d: Explain return-generating models (including the market model) and their uses.

Return-generating models provide an estimate of the expected return of a security given certain input parameters called factors.

Multi-factor models include macroeconomic, fundamental, and statistical models.

In a single-factor model, only one factor is considered. A classic single-factor model is the market model which is given by this equation:

$R_i = \alpha_i + \beta R_m + e_i$

LO.e: Calculate and interpret beta.

Beta is a measure of systematic (or market) risk. It is calculated using the following equation:

$$\beta = \frac{\mathrm{Cov}(i, M)}{\sigma_M^2} = \frac{\rho_{iM} * \sigma_i}{\sigma_M}$$

- $\beta > 0$: Return of the asset follows the market trend.
- $\beta < 0$: Return of the asset moves in an opposite direction to the market trend (negatively correlated with the market).
- $\beta = 0$: An asset's return has no correlation with the market. For example, a risk-free asset has a beta of zero.
- $\beta = 1$: Beta of the market is equal to 1. If a stock has a beta of 1, it means it has the same volatility as that of the market.

LO.f: Explain the capital asset pricing model (CAPM), including its assumptions, and the security market line (SML).

LO.g: Calculate and interpret the expected return of an asset using the CAPM.

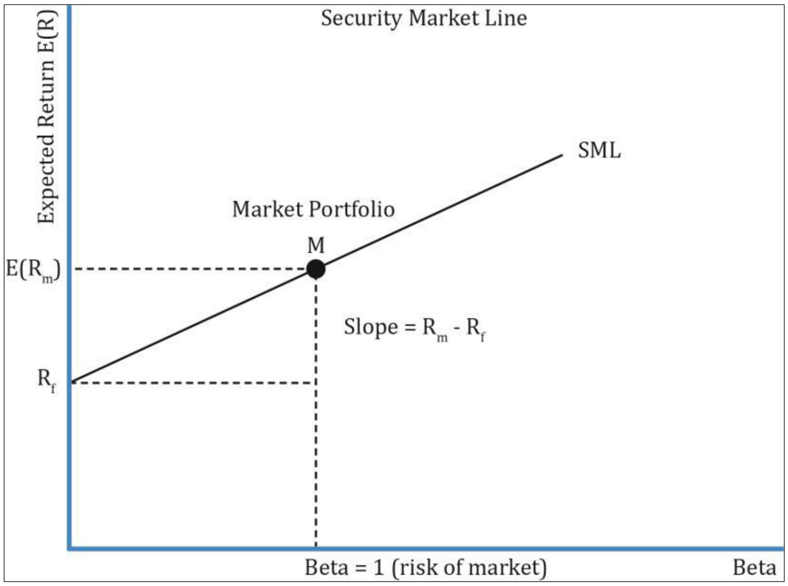
$$\text{CAPM: } r_e = R_f + \beta[E(R_{mkt}) - R_f]$$

The assumptions of CAPM are:

- Investors are risk-averse, utility maximizing, rational individuals.
- Markets are frictionless. There are no transaction costs and no taxes.
- All investors plan for the same, single holding period.
- Investors have homogeneous expectations.
- Investments are infinitely divisible.
- Investors are price takers.

LO.h: Describe and demonstrate applications of the CAPM and the SML.

The security market line (SML) is a graphical representation of the capital asset pricing model and applies to all securities, whether they are efficient or not. The security is undervalued if it plots above the SML. The security is overvalued if it plots below the SML. The security is fairly priced if it plots on the SML.



Differences between CML and SML	
CML	**SML**
Does not apply to all securities; applies to only efficient portfolios. Only systematic risk is priced. So CAL/CML can only be applied to those portfolios whose total risk is equal to systematic risk.	Applies to any security. It may include inefficient portfolios as well.
X-axis has the standard deviation of the portfolio.	X-axis has beta of the asset or portfolio.
$R_p = R_f + (\frac{R_m - R_f}{\sigma_m})^* \sigma_p$ where σ_p is the total risk of the portfolio.	$r_e = R_f + \beta[E(R_{mkt}) - R_f]$ Where β is the systematic risk. SML is just a graphical representation of CAPM.
Slope of the CML is the Sharpe ratio: $\frac{R_m - R_f}{\sigma_m}$	Slope of the SML is the market risk premium: $E(R_{mkt}) - R_f$
Similarities between CML and SML	
Both CML and SML have expected portfolio return on the y-axis.	
The line stretches from the risk-free asset to the market portfolio in both the cases.	

LO.i: Calculate and interpret the Sharpe ratio, Treynor ratio, M2, and Jensen's alpha.

The four measures commonly used in performance evaluation are:

Sharpe Ratio	$\frac{\text{Portfolio risk premium}}{\text{portolio total risk}} = \frac{R_p - R_f}{\sigma_p}$

M-Squared	$M^2 = \frac{(R_p - R_f)\sigma_m}{\sigma_p} - (R_m - R_f)$
Treynor Ratio	$\frac{\text{Portfolio risk premium}}{\text{beta risk}} = \frac{R_p - R_f}{\beta_p}$
Jensen's Alpha	Actual portfolio return – expected return $= R_p - [R_f + \beta(R_m - R_f)]$

Practice Questions

1. Jeff Thomas only invests in risky assets, while Lisa Jones invests in a combination of risky and the risk-free asset. Which of the following is *most accurate*?
   A. At any given level of risk, the maximum return for both Jeff and Lisa would be denoted by the efficient frontier.
   B. At any given level of risk, Jeff's maximum return is denoted by the CAL and Lisa's maximum return is denoted by the efficient frontier.
   C. At any given level of risk, Jeff's maximum return is denoted by the efficient frontier and Lisa's maximum return is denoted by the CAL.

2. The capital market line (CML) plots the risk and return of portfolio combinations consisting of risk-free asset and:
   A. the market portfolio.
   B. any risky asset.
   C. the leveraged portfolio.

3. If the borrowing rate is higher than the lending rate:
   A. the slope of the borrowing segment of CML will be less than the slope of the lending segment of the CML.
   B. the slope of the borrowing segment of CML will be equal to the slope of the lending segment of the CML.
   C. the slope of the borrowing segment of CML will be greater than the slope of the lending segment of the CML.

4. Which of the following is *most likely* to be synonymous with firm-specific risk?
   A. unsystematic risk.
   B. systematic risk.
   C. non-diversifiable risk.

5. With respect to return-generating models, the intercept term and the slope term of the market model is:

	Intercept	Slope
A.	alpha	systematic risk
B.	beta	nonsystematic risk
C.	variance	total risk

6. An analyst has compiled the following data:

Risk-free rate	5%
Expected market return	12%
Beta of stock ABC	1.3

Current price of stock ABC	$20
Year-end forecasted price of stock ABC	$22
Dividend anticipated to be paid over the year	$2

Based on his price and dividend forecast, the analysts should:

A. buy the stock.
B. sell the stock.
C. stay neutral.

7. Carol Davis, a portfolio manager is analyzing three securities A, B, and C for an investment opportunity. She has compiled the following data:

Stock	**A**	**B**	**C**
Expected Return	10.68%	16.30%	12.16%
Beta	1.3	1.6	0.8

The risk-free rate is 3.5% and market return is 11.5%. In her analysis, Carol makes the following statements:
Statement 1: "Security A is overvalued."
Statement 2: "Security C is undervalued."
Which of her statements is *most likely* true?

A. Statement 1.
B. Statement 2.
C. Both statement 1 and statement 2.

8. George Baker, a portfolio manager, earned a return of 15% on his portfolio over the past year. The market return over the same period was 8.5% and the risk-free rate was 2%. The portfolio had a beta of 0.75.
Jensen's alpha for George's portfolio is *closest* to:

A. 6.5%.
B. 8.1%.
C. 6.9%.

9. An analyst gathers the following information:

Stock	Expected annual return (%)	Expected standard deviation (%)	Correlation between stock and market
A	12	20	0.6
B	9	15	0.7
C	16	25	0.3
Market	6	10	1.0

The stock that has the highest total risk and market risk are:

Total risk **Market risk**

A.	Stock B	Stock C
B.	Stock C	Stock A
C.	Stock A	Stock B

10. Which of the following statements about the Security Market Line is *least accurate*?
    A. SML prices securities based on total risk.
    B. SML assists in identifying mispriced securities.
    C. The slope of SML equals the market risk premium.

11. Which of the following is *least likely* an assumption of the Capital Asset Pricing Model (CAPM)?
    A. There are no transaction costs, taxes, and other hurdles in trading.
    B. Investments can be made in fractions.
    C. Investors plan for multiple holding periods.

Solutions

1. C is correct. Since Jeff only invests in risky assets, the maximum return that he is likely to get at any given level of risk is denoted by the efficient frontier. As Lisa invests in a combination of risk-free and risky assets, the maximum return at any given level of risk that she is likely to get would be denoted by the capital allocation line (CAL).

2. A is correct. The capital allocation line includes all possible combinations of the risk-free asset and any risky portfolio.
   The capital market line is a special case of the capital allocation line, which uses the market portfolio as the optimal risky portfolio.

3. A is correct. The CML is divided into two parts: lending part and borrowing part. The point that divides the CML into the lending part and borrowing part is the market portfolio.
   If the borrowing rate is higher than the risk-free rate, then the additional return for each additional unit of risk for the borrowing portfolio will be lower than the additional return for each additional unit of risk for the lending portfolio.
   Hence, if the borrowing rate is higher than the risk-free rate, the slope of the borrowing segment will be lower than the lending segment.

4. A is correct**.** The firm-specific or industry-specific risk is known as unsystematic risk. Unsystematic risk can be diversified, unlike the systematic or market risk. The sum of systematic variance and nonsystematic variance equals the total variance of the asset.

5. A is correct**.** In the market model, $\mathrm{Ri} = \alpha\mathrm{i} + \beta\mathrm{iRm} + \mathrm{ei}$, the intercept, αi, and slope coefficient, βi, are estimated using historical security and market returns. The security characteristic line plots the excess return of the security on the excess return of the market. In this graph, Jensen's alpha is the intercept and the beta is the slope.

6. A is correct.
   Using CAPM,
   Required return for stock ABC $= 0.05 + 1.3 * (0.12 - 0.05) = 14.1\%$
   Forecasted return for stock ABC $= (22 - 20 + 2)/20 = 20.0\%$
   Based on this, the stock ABC plots above SML and hence is undervalued. The analyst should buy the stock.

7. C is correct.
   For a stock to be undervalued, its expected return should be greater than the required return (from CAPM).
   Using CAPM,

Required return for stock A $= 0.035 + 1.3 * (0.115 - 0.035) = 13.9\%$
Required return for stock B $= 0.035 + 1.6 * (0.115 - 0.035) = 16.3\%$
Required return for stock C $= 0.035 + 0.8 * (0.115 - 0.035) = 9.9\%$
Thus,
Stock A is overvalued.
Stock B is on par with its value.
Stock C is undervalued.

8. B is correct.
Jensen's alpha $= \alpha_p = R_p - [R_f + \beta_p(R_m - R_f)]$
Jensen's alpha = 0.15 – [0.02 + 0.75(0.085 – 0.02)] = 0.081 or 8.1%.

9. B is correct.
<u>Total risk</u>:
Total risk is defined by the expected standard deviation.
Stock C has the highest total risk.

<u>Market risk</u>:
Market risk is defined by beta.

$$\beta = \frac{\rho_{im}\,\sigma_i\,\sigma_m}{\sigma_m^2}$$

$$\beta_{\text{Stock A}} = \frac{0.6 \times 0.20}{0.10} = 1.2$$

$$\beta_{\text{Stock B}} = \frac{0.7 \times 0.15}{0.10} = 1.05$$

$$\beta_{\text{Stock C}} = \frac{0.3 \times 0.25}{0.10} = 0.75$$

Stock A has the highest market risk.

10. A is correct. The total risk for a security is a combination of market risk and firm-specific risk. The security market line is only based on the market risk or systematic risk, as the firm-specific risk is diversifiable. The slope of the SML is the market premium and the SML can also assist in identifying fairly priced securities.

11. C is correct. The assumptions of CAPM are:
- Risk-aversion: With greater risk, investors require greater returns.
- Frictionless markets: There are no transaction costs, taxes, and other hurdles in trading.
- Utility maximizing investors: Investors select investments according to their preferences that maximize their utility.
- One-period horizon: All investors have the same one-period time horizon.
- Divisible assets: All investments are infinitely divisible.

- Competitive markets: Market prices cannot be influenced.
- Homogenous expectations: All investors have the same expectations on assets.

