R46 Basics of Derivative Pricing and Valuation

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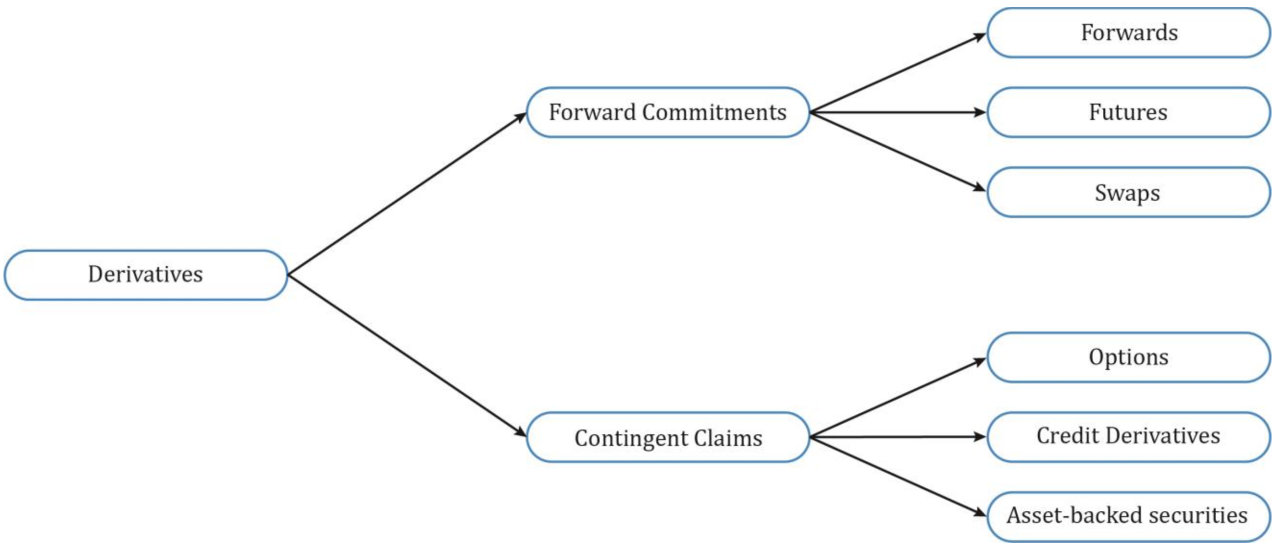
1. Introduction

The major questions addressed in this reading are:

- How does the pricing of the underlying asset affect the pricing of derivatives?
- How are derivatives priced using the principle of arbitrage?
- How are the prices and values of forward contracts determined?
- How are futures contracts priced differently from forward contracts?
- How are the prices and values of swaps determined?
- How are the prices and values of European options determined?
- How does American option pricing differ from European option pricing?

2. Basic Derivative Concepts, Pricing the Underlying



2.1 Basic Derivative Concepts

This is a recap of what we saw in the previous reading.

Forward contract: It is a customized over-the-counter derivative contract in which two parties agree that one party, the buyer, will purchase an underlying asset from the other party, the seller, at a later date, at a fixed price they agree upon when the contract is signed.

Futures contract: It is a standardized derivative contract created and traded on a futures exchange in which two parties agree that one party, the buyer, will purchase an underlying asset from the other party, the seller, at a later date at a price agreed upon by the two parties when the contract is initiated. There is also a daily settling of gains and losses and a credit guarantee by the futures exchange through its clearinghouse.

Swap contract: It is an over-the counter derivate contract in which two parties agree to exchange a series of cash flows whereby one party pays a variable series that will be determined by an underlying asset or rate, and the other party pays either 1) a variable series determined by a different underlying asset or rate or 2) a fixed series.

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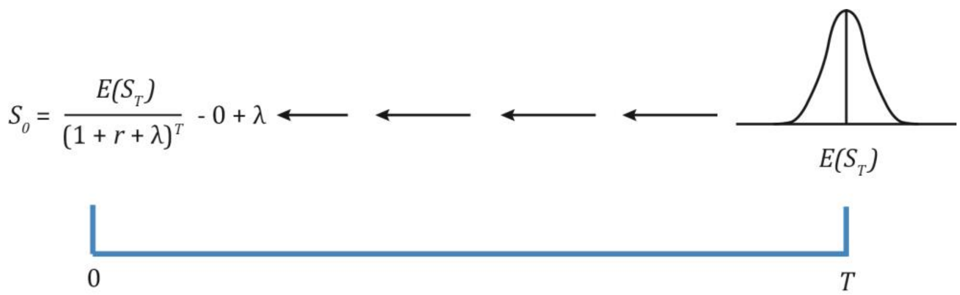
Option contract: It is a derivative contract in which one party, the buyer, pays a sum of money to the other party, the seller or writer, and receives the right to buy or sell an underlying asset at a fixed price either on a specific expiration date or at any time prior to the expiration date.

2.2 Pricing the Underlying

The price or value of a financial asset is the expected future price plus any benefits such as dividends or coupon interest minus costs discounted at a rate appropriate for the risk assumed. The exhibit below shows how to get the current price S_0 of an asset by discounting its expected future price $E(S_T)$, by the risk-free rate *r,* plus the risk premium λ, over the period from 0 to T.



Source: CFA Program Curriculum, Basics of Derivative Pricing and Valuation

The price of the underlying at time t = 0 is given by $S_0 = \frac{E\,(S_T)}{(1 + r + \lambda)^T} - \theta + \gamma$

Let us now decompose this expression and understand what each of the terms means.

- *Expected future value* $E(S_T)$: Let's say that the expected future value of the underlying asset a year later at t = 1 is 100. It cannot be known for sure and its value a year later, given by the normal distribution $E(S_T)$, is the expected value.
- *Discount rate*: The first term $\frac{E\,(S_T)}{(1 + r + \lambda)^T}$ is the present value of expected future price of an asset with no interim cash flows (in our case, 100) where *r* = risk-free rate. Given that the value in the future is uncertain, it is not appropriate to discount it at the risk-free rate, so a risk premium λ is added to the risk-free rate. If the riskiness of the asset is high, then λ is high.
- The present value of the asset is then given by $\frac{E\,(S_T)}{(1 + r + \lambda)^T}$
- γ is the present value of any benefits of holding the asset between t = 0 and t = 1. γ is added to the current price. It is assumed that the benefits are certain, so their future value is discounted at the risk-free rate to calculate γ. γ includes both monetary and non-monetary benefits. For an asset like stocks, the dividend paid is an example of monetary benefit. **Convenience yield** is the non-monetary benefit of holding an asset. The pleasure one derives from looking at one's art collection or gold is an example of non-monetary benefit.

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- θ is the present value of the costs of holding an asset between t = 0 and t = 1. It is subtracted from the current price. For example, if the asset is gold and you rent a locker to store it safely, then this cost incurred that must be subtracted from the current price.
- **Cost of carry** is the net cost of holding an asset. Cost of carry = θ − γ

3. The Principle of Arbitrage

Arbitrage is a type of transaction undertaken when two assets or portfolios produce identical results but sell for different prices. Let us take the example below:

Given: Assets A and B produce the same values at time T but at time 0, A is selling for less than B.

0	T
$S_0{}^A < S_0{}^B$:	$S_T{}^A = S_T{}^B$:
Buy A at $S_0{}^A$	Sell A for $S_T{}^A$
Sell B at $S_0{}^B$	Buy B for $S_T{}^B$
Cash flow = $S_0{}^B - S_0{}^A (> 0)$	Cash flow = $S_T{}^A - S_T{}^B (= 0)$

Source: CFA Program Curriculum, Basics of Derivative Pricing and Valuation

Interpretation**:**

- The exhibit above is self-explanatory. Assets A and B have the same values at time T, but initially at time t = 0, asset A sells for lesser than asset B. This leads to an arbitrage opportunity.
- For example, at t = 0, assume A is priced at 100 and B is priced at 101. If you borrow B and sell it, and buy A for 100, the net cash is 101 – 100 = 1.
- At time t = T, both A and B are priced the same. Sell asset A. Use the proceeds to buy B and return it since it was borrowed initially. The net cash flow is 0.

Arbitrage opportunities are exploited quickly and hence do not last for long. In the above example, when the arbitrage opportunity exists, the demand for A will go up causing its price to go up. Similarly, when more people sell B, its demand goes down, leading to a decrease in its price causing the prices of both the assets to come to the same level. It is not just the difference in price that matters; one needs to consider the transaction costs as well. If the transaction costs exceed the benefit from an arbitrage opportunity, then it is not worth exploiting.

Law of one price: If two assets have the same expected return in the future, then their market price today must be the same.

Arbitrage and Derivatives

The price of a derivative is tied to the price of the underlying. For example, if the price of a stock goes up, then a call option on the stock also goes up. Therefore, a derivative can be

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used to hedge an underlying, or vice versa. For example, if you are long on Google stock, then you enter into a forward contract to sell the stock at a future date at a fixed forward price to eliminate your risk.

Since the risk is eliminated, the expected return on the combined position is the risk-free return.

A derivative must be priced such that no arbitrage opportunities exist, and there can be only one price for the derivative that earns the risk-free return. If it earns a return in excess of the risk-free rate, then arbitrage opportunities exist for (underlying + derivative) position.

Arbitrage and Replication

An asset and a derivative can be combined to produce a risk-free bond in one of the ways shown below. Conversely, an asset and the risk free asset can be combined to produce a derivative.

Asset	+	Derivative	=	Risk-free asset
Asset	-	Risk-free asset	=	-Derivative
Derivative	-	Risk-free asset	=	-Asset

For example, a derivative on a stock index combined with the risk-free asset [Long derivative (Stock index futures) + Long risk-free asset (Lending) = Long asset (Stock index)

<u>Replication</u> is the creation of an asset or portfolio from another asset, portfolio, and/or derivative. Why is replication needed? Isn't it easier to just buy a government security to earn the risk-free rate instead of buying the asset and a derivative? There are some situations under which replication is valuable:

- When transaction costs are lower.
- When one of the components, say the derivative, is not rightly priced resulting in an arbitrage opportunity.

Risk Aversion, Risk Neutrality, and Arbitrage-Free Pricing

Risk aversion: Most investors are risk-averse and expect a compensation (risk premium) for assuming risk. As we have seen earlier, a derivative *can* be combined with an asset to mitigate the risk and produce a risk-free asset. We saw in the previous section what factors affect a derivative's pricing. Since the risk aversion of an investor does not impact derivative pricing, one can derive the derivative price assuming investors are risk-neutral. When

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pricing derivatives, use the risk-free rate, i.e., the price of a derivative can be calculated by discounting it at the risk-free rate rather than the risk-free rate plus a risk premium.

When pricing assets, a risk premium is added to the risk-free rate. Recall, the first term of the current price S_0 of an asset is $\frac{E\,(S_T)}{(1+r+\lambda)^T}$ where λ is the risk premium. This means that the asset's price in the spot market factors in the risk aversion of an investor. Since the risk aversion is already captured in the asset pricing, it is not included in a derivative's pricing.

Risk-neutral pricing: Derivative pricing is sometimes called *risk-neutral pricing* because there is only one derivative price, which combined with the underlying asset, can earn the risk-free rate.

Arbitrage-free pricing: The overall process of pricing derivatives by arbitrage and risk-neutrality is called *arbitrage-free pricing*. It is also called the principle of no arbitrage. Limitations to execute arbitrage transactions include:

- It may not be possible to short sell assets.
- Information on arbitrage opportunities/volatility of the asset may not be easily accessible, and it can be risky to execute if one lacks accurate information on the inputs.
- Arbitrage requires capital, which may not be easily available to everyone to maintain positions. For instance, if exploiting an arbitrage opportunity requires $100 million, it is not easy to borrow so much money to engage in a transaction.

Clearinghouses do not place any restrictions on transactions that can be arbitraged.

A *hedge portfolio* is one that eliminates arbitrage opportunities and implies a unique price for a derivative.

4. Pricing and Valuation of Forward Contracts: Pricing vs. Valuation; Expiration; Initiation

This section lays the foundation for the subsequent sections. We look at two terms, price and value, and what they mean in context to different assets/derivatives.

Stock price vs. value

- The price of a stock is its market price while the value of a stock is its intrinsic value often determined by fundamental analysis (by analyzing a company's financial statements, and discounting future cash flows). Alternatively, the book value of a company is compared to its market value.
- For instance, if the intrinsic value of a stock is 110 and its market price is 100, then it means investors must buy the stock.

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Option contract price versus value

- The price is similar to what we saw above for a stock. For instance, the price of a call option is its market price or what you would pay to buy the call option. The value of a call option is based on the underlying and other variables that impact the option.

Forwards, futures, and swaps

- Unlike stocks, bonds, or options, these do not require an initial cash outlay.
- Their price and value cannot be compared to each other.
- The price of a forward contract is the fixed price or rate that is embedded in the contract; it represents the price or rate at which the underlying will be purchased at a later date.
- The value changes over the life of the contract. It is zero at initiation. Over time, as spot price or rate changes, the value to each party changes. If the price of the underlying increases, then the value to the long also increases.

4.1 Pricing and Valuation of Forward Commitments

Pricing and Valuation of Forward Contracts

Let us take an example. Assume Leo owns a share of GE, whose spot price today (S_0) is $100. Rachel enters into a forward contract with Leo today to buy a share of GE at $101 after 1 month.

The terms, parties, and ways to settle the contract are described below:

- $101 is the forward price; the contract is for a period of 1 month.
- Neither Rachel nor Leo pays any money to each other at t = 0 when they enter into the contract.
- Rachel is the long party and Leo is the short party.
- When the contract expires after 1 month, the forward contract can be settled in two ways: 1) Rachel pays $101 to Leo and receives the share in return; 2) if GE trades at $103 after 1 month, then Leo pays $2 (the difference between the agreed-upon forward price and the current stock price) to Rachel.
- There is a risk of default in forward contracts. If the stock price increases to $103, Rachel faces the risk that Leo fails to deliver the share as agreed. Conversely, if the stock price decreases to $99, Leo faces the risk that Rachel does not buy the share as agreed upon.
- Is there an alternative way for Rachel to buy this asset? Yes, to wait until time T and buy the asset at its then spot price S_T. The drawback is that one cannot be sure what the price of the asset will be then; it may be more, or less.

The above example is shown in a generic way in the diagram below. It shows the transactions from a buyer's perspective at times t = 0 when the contract is initiated and time t = T, when the contract expires.

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Interpreting the notation and key features of a forward contract:

- *Forward price*: It is denoted by F_0 (T) where subscript 0 indicates the price that is agreed upon at time t = 0 and T indicates the contract ends at time T.
- *Spot price*: It is denoted by S_t. At t = 0, spot price is S_0. When the forward contract expires at t = T, the spot price is S_T. At any time during the life of the contract, it is denoted by S_t.
- *Value of the contract at time t* is denoted by $V_t(T)$.
- The price to be paid at time T when the contract expires is fixed at time t = 0.
- No money exchanges hands at time t = 0, i.e., no party pays anything to the other at t = 0.

The forward price agreed at the initiation date of the contract is the spot rate compounded at the risk-free rate over the life of the contract.

$$F_0(T) = S_0 \text{ x } (1 + r)^T$$

Assuming a risk-free rate of 10%, spot price of $S_0 =$ \$100, and time period of 1 month, we calculate the forward price as:

$$T = \frac{1}{12} = 0.0833;\ F_0\ (T) = S_0 \text{ x } (1 + r)^T = 100 \times (1.1)^{0.0833} = 101$$

Why is the forward price equal to the spot rate compounded at the risk-free rate over the contract period?

In theory, if Rachel did not enter into the forward contract, then she could invest \$100 for period T in a risk-free asset and earn 10% or \$101 after 1 month. So, the forward contract must earn the risk-free rate as an excess return will result in an arbitrage opportunity.

Value of a forward contract at initiation

Value of a forward contract at initiation = 0

The value of a forward contract at initiation is zero because neither party pays any money to the other. There is no value to either party.

- At the end of the contract, Leo, as the short party, is obligated to sell the asset to the buyer (Rachel) for \$101.
- Given a spot price of \$100, a risk-free rate of 10%, and a forward price of \$101, there is no advantage to either party at t = 0. The forward price is such that no party can earn an arbitrage profit.

Value at expiration for the long party:

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$V_t(T) = S_T - F_0(T)$

where:
S_T = spot price of the underlying

$F_0(T)$ = forward price agreed in the contract

The value of a forward contract is positive to the long party if $S_T > F_0(T)$ and negative if $S_T < F_0(T)$ at expiration.

Value at expiration for the short party:

$V_t(T) = -(S_T - F_0(T))$

The value of a forward contract is positive to the short party if $F_0(T) > S_T$ and negative if $F_0(T) < S_T$ at expiration.

If the spot price at time T is 105, then Rachel (long) gets the asset by paying 101 and can immediately sell it for 105 at the then market price. Value to the long = $S_T - F$ = 105 - 101 = 4.

5. Pricing and Valuation of Forward Contracts: Between Initiation and Expiration; Forward Rate Agreements

Value of a forward contract during the life of the contract

Value during the life of the contract $V_t = S_t - \frac{F}{(1+r)^{T-t}}$

This is the difference between the spot price at time t, and the present value of forward price for the remaining life of the contract. In our example, at t = 15 days, if S_t = 106, then $V_t = 106 - \frac{101}{(1.1)^{0.5}} = 9.7$. Since the price has gone up, the value to the long party is positive.

Instructor's Note

As market conditions change, only the value of a forward contract changes. Its price does not change as it is fixed at contract initiation.

Forward Price with Benefits and Costs

Now, let us consider an asset with benefits and costs. How is the forward price for such an asset calculated? The forward price of an asset with benefits and/or costs is the spot price compounded at the risk-free rate over the life of the contract minus the future value of those benefits and costs.

$F_0(T) = (S_0 - \gamma + \theta) \times (1 + r)^T$

It can be rewritten as:

$F_0(T) = (S_0) \times (1 + r)^T - (\gamma - \theta) \times (1 + r)^T$

where:

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γ = present value of the benefit. It is subtracted from the spot price because if you own the asset, you receive any benefits associated with the asset, during the life of the contract.

θ = present value of costs incurred on the asset during the life of the contract. These costs make it more expensive to hold the asset and hence increase the forward price.

In the previous example, assume $10 is the present value of benefits from the asset and $20 is the present value of costs associated with holding the asset. The forward price at time 0 when Rachel enters into the contract is:

$F_0(T) = (100 - 10 + 20)(1.1)^{0.0833} = 110.9.$

The value of a forward contract is the spot price of the underlying asset minus the present value of the forward price. The value of the contract at time t is given by the expression below:

<u>Value of the forward contract</u>

$V_t(T) = S_t - \frac{F_0(T)}{(1+r)^{T-t}}$ (without benefits and costs)

$V_t(T) = S_t - (\gamma - \theta)(1 + r)^t - \frac{F_0(T)}{(1+r)^{T-t}}$ (with benefits and costs)

In other words, the value of the contract is the spot price minus the net benefit for the remaining period, minus the present value of the forward price. Note that $(\gamma - \theta)$ is the net benefit.

Instructor's Note

The forward price can be lower than the spot price though it is not common.

$F = S(1 + r)^t + FV(\text{costs}) - FV(\text{benefits})$

When the future value of benefits is higher than the future value of costs and the compounded spot price, then the forward price is lower than the spot price. If a commodity is in short supply, then its non-monetary benefits/convenience yield may be higher.

5.1 A Word about Forward Contracts on Interest Rates

Forward Rate Agreement (FRA)

So far, the forward contracts we have seen had an asset as the underlying. But some forward contracts, which have an interest rate as the underlying, are called forward rate agreements. FRAs allow us to lock in a rate today for a loan in the future and act as a hedge against interest rate risk. They are forward contracts that allow participants to make a known interest payment at a later date and receive, in return, an unknown interest payment.

The payoffs on an FRA are determined by market interest rates at expiration:

- If reference rate at FRA expiration is greater than the FRA rate, then the long benefits.
- If reference rate at FRA expiration is lower than the FRA rate, then the short benefits.

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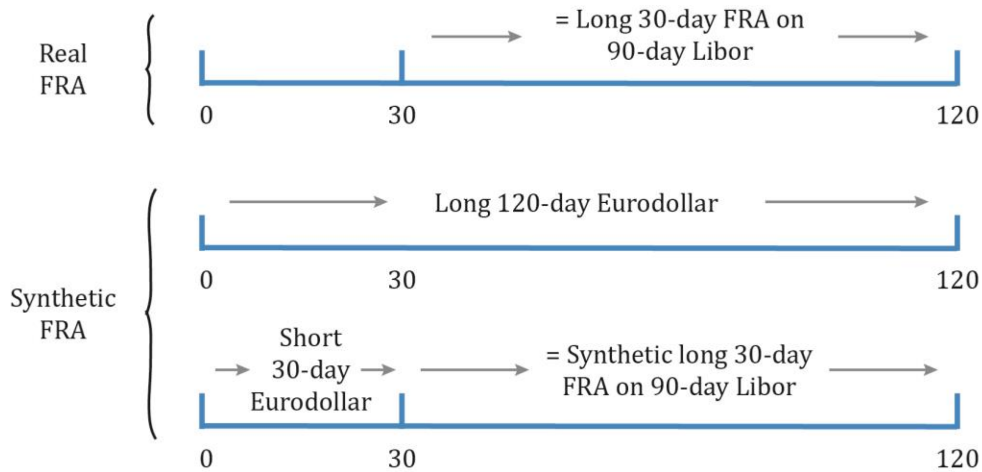
Example of an FRA

The exhibit below shows going long a 30-day FRA in which the underlying is 90-day Libor.



Source: CFA Program Curriculum, Basics of Derivative Pricing and Valuation

Interpretation:

- FRAs are usually based on LIBOR (London Interbank Offered Rate). The FRA described above is a 30-day FRA in which the underlying is the 90-day LIBOR.
- Assume the time now is t = 0 and you need to borrow $100 million after 30 days. You are concerned that the interest rate represented by LIBOR might increase between now and 30 days, and would like to lock in a rate available now of 5%. This is the rate that will be applicable on the loan 30 days later. This removes any uncertainty regarding the interest payment on the loan, as it is fixed today. The rate is for a period, in this case for 90 days. You enter into an agreement today to borrow $100 million 30 days later at a rate of 5% for 90 days.
- After 90 days, if the rate increases to 6%, then you benefit as your outflow is based on 5%, but you receive an inflow based on 6%. However, you lose if the rate decreases to 4% as the outflow will be higher than what you receive.
- Are FRAs of interest only to parties that borrow money? No, even parties that lend money engage in FRAs to lock in an interest rate if they believe the rates will go down in the future.

Synthetic FRA: Instead of engaging in a real FRA, a synthetic FRA can be constructed by lending a 30-day Eurodollar time deposit and buying a 120-day Eurodollar time deposit.

Instructor's Note

A quick summary of the concepts discussed in earlier readings.

1. The rate on a zero-coupon bond is also a spot rate.

2. Term structure of interest rates: interest rates on loans of different maturities. For example, the 30-day rate is 6%, 60-day rate is 7% and so on.

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6. Pricing and Valuation of Futures Contracts

Futures contracts have standard terms, are traded on a futures exchange, and are more heavily regulated than forward contracts. Some of its characteristics are listed below:

- *Marking to market*: Futures contracts are marked to market on a daily basis. What this means is that if there is a gain or loss on the position relative to the previous day's closing price, then the gain is credited to the winning account by deducting the amount from the losing account.
- *Credit guarantee*: We saw that there was a default risk inherent in every forward contract. In futures contracts, there is a credit guarantee by the futures exchange through the clearinghouse.
- *Clearinghouse*: The clearinghouse is the counterparty to every trade on the exchange.
- *Daily cash flow*: In a forward contract, the gain or loss is realized at the end of the contract period. But, in a futures contract, there is a cash flow on a daily basis.

Relationship between futures prices and interest rates

- If futures prices are positively correlated with interest rates, futures contracts are more desirable to holders of long positions than are forwards because there are intermediate cash flows on which interest can be earned. This cash flow can be invested at higher interest rates.
- If interest rates are constant, or have zero correlation with futures prices, then forwards and futures prices will be the same.
- If futures prices are negatively correlated with interest rates, then it is more desirable to have forwards than futures to holders of long positions.

Payoffs and Valuation of Futures Contracts

Let us take a simple example to compare the payoffs and valuation of a forward contract with a futures contract over three days. The futures price at the end of every day is given below with the price at initiation being 100.

Gain to Long in a Forward Contract

	Forward contracts	Futures contracts
Payoff	Ignoring the time value of money, total payoff is the same for forward and futures contract.	If the time value of money is considered, then it depends on the correlation of futures and interest rate.

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Value	Zero at initiation. Keeps on increasing until expiration of contract.	Zero at initiation. Keeps rising until settlement on the next trading day. Becomes zero once marked to market. For instance, increase from 0 to 1 on day 1.
Credit risk	Higher	Lower because of daily settlement

7. Pricing and Valuation of Swap Contracts

To understand how swaps work, let us consider a 3-year plain vanilla interest rate swap with annual settlement where the fixed-rate payer pays 10% and the floating rate payer pays an interest based on LIBOR. LIBOR rates at t = 0, 1, and 2 are 9%, 10%, and 11%.

Net gain to Fixed	-1%	0%	1%
Fixed payment	10%	10%	10%
Floating payment	9%	10%	11%

<u>Interpretation:</u>

- At times 0, 1, and 2, the two parties exchange a series of payments. The fixed rate payer makes a fixed payment of 10% and receives a floating payment based on the value of the underlying at that point in time. Here is it 9%, 10%, and 11% respectively.
- Like a forward rate agreement, rates are locked in for future. So, a swap is similar to a series of forward contracts, with each contract expiring at specific times, where one party agrees to make a fixed payment and receive a variable payment. But, the prices of the implicit forward contracts embedded in a swap are not equal.
- It is like getting into multiple forward rate agreements to lock in rates for different periods in the future at a different forward price.

Swap is a series of off-market forward contracts where:

- Each forward contract is created at a price and maturity equal to the fixed price of the swap with the same maturity and payment dates respectively.
- This means that the series of FRAs built into a swap are all off-market FRAs - some with positive values and some with negative values.
- The combined value of the off-market FRAs is zero.

Now, how do we determine the price of a swap that would result in a combined value of its FRAs to be zero? This is discussed in the following section.

Difference between Price and Value of a Swap Contract

Some key points regarding the price and value of a swap or as follows:

- The value of a swap at initiation is typically equal to zero.

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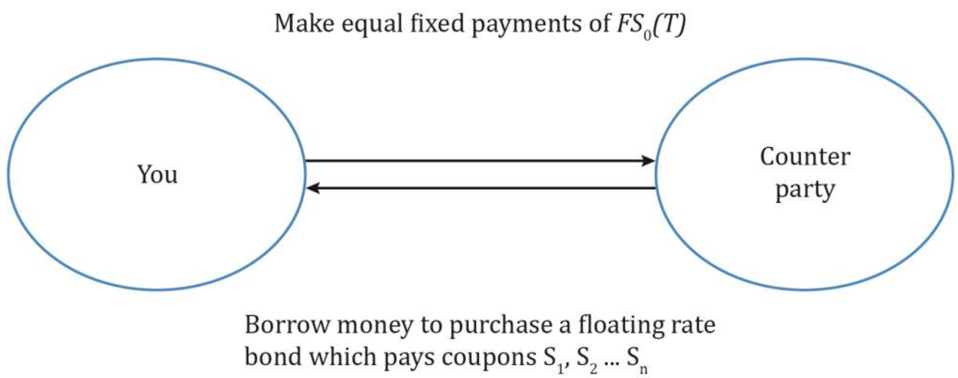
- The swap price is determined at initiation through a process known as *replication*. No-arbitrage pricing is the key to pricing a swap. *Replication* implies that the valuation of a swap price is the present value of all the net cash flow payments from the swap.
- The value of a swap changes during the life of the contract.

Example of a Swap

Consider a 3-year plain vanilla interest rate swap with annual settlement where the fixed-rate payer pays 10% and the floating-rate payer pays based on LIBOR. LIBOR rates at t = 0, 1, and 2 are 9%, 10%, and 11%.

The fixed rate of the swap is referred to as its price. In this case, it is 10%. A swap can be replicated as follows:



- Step 1: Buy a floating-rate bond or any asset that pays coupons of unknown value S_0, $S_{1,}$ q….S_N at times t = 1, 2 ..N.
- Step 2: Borrow money to purchase this floating rate bond (equivalent to issuing a fixed-rate bond); the payments for the money borrowed are equal fixed-payments of FS_0 (t) at t=1, 2, …N. This must have the same cash flow as the swap.
- The rate at which the money for the floater was borrowed is the price of the swap. Given the no-arbitrage pricing, the fixed rate on the swap must be equal to the fixed rate at which the fixed-rate bond was issued in step 2.
- The value of the swap during the life of a swap is based on the present value of the expected future cash flows. The cash flows, floating payments in particular, are based on the market price of the underlying.

8. Pricing and Valuation of Options

Let us look at the different types of options.

There are two parties in any option:

- Long party: The buyer of an option or the party that holds the position is the long party.
- Short party: The seller of an option or the party that writes the position is the short party.

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Types of options based on purpose:

- Call option: Gives the buyer the right, not the obligation, to buy the underlying asset at a given price on a specified expiration date. The seller of the option has an obligation to sell the underlying asset. The given price is called the strike price or exercise price. For example, if the exercise price is 25, then the buyer has the right to buy the underlying at 25.
- Put option: Gives the buyer the right, not the obligation, to sell the underlying asset at a given price on a specified expiration date. The seller of the option has an obligation to buy the underlying asset.

Types of options based on when they can be exercised:

- European option: This type of option can be exercised only on the expiration date.
- American option: This type of option can be exercised on or any time before the option's expiration date.

8.1 European Option Pricing

Value of European Option at Expiration

Call option

- At expiration, the value of a call option is the greater of zero or the value of the underlying asset minus the exercise price.
- $C_T = \max(0, S_T - X)$

Put option

- At expiration, the value of a put option is the greater of zero or the exercise price minus the value of the underlying.
- $P_T = \max(0, X - S_T)$

Relationship between the value of the option and the value of the underlying

- The value of a European call option is directly related to the value of the underlying.
- The value of a European put option is inversely related to the value of the underlying.

Effect of the Exercise Price

Consider two call options with similar attributes (time to expiration, underlying) but different exercise prices. Assume the exercise price for call option 1 is 25 and that for call option 2 is 30. The right to buy at a lower price of 25 will be more valuable than the right to buy at a higher price of 30.

Similarly, in the case of a put option, the right to sell for a higher price is more valuable than the right to sell for a lower price.

Relationship between the value of the option and the exercise price

- The value of a European call option is inversely related to the exercise price.
- The value of a European put option is directly related to the exercise price.

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Moneyness of an option: Indicates whether an option is in, at, or out of the money. The table below shows the moneyness of an option for various relative values of S and X.

	Call Option	**Put Option**
In the money	S > X	S < X
At the money	S = X	S = X
Out of the money	S < X	S > X

Effect of Time to Expiration

Longer-term options should be worth more than shorter-term options. For instance, you have a bought a call option on a stock at an exercise price of $25. Which one would be more valuable to you; an option that expires in a week or an option that expires a year later? Without doubt, the option that expires a year later as it gives enough time for the stock price to increase and make the option valuable.

But, is it true for a put option? Partly, yes. The more the time to expiration, the more the opportunity for the stock to fall below its exercise price. What does the put option holder get in return when the underlying falls below the exercise price? Just the exercise price. The longer the holder waits to exercise the option, the lower the present value of the payoff. If the discount rate is high, then a longer time to expiration results in a lower present value for the payoff.

Relationship between the value of the option and the time to expiration

- The value of a European call option is directly related to the time to expiration.
- The value of a European put option can be directly or inversely related to the time to expiration. Inversely related under these conditions: the longer the time to expiration, the deeper the option is in the money, and the higher the risk-free rate.

Effect of the Risk-free Rate of Interest

If the risk-free rate is high, then the call price is higher. For example, you want to invest in a stock whose price is $100. You can either buy the stock for $100 or buy the call option on the stock for $5. Both these actions give you exposure to the stock. If you buy the call option, then you can invest the remaining $95. If the interest rates are high, then buying the call option is more valuable because of the interest earned.

In the case of puts, the longer time to expiration with a higher risk-free rate lowers the present value of the exercise price when the option is exercised. That is, when the time to expiration is longer you get the money later and, if the risk-free rate is high, it is discounted by a higher number which lowers the value of the payoff.

Relationship between the value of the option and the risk-free interest rate

- The value of a European call option is directly related to the risk-free interest rate.
- The value of a European put option is inversely related to the risk-free interest rate.

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Effect of Volatility of the Underlying

Volatility is good for both call and put options. If the underlying stock becomes more volatile, then the probability of the options expiring deep in the money becomes greater.

Relationship between the value of the option and the volatility of the underlying

- The value of a European call option is directly related to the volatility of the underlying.
- The value of a European put option is directly related to the volatility of the underlying.

Effect of Payments on the Underlying and the Cost of Carry

		Call option	**Put option**
Benefits (dividends on stocks, interest on bonds, convenience yield on commodities)	Stocks and bonds fall in value when dividends and interest are paid.	Decreases because call holders do not get these benefits.	Increases
Carrying costs		Increases as call holders do not incur these costs.	Decreases

Relationship between the value of the option and benefits/costs of the underlying

- A European call option is worth less the more benefits that are paid by the underlying. The call option is worth more the more the costs that are incurred in holding the underlying.
- A European put option is worth more the more benefits that are paid by the underlying. The put option is worth less the more costs that are incurred in holding the underlying.

9. Lower Limits for Prices of European Options

In this section, we look at the least price one would be interested in paying for a call option. Let us consider a call option on a stock with a strike price of X that expires at time T. The initial price of the underlying at time t = 0 is S_0, the underlying price at expiration is S_T, and the risk-free rate is r%. If the option is in the money, the payoff at expiration will be $S_T - X$. The lowest price of the call option is given by:

$$c_0 >= \max\left(0, S_0 - \frac{X}{(1+r)^T}\right)$$

<u>Interpreting the above equation:</u>

- It is based on the premise that a call option is equivalent to buying the asset by borrowing the present value of X at a risk-free rate of r%.
- It is the present value of X because you would have to pay X and take delivery of the asset at time T. The payoff for this transaction is S_T - X (this number can even be negative). In the case of a call option, it can either be zero (out-of-the-money) or S_T-X.

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- The price of a call option must be at least zero and not a negative value because if it is less than zero, then it cannot be exercised.
- $\frac{X}{(1+r)^T}$ is the present value of the exercise price X. So, the lowest price of a call option is
  $c_0 >= \max(0, S_0 - \frac{X}{(1+r)^T})$.

Similarly, a put option is equivalent to selling short an asset and investing the proceeds (present value of X) in a risk-free bond at r% for T periods. It will pay X at expiration. A put can never be worth less than zero as the holder cannot be forced to exercise it. The lowest price for a put option is given by:

$$p_0 >= \max(0, \frac{X}{(1+r)^T} - S_0)$$

10. Put-Call Parity, Put-Call-Forward Parity

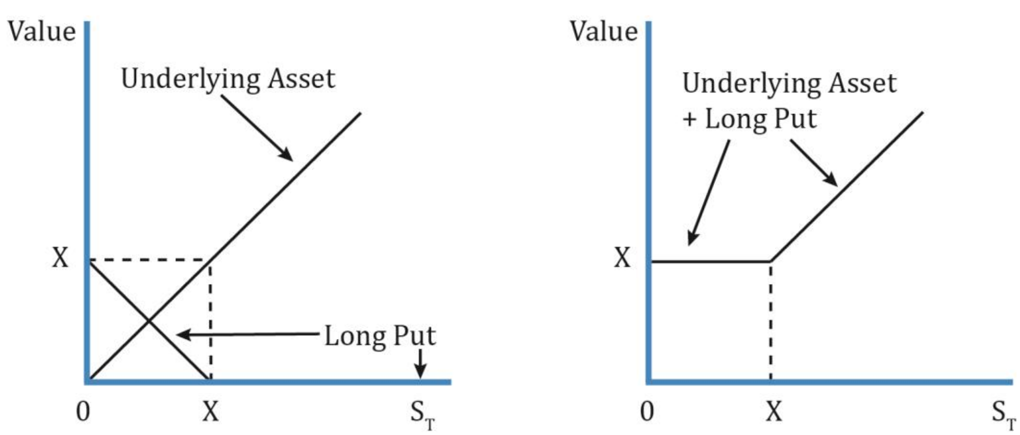
The four instruments in a put-call parity are as follows:

- Asset: An asset such as a stock with no carrying costs and no interim cash flows. At time 0, the price of the stock is S_0.
- Zero-coupon bond: A zero-coupon bond with a face value of X that matures at time T. At time 0, the value of this bond is $\frac{X}{(1+r)^T}$.
- Call option: A call option on the stock with a strike price of X that expires at time T. S_T is the underlying price at expiration T.
- Put option: A put option on the stock with a strike price of X that expires at time T.

Note: the strike price, X, of the options is the same as the par value of the bond.

According to put-call parity, fiduciary call = protective put.

A protective put is like buying insurance for an asset you own, specifically a stock, to protect against a downside. A protective put is to buy the underlying and buy a put option on it. For instance, you own 1,000 shares of Apple, the market is highly volatile and you are worried about a huge decline in the near term. To limit the downside risk, you can buy a put option on the stock by paying a premium. This is called a protective put.



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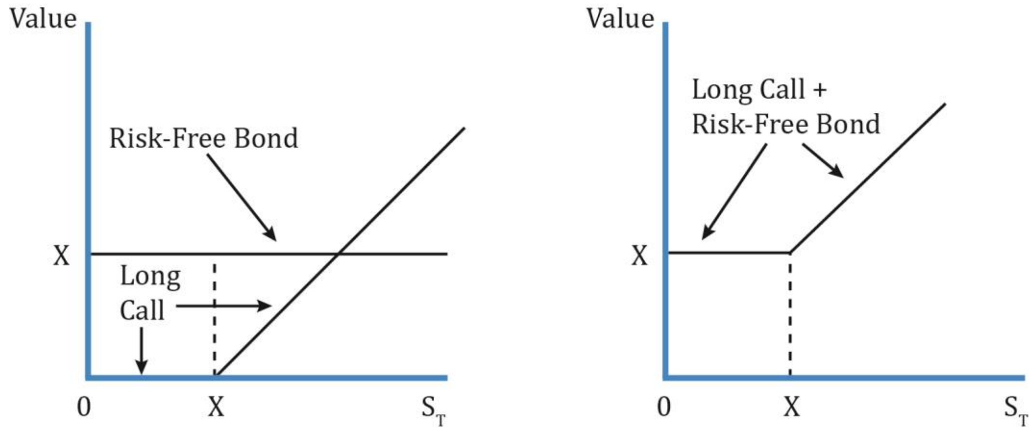
<u>Interpretation</u>:

- As you can see, the downside risk is limited to the premium paid for the put. If the underlying price falls below X, it can be still be sold at X by exercising the put option.
- If the underlying rises above X, then it can be sold at the market price and the put option expires worthless.

A fiduciary call is a long call plus a risk-free bond. The diagram below shows the payoff for a fiduciary call:



The table below shows how the fiduciary call is equal to the protective put under two possible scenarios: when the stock is above the exercise price (call is in the money) and the stock is below the exercise price (put is in the money).

Outcome at time T when: →	**Put expires in the money ($S_T < X$)**	**Call expires in the money ($S_T >= X$)**
Protective put		
Asset	S_T	0
Long puts	$X - S_T$	0
Total	X	S_T
Fiduciary call		
Long call	0	$S_T - X$
Risk-free bond	X	X
Total	X	S_T

	Fiduciary Call	**Protective Put**
Constituents	Long call + risk-free bond	Long put + stock
Equation	$c_0 + \frac{X}{(1+r)^T}$	$p_0 + S_0$
Payoff at T if call expires in the money ($S_T >= X$)	S_T	S_T
Payoff at T if put expires in the money	X	X

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($S_T < X$)		
Easy way to remember (alphabetical order BCPS)	B + C	P + S
Put-call parity: $c_0 + \frac{X}{(1+r)^T} = p_0 + S_0$		

If, at time 0, the fiduciary call is not priced the same as the protective put, then there is an arbitrage opportunity. The put-call parity relationship can be rearranged in the following ways:

Synthetic call: $c_0 = p_0 + S_0 - \frac{X}{(1+r)^T}$

Synthetic bond: $\frac{X}{(1+r)^T} = p_0 + S_0 - c_0$

Synthetic stock: $S_0 = c_0 + \frac{X}{(1+r)^T} - p_0$

Synthetic put: $p_0 = c_0 + \frac{X}{(1+r)^T} - S_0$

10.1 Put-Call Forward Parity

In this section, we see how to create protective put with a forward contract. Recall we saw in an earlier section that:

Asset – Forward = Risk-free Bond (long asset + short forward = risk-free bond)

Rearranging, we get: Asset = Forward + risk-free bond

Now, if we substitute the asset part in a protective put with the above equation, then we get:

Protective put: Asset + put = Forward + risk-free bond + put

The exhibit below shows that the payoff is the same for a protective put with an asset as well as a forward contract when the put expires in and out of the money.

	Outcome at T	
	Put Expires In the Money ($S_T < X$)	**Put Expires Out of the Money ($S_T \geq X$)**
Protective put with asset		
Asset	S_T	S_T
Long put	$X - S_T$	0
Total	X	S_T
Protective put with forward contract		
Risk-free bond	$F_0(T)$	$F_0(T)$
Forward contract	$S_T - F_0(T)$	$S_T - F_0(T)$
Long put	$X - S_T$	0
Total	X	S_T

Source: CFA Program Curriculum, Basics of Derivative Pricing and Valuation

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Interpretation:

- The risk-free bond has a par value equal to the forward price of F_0 (T).
- Value of the forward contract at expiration = S_T – F_0 (T).
- Notice that the value of the forward contract at expiration is the same irrespective of whether the put expires in or out of the money.

According to put-call parity, since a fiduciary call = protective put, a fiduciary call must be equal to a protective put with a forward contract.

Bond + Call = Long Put + Asset

Replacing S in the above equation with a forward contract, we get:

Bond + Call = Long Put + Long forward + Risk-free bond

The exhibit below shows that the payoffs for fiduciary call is equal to protective put when call and put expire in the money.

	Outcome at T	
	Put Expires In the Money ($S_T < X$)	**Put Expires Out of the Money ($S_T \geq X$)**
Protective put with forward contract		
Risk-free bond	$F_o\ (T)$	$F_o\ (T)$
Forward contract	$S_T - F_o\ (T)$	$S_T - F_o\ (T)$
Long put	X - S_T	0
Total	X	S_T
Fiduciary call		
Call	0	$S_T - X$
Risk-free bond	X	X
Total	X	S_T

11. Binomial Valuation of Options

This section deals with the payoff of an option. The payoff of an option depends on the value of the underlying at expiration, which cannot be known with certainty ahead of time. What matters from an option's perspective is whether it expires in or out of the money, i.e., whether the stock price was above or below the strike price at expiration.

The binomial model is a simple model for valuing options based on only two possible outcomes for a stock's movement: going up and going down. The exhibit below shows the binomial option-pricing model.

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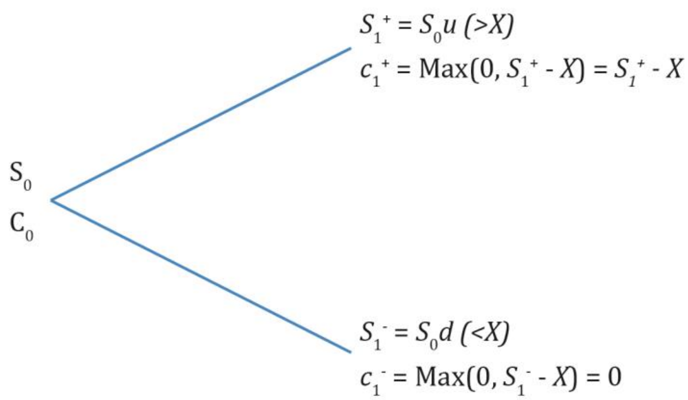

Source: CFA Program Curriculum, Basics of Derivative Pricing and Valuation

The notations used in the model are explained below:

Notation	Explanation
S_0	Value of the stock at time t = 0
C_0	Price of the call option at time t = 0
S_1^+	Price of the stock at time t = 1; + indicates the price of the stock went up
S_1^-	Price of the stock at time t = 1; - indicates the price of the stock went down
c_1^+	Value of the call option at t = 1
c_1^-	Value of the call option at t = 1
u	Up factor = S_1^+/S_0
D	Down factor = S_1^-/S_0

Π and 1- π are called the synthetic probabilities; they represent the weighted average of producing the next two possible call values, a type of expected future value. By discounting the future expected call values at the risk-free rate, we can get the current call value.

Risk-neutral probability

$\Pi = \frac{1+r-d}{u-d}$

where:
r = risk-free rate
u = up factor; d = down factor

Let us do a numerical example to calculate the call price using a 1 – period binomial model now. Data already given to us is in normal font, while the calculated ones are in **bold.**

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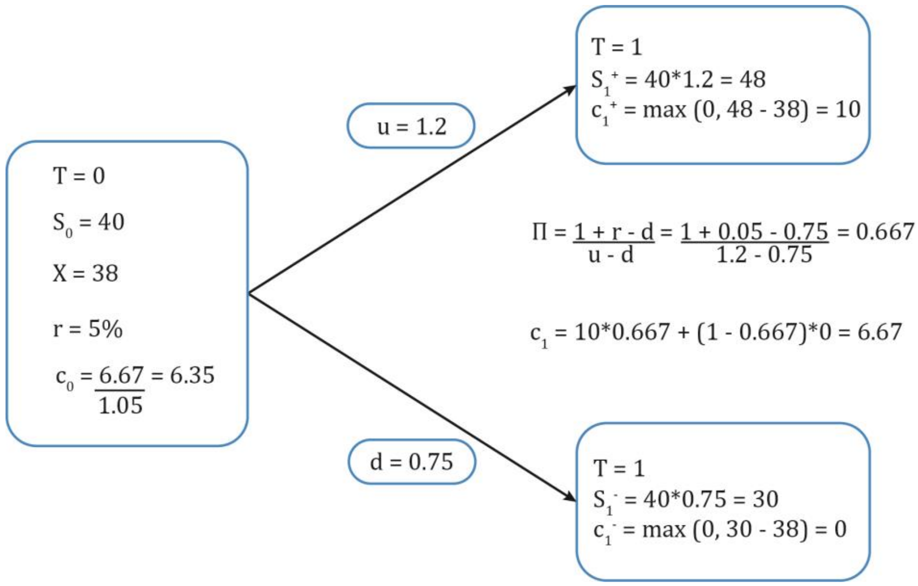

The value of the call option at time 0 using the 1-period binomial model is 6.35.

Call price using the binomial model

$$c_0 = \frac{\pi c_1^+ + (1 - \pi) c_1^-}{1 + r}$$

where:
π = risk-neutral probability
r = risk-free rate
c_0 = price of call option at time 0

Some important points about the binomial pricing model:

1. Volatility of the underlying: The volatility of the underlying is important in determining the value of the option. It is shown in the difference between S_1^+ and S_1^-. The higher the volatility, the greater the difference between the stock prices S_1^+ and S_1^-, and c_1^+ and c_1^-.
2. The probabilities of the up and down moves do not appear in the formula.
3. Instead, what is used is the synthetic or pseudo probability, π, to determine the expected future value. $\Pi = \frac{1+r-d}{u-d}$
4. The formula for calculating c_0 is the expected future value discounted at the risk-free rate.

Binomial Value of Put Options

What is the value of the put option using the same principle discussed in the previous section?

Assume the following data is given:

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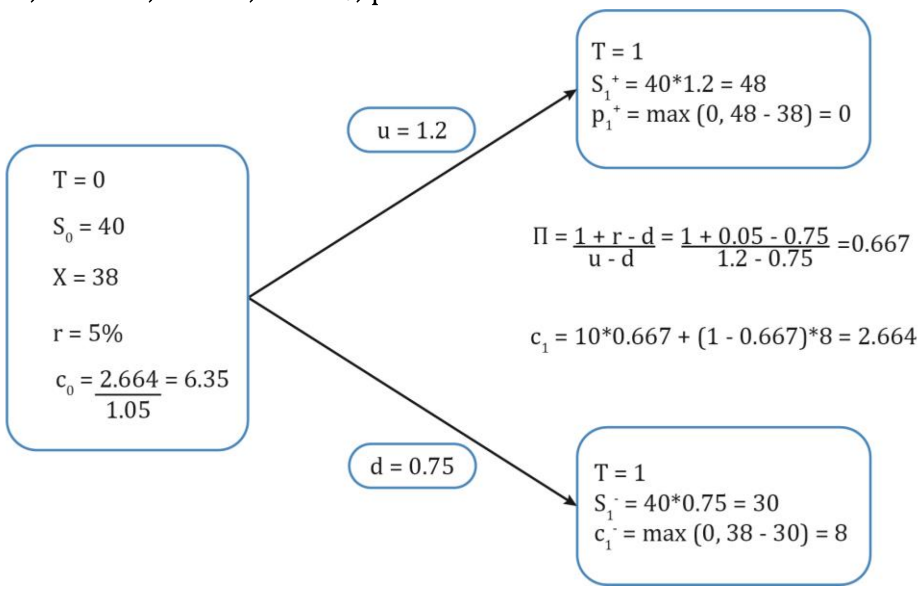
$S_0 = 40$; $u = 1.2$; $d= 0.75$; $X = 38$; $r = 5\%$; $p_0 = ?$



The value of the put option at time 0 using the 1-period binomial model is 2.531.

12. American Option Pricing

So far we looked at the payoff and valuation of European options. Now we will discuss American options.

- The primary difference between the two is that American options can be exercised any time before expiration or exercise date. So, they are more valuable than European options since the holder can exercise the option before maturity date.
- Early exercise is not mandatory (required) so the right to exercise early cannot have a negative value.
- American options cannot sell for less than European options.

Some important points on American/European options:

- If there are no interim cash flows on the underlying asset before the call expires like dividends on stocks, then it is not prudent to exercise the option.
- American call prices can differ from European call prices only if there are cash flows on the underlying, such as dividends or interest; these cash flows are the only reason for early exercise of a call.
- American put prices can differ from European put prices, because the right to exercise early always has value for a put, which is because of a lower limit on the value of the underlying.
- If the underlying stock pays a dividend, then it is prudent for a put option holder not to exercise the option until its expiration. This ensures he receives the dividend on the stock.

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Summary

LO.a: Explain how the concepts of arbitrage, replication, and risk neutrality, are used in pricing derivatives.

Arbitrage is a type of transaction undertaken when two assets or portfolios produce identical results but sell for different prices.

A derivative must be priced such that no arbitrage opportunities exist, and there can only be one price for the derivative that earns the risk-free return.

Asset + Derivative = Risk-free asset

Replication is the creation of an asset or portfolio from another asset, portfolio, and/or derivative.

Risk aversion of the investor does not impact derivative pricing. Risk-free rate is used for pricing derivatives.

The overall process of pricing derivatives by arbitrage and risk neutrality is called arbitrage-free pricing.

LO.b: Explain the difference between value and price of forward and futures contracts.

Price	**Value**
The price of a stock is its market price.	The value of a stock is its intrinsic value, often determined by a fundamental analysis.
The price of an option is similar to what we saw above for stocks.	The value of a call option is based on the underlying and other variables that impact the option.
The price of a forward contract is the fixed price or rate that is embedded in the contract; it represents the price or rate at which the underlying will be purchased at a later date.	Value changes over the life of the contract. It is zero at initiation. Over time, as spot price or rate changes, the value to each party changes.

LO. c. Calculate a forward price of an asset with zero, positive, or negative net cost of carry.

The forward price of an asset with benefits and/or costs is the spot price compounded at the risk-free rate over the life of the contract minus the future value of those benefits and costs.

$$F_0\,(T) = (S_0 - \gamma + \theta) \text{ x } (1 + r)^T$$

It can be rewritten as:

$$F_0\,(T) = (S_0) \text{ x } (1 + r)^T - (\gamma - \theta) \text{ x } (1 + r)^T$$

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Value of the forward contract

$V_t\ (T) = S_t - \frac{F_0(T)}{(1+r)^{T-t}}$ (without benefits and costs)

$V_t\ (T) = S_t - (\gamma - \theta)\ (1 + r)^t - \frac{F_0(T)}{(1+r)^{T-t}}$ (with benefits and costs)

LO.d: Explain how the value and price of a forward contract are determined at expiration, during the life of the contract, and at initiation.

- At initiation the value of forward contract is zero.
- Value at expiration: S_T - F
- Value during the life of the contract**:** $V_t = S_t - \frac{F}{(1+r)^{T-t}}$

LO.e: Describe monetary and nonmonetary benefits and costs associated with holding the underlying asset, and explain how they affect the value and price of a forward contract.

The forward price of an asset with benefits and/or costs is the spot price compounded at the risk-free rate over the life of the contract minus the future value of benefits and plus the future value of costs.

The value of a forward contract is the spot price of the underlying asset minus the present value of the forward price.

LO.f Define a forward rate agreement and describe its uses.

FRAs have an interest rate as the underlying. They allow us to lock in a rate today for a loan in the future. They allow us to make a known interest payment at a later date and receive, in return, an unknown interest payment.

LO.g: Explain why forward and futures prices differ.

- Futures contracts have standard terms, are traded on a futures exchange, and are more heavily regulated than forward contracts.
- Futures contracts are marked to market on a daily basis. Whereas in a forward contract, the gain or loss is realized at the end of the contract period.
- If interest rates are constant, or have zero correlation with futures prices, then forwards and futures prices will be the same.
- If futures prices are negatively correlated with interest rates, then it is more desirable to buy forwards than futures for a long position.

LO.h: Explain how swap contracts are similar to but different from a series of forward contracts.

A normal forward has a zero value at the start because of no-arbitrage pricing. In a swap, since the fixed price is priced different than the market price, it has a non-zero value at the start and is called an off-market forward. **A** swap is a series of off-market forward contracts where:

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- Each forward contract is created at a price and maturity equal to the fixed price of the swap with the same maturity and payment dates respectively.
- This means that the series of FRAs built into a swap are all off-market FRAs: some with positive values and some with negative values.
- The combined value of the off-market FRAs is zero.

LO.i: Explain the difference between value and price of swaps.

- The value of the swap: As with forwards and futures, the value of a swap at initiation is equal to zero.
- Swap price: The fixed rate of the swap is referred to as its price.
- The value of the swap during the life of a swap is based on the present value of the expected future cash flows. The cash flows, floating payments in particular, are based on the market price of the underlying.

LO.j: Explain the exercise value, time value, and moneyness of an option.

Moneyness

- Refers to whether an option is in the money or out of the money.
- If immediate exercise of the option would result in a positive payoff, then the option is in the money.
- If immediate exercise would result in loss, then the option is out of the money.
- If immediate exercise would result in neither a gain nor a loss, then the option is at the money.

	Call Option	**Put Option**
In the money	S > X	S < X
At the money	S = X	S = X
Out of the money	S < X	S > X

Exercise value

- Exercise value of an option is the maximum of zero and the amount that the option is in the money.

Time value

- Prior to expiration an option also has time value in addition to exercise value.
- The time value of an option is the amount by which the option premium exceeds the exercise value.
- When an option reaches expiration, the time value is zero.

LO.k: Identify the factors that determine the value of an option, and explain how each factor affects the value of an option.

Increase in	**Value of call option will**	**Value of put option will**
Value of the underlying	Increase	Decrease
Exercise price	Decrease	Increase

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Risk-free rate	Increase	Decrease
Time to expiration	Increase	Increase (exception: a few European puts)
Volatility of the underlying	Increase	Increase
Costs incurred while holding the underlying	Increase	Decrease
Benefits received while holding the underlying	Decrease	Increase

LO.l: Explain put–call parity for European options.

According to put-call parity, fiduciary call = protective put.

Long call + risk-free bond = Long put + stock

$c_0 + \frac{X}{(1+r)^T} = p_0 + S_0$

LO.m: Explain put–call–forward parity for European options.

According to put-call parity, since a fiduciary call = protective put, a fiduciary call must be equal to a protective put with a forward contract.

Bond + Call = Long Put + Asset

Replacing asset in the above equation with a forward contract + risk free bond, we get:

Bond + Call = Long Put + Long forward + Risk-free bond

LO.n: Explain how the value of an option is determined using a one-period binomial model.

The binomial model is a simple model for valuing options based on only two possible outcomes for a stock's movement: going up and going down.

Π and 1- π are called the synthetic probabilities; they represent the weighted average of producing the next two possible call values. By discounting the future expected call values at the risk-free rate, we can get the current call value.

$$\Pi = \frac{1 + r - d}{u - d}$$

$$c_0 = \frac{\pi c_1^+ + (1 - \pi)c_1^-}{1 + r}$$

LO.o: Explain under which circumstances the values of European and American options differ.

- If the underlying asset has a cash flow such as dividend or interest, then American call prices will be more than European call prices, because the option can be exercised early to collect this cash flow.

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- American put prices will always be higher than European put prices, because the right to exercise early always has value for a put.

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Practice Questions

1. An arbitrage transaction generates a net inflow of funds:
   A. throughout the holding period.
   B. at the end of the holding period.
   C. at the start of the holding period.

2. An arbitrage opportunity is *least likely* to be exploited when:
   A. the price differential between assets is large.
   B. when large volume transactions can be executed.
   C. when small volume transactions can be executed.

3. Risk-seeking investors:
   A. give away a risk premium because they enjoy taking risk.
   B. expect a risk premium to compensate for the risk.
   C. require no premium to compensate for assuming risk.

4. The price of a forward contract:
   A. is the amount paid at initiation.
   B. is the amount paid at expiration.
   C. fluctuates over the term of the contract

5. At the initiation of a forward contract on an asset that neither receives benefits nor incurs carrying costs during the term of the contract, the forward price is equal to the:
   A. spot price.
   B. future value of the spot price.
   C. present value of the spot price.

6. Which of the following factors *most likely* explains why the spot price of a commodity in short supply can be greater than its forward price?
   A. Opportunity cost
   B. Lack of dividends
   C. Convenience yield

7. To the holder of a long position, it is more desirable to own a forward contract than a futures contract when interest rates and futures prices are:
   A. negatively correlated.
   B. uncorrelated.
   C. positively correlated.

8. The value of a swap is equal to the present value of the:

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A. fixed payments from the swap.
B. net cash flow payments from the swap.
C. underlying at the end of the contract

9. Which of the following statements is *least likely* correct?
   A. For a correctly priced swap the price is the same as value.
   B. The price of a swap established at the start of the swap and remains fixed.
   C. The value of swap changes during the life of the swap.

10. At expiration, a European put option will be valuable if the exercise price is:
    A. less than the underlying price.
    B. equal to the underlying price.
    C. greater than the underlying price

11. For a European call option with two months until expiration, if the spot price is below the exercise price, the call option will *most likely* have:
    A. zero time value.
    B. positive time value.
    C. positive exercise value

12. If the risk-free rate increases, the value of an in-the-money European put option will *most likely*:
    A. decrease.
    B. remain the same.
    C. increase.

13. The value of a European call option is inversely related to the:
    A. exercise price.
    B. time to expiration.
    C. volatility of the underlying.

14. Based on put-call parity, a trader who combines a long asset, a long put, and a short call will create a synthetic:
    A. long bond.
    B. fiduciary call.
    C. protective put.

15. Combining a protective put with a forward contract generates equivalent outcomes at expiration to those of a:
    A. fiduciary call.
    B. long call combined with a short asset.

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C. forward contract combined with a risk-free bond.

16. Which of the following is *least likely* to be required by the binomial option-pricing model?
    A. Spot price.
    B. Two possible prices one period later.
    C. Actual probabilities of the up and down moves.

17. Which of the following circumstances will *most likely* affect the value of an American call option relative to a European call option?
    A. Dividends are declared.
    B. Expiration date occurs.
    C. The risk-free rate changes.

18. Which of the following is *most likely* correct about American options?
    A. European options are more valuable than American options.
    B. Early exercise is mandatory in American options so the right to exercise early cannot have a negative value.
    C. American options cannot sell for less than European options

19. American put prices can differ from European put prices because:
    A. the right to exercise early always has value for a put.
    B. there is an upper limit on the value of the underlying.
    C. the right to exercise early does not necessarily has value for a put.

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Solutions

1. C is correct. An arbitrage transaction is undertaken when two assets or portfolios produce identical results but sell for different prices. A trader buys the lower priced asset and sells the higher priced asset. This generates a net inflow of funds at the start of the holding period. Because the two assets or portfolios produce identical results and the payoffs offset each other, there is no money gained or lost at the end of the holding period.

2. C is correct. An arbitrage opportunity is more likely to be exploited when the price differential between assets is large or when large volume transactions can be executed.

3. A is correct. Risk- seeking investors give away a risk premium because they enjoy taking risk. Risk- averse investors expect a risk premium to compensate for the risk. Risk-neutral investors neither give nor receive a risk premium because they have no feelings about risk.

4. B is correct. The forward price is agreed upon at the start of the contract and is the fixed price at which the underlying will be purchased (or sold) at expiration. This price stays fixed over the term of the contract.

5. B is correct. At initiation, the forward price is the future value of the spot price.

6. C is correct. Since the commodity is in short supply, there is a convenience in holding the asset. This convenience yield may result in lower forward prices.

7. A is correct. If futures prices and interest rates are negatively correlated, forwards are more desirable. If futures prices and interest rates are uncorrelated, forward and futures prices will be the same. If futures prices are positively correlated with interest rates, futures contracts are more desirable.

8. B is correct. The value of a swap is the present value of all the net cash flow payments from the swap.

9. A is correct. Both B and C are correct statements. The price of a swap is established at the start of the swap and remains fixed. The value of a swap is typically zero at the start of the swap and changes during the life of the swap.

10. C is correct. A European put option will be valuable at expiration if the exercise price is greater than the underlying price.

11. B is correct. A European call option with two months until expiration will typically have positive time value. Since the spot price is below the exercise price it will have zero

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exercise value.

12. A is correct. An in-the-money European put option decreases in value with an increase in the risk-free rate. This is because a higher risk free rate reduces the present value of any proceeds received on exercise.

13. A is correct. The value of a European call option is inversely related to the exercise price. Both the time to expiration and the volatility of the underlying are positively related to the value of a European call option.

14. A is correct. A long bond can be synthetically created by combining a long asset, a long put, and a short call.

15. A is correct. Put–call forward parity demonstrates that the outcome of a protective put with a forward contract equals the outcome of a fiduciary call.

16. C is correct. In a binomial option-pricing model, we use pseudo or "risk-neutral" probabilities. Therefore, the actual probabilities of the up and down moves in the underlying are not required. Both the spot price of the underlying and two possible prices one period later are required by the binomial option-pricing model.

17. A is correct. When a dividend is declared, an American call option will have a higher value than a European call option because an American call option holder can exercise early to capture the value of the dividend.

18. C is correct. The other two statements are incorrect. The primary difference between the two is that American options can be exercised any time before expiration or exercise date. So, they are more valuable than European options since the holder can exercise the option before maturity date. Early exercise is not mandatory (required) so the right to exercise early cannot have a negative value.

19. A is correct. American put prices can differ from European put prices, because the right to exercise early always has value for a put, which is because of a lower limit on the value of the underlying.

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