Model-Based LQR Control of Two-Wheeled Balancing Robot

Aung Kyaw Nyein
West Yangon Technological University, Yangon, Myanmar

Model-Based LQR Control of Two-Wheeled Balancing Robot

[1] Khin Wai Hlaing Phyo, [2] Aung Kyaw Nyein

[1,2] Department of Mechatronic Engineering (West Yangon Technological University),

Yangon, Myanmar

khinwaihlaingphyo001@gmail.com, aungkyawnyein.mtu@gmail.com

Abstract— Two-wheeled balancing robot has working principle like inverted pendulum that the controller in the mobile robot plays a critical role in self-balancing and stabilizing. This paper inscribes the design, construction and control of a two-wheeled balancing robot. There are two DC geared motor, MPU 6050 and Arduino Uno in this system. The focus of this paper is to control the two-wheeled balancing robot by using Linear-Quadratic Regulator (LQR) controller. In order to measure the angle of robot, MPU 6050 is used and its measurement data noise is filtered using complementary filter. With the help of encoder feedback, the control of two-wheeled balancing robot uses LQR controller included with adjusting motor speed and spinning direction. Kinematic and electrical parameters are determined experimentally, LQR control modelling based upon the linearized equations of motion. The experimental results show that the two-wheeled balancing robot is able to maintain robot position at its balanced condition.

Keywords: Two-Wheeled Balancing Robot (TWBR), LQR controller, MPU 6050, Complementary Filter, DC gear motor with encoder.

I. INTRODUCTION

Robotic technology to help human's work is growing fast nowadays. There are many researches that develop many kinds of robot with different size, shape, and movement. Starting from stationary robot that does not move its position, to mobile robot that moves freely to any place. One of the applications of using these robots can be a service robot platform like Segway. Different forms and uses, mobile robots have been designed and are in the market worldwide. As the result, balancing robots that serve various purposes have been developed. TWBR is an open loop unstable system with highly nonlinear dynamics. TWBR problem is common in the field of control engineering. One example for stationary robot is arm robot which is commonly used as industrial robot that resembles an inverted human arm mounted on a base [1]. The optimized SFC can only work in the certain operating range and cannot react to system uncertainty. When the operating point is changed, the control gains have to be re-tuned in order to stabilize the robot, which is not very practical in the operation. Recently, the soft computing techniques such as fuzzy logic and neural networks have been used to control TWIP robot [2]. Two fuzzy controllers based on Mamdani and Takagi-Sugeno are designed for an inverted pendulum subjected to disturbance. Moreover, artificial neural network-based real-time switching dynamic controller is designed to solve balancing problem on various loose surfaces such as sand, pebble and soil [3]. The control techniques used are PID, Linear-Quadratic Regulator (LQR), and Sliding Mode Control .The comparison results of the three controllers are for angular wheel position tracking, for external disturbances, and parameter uncertainties in the model[4]. They proposed two control methods, an innovative double PID control method and a modern LQR (liner quadratic regulator) control method. Dynamic performance and steady state performance are investigated and compared of the two controllers [5]. Reference [6] proposed that the presence of friction produces limits cycles. Considering one of the practical use of inverted pendulum, the electric two wheeled self-balancing vehicle, lots of efforts have been spent to reduce the existed vibration by improved control algorithm [7] . The balancing robot principle also has been implemented for transportation application. One of them is Segway PT, a commercial personal transporter product founded by Dean Kamen. It uses the same principle as balancing robot but the movement depends on user's body. To move forward/backward, user needs to lean towards the wanted direction and turning is controlled by the steer [8-9]. But many researchers related to balancing robot topics are still focused on the balance control itself. In this paper, the robot is composed of DC gear motors driving two wheels, a structure, battery, MPU 6050 and microcontroller. It is a highly nonlinear and unstable system. LQR controller has been designed for balancing and stabilizing the robot.

II. SYSTEM DESCRIPTION

A robot must employ a suitable control method to obtain a good stability. The Two-wheeled balancing robot is built to serve as balancing robot for Linear Quadratic Regulator (LQR) controller in this paper. Two-wheeled balancing robot comprises of three main parts including sensors, microcontroller and motor as shown in Fig. 1.

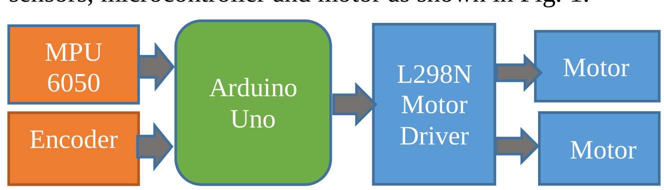

Fig. 1 Block diagram of system components

In this block diagram, MPU-6050 sensor combines a 3-axis accelerometer and a 3-axis gyroscope with Micro Electro Mechanical System (MEMS).Accelerometer is used to detect angle of tilt or inclination along the X, Y and Z axes. Gyroscope is used to detect rotational velocity along the X, Y and Z axes. The complementary filter provides data both for short and long term from gyroscope and accelerometer respectively.The motor drive system with encoder consists of motor controllers, and two DC motors. The robot uses dual H-bridge motor controller to control two DC motors. For controlling motor in both directions H bridge circuit is used.Prototype of robot is shown in Fig. 2.

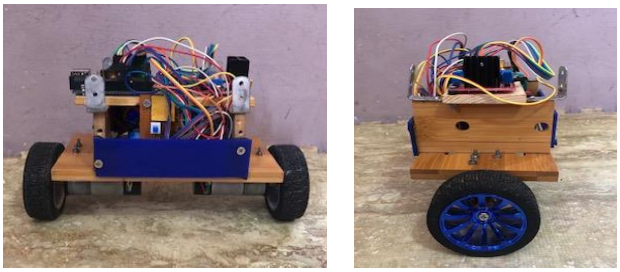

Fig. 2 Prototype of Two-wheeled balancing robot

A. *Dynamic Modelling*

The Dynamic model of two-wheeled balancing robot is shown in Fig. 3 and the parameters description has been provided in Table I.

TABLE I
LIST OF SYMBOLS AND SPECIFICATIONS

Parameters	**Description**
g =9.81	Gravity acceleration (m/s^2)
M=0.0227	Wheel mass(kg)
R=0.035	Wheel radius(m)
m=0.84	Chassis mass(kg)
l=0.032	Distance of the center of the mass from the wheel axle(m)
$J_{w=}$4.593E-05	Wheel inertia moment(kg*m^2)
Jc=0.0032	Chassis inertia moment(kg*m^2)
u0=0.1	coefficient of friction between wheel and ground
u1=0	coefficient of friction between chassis and wheel

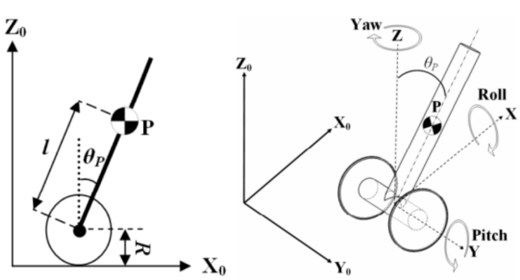

Fig. 3 Dynamic model of two-wheeled balancing robot

Equations 1 and 2 illustrate the displacement of point P along X_o and Z_o axes;

$$X_p = x + l \sin\theta_p \tag{1}$$

$$Z_p = l \cos\theta_p \tag{2}$$

Lagrangian approach was used to generate the ordinary differential equations (ODE) of system. Equation 3 shows the general form of Lagrangian equation.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_r}\right) - \frac{\partial L}{\partial q_r} = Q_r \tag{3}$$

Equation 4 illustrates the Lagrangian equation of two-wheeled balancing robot where, E is kinetic energy, U is potential energy, F is dissipation energy and τ_r is required torque for left and right wheels.

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_r}\right) - \frac{\partial E}{\partial q_r} + \frac{\partial F}{\partial \dot{q}r} + \frac{\partial U}{\partial q_r} = \tau_r \tag{4}$$

Total kinetic energy of chassis (E_{KC}) was observed by adding the translational and rotational kinetic energy of chassis.

$$E_{KC} = E_{KC}E_{TKC} + E_{RKC}$$

$$E_{KC} = \frac{1}{2}m\left(v^2 + l^2{\omega_p}^2\right) + mvl\omega_p \cos\theta_p + \frac{1}{2}J_c{\omega_p}^2 \tag{5}$$

Total kinetic energy of wheels (E_{KW}) was observed by adding the translational and rotational kinetic energy of wheels.

$$E_{KW} = E_{TKW} + E_{RKW}$$

$$E_{KW} = Mv^2 + J_w\frac{v^2}{R^2} \tag{6}$$

Total kinetic energy (E) of two-wheeled balancing robot was observed by adding equations 5 and 6.

$$E = E_{KC} + E_{KW}$$

$$E = \frac{1}{2}m(v^2 + l^2{\omega_P}^2) + mvl\cos\theta_P\,\omega_P + \frac{1}{2}J_C{\omega_P}^2 + Mv^2 + J_W\frac{v^2}{R^2}$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}ml^2{\omega_P}^2 + mvl\cos\theta_P\,\omega_P + \frac{1}{2}J_C{\omega_P}^2 + Mv^2 + J_W\frac{v^2}{R^2} \tag{7}$$

There are no changes in potential energy of wheels while robot moves. Therefore, total potential energy of system (U) was defined by equation 8.

$$U = mgl\cos\theta_P \tag{8}$$

Total dissipation energy of system (F) was defined by equation 9.

$$F = \mu_0 v^2 + \mu_1{\omega_P}^2 \tag{9}$$

Displacement of balancing robot was selected as the first general variable (q_1) in Lagrangian equation of system.

q_1=x. The first differential equation of system equations 10 is

$$\left(m + 2M + \frac{2Jw}{R^2}\right)\ddot{x} + ml(\cos\theta_P)\ddot{\theta}_P + 2\mu_0 v = 0 \qquad (10)$$

Pitch angle of balancing robot was selected as the second general variable (q_2) in Lagrangian equation of system.

$q_2 = \theta_P$.The second differential equation of system equations 11 is;

$$ml(\cos\theta_P)\ddot{x} + (ml^2 + J_C)\ddot{\theta}_P + (mv\omega_P - mg)l(\sin\theta_P) + 2\mu_1\omega_P = \tau_L + \tau_R \qquad (11)$$

Equations 10 and 11 are two non-linear equations of two-wheeled balancing robot. There is only one equilibrium point for balancing robot and it is when robot is balanced or up-right. Therefore, it was assumed that θ_P = 0.

$\sin\theta_P \approx \theta_P$

$\cos\theta_P$=1

Linear equations of two-wheeled balancing robot were observed by substituting equations 10 and 11 into non-linear equations. Equations 12 and 13 show the linear equations of system.

$$\left(m + 2M + \frac{2Jw}{R^2}\right)\ddot{x} + ml\ddot{\theta}_P + 2\mu_0 v = 0 \qquad (12)$$

$$ml(\cos\theta_P)\ddot{x} + (ml^2 + J_C)\ddot{\theta}_P + (mv\omega_P - mg)l\theta_P + 2\mu_1\omega_P = \tau_L + \tau_R \qquad (13)$$

***B**. State-Space Modelling*

Equations 14 and 15 are two equations of general state-space model for a dynamic system. Fig.4 illustrates the related block diagram of the state-space equations.

$$\dot{X} = AX + Bu \qquad (14)$$

$$Y = CX + Du \qquad (15)$$

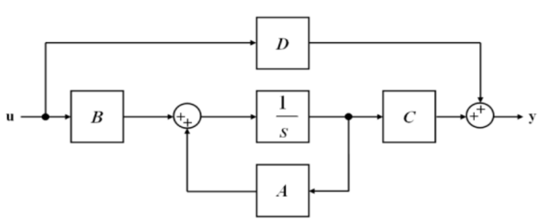

Fig. 4 Block Diagram of state space equation

Four state variables were chosen for dynamic system of two-wheeled balancing robot. Equation 16 shows the state vector of dynamic system (X). Four elements of state vectors are:

Chassis displacement, x
Chassis pitch angle, θ_P
Chassis velocity, v
Chassis angular velocity, ω_P

State variables of velocity and angular velocity are derivative of displacement and pitch angle respectively in equation 16.

$$X = [x \;\; \theta_P \;\; v \;\; \omega_P]^T \qquad (16)$$

Equation 17 shows the expanded version of state equation.

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\\ \dot{X}_4\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12} & A_{13} & A_{14}\\ A_{21} & A_{22} & A_{23} & A_{24}\\ A_{31} & A_{32} & A_{33} & A_{34}\\ A_{41} & A_{42} & A_{43} & A_{44}\end{bmatrix} \times \begin{bmatrix}X_1\\ X_2\\ X_3\\ X_4\end{bmatrix} + \begin{bmatrix}B_1\\ B_2\\ B_3\\ B_4\end{bmatrix} \times u \qquad (17)$$

Therefore, A, B, C and D matrices are shown below;

$$A = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -\frac{(ml)^2 g}{den} & -\frac{2\mu_0(ml^2+J_C)}{den} & \frac{2ml\mu_1}{den}\\ 0 & \frac{\left(m+2M+2\frac{Jw}{R^2}\right)mgl}{den} & \frac{2\mu_0 ml}{den} & -\frac{2\mu_1\left(m+2M+2\frac{Jw}{R^2}\right)}{den}\end{bmatrix} \qquad (18)$$

$$B = \begin{bmatrix}0\\ 0\\ -\frac{ml}{den}\\ \frac{m+2M+2\frac{Jw}{R^2}}{den}\end{bmatrix} \qquad (19)$$

$$C = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix} \qquad (20)$$

$$D = \begin{bmatrix}0\\ 0\end{bmatrix} \qquad (21)$$

Where ;

$$den = \left(m + 2M + 2\frac{Jw}{R^2}\right)(ml^2 + J_C) - (ml)^2 \qquad (22)$$

The state equation express by substituting parameters;

$$\dot{X} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -2.2150 & -2.5376 & 0\\ 0 & 79.1397 & 16.8 & 0\end{bmatrix} \times \begin{bmatrix}x\\ \theta_P\\ v\\ \omega_P\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -8.46\\ 302.31\end{bmatrix} u$$

The output equation express;

$$Y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix} \times \begin{bmatrix}x\\ \theta_P\\ v\\ \omega_P\end{bmatrix}$$

III. PROPOSE METHOD

In this paper, Linear Quadratic Regulator is used to stabilize a Two-wheeled balancing robot. LQR controller is useful to handle multivariable systems that is also called a state-feedback control system. The optimal control will be written as a state feedback, given by the equation 23.

$$U(k) = -K\,x(k) \qquad (23)$$

To determine the gain matrix K, the performance index, cost function J should be minimized by equation 24.

$$J = \sum_{k=k_o}^{k_1-1} x^T(k+1)Qx(k+1) + u^T(k)Ru(k) \qquad (24)$$

Q and R are weight function for the states relates to controller accuracy and the inputs relates to controller effort. Q is a positive semidefinite matrix and R is a positive definite matrix. The key for optimal controller design is to select the appropriate weighting matrix Q and R. The widespread method used to choose *Q* and R by means of simulation and trial. The larger weight matrix Q will increase the gain matrix, K which can make fast response system to achieve intermediate state cost function. In this section, the LQR controller is tuned by changing the nonzero elements of the ***Q11*** and ***Q22*** elements, which, corresponding to and closely influence the two concerned states: position and angle. **R** value is set to 1.We choose the weighting matrix;

$$Q = \begin{bmatrix} 5000 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad , \quad R = 1$$

In MATLAB, we can use the command **lqr(A,B,Q,R)** to obtain the linear state feedback controller of gain matrix K. This controller had the gain, ***K*** = [0.001 3.43 201.68 5.79] for balancing robot. Fig. 5 and 6 illustrate the closed-loop impulse response of displacement and pitch angle of system respectively.

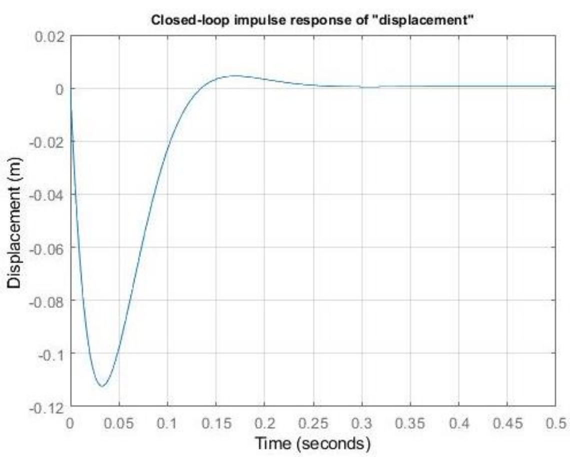

Fig. 5 Closed-loop Simulation Result for Displacement

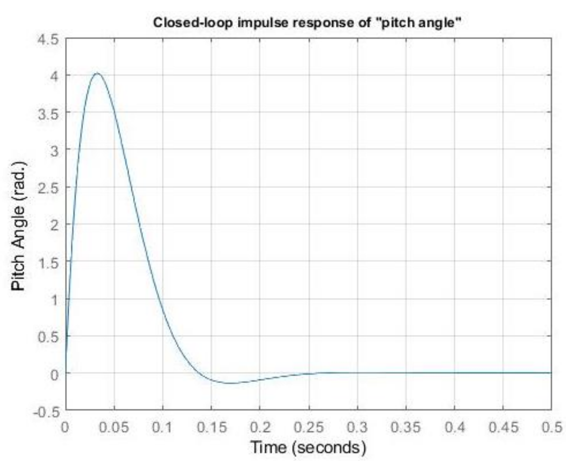

Fig. 6 Closed-loop Simulation Result for Pitch Angle

The state feedback controller with LQR control method is used as a closed loop system to get the good responses as well as to reach stable state of the system. As the simulation results, the robot moved forward and balanced itself after 0.25 seconds.

IV. EXPERIMENTAL RESULTS

This paper tends to design and develop the prototype for two wheeled balancing robot with LQR controller. The control circuit and the structure of the robot are successfully constructed. The robot has achieved results at a satisfactory level and the testing results of the experimental system which is implemented are described. The gain value of position data is not use for balancing condition. The following experimental result test1 of Fig. 7 and 8 are listed using the value obtained gain, K= [0 3.43 0 5.79].

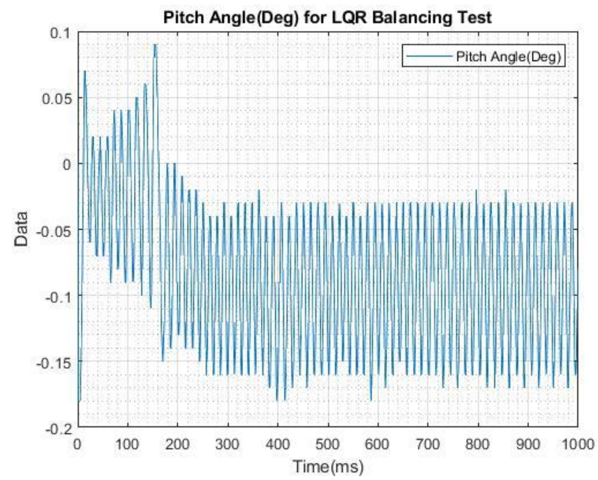

Fig. 7 Pitch Angle and PWM for LQR Balancing Test1

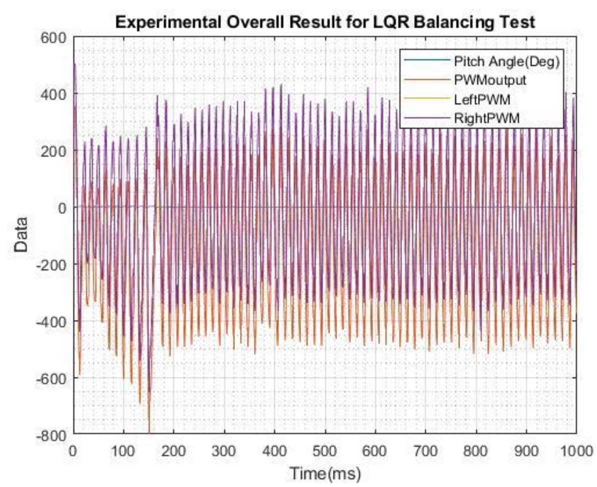

Fig. 8 Experimental Overall Result for LQR Balancing Test1

In this experiment with the value obtained gain K, vehicle start capable to be balanced but the response is oscillation. LQR control is calculated based on a linear model of the plant under control. If the linear model represents plant exactly, then the controller is optimal. However, there are plant changes or nonlinearities and then the resulting controller will degrade and the system may even become unstable. The states of a system can have some physical meaning (e.g. velocity, acceleration), but sometimes they have no physical interpretation at all. Consequently, there may be difficulty in obtaining the states to use for feedback. Therefore, we adjusted the gains by means of intuition manual tuning. The following experimental result test2 of Fig. 9 and 10 are listed using the reduce value of gain, K= [0.15 2.35 0.1 5.5].

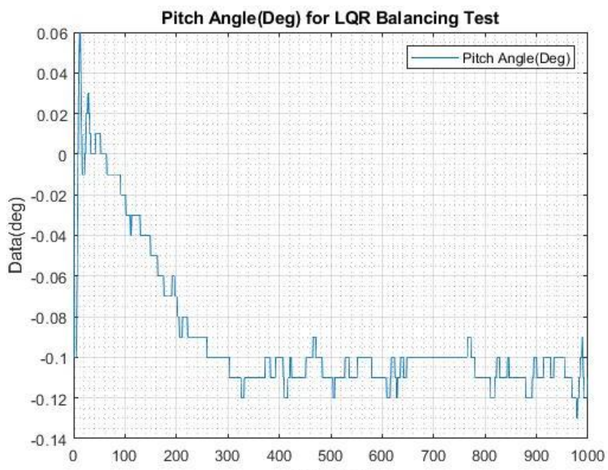

Fig. 9 Pitch Angle and PWM for LQR Balancing Test2

The value of reduce gain, K is the best response when compared with the testing. The robot is able to maintain the upright standing balance position in Fig. 9.

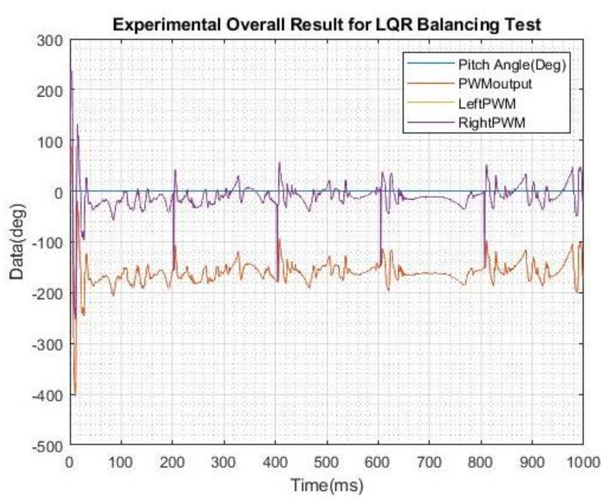

Fig.10 Experimental Overall Result for LQR Balancing Test2

Overall result contain pitch angle, pulse width modulation PWM, left and right revolution per minute RPM. Experimental overall result for LQR balancing test as shown in Fig. 10.

V. CONCLUSION

We have constructed two-wheeled balancing robot, and implemented a pitch angle estimator and stabilizing LQR controller for the balancing motion. Optimization of mechanical design will be required design improvement for balancing motion. The two-wheeled balancing robot system gets a stable and smooth condition by this control method, which provides a stable and reliable balance condition for the motion control. The robot can balance the upright position only by using two wheels. As future work, it is considered how can compare robot position at its balanced condition with another methods.

ACKNOWLEDGMENT

Specials thanks are due to my supervisor Dr. Aung Kyaw Nyein (Professor), Department of Mechatronic Engineering (West Yangon Technological University) and my co-supervisor Dr. Aemi Aung (Professor), Department of Mechatronic Engineering (West Yangon Technological University) for their kindness and guidelines for this paper. I wish to thank my teachers to help me through this paper. I would like to express my special thanks to my parents for their noble.

REFERENCES

[1] Schilling, Robert. "Fundamentals of robotics." (2013).

[2] Marzi, Hosein. "Fuzzy control of an inverted pendulum using AC induction motor actuator." In Computational Intelligence for Measurement Systems and Applications, Proceedings of 2006 IEEE International Conference on, pp. 109-114. IEEE, 2006.

[3] Unluturk, Ali, and Omer Aydogdu. "Adaptive control of two-wheeled mobile balance robot capable to adapt different surfaces using a novel artificial neural network–based real-time switching dynamic controller." International Journal of Advanced Robotic Systems 14, no. 2 (2017): 1729881417700893.

[4] Juan Villacrés, Michelle Viscaíno, Marco Herrera, Oscar Camacho "Real-Time Implementation of Different Controllers for a Two-wheeled Inverted Pendulum" INTERNATIONAL JOURNAL OF CIRCUITS, SYSTEMS AND SIGNAL PROCESSING ,Volume 10, 2016 .

[5] Wende Li, Hui Ding , Kai Cheng "An Investigation on the Design and Performance Assessment of double-PID and LQR Controllers for the Inverted Pendulum" UKACC International Conference on Control 2012 Cardiff, UK, 3-5 September 2012 .

[6] H. T. Teixeira, Victor Semedo de Mattos Siqueira, Celso Jose Munaro, "Comparison of Methods for Estimation and Compensation of Friction Applied to an Inverted Pendulum," Proceedings of the IEEE, International Conference on Control and Automation, Santiago, Chile, pp. 818-823, December 2011.

[7] D. Chol, J. Oh, "Human-friendly Motion Control of a Wheeled Inverted Pendulum by Reduced-order Disturbance Observer," Proceedings of IEEE, International Conference on Robotics and Automation, Pasadena, USA, pp. 2521-2526May 2008.

[8]"Segway PT." Wikipedia. Wikimedia Foundation. Web. 30 Apr. 2015. <https://en.wikipedia.org/wiki/Segway_PT>.

[9] "Segway - The Leader in Personal, Green Transportation." Segway - The Leader in Personal, Green Transportation.Web.11Mar.2015.<http://www.segway.com/.