Assignmenet 1

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26/3/2020

The problem

A trading company is looking for a way to maximize profit per transportation of their goods. The company has a train available with 3 wagons. When stocking the wagons they can choose among 4 types of cargo, each with its own specifications. How much of each cargo type should be loaded on which wagon in order to maximize profit?

The decision variables

x_{ij} $i \in [1;4]$, $j \in [1;3]$

For example x_{11} is the quantity of cargo 1 in wagon 1.

i = cargo type

j = #wagon

The objective function

Maximize final transportation profit:

MAX $2000x_{11}+2000x_{12}+2000x_{13}+2500x_{21}+2500x_{22}+2500x_{23}+5000x_{31}+5000x_{32}+5000x_{33}+3500x_{41}+3500x_{42}+3500x_{43}$

The constraints

weight capacity per wagon:

wagon 1: $x_{11}+x_{21}+x_{31}+x_{41}<=10$

wagon 2: $x_{12}+x_{22}+x_{32}+x_{42}<=8$

wagon 3:$x_{13}+x_{23}+x_{33}+x_{43}<=12$

cargo weight availability:

cargo 1: $x_{11}+x_{12}+x_{13}<=18$

cargo 2: $x_{21}+x_{22}+x_{23}<=10$

cargo 3: $x_{31}+x_{32}+x_{33}<=5$

cargo 4: $x_{41}+x_{42}+x_{43}<=20$

wagon volume capacity:

wagon 1: $400x_{11}$+$300x_{21}$+$200x_{31}$+$500x_{41}$<=5000

wagon 2: $400x_{12}$+$300x_{22}$+$200x_{32}$+$500x_{42}$<=4000

wagon 3: $400x_{13}$+$300x_{23}$+$200x_{33}$+$500x_{43}$<=8000

Non negativity:

x_{ij} >= 0 ∀ i,j

Building the model

```
model = make.lp(0,12) # 0 constraints, 12 variables
lp.control(model, sense="max") # original maximization problem
set.objfn(model,obj=c(2000,2000,2000,2500,2500,2500,5000,5000,5000,3500,3500,3500))
# definition of the objective function using profit coef

# Constraint of wagon 1 weight capacity
add.constraint(model,
               xt=c(1,1,1,1),
               type="<=",rhs=10,
               indices=c(1,4,7,10))


# Constraint of wagon 2 weight capacity
add.constraint(model,
               xt=c(1,1,1,1),
               type="<=",rhs=8,
               indices=c(2,5,8,11))


# Constraint of wagon 3 weight capacity
add.constraint(model,
               xt=c(1,1,1,1),
               type="<=",rhs=12,
               indices=c(3,6,9,12))

# Constraint of cargo 1 availability
add.constraint(model,
               xt=c(1,1,1),
               type="<=",rhs=18,
               indices=c(1,2,3))

# Constraint of cargo 2 availability
add.constraint(model,
               xt=c(1,1,1),
               type="<=",rhs=10,
               indices=c(4,5,6))

# Constraint of cargo 3 availability
add.constraint(model,
               xt=c(1,1,1),
               type="<=",rhs=5,
               indices=c(7,8,9))
```

```r
# Constraint of cargo 4 availability
add.constraint(model,
               xt=c(1,1,1),
               type="<=",rhs=20,
               indices=c(10,11,12))

#constraint of wagon 1 volume capacy
add.constraint(model,
               xt=c(400,300,200,500),
               type="<=",rhs=5000,
               indices=c(1,4,7,10))


# Constraint of wagon 2 volume capacity
add.constraint(model,
               xt=c(400,300,200,500),
               type="<=",rhs=4000,
               indices=c(2,5,8,11))


# Constraint of wagon 3 wvolume capacity
add.constraint(model,
               xt=c(400,300,200,500),
               type="<=",rhs=8000,
               indices=c(3,6,9,12))

# lower bound for the decision variables
set.bounds(model,lower=c(0,0,0,0,0,0,0,0,0,0,0,0))
```

```r
model
```

```
## Model name:
##   a linear program with 12 decision variables and 10 constraints
```

```r
solve(model)
```

```
## [1] 0
```

```r
get.constraints(model)
```

```
##  [1]   10    8   12    0    5    5   20 2500 4000 6000
```

```r
get.variables(model) #optimal solution
```

```
##  [1]  0  0  0  5  0  0  5  0  0  0  8 12
```

The Optimal solution result in the following wagon composition

Wagon 1: 5 TONNE of cargo 2 + 5 TONNE of cargo 3

Wagon 2: 8 TONNE of cargo 4

Wagon 3: 12 TONNE of cargo 4

```r
get.objective(model)
```

```
## [1] 107500
```

Optimal Value result is a final profit of $107500

```r
get.basis(lprec = model, nonbasic = FALSE)
```

```
##  [1] -14 -21 -22  -4 -15 -17  -5  -8  -9 -10
```

Optimal basis is composed as follow:

(c4, c11, c12, s4, c5, c7, s5, s8, s9, s10)

```r
get.dual.solution(model) #shadow price NB: 1 have not to be conidered
```

```
##  [1]    1 2500 2500 2500    0    0 2500 1000    0    0    0 -500 -500 -500
## [15]    0    0    0    0    0    0    0    0    0
```

SHADOW PRICES

The array [2500, 2500, 25000,0,0, 2500,1000,0,0,0, -500, -500, -500, 0,0,0,0,0,0,0,0,0] represents the shadow prices for the dual variables (that are 22 because 22 are the constraints in the problem). First 10 elements refers to constraints about wagon weight capacity (1:3), cargos weight availability (4:7) and wagon volume capacity (8:10). Others 12 elements refer to non negativity constraints.

Increasing RHS of the first constraint (wagon 1 weight capacity) of 1 unit, from 10 to 11, will increase the result of final profit of $2500. An increase of 2 unit in RHS of first constraint, from 10 to 12, will increase the result of final profit of $5000 (2 times $2500). A decrease of 1 unit in RHS of first constraint, from 10 to 12, will decrease the result of final profit of $2500.

Small changes in RHS of constraint about weight availability of cargo 1 and 2, will not affect the final profit. Small changes in RHS of constraints about wagon volume capacity, will not affect the final profit.

A decision of unsing 1 unit of x_{11} or x_{12} or x_{13}, instead of 0, will decrease the result of final profit of $500. A decision of unsing 2 unit of x_{11} or x_{12} or x_{13}, insead of 0, will decrease the result of final profit of $1000 (2 times 500).

Shadow prices equal to 0 means that corrensponding constraints is non binding. If the RHS of constraints which shadow prices equal to zero (# 4,5,8,9,10,14:22) increase/decrease there will not be change in the result of final profit.

Sensitivity analysis

```r
printSensitivityObj(model) # get pretty-printed sensitivity analysis for the obj func.
```

```
##    Objs         Sensitivity
## 1    C1   -inf <= C1 <= 2500
## 2    C2   -inf <= C2 <= 2500
## 3    C3   -inf <= C3 <= 2500
## 4    C4   2500 <= C4 <= 2500
```

```
## 5    C5  -inf <= C5 <= 2500
## 6    C6  -inf <= C6 <= 2500
## 7    C7   5000 <= C7 <= inf
## 8    C8  -inf <= C8 <= 5000
## 9    C9  -inf <= C9 <= 5000
## 10  C10 -inf <= C10 <= 3500
## 11  C11 3500 <= C11 <= 3500
## 12  C12  3500 <= C12 <= inf
```

OPTIMALITY RANGES

The objective coefficent C1 (profit of cargo 1 on wagon 1, equal to 2000) can increase till 2500 or decrease till -∞ witouth changing the optimal solution, assuming all other coefficient remain constant. [Same for C2, C3]

Any changes in C4 (profit of cargo 2 on wagon 1, qual to 2500) may cause a change of the optimal soluiton, since it has range equal to zero. [Same for C11]

The objective coefficent C5 (profit of cargo 2 on wagon 2, equal to 2500) can decrease till -∞ witouth changing the optimal solution, assuming all other coefficient remain constant. [Same for C6]

The objective coefficent C7 (profit of cargo 3 on wagon 1, equal to 5000) can increase till +∞ witouth changing the optimal solution, assuming all other coefficient remain constant.

The objective coefficent C8 (profit of cargo 3 on wagon 2, equal to 5000) can decrease till -∞ witouth changing the optimal solution, assuming all other coefficient remain constant. [Same for C9]

The objective coefficent C10 (profit of cargo 4 on wagon 1, equal to 3500) can decrease till -∞ witouth changing the optimal solution, assuming all other coefficient remain constant.

The objective coefficent C12 (profit of cargo 4 on wagon 3, equal to 3500) can increase till +∞ witouth changing the optimal solution, assuming all other coefficient remain constant.

```r
printSensitivityRHS(model) # get pretty-printed sensitivity analysis for the rhs
```

```
##    Rhs                                  Sensitivity
## 1   B1                                5 <= B1 <= 15
## 2   B2                                 8 <= B2 <= 8
## 3   B3                              12 <= B3 <= 16
## 4   B4                           -inf <= B4 <= inf
## 5   B5                           -inf <= B5 <= inf
## 6   B6 0.0000000000000000888178419700125 <= B6 <= 10
## 7   B7                              15 <= B7 <= 20
## 8   B8                           -inf <= B8 <= inf
## 9   B9                           -inf <= B9 <= inf
## 10 B10                         -inf <= B10 <= inf
## 11 B11                              -5 <= B11 <= 5
## 12 B12                              -5 <= B12 <= 0
## 13 B13                              -5 <= B13 <= 0
## 14 B14                         -inf <= B14 <= inf
## 15 B15                         -inf <= B15 <= inf
## 16 B16                              -8 <= B16 <= 0
## 17 B17                         -inf <= B17 <= inf
## 18 B18                               0 <= B18 <= 0
## 19 B19                              -5 <= B19 <= 0
## 20 B20                               0 <= B20 <= 5
## 21 B21                         -inf <= B21 <= inf
## 22 B22                         -inf <= B22 <= inf
```

RHS RANGES

This 22 Ranges represent the range between which RHSs can vary without changing the corrensponding shadow prices. (one for each constraints, including non negativity constraints)

For example B1 RHS (weight capacity of wagon 1, equal to 10) can decrease/increase of 5 unit and the corrensponding shadow prices ($2500) will not change.

B4, B5, B8, B9,B10, B14, B15, B17, B21, B22 RHSs can vary in any way without changing corrensponding shadow prices.

B18 RHS have to remain equal to zero.

Questions about LP

1. Can an LP model have more than one optimal solution. Is it possible for an LP model to have exactly two optimal solutions? Why or why not?

An LP model can have more than one optimal solution. An Lp model can have 0,1, or ∞ solutions.It is not possible for an Lp problem to have exactly 2 optimal solution beacause if 2 solutions exist than a segment between them exist and so ∞ solutions exist.

2. Are the following objective functions for an LP model equivalent? That is, if they are both used, one at a time, to solve a problem with exactly the same constraints, will the optimal values for x1 and x2 be the same in both cases? Why or why not?

max 2x1+3x2

min -2x1-3x2

This 2 funtion are opposite function [f(x), -f(x)], so maximizing function 1 is equivalent to minimizing function 2. The optimal solution founded for functoin 1 will result in being the optimal solution also for function 2.

3. Which of the following constraints are not linear or cannot be included as a constraint in a linear programming problem?

B, D, E (because they does not respect additivity and proportionality assumptions)