1. Min $x_1^2 + x_1 x_2 + x_2^2 - x_1 - 1$ s.a $5x_1^2 + x_2^2 \le 1$

$2x_1 - x_2 \le 0$

a). $\hat{x} = [1/14, 1/7]$ cumple con CNPO.

$\nabla f(x) = \begin{bmatrix} 2x_1 + x_2 - 1 \\ x_1 + 2x_2 \end{bmatrix}$, $\nabla g_1(x) = \begin{bmatrix} 10x_1 \\ 2x_2 \end{bmatrix}$, $\nabla g_2(x) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

① $2x_1 + x_2 - 1 + 10\mu_1 x_1 + 2\mu_2 = 0$

② $x_1 + 2x_2 + 2\mu_1 x_2 - \mu_2 = 0$

③ $5x_1^2 + x_2^2 \le 1$

④ $2x_1 - x_2 \le 0$

⑤ $\mu_1(5x_1^2 + x_2^2 - 1) = 0$

⑥ $\mu_2(2x_1 - x_2) = 0$

⑦ $\mu_1 \ge 0$

⑧ $\mu_2 \ge 0$

$\rightarrow 5(1/14)^2 + (1/7) - 1 =$
$= 5/196 - 6/7 \ne 0.$

$\rightarrow 2(1/14) - 1/7$
$= 1/7 - 1/7 = 0 \rightarrow g_2$ activa.

• $x_1 = 1/4$, $x_2 = 1/7$, $\mu_1 = 0$, $\mu_2 > 0$.

$\ne 0$
$\nabla g_1(\hat{x})$ indep $\nabla g_2(\hat{x})$.
punto regular

② $\mu_2 = 1/4 + 2(1/7) \ge 0$
$= 15/28 = 0.53$

cumple CNPO.

b). $\hat{x} = [1/14, 1/7]$ ¿? mínimo local.

• $H(x) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, $G_1(x) = \begin{bmatrix} 10 & 0 \\ 0 & 2 \end{bmatrix}$, $G_2(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

• $\mathcal{L}(\hat{x}, \hat{\mu}) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} + \cancel{\frac{15}{28} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}^{0} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$y_1 = \alpha$
$y_2 = 2y_1 = 2\alpha.$

• $T(\hat{x}) = \left\{ y : \begin{bmatrix} 2 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 0 \right\} = \{ y : 2y_1 - y_2 = 0 \} = \left\{ y : \alpha \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \alpha \in \mathbb{R} \right\}$

• $y' \mathcal{L} y = \alpha^2 \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \alpha^2 14. \ge 0,$ CNSO ✓ mínimo local

2. máx $x_1 + 2x_2$ s.a $x_2 \leq (x_1-2)(x_1-1)(x_1+1)(x_1+2)$

$x_1 \geq -2 \rightarrow -2 - x_1 \leq 0$

$x_1 \leq 1 \rightarrow x_1 - 1 \leq 0$

$\nabla f(x) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\nabla g_1 = \begin{bmatrix} -4x_1^3 + 10x_1 \\ 1 \end{bmatrix}$, $\nabla g_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, $\nabla g_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

∇g_2 y ∇g_3 son linealmente dependientes

$\hat{x} = (0,4)$

$g_1 \rightarrow -(-2)(-1)(1)(2) + 4 = 0,$

$g_2 \rightarrow -2 - 0 = -2$

$g_3 \rightarrow 0 - 1 = -1$

solo g_1 está activa.

$\bar{x} = (1,0)$

$g_1 \rightarrow -(-1)(0)(2)(3) + 0 = 0$

$g_2 \rightarrow -2 - 1 = -3$

$g_3 \rightarrow 1 - 1 = 0$

g_1 y g_3 activas.

3. min $x_1^2 - x_2$ s.a $x_1 + x_2 = 2$ $x_0 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} = [2, 0]$

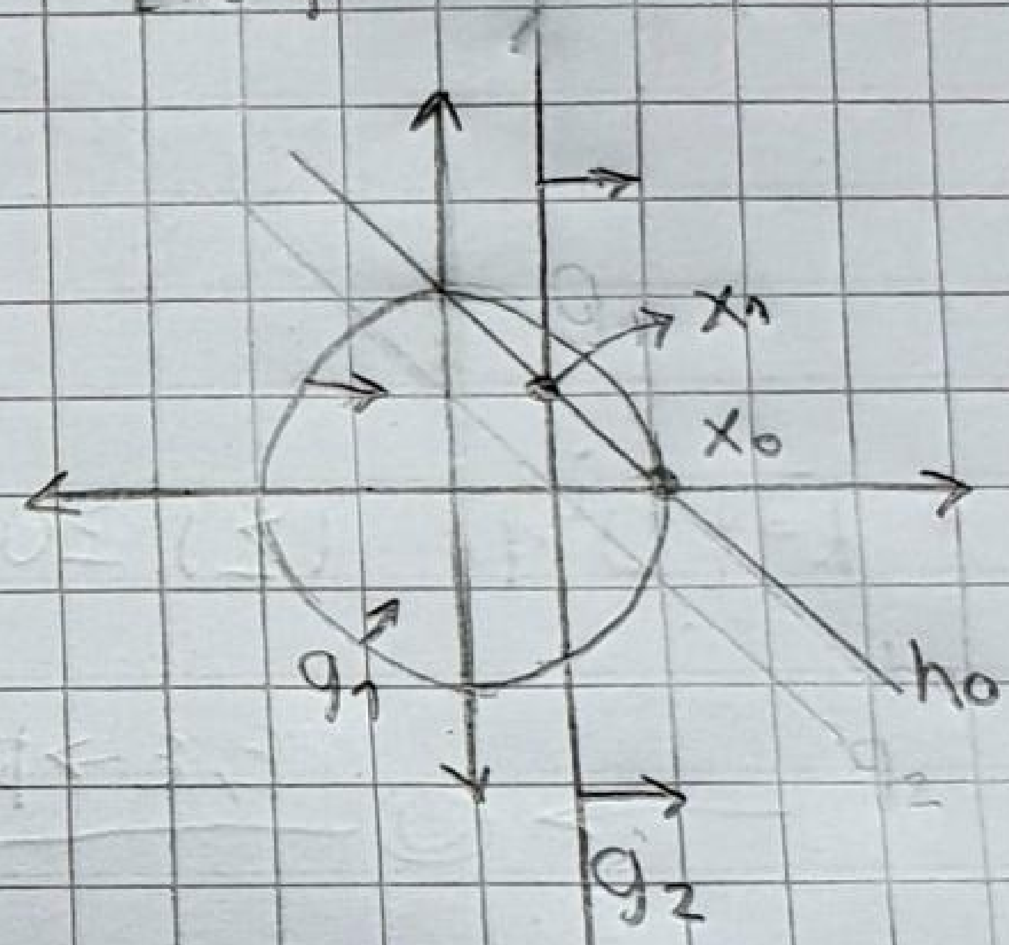
$x_1^2 + x_2^2 \leq 4$

$-x_1 + 1 \leq 0$

conjuntos activos:

- $h_0 = x_1 + x_2 - 2$ • $g_1 = x_1^2 + x_2^2 - 4$ • $g_2 = -x_1 + 1$
- con $[2, 0]$: $h_0 = 0$, $g_1 = 0$, $g_2 = -2$
- $\nabla f(x) = \begin{bmatrix} 2x_1 \\ -1 \end{bmatrix}$, $\nabla h_0(x) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\nabla g_1(x) = \begin{bmatrix} 2x_1 \\ 2x_2 \end{bmatrix}$, $\nabla g_2(x) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

① $2x_1 + \lambda + 2\mu_1 x_1 - \mu_2 = 0$
② $-1 + \lambda + 2\mu_1 x_2 = 0$
③ $x_1 + x_2 = 2$
④ $x_1^2 + x_2^2 \leq 4$
⑤ $-x_1 + 1 \leq 0$
⑥ $\mu_1 (x_1^2 + x_2^2 - 4) = 0$
⑦ $\mu_2 (-x_1 + 1) = 0$
⑧ $\mu_1 \geq 0$
⑨ $\mu_2 \geq 0$



- $W = \{0, 1\}$, $\mu_1 > 0$, $\mu_2 = 0$

$x_1 = 2$, $x_2 = 0$ ① - ②. $2x_1 + \cancel{\lambda} + 2\mu_1 x_1 - \overset{0}{\cancel{\mu_2}} + 1 - \cancel{\lambda} - 2\mu_1 \overset{0}{\cancel{x_2}} = 0$

$4 + 4\mu_1 + 1 = 0$, $\mu_1 = -\frac{5}{4}$ no cumple ⑧ ∴ X

saco g_1.

→ ∴ por gráfica : h_0, g_1, g_2 → nunca activas al tiempo.

- $W = \{0, 2\}$, $\mu_1 = 0$, $\mu_2 > 0$

⑦. $-x_1 + 1 = 0$, $x_1 = 1$ ③. $x_2 = 2 - 1 = 1$

$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

② - ①. $-1 + \cancel{\lambda} + 2\overset{0}{\cancel{\mu_1}} x_2 - 2x_1 - \cancel{\lambda} - 2\overset{0}{\cancel{\mu_1}} x_1 + \mu_2 = 0 \rightarrow \mu_2 = 2x_1 + 1 = 3. > 0$ ✓

①. $\lambda = \mu_2 - 2\overset{0}{\cancel{\mu_1 x_1}} - 2x_1 = 3 - 2 = 1$

4. $\min\ x_1^2 + 2x_2^2$ s.a $-x_1 - x_2 + 1 \leq 0$.

$Q(x, C) = f(x) + C \sum g_i^2(x)$.

$= x_1^2 + 2x_2^2 + C(-x_1 - x_2 + 1)^2$

$\nabla Q(x, C) = \begin{bmatrix} 2x_1 - 2C(-x_1 - x_2 + 1) \\ 4x_2 - 2C(-x_1 - x_2 + 1) \end{bmatrix} = 0$

① $2x_1 - 2C(-x_1 - x_2 + 1) = 0$
② $4x_2 - 2C(-x_1 - x_2 + 1) = 0$.

② − ①. $4x_2 - \cancel{2C(-x_1 - x_2 + 1)} - 2x_1 + \cancel{2C(-x_1 - x_2 + 1)} = 0$
$4x_2 = 2x_1$
$2x_2 = x_1$

①. $2(2x_2) + 2C(2x_2) + 2Cx_2 - 2C = 0$.
$4x_2 + 4Cx_2 + 2Cx_2 - 2C = 0$
$2x_2 + 3Cx_2 = C \rightarrow x_2(2+3C) = C \rightarrow x_2 = \dfrac{C}{2+3C}$, $x_1 = \dfrac{2C}{2+3C}$

• $\lambda = 2C \cdot g(x) = 2C\left(\dfrac{-2C}{2+3C} - \dfrac{C}{2+3C} + 1\right) = \dfrac{-4C^2}{2+3C} - \dfrac{2C^2}{2+3C} + 2C$

$= \dfrac{-6C^2}{2+3C} + 2C = \dfrac{-6C^2 + 2C(2+3C)}{2+3C} = \dfrac{\cancel{-6C^2} + 4C + \cancel{6C^2}}{2+3C}$

$= \dfrac{4C}{2+3C}$

C	1	2	5	10	50	200	500
λ	1	1	20/17	5/4	200/152	800/602	2000/1502
			1,1	1,25	1,3	1,3	1,33

$\lim_{C \to \infty} \dfrac{4C}{2+3C} = \dfrac{4}{3} = 1{,}3$