TDT4136 - Introduction to Artificial Intelligence

Assignment 5

Propositional and First Order Logic

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1. Models and entailment in propositional logic

1.1. Validity and Soundness

(a) Generate the vocabulary of the following argument.

(b) Translate the argument into propositional logic statements.

(c) Add a premise (P4) to make the conclusion of the argument valid.

P1 to P3 are the premises, C is the conclusion:

- (P1) If Peter's argument is valid and all the premises of Peter's argument are true, then Peter's argument is sound.
- (P2) If the premises of Peter's argument entail the conclusion of Peter's argument, then Peter's argument is valid.
- (P3) The premises of Peter's argument entail the conclusion of Peter's argument.
- (C) Peter's argument is sound.

1.2. Modelling

A sentence is satisfiable if it is true in or satisfied by some model.

(a) $(p \Rightarrow q) \Rightarrow (p \Rightarrow r) \Rightarrow (q \Rightarrow r)$

p	q	r	$p \Rightarrow q$	$p \Rightarrow r$	$q \Rightarrow r$	$(p \Rightarrow r) \Rightarrow (q \Rightarrow r)$	Total
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	0	0	0
0	1	1	1	1	1	1	1
1	0	0	0	0	1	1	1
1	0	1	0	1	1	1	1
1	1	0	1	0	0	1	0
1	1	1	1	1	1	1	1

This complete model is not a tautology, as not all instances of the model is true.

(b) $(p \vee (\neg q \Rightarrow r)) \Rightarrow (q \vee (\neg p \Rightarrow r))$

p	q	r	$\neg q \Rightarrow r$	$p \vee (\neg q \Rightarrow r)$	$\neg p \Rightarrow r$	$q \vee (\neg p \Rightarrow r)$	Total
0	0	0	0	0	0	0	1
0	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1

This complete model is a tautology, as all instances of the model is true.

(c) $\neg(p \vee (q \Rightarrow \neg r)) \Rightarrow ((p \Rightarrow q) \wedge (p \Rightarrow r))$

p	q	r	$q \Rightarrow \neg r$	$\neg(p \vee (q \Rightarrow \neg r))$	$p \Rightarrow q$	$p \Rightarrow r$	Total
0	0	0	1	0	1	1	1
0	0	1	0	1	1	1	1
0	1	0	1	0	1	1	1
0	1	1	1	0	1	1	1
1	0	0	1	0	0	0	1
1	0	1	1	0	0	1	1
1	1	0	0	0	1	0	1
1	1	1	1	0	1	1	1

This complete model is a tautology.

(d) $\neg(\neg p \vee (q \wedge r)) \Rightarrow (\neg(p \vee q) \wedge r)$

p	q	r	$q \wedge r$	$\neg(\neg p \vee (q \wedge r))$	$\neg(p \vee q)$	$\neg(p \vee q) \wedge r$	Total
0	0	0	0	0	1	0	1
0	0	1	0	0	1	1	1
0	1	0	0	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	0	1	0	0	0
1	0	1	0	1	0	0	0
1	1	0	0	1	0	0	0
1	1	1	1	0	0	0	1

This complete model is not a tautology.

1.3. Modelling 2

p	q	r	ϕ	$(r \Rightarrow q)$
0	0	0	1	1
0	0	1	0	0
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	0	0
1	1	0	1	1
1	1	1	1	1

The expression is independent of p, meaning it doesnt affect the value of ϕ. ϕ is equvialent to $(r \Rightarrow q)$. If we want to include p, we can write $(r \Rightarrow q) \wedge (p \vee \neg p)$.

2. Resolution in propositional logic

2.1. Conjunctive Normal Form

2.2. Inference in propositional logic

3. Representation in First-Order Logic (FOL)

3.1. a) Argument A

3.2. b) Argument B

4. Resolution in FOL