NTNU

Kunnskap for en bedre verden

TDT4136 - INTRODUCTION TO ARTIFICIAL INTELLIGENCE

ASSIGNMENT 5

Propositional and First Order Logic

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18.10.23

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1. Models and entailment in propositional logic

1.1. Validity and Soundness

P1 to P3 are the premises, C is the conclusion:

- **(P1) If Peter's argument is valid and all the premises of Peter's argument are true, then Peter's argument is sound.**
- **(P2) If the premises of Peter's argument entail the conclusion of Peter's argument, then Peter's argument is valid.**
- **(P3) The premises of Peter's argument entail the conclusion of Peter's argument.**
- **(C) Peter's argument is sound.**

(a) **Generate the vocabulary of the following argument.**

- (P1) Peter's argument is valid.
- (P2) The premises of Peter's argument entail the conclusion.
- (P3) Peter's argument is sound.
- (C) The conculsion is true

(b) **Translate the argument into propositional logic statements.**

- (P1) P
- (P2) $P \Rightarrow C$
- (P3) C
- (C) C

(c) **Add a premise (P4) to make the conclusion of the argument valid.**
Adding (P4) Peter's argument is valid, ensures that Peter's argument is both sound and valid, and thus the conclusion is valid.

1.2. Modelling

For each of the following statements, determine if they are satisfiable by building the complete model (truth table) and mark tautologies. A sentence is satisfiable if it is true in or satisfied by some model.

(a) $(\mathbf{p} \Rightarrow \mathbf{q}) \Rightarrow (\mathbf{p} \Rightarrow \mathbf{r}) \Rightarrow (\mathbf{q} \Rightarrow \mathbf{r})$

p	q	r	$p \Rightarrow q$	$p \Rightarrow r$	$q \Rightarrow r$	$(p \Rightarrow r) \Rightarrow (q \Rightarrow r)$	Total
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	0	0	0
0	1	1	1	1	1	1	1
1	0	0	0	0	1	1	1
1	0	1	0	1	1	1	1
1	1	0	1	0	0	1	0
1	1	1	1	1	1	1	1

This complete model is not a tautology, as not all instances of the model is true.

(b) $(\mathbf{p} \vee (\neg \mathbf{q} \Rightarrow \mathbf{r})) \Rightarrow (\mathbf{q} \vee (\neg \mathbf{p} \Rightarrow \mathbf{r}))$

p	q	r	$\neg q \Rightarrow r$	$p \vee (\neg q \Rightarrow r)$	$\neg p \Rightarrow r$	$q \vee (\neg p \Rightarrow r)$	Total
0	0	0	0	0	0	0	1
0	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1

This complete model is a tautology, as all instances of the model is true.

(c) $\neg(\mathbf{p} \vee (\mathbf{q} \Rightarrow \neg\mathbf{r})) \Rightarrow ((\mathbf{p} \Rightarrow \mathbf{q}) \wedge (\mathbf{p} \Rightarrow \mathbf{r}))$

p	q	r	$q \Rightarrow \neg r$	$\neg(p \vee (q \Rightarrow \neg r))$	$p \Rightarrow q$	$p \Rightarrow r$	Total
0	0	0	1	0	1	1	1
0	0	1	0	1	1	1	1
0	1	0	1	0	1	1	1
0	1	1	1	0	1	1	1
1	0	0	1	0	0	0	1
1	0	1	1	0	0	1	1
1	1	0	0	0	1	0	1
1	1	1	1	0	1	1	1

This complete model is a tautology.

(d) $\neg(\neg\mathbf{p} \vee (\mathbf{q} \wedge \mathbf{r})) \Rightarrow (\neg(\mathbf{p} \vee \mathbf{q}) \wedge \mathbf{r})$

p	q	r	$q \wedge r$	$\neg(\neg p \vee (q \wedge r))$	$\neg(p \vee q)$	$\neg(p \vee q) \wedge r$	Total
0	0	0	0	0	1	0	1
0	0	1	0	0	1	1	1
0	1	0	0	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	0	1	0	0	0
1	0	1	0	1	0	0	0
1	1	0	0	1	0	0	0
1	1	1	1	0	0	0	1

This complete model is not a tautology.

1.3. Modelling 2

Let ϕ be a sentence that contains three atomic constituents and let the truth conditions of ϕ be defined by the truth table below. Write a propositional logic statement that contains p, q, and r as constituents, and that is equivalent to ϕ.

p	q	r	ϕ	$(r \Rightarrow q)$
0	0	0	1	1
0	0	1	0	0
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	0	0
1	1	0	1	1
1	1	1	1	1

The expression is independent of p, meaning it doesnt affect the value of ϕ. ϕ is equvialent to $(r \Rightarrow q)$. If we want to include p, we can write $(r \Rightarrow q) \wedge (p \vee \neg p)$.

2. Resolution in propositional logic

2.1. Conjunctive Normal Form

Convert each of the following sentences to their Conjunctive Normal Form (CNF). A CNF formula is composed of a conjunction (AND) of one or more clauses, where each clause is a disjunction (OR) of literals.

1. $\mathbf{p} \iff \mathbf{q}$

   $(p \iff q) \equiv ((p \wedge q) \vee (\neg p \wedge \neg q)) \equiv \underline{\underline{((p \vee \neg q) \wedge (\neg p \vee q))}}$

2. $\neg((\mathbf{p} \Rightarrow \mathbf{q}) \wedge \mathbf{r})$

   $(p \Rightarrow q) \equiv (p \wedge \neg q)$

   $((p \Rightarrow q) \wedge r) \equiv (p \wedge \neg q \wedge r)$

   $\neg((p \Rightarrow q) \wedge r) \equiv \underline{\underline{(\neg p \vee q \vee \neg r)}}$

3. $((\mathbf{p} \vee \mathbf{q}) \vee (\mathbf{r} \wedge \neg(\mathbf{q} \Rightarrow \mathbf{r})))$

   $\neg(q \Rightarrow r) \equiv \neg(q \wedge \neg r) \equiv (\neg q \vee r)$

   $(r \wedge \neg(q \Rightarrow r)) \equiv (r \wedge (\neg q \vee r))$

   $((p \vee q) \vee (r \wedge \neg(q \Rightarrow r))) \equiv ((p \vee q) \vee (r \wedge (\neg q \vee r))) \equiv \underline{\underline{((p \vee q \vee r) \wedge (p \vee r))}}$

4. Yes, the expression above is in CNF.

2.2. Inference in propositional logic

Use resolution to conclude r from the following statements.

1. $(\mathbf{p} \Rightarrow \mathbf{q}) \Rightarrow \mathbf{q}$

2. $\mathbf{p} \Rightarrow \mathbf{r}$

3. $(\mathbf{r} \Rightarrow \mathbf{s}) \Rightarrow (\neq (\mathbf{s} \Rightarrow \mathbf{q}))$

Convert each statement into CNF:

1. $((p \Rightarrow q) \Rightarrow q) \equiv ((p \wedge \neg q) \vee q)$
2. $(p \Rightarrow r) \equiv (\neg p \vee r)$
3. $((r \Rightarrow s) \Rightarrow (\neq (s \Rightarrow q))) \equiv ((s \wedge \neg q))$

By performing resolution on clause 1 and 2 on q, we get: $(p \wedge \neg q) \vee (\neg p \vee r)$, which is equivalent to $p \vee r$, which we will call clause 4 from here on. By performing resolution on clause 4 and 3 on q, we get: $(s \wedge (\neg p \vee r))$, which is equivalent to $(s \wedge p) \vee (s \wedge r)$, which we will call clause 5 from here on. This result tells us that either $p \vee r$ or s must be true. If $p \vee r$ is true, we have $(\neg p \vee r)$, which means r must be true. If s is true, then r may be either true or false.

3. Representation in First-Order Logic (FOL)

Consider the following baseball vocabulary:

1. $Pitcher(p)$ **is a predicate where person** p **is a pitcher**.
2. $flies(p_1, p_2)$ **is a predicate where person** p_1 **flies**[1] **out to person** p_2.
3. $Centerfielder(p)$ **is a predicate where person** p **is a centerfielder.**
4. $scores(p)$ **is a predicate where person** p **scores.**
5. $friend(p_1, p_2)$ **is a predicate where person** p_1 **is the friend of person** p_2 **(but not vice versa).**
6. **Robinson, Crabb, Samson, Jones are constants denoting persons.**

Now look at the following translations of natural language into first order logic statements describing a baseball game. Using the provided vocabulary, translate the conclusion of each of the following arguments into an FOL statement.

3.1. Argument A

Only pitchers fly out to Robinson. Crabb scores only if Samson flies out to Robinson and Robinson is a centerfielder. Crabb scores. Conclusion: Samson is a pitcher.

- $\forall \mathbf{x} : [\mathbf{flies}(\mathbf{x}, \mathbf{Robinson}) \rightarrow \mathbf{Pitcher}(\mathbf{x})]$
- $\mathbf{scores}(\mathbf{Crabb}) \rightarrow (\mathbf{flies}(\mathbf{Samson}, \mathbf{Robinson}) \wedge \mathbf{Centerfielder}(\mathbf{Robinson}))$
- $\mathbf{scores}(\mathbf{Crabb})$

Translated into FOL we get:

[1] A flyout occurs when a batter hits the ball in the air, and a fielder catches it before it touches the ground.

$\forall x : (flies(x, Robinson) \rightarrow Pitcher(x))$
$\wedge (scores(Crabb) \rightarrow (flies(Samson, Robinson) \wedge Centerfielder(Robinson)))$
$\wedge\, scores(Crabb)$
$\rightarrow Pitcher(Samson)$

3.2. Argument B

No centerfielder who does not score has any friends. Robinson and Jones are both centerfielders. Any centerfielder who flies out to Jones does not score. Robinson flies out to Jones. Conclusion: Jones is not a friend of Robinson.

- $\forall \mathbf{x} : [(\mathbf{Centerfielder}(\mathbf{x}) \wedge \neg \mathbf{scores}(\mathbf{x})) \rightarrow \neg \exists \mathbf{y} : [\mathbf{friend}(\mathbf{y}, \mathbf{x})]]$
- $\mathbf{Centerfielder}(\mathbf{Robinson}) \wedge \mathbf{Centerfielder}(\mathbf{Jones})$
- $\forall \mathbf{x} : [(\mathbf{Centerfielder}(\mathbf{x}) \wedge \mathbf{flies}(\mathbf{x}, \mathbf{Jones})) \rightarrow \neg \mathbf{scores}(\mathbf{x})]$
- $\mathbf{flies}(\mathbf{Robinson}, \mathbf{Jones})$

Square brackets [] are used to stress which parts of the statement belong under which quantifier.

Translated into FOL we get:

$\forall x : ((Centerfielder(x) \wedge \neg scores(x)) \rightarrow \neg \exists y : [friend(y, x)])$
$\wedge (Centerfielder(Robinson) \wedge Centerfielder(Jones))$
$\wedge [\forall x : [(Centerfielder(x) \wedge flies(x, Jones)) \rightarrow \neg scores(x)]]$
$\wedge\, flies(Robinson, Jones)$
$\rightarrow \neg friend(Robinson, Jones)$

4. Resolution in FOL

Using resolution, prove the conclusions from Arguments A and B from exercise 3.

4.1. Argument A

1. $\forall \mathbf{x} : [\mathbf{flies}(\mathbf{x}, \mathbf{Robinson}) \rightarrow \mathbf{Pitcher}(\mathbf{x})]$
2. $\mathbf{scores}(\mathbf{Crabb}) \rightarrow (\mathbf{flies}(\mathbf{Samson}, \mathbf{Robinson}) \wedge \mathbf{Centerfielder}(\mathbf{Robinson}))$
3. $\mathbf{scores}(\mathbf{Crabb})$

Proof by contradiction: $\neg Pitcher(Samson)$.

Insert (2) into (3):

(4) $scores(Crabb) \rightarrow (flies(Samson, Robinson) \wedge Centerfielder(Robinson))$

Replace x with Samson in (1):

(5) $flies(Samson, Robinson) \rightarrow Pitcher(Samson)$

As we assume $\neg Pitcher(Samson)$, (5) must be false.

Therefore $flies(Samson, Robinson)$ must be false. Consequently, the right hand side of (4) must also be false.

Thus, we have proven by contradiction: $Pitcher(Samson)$.

4.2. Argument B

1. $\forall \mathbf{x} : [(\mathbf{Centerfielder}(\mathbf{x}) \wedge \neg \mathbf{scores}(\mathbf{x})) \rightarrow \neg \exists \mathbf{y} : [\mathbf{friend}(\mathbf{y}, \mathbf{x})]]$
2. $\mathbf{Centerfielder}(\mathbf{Robinson}) \wedge \mathbf{Centerfielder}(\mathbf{Jones})$
3. $\forall \mathbf{x} : [(\mathbf{Centerfielder}(\mathbf{x}) \wedge \mathbf{flies}(\mathbf{x}, \mathbf{Jones})) \rightarrow \neg \mathbf{scores}(\mathbf{x})]$

4. $\mathbf{flies}(\mathbf{Robinson}, \mathbf{Jones})$

Proof by contradiction: $friend(Jones, Robinson)$.

By combining (3) and (4), thus replacing x with Robinson, we get: $(Centerfielder(Robinso$
$flies(Robinson, Jones)) \rightarrow \neg scores(Robinson)$

Thus we get that Robinson is centerfield and doesn't score. By inserting this into (1), we get:

(5) $(Centerfielder(Robinson) \wedge \neg scores(Robinson)) \rightarrow \neg \exists y : [friend(y, Robinson)]$

As y can be any person, we can insert Jones into (5), which gives $\neg friend(Jones, Robinson)$
This is a contradiction to what we desired to prove, thus $friend(Jones, Robinson)$ must be false.