

TDT4136 - INTRODUCTION TO ARTIFICIAL INTELLIGENCE  
ASSIGNMENT 5

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# Propositional and First Order Logic

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## Innholdsfortegnelse

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# 1. Models and entailment in propositional logic

## 1.1. Validity and Soundness

**P1 to P3 are the premises, C is the conclusion:**

- (P1) If Peter's argument is valid and all the premises of Peter's argument are true, then Peter's argument is sound.
- (P2) If the premises of Peter's argument entail the conclusion of Peter's argument, then Peter's argument is valid.
- (P3) The premises of Peter's argument entail the conclusion of Peter's argument.
- (C) Peter's argument is sound.

**(a) Generate the vocabulary of the following argument.**

- (P1) Peter's argument is valid, then Peter's argument is sound.
- (P2) Peter's argument is valid, then Peter's argument entail the conclusion of Peter's argument.
- (P3) Peter's argument entail the conclusion of Peter's argument.
- (C) Peter's argument is sound.

**(b) Translate the argument into propositional logic statements.**

- (P1)  $A_{Valid} \Rightarrow A_{Sound}$
- (P2)  $A_{Valid} \Rightarrow (A \Rightarrow C)$
- (P3)  $A \Rightarrow C$
- (C)  $A_{Sound}$

**(c) Add a premise (P4) to make the conclusion of the argument valid.**

Adding (P4) Peter's argument is valid, ensures that Peter's argument is both sound and valid, and thus the conclusion is valid.

## 1.2. Modelling

**For each of the following statements, determine if they are satisfiable by building the complete model (truth table) and mark tautologies.** A sentence is satisfiable if it is true in or satisfied by some model.

1.  $(\mathbf{p} \Rightarrow \mathbf{q}) \Rightarrow ((\mathbf{p} \Rightarrow \mathbf{r}) \Rightarrow (\mathbf{q} \Rightarrow \mathbf{r}))$

p	q	r	$p \Rightarrow q$	$p \Rightarrow r$	$q \Rightarrow r$	$(p \Rightarrow r) \Rightarrow (q \Rightarrow r)$	Total
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	0	0	0
0	1	1	1	1	1	1	1
1	0	0	0	0	1	1	1
1	0	1	0	1	1	1	1
1	1	0	1	0	0	1	1
1	1	1	1	1	1	1	1

This complete model is not a tautology, as not all instances of the model is true.

2.  $(\mathbf{p} \vee (\neg \mathbf{q} \Rightarrow \mathbf{r})) \Rightarrow (\mathbf{q} \vee (\neg \mathbf{p} \Rightarrow \mathbf{r}))$

p	q	r	$\neg q \Rightarrow r$	$p \vee (\neg q \Rightarrow r)$	$\neg p \Rightarrow r$	$q \vee (\neg p \Rightarrow r)$	Total
0	0	0	0	0	0	0	1
0	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1

This complete model is a tautology, as all instances of the model is true.

3.  $\neg(\mathbf{p} \vee (\mathbf{q} \Rightarrow \neg \mathbf{r})) \Rightarrow ((\mathbf{p} \Rightarrow \mathbf{q}) \wedge (\mathbf{p} \Rightarrow \mathbf{r}))$

p	q	r	$q \Rightarrow \neg r$	$\neg(p \vee (q \Rightarrow \neg r))$	$p \Rightarrow q$	$p \Rightarrow r$	Total
0	0	0	1	0	1	1	1
0	0	1	1	0	1	1	1
0	1	0	1	0	1	1	1
0	1	1	0	1	1	1	1
1	0	0	1	0	0	0	1
1	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1
1	1	1	0	0	1	1	1

This complete model is a tautology.

4.  $\neg(\neg \mathbf{p} \vee (\mathbf{q} \wedge \mathbf{r})) \Rightarrow (\neg(\mathbf{p} \vee \mathbf{q}) \wedge \mathbf{r})$

p	q	r	$q \wedge r$	$\neg(\neg p \vee (q \wedge r))$	$\neg(p \vee q)$	$\neg(p \vee q) \wedge r$	Total
0	0	0	0	0	1	0	1
0	0	1	0	0	1	1	1
0	1	0	0	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	0	1	0	0	0
1	0	1	0	1	0	0	0
1	1	0	0	1	0	0	0
1	1	1	1	0	0	0	1

This complete model is not a tautology.

### 1.3. Modelling 2

Let  $\phi$  be a sentence that contains three atomic constituents and let the truth conditions of  $\phi$  be defined by the truth table below. Write a propositional logic statement that contains p, q, and r as constituents, and that is equivalent to  $\phi$ .

p	q	r	$\phi$	$(r \Rightarrow q)$
0	0	0	1	1
0	0	1	0	0
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	0	0
1	1	0	1	1
1	1	1	1	1

The expression is independent of p, meaning it doesn't affect the value of  $\phi$ .  $\phi$  is equivalent to  $(r \Rightarrow q)$ . If we want to include p, we can write  $(r \Rightarrow q) \wedge (p \vee \neg p)$ .

## 2. Resolution in propositional logic

### 2.1. Conjunctive Normal Form

Convert each of the following sentences to their Conjunctive Normal Form (CNF). A CNF formula is composed of a conjunction (AND) of one or more clauses, where each clause is a disjunction (OR) of literals.

1.  $p \iff q$

$$(p \iff q) \equiv ((p \wedge q) \vee (\neg p \wedge \neg q)) \equiv \underline{\underline{((p \vee \neg q) \wedge (\neg p \vee q))}}$$

2.  $\neg((p \Rightarrow q) \wedge r)$

$$(p \Rightarrow q) \equiv (p \wedge \neg q)$$

$$((p \Rightarrow q) \wedge r) \equiv (p \wedge \neg q \wedge r)$$

$$\neg((p \Rightarrow q) \wedge r) \equiv \underline{\underline{(\neg p \vee q \vee \neg r)}}$$

3.  $((p \vee q) \vee (r \wedge \neg(q \Rightarrow r)))$

$$\neg(q \Rightarrow r) \equiv \neg(q \wedge \neg r) \equiv (\neg q \vee r)$$

$$(r \wedge \neg(q \Rightarrow r)) \equiv (r \wedge (\neg q \vee r))$$

$$((p \vee q) \vee (r \wedge \neg(q \Rightarrow r))) \equiv ((p \vee q) \vee (r \wedge (\neg q \vee r))) \equiv \underline{\underline{((p \vee q \vee r) \wedge (p \vee r))}}$$

4. Yes, the expression above is in CNF.

### 2.2. Inference in propositional logic

Use resolution to conclude  $r$  from the following statements.

1.  $(p \Rightarrow q) \Rightarrow q$

2.  $p \Rightarrow r$

3.  $(r \Rightarrow s) \Rightarrow (\neg(s \Rightarrow q))$

Convert each statement into CNF:

1.  $((p \Rightarrow q) \Rightarrow q) \equiv ((p \wedge \neg q) \vee q)$
2.  $(p \Rightarrow r) \equiv (\neg p \vee r)$
3.  $((r \Rightarrow s) \Rightarrow (\neg(s \Rightarrow q))) \equiv ((s \wedge \neg q))$

By performing resolution on clause 1 and 2 on  $q$ , we get:  $(p \wedge \neg q) \vee (\neg p \vee r)$ , which is equivalent to  $p \vee r$ , which we will call clause 4 from here on. By performing resolution on clause 4 and 3 on  $q$ , we get:  $(s \wedge (\neg p \vee r))$ , which is equivalent to  $(s \wedge p) \vee (s \wedge r)$ , which we will call clause 5 from here on. This result tells us that either  $p \vee r$  or  $s$  must be true. If  $p \vee r$  is true, we have  $(\neg p \vee r)$ , which means  $r$  must be true. If  $s$  is true, then  $r$  may be either true or false.



### 3. Representation in First-Order Logic (FOL)

Consider the following baseball vocabulary:

1. *Pitcher*( $p$ ) is a predicate where person  $p$  is a pitcher.
2. *flies*( $p_1, p_2$ ) is a predicate where person  $p_1$  flies<sup>1</sup> out to person  $p_2$ .
3. *Centerfielder*( $p$ ) is a predicate where person  $p$  is a centerfielder.
4. *scores*( $p$ ) is a predicate where person  $p$  scores.
5. *friend*( $p_1, p_2$ ) is a predicate where person  $p_1$  is the friend of person  $p_2$  (but not vice versa).
6. Robinson, Crabb, Samson, Jones are constants denoting persons.

Now look at the following translations of natural language into first order logic statements describing a baseball game. Using the provided vocabulary, translate the conclusion of each of the following arguments into an FOL statement.

#### 3.1. Argument A

Only pitchers fly out to Robinson. Crabb scores only if Samson flies out to Robinson and Robinson is a centerfielder. Crabb scores. Conclusion: Samson is a pitcher.

- $\forall x : [\text{flies}(x, \text{Robinson}) \rightarrow \text{Pitcher}(x)]$
- $\text{scores}(\text{Crabb}) \rightarrow (\text{flies}(\text{Samson}, \text{Robinson}) \wedge \text{Centerfielder}(\text{Robinson}))$
- $\text{scores}(\text{Crabb})$

Translated into FOL we get:

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<sup>1</sup>A flyout occurs when a batter hits the ball in the air, and a fielder catches it before it touches the ground.

$$\begin{aligned} &\forall x : (\text{flies}(x, \text{Robinson}) \rightarrow \text{Pitcher}(x)) \\ &\wedge (\text{scores}(\text{Crabb}) \rightarrow (\text{flies}(\text{Samson}, \text{Robinson}) \wedge \text{Centerfielder}(\text{Robinson}))) \\ &\wedge \text{scores}(\text{Crabb}) \\ &\rightarrow \text{Pitcher}(\text{Samson}) \end{aligned}$$

### 3.2. Argument B

**No centerfielder who does not score has any friends. Robinson and Jones are both centerfielders. Any centerfielder who flies out to Jones does not score. Robinson flies out to Jones. Conclusion: Jones is not a friend of Robinson.**

- $\forall x : [(\text{Centerfielder}(x) \wedge \neg \text{scores}(x)) \rightarrow \neg \exists y : [\text{friend}(y, x)]]$
- $\text{Centerfielder}(\text{Robinson}) \wedge \text{Centerfielder}(\text{Jones})$
- $\forall x : [(\text{Centerfielder}(x) \wedge \text{flies}(x, \text{Jones})) \rightarrow \neg \text{scores}(x)]$
- $\text{flies}(\text{Robinson}, \text{Jones})$

**Square brackets [] are used to stress which parts of the statement belong under which quantifier.**

Translated into FOL we get:

$$\begin{aligned} &\forall x : ((\text{Centerfielder}(x) \wedge \neg \text{scores}(x)) \rightarrow \neg \exists y : [\text{friend}(y, x)]) \\ &\wedge (\text{Centerfielder}(\text{Robinson}) \wedge \text{Centerfielder}(\text{Jones})) \\ &\wedge [\forall x : [(\text{Centerfielder}(x) \wedge \text{flies}(x, \text{Jones})) \rightarrow \neg \text{scores}(x)]] \\ &\wedge \text{flies}(\text{Robinson}, \text{Jones}) \\ &\rightarrow \neg \text{friend}(\text{Robinson}, \text{Jones}) \end{aligned}$$

## 4. Resolution in FOL

Using resolution, prove the conclusions from Arguments A and B from exercise 3.

### 4.1. Argument A

1.  $\forall x : [\text{flies}(x, \text{Robinson}) \rightarrow \text{Pitcher}(x)]$
2.  $\text{scores}(\text{Crabb}) \rightarrow (\text{flies}(\text{Samson}, \text{Robinson}) \wedge \text{Centerfielder}(\text{Robinson}))$
3.  $\text{scores}(\text{Crabb})$

Proof by contradiction:  $\neg \text{Pitcher}(\text{Samson})$ .

Insert (2) into (3):

$$(4) \text{scores}(\text{Crabb}) \rightarrow (\text{flies}(\text{Samson}, \text{Robinson}) \wedge \text{Centerfielder}(\text{Robinson}))$$

Replace x with Samson in (1):

$$(5) \text{flies}(\text{Samson}, \text{Robinson}) \rightarrow \text{Pitcher}(\text{Samson})$$

As we assume  $\neg \text{Pitcher}(\text{Samson})$ , (5) must be false.

Therefore  $\text{flies}(\text{Samson}, \text{Robinson})$  must be false. Consequently, the right hand side of (4) must also be false.

Thus, we have proven by contradiction:  $\text{Pitcher}(\text{Samson})$ .

### 4.2. Argument B

1.  $\forall x : [(\text{Centerfielder}(x) \wedge \neg \text{scores}(x)) \rightarrow \neg \exists y : [\text{friend}(y, x)]]$
2.  $\text{Centerfielder}(\text{Robinson}) \wedge \text{Centerfielder}(\text{Jones})$
3.  $\forall x : [(\text{Centerfielder}(x) \wedge \text{flies}(x, \text{Jones})) \rightarrow \neg \text{scores}(x)]$

**4. flies(Robinson, Jones)**

Proof by contradiction:  $friend(Jones, Robinson)$ .

By combining (3) and (4), thus replacing  $x$  with Robinson, we get:

$$(Centerfielder(Robinson) \wedge flies(Robinson, Jones)) \\ \rightarrow \neg scores(Robinson)$$

Thus we get that Robinson is centerfield and doesn't score. By inserting this into (1), we get:

$$(5) (Centerfielder(Robinson) \wedge \neg scores(Robinson)) \\ \rightarrow \neg \exists y : [friend(y, Robinson)]$$

As  $y$  can be any person, we can insert Jones into (5), which gives  $\neg friend(Jones, Robinson)$ . This is a contradiction to what we desired to prove, thus  $friend(Jones, Robinson)$  must be false.