NTNU
Kunnskap for en bedre verden

TDT4136 - INTRODUCTION TO ARTIFICIAL INTELLIGENCE

ASSIGNMENT 5

# Propositional and First Order Logic

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# 1. Models and entailment in propositional logic

## 1.1. Validity and Soundness

P1 to P3 are the premises, C is the conclusion:

- (P1) If Peter's argument is valid and all the premises of Peter's argument are true, then Peter's argument is sound.
- (P2) If the premises of Peter's argument entail the conclusion of Peter's argument, then Peter's argument is valid.
- (P3) The premises of Peter's argument entail the conclusion of Peter's argument.
- (C) Peter's argument is sound.

(a) Generate the vocabulary of the following argument.

- (P1) Peter's argument is valid.
- (P2) The premises of Peter's argument entail the conclusion.
- (P3) Peter's argument is sound.
- (C) The conculsion is true

(b) Translate the argument into propositional logic statements.

- (P1) P
- (P2) $P \Rightarrow C$
- (P3) C
- (C) C

(c) Add a premise (P4) to make the conclusion of the argument valid. Adding (P4) Peter's argument is valid, ensures that Peter's argument is both sound and valid, and thus the conclusion is valid.

## 1.2. Modelling

For each of the following statements, determine if they are satisfiable by building the complete model (truth table) and mark tautologies.

A sentence is satisfiable if it is true in or satisfied by some model.

(a) $(p \Rightarrow q) \Rightarrow (p \Rightarrow r) \Rightarrow (q \Rightarrow r)$

| p | q | r | $p \Rightarrow q$ | $p \Rightarrow r$ | $q \Rightarrow r$ | $(p \Rightarrow r) \Rightarrow (q \Rightarrow r)$ | Total |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

This complete model is not a tautology, as not all instances of the model is true.

(b) $(p \vee (\neg q \Rightarrow r)) \Rightarrow (q \vee (\neg p \Rightarrow r))$

| p | q | r | $\neg q \Rightarrow r$ | $p \vee (\neg q \Rightarrow r)$ | $\neg p \Rightarrow r$ | $q \vee (\neg p \Rightarrow r)$ | Total |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

This complete model is a tautology, as all instances of the model is true.

(c) $\neg(p \vee (q \Rightarrow \neg r)) \Rightarrow ((p \Rightarrow q) \wedge (p \Rightarrow r))$

| p | q | r | $q \Rightarrow \neg r$ | $\neg(p \vee (q \Rightarrow \neg r))$ | $p \Rightarrow q$ | $p \Rightarrow r$ | Total |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |

This complete model is a tautology.

(d) $\neg(\neg p \vee (q \wedge r)) \Rightarrow (\neg(p \vee q) \wedge r)$

| p | q | r | $q \wedge r$ | $\neg(\neg p \vee (q \wedge r))$ | $\neg(p \vee q)$ | $\neg(p \vee q) \wedge r$ | Total |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |

This complete model is not a tautology.

## 1.3. Modelling 2

Let $\phi$ be a sentence that contains three atomic constituents and let the truth conditions of $\phi$ be defined by the truth table below. Write a propositional logic statement that contains p, q, and r as constituents, and that is equivalent to $\phi$.

| p | q | r | $\phi$ | $(r \Rightarrow q)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

The expression is independent of p, meaning it doesnt affect the value of $\phi$. $\phi$ is equvialent to $(r \Rightarrow q)$. If we want to include p, we can write $(r \Rightarrow q) \wedge (p \vee \neg p)$.

# 2. Resolution in propositional logic

## 2.1. Conjunctive Normal Form

Convert each of the following sentences to their Conjunctive Normal Form (CNF). A CNF formula is composed of a conjunction (AND) of one or more clauses, where each clause is a disjunction (OR) of literals.

1. $p \iff q$

   $(p \iff q) \equiv ((p \wedge q) \vee (\neg p \wedge \neg q)) \equiv \underline{\underline{((p \vee \neg q) \wedge (\neg p \vee q))}}$

2. $\neg((p \Rightarrow q) \wedge r)$

   $(p \Rightarrow q) \equiv (p \wedge \neg q)$

   $((p \Rightarrow q) \wedge r) \equiv (p \wedge \neg q \wedge r)$

   $\neg((p \Rightarrow q) \wedge r) \equiv \underline{\underline{(\neg p \vee q \vee \neg r)}}$

3. $((p \vee q) \vee (r \wedge \neg(q \Rightarrow r)))$

   $\neg(q \Rightarrow r) \equiv \neg(q \wedge \neg r) \equiv (\neg q \vee r)$

   $(r \wedge \neg(q \Rightarrow r)) \equiv (r \wedge (\neg q \vee r))$

   $((p \vee q) \vee (r \wedge \neg(q \Rightarrow r))) \equiv ((p \vee q) \vee (r \wedge (\neg q \vee r))) \equiv \underline{\underline{((p \vee q \vee r) \wedge (p \vee r))}}$

4. Yes, the expression above is in CNF.

## 2.2. Inference in propositional logic

Use resolution to conclude r from the following statements.

1. $(p \Rightarrow q) \Rightarrow q$
2. $p \Rightarrow r$
3. $(r \Rightarrow s) \Rightarrow (\neq (s \Rightarrow q))$

Convert each statement into CNF:

1. $((p \Rightarrow q) \Rightarrow q) \equiv (p \vee q)$

2. $(p \Rightarrow r) \equiv (\neg p \vee r)$

3. $((r \Rightarrow s) \Rightarrow (\neq (s \Rightarrow q))) \equiv ((r \wedge \neg s) \vee (s \wedge \neg q))$

# 3. Representation in First-Order Logic (FOL)

## 3.1. a) Argument A

## 3.2. b) Argument B

# 4. Resolution in FOL