Week 16.

① Show that the set of all integers is countable
List all of the integers correctly.
0, 1, -1, 2, -2, 3, -3, 4, -4, 5, -5, ... → in this way
we can predictably list all integers, therefore,
the set of all integers is countable.

② Prove that the set of all rational numbers is countable.
The key is to list all rational numbers correctly:

$$\begin{array}{cccccccccccc}
\frac{1}{1} & -\frac{1}{1} & \frac{2}{1} & -\frac{2}{1} & \frac{3}{1} & -\frac{3}{1} & \frac{4}{1} & -\frac{4}{1} & \frac{5}{1} & -\frac{5}{1} & \cdots \\
\frac{1}{2} & -\frac{1}{2} & \frac{2}{2} & -\frac{2}{2} & \frac{3}{2} & -\frac{3}{2} & \frac{4}{2} & -\frac{4}{2} & \frac{5}{2} & -\frac{5}{2} & \cdots \\
\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & -\frac{2}{3} & \frac{3}{3} & -\frac{3}{3} & \frac{4}{3} & -\frac{4}{3} & \frac{5}{3} & -\frac{5}{3} & \cdots \\
\frac{1}{4} & -\frac{1}{4} & \frac{2}{4} & -\frac{2}{4} & \frac{3}{4} & -\frac{3}{4} & \frac{4}{4} & -\frac{4}{4} & \frac{5}{4} & -\frac{5}{4} & \cdots
\end{array}$$

③ Prove that the power set of a set of all natural numbers (Z^+) is uncountable
Lets assume that it countable
{1, 2, 3, 4, 5, 6, 7, ...}

1 → N Y Y N N Y Y .....
2 → Y Y Y Y Y Y Y .....
3 → N N N N N N N ....
4 → N Y N Y N Y N ....
⋮

Using Cantor's diagonal arguement we can build a subset that will be different in at least one way from every other subset on N_0 sized list.

$|P(Z^+)| > |Z^+|$

4. Hilberts Hotel – infinite buses w/ infinite number of people

Hotel	R_1	R_2	R_3	R_4	R_5, . . .
B_1	B_1S_1	B_1S_2	B_1S_3	B_1S_4	B_1S_5, . .
B_2	B_2S_1	B_2S_2	B_2S_3	B_2S_4	B_2S_5, . .
B_3	B_3S_1	B_3S_2	B_3S_3	B_3S_4	B_3S_5, . .
⋮					

5. Hilberts Hotel – single bus w/ infinite number of people w/ infinitely long names consist. of A & B

No room BAAA, ...

Room 1 ABBAAAA, ... ← Cantoris diagonalization

Room 2 ABBBBBAA, ...

Room 3 BABABAAA, ...

Room 4 BBBBAAA, ...

⋮

We have uncountable infinity of people w/ infinitely long names consisting of A & B.