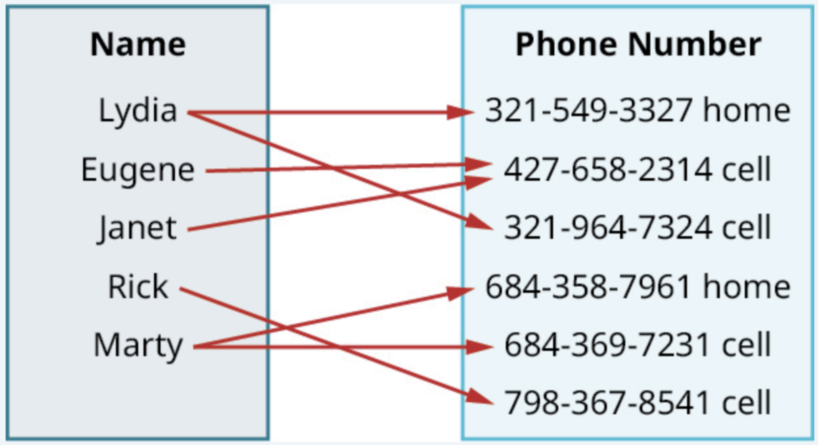
Math_HW_1

1.

**Determining If a Relation Is a Function with Mapping**

Use the mapping in Figure 5.60 to determine whether the relation is a function.




This is not a function because one input (ex. Lydia, Marty) can produce two outputs.

2.

Determine if each of the following equations are functions:

a. $y = x^2 + 1$

b. $y^2 = x + 1$

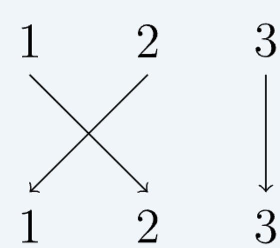
a.yes, any input produces only 1 output.

b.no, any input will produce 2 possible outputs

3.

Which functions are surjective (i.e., onto)?

1. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 3n$.
2. $g : \{1,2,3\} \to \{a,b,c\}$ defined by $g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$.
3. $h : \{1,2,3\} \to \{1,2,3\}$ defined as follows:



Only 3rd function is surjective

4.

Which functions are injective (i.e., one-to-one)?

1. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 3n$.
2. $g : \{1, 2, 3\} \to \{a, b, c\}$ defined by $g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$.
3. $h : \{1, 2, 3\} \to \{1, 2, 3\}$ defined as follows:

$$\begin{tikzcd} 1 \arrow[dr] & 2 \arrow[dl] & 3 \arrow[d] \\ 1 & 2 & 3 \end{tikzcd}$$

1st and 3rd are injective

5.

If $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{1}{x} - 2$, is $g = f^{-1}$?

Yes, because f^-1 = 1/x -2

6.

Find the inverse of the function $f(x) = 2 + \sqrt{x - 4}$.

Y = 2 + sqrt(x-4) → y-2 = sqrt(x-4) → (y-2)^2 = x-4 → (y-2)^2 +4 = x;

7.

Find a formula for the inverse function that gives Fahrenheit temperature as a function of Celsius temperature.

$$C = \frac{5}{9}(F - 32)$$

9/5C = F-32 → 9/5C + 32 = F;

8.

Find the domain and range of the following function:

$g(x) = 2\sqrt{x - 4}$

Domain: [4; +∞)
Range: R+

9.

Find the domain and range of the following function:

$$h(x) = -2x^2 + 4x - 9$$

Domain: R
Range: [-7; -∞)
parabola opens down because a = -2. Vertex -b/2a = - 4/-4 = 1. h(1) = -7
10.

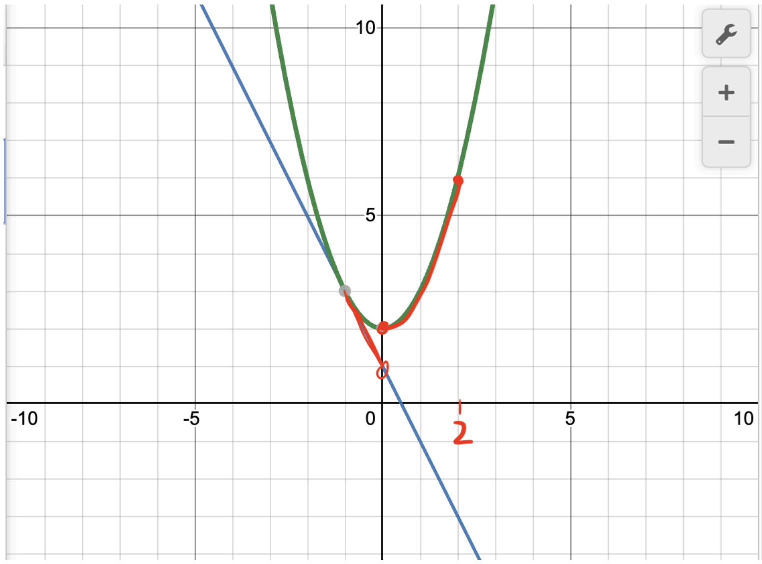
Find the domain of the following functions:

$$f(x) = \frac{x - 4}{x^2 - 2x - 15}$$

Domain: (-∞; -3) U (-3; 5) U (5; +∞)
x^2 - 2x -15 != 0
(x-5)(x+3) !=0
x!= 5; x!= -3
11.

Evaluate the following piecewise-defined function for the given values of $x$, and graph the function:

$$f(x) = \begin{cases} -2x + 1 & -1 \le x < 0 \\ x^2 + 2 & 0 \le x \le 2 \end{cases}$$



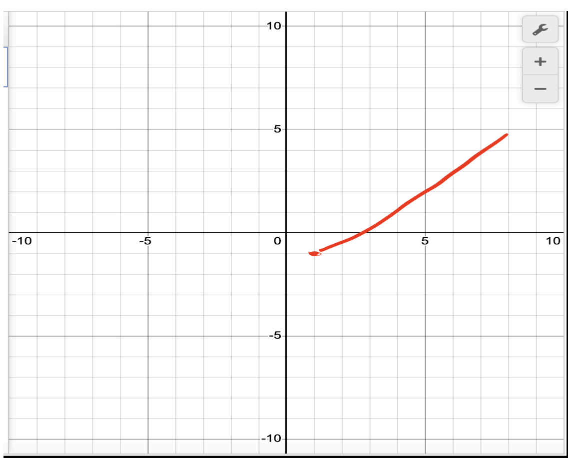

12.

Find the slope of the line that passes through the points $(-1, 2)$ and $(3, -4)$. Plot the points and graph the line.

Slope = (-4-2)/(3-(-1)) = -6/4 = -3/2. The line is decreasing

13.

Graph the line passing through the point $(1, -1)$ whose slope is $m = \frac{3}{4}$.



14.

Given the function $g(t)$ shown in Figure 1.3.1, find the average rate of change on the interval $[-1, 2]$.

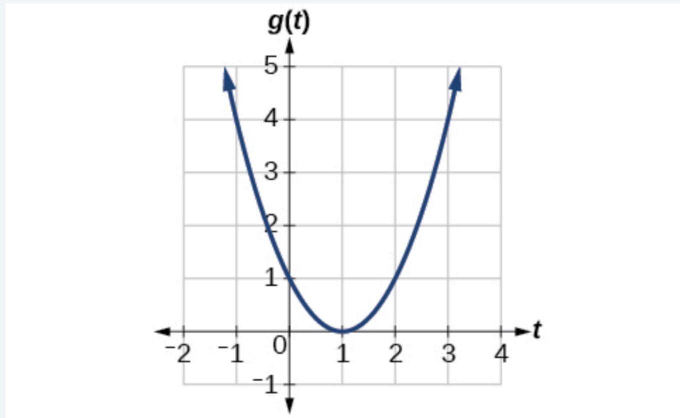



Figure 1.3.1: Graph of a parabola.

Lets take 2 points on provided interval: (-1;4) and (2;1)
Average rate of change = Δy/Δx =(1-4) / (2- (-1)) = -3/3 = 1
15.

Compute the average rate of change of $f(x) = x^2 - \frac{1}{x}$ on the interval $[2, 4]$.

f(2) = 2^2 -½ = 3.5
f(4) = 4^2 - ¼ = 15.75
Average rate of change = (15.75 - 3.5) / 4-2 = 12.25 / 2 = 6.125

16.

Given $f(t) = t^2 - t$ and $h(x) = 3x + 2$, evaluate $f(h(1))$.

First, lets evaluate h(1) = 3 * 1 + 2 = 5.
Next f(5) = 5^2 - 5 = 20

17.

Find the domain of

$$(f \circ g)(x) \text{ where } f(x) = \frac{5}{x-1} \text{ and } g(x) = \frac{4}{3x-2}$$

3x - 2 != 0 —> x != ⅔
4/(3x-2) ! = 1 → 4 != 3x-2 → 6 != 3x → x != 2;

Domain: (-infinity; ⅔) U (⅔; 2) U (2; +infinity);

18.

Find and simplify the functions $(g - f)(x)$ and $\left(\frac{g}{f}\right)(x)$, given $f(x) = x - 1$ and $g(x) = x^2 - 1$. Are they the same function?

(g-f)(x) = g(x) - f(x) = x^2 -1 -x+1 = x^2 -x;

(g/f)(x) = g(x) / f(x) = x^2-1/x-1 = x+1;
These functions are not the same

19.

Write a formula for the graph shown in Figure 1.5.12, which is a transformation of the toolkit square root function.

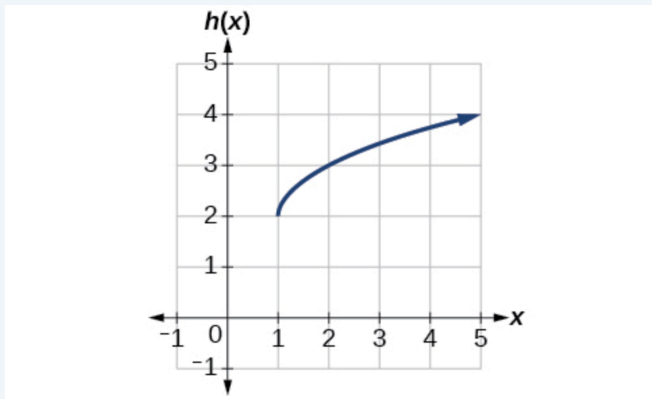



Figure 1.5.12: Graph of a square root function transposed right one unit and up 2.

Sqrt (x-1) +2

20.

Write a formula for a transformation of the toolkit reciprocal function $f(x) = \frac{1}{x}$ that shifts the function's graph one unit to the right and one unit up.

f(x) = 1/(x-1) + 1

21.

Is the function $f(x) = x^3 + 2x$ even, odd, or neither?

f(-x) = (-x)^3 + 2 * (-x) = -x^3 -2x —> the function is definitely not even.
-f (-x) = -(-x^3 -2x) = x^3 + 2x —> this is odd function

22.

Is the function $f(s) = s^4 + 3s^2 + 7$ even, odd, or neither?

f(-s) = (-s)^4 + 3(-s)^2 +7 = s^4 + 3s^2 +7 —> this is even function.

23.

## Point-Slope Form of a Linear Equation

The **point-slope form** of a linear equation takes the form

$$y - y_1 = m(x - x_1)$$

where $m$ is the slope, $x_1$ and $y_1$ are the $x$-and$y$- coordinates of a specific point through which the line passes.

Write the point-slope form of an equation of a line that passes through the points (5, 1) and (8, 7). Then rewrite it in the slope-intercept form.

Slope = 7-1 / 8-5 = 6/3 = 2
Point-slope form: y - 1 = 2(x - 5)
Slope-intercept form: y = 2x - b. Lets solve for b.
1 = 2*5 - b
B =9
Slope intercept form : y=2x - 9;

24

If $f(x)$ is a linear function, and $(3,–2)$ and $(8,1)$ are points on the line, find the slope. Is this function increasing or decreasing?

Slope = 1-(-2) / 8-3 = ⅗. Function is increasing because slope is a positive number.
25.

For the function f shown in Figure 3.4.14, find all absolute maxima and minima.

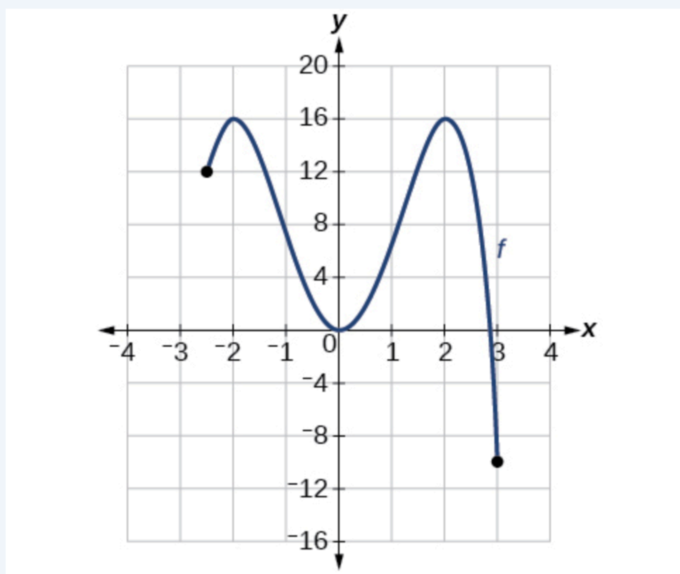



Figure 3.4.14: Graph of a polynomial.

Absolute maxima 16
Absolute minima -10

26.

For the function f whose graph is shown in Figure 3.4.9, find all local maxima and minima.

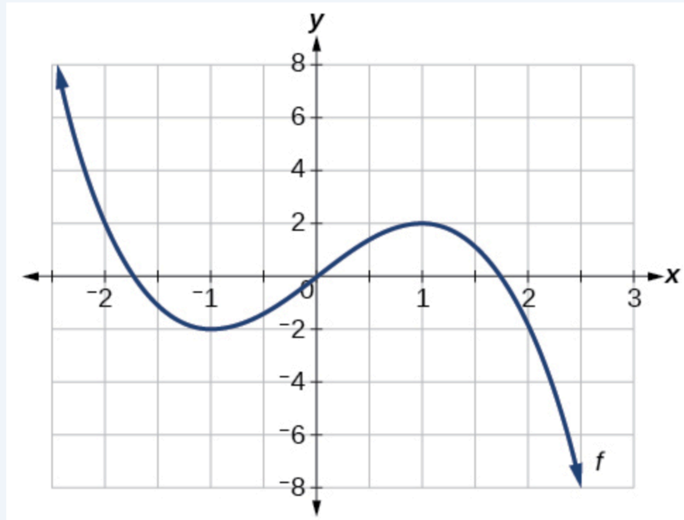



Figure 3.4.9: Graph of a polynomial.

Local maxima: (1;2)
Local minima: (-1;-2)

27.

Given the functions below, identify the functions whose graphs are a pair of parallel lines and a pair of perpendicular lines.

$$f(x) = 2x + 3 \qquad h(x) = -2x + 2$$
$$g(x) = \tfrac{1}{2}x - 4 \qquad j(x) = 2x - 6$$

f(x) and j(x) are parallel
g(x) and h(x) are perpendicular

28.

$$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$$

Y = 7 - 2x;
X - 2(7-2x) = 6 → x - 14 +4x = 6 → 5x = 20 → x = 4;

Y = 7-2*4= -1.
Solution (4; -1) works for both equations

29. $\begin{cases} 4x + 2y = 4 \\ 6x - y = 8 \end{cases}$

6x -8 = y

4x + 2(6x -8) = 4→ 4x + 12x - 16 = 4 → 16x = 20 → x = 20/16 = 5/4
Y = 6*5/4 -8 = -1/2

Lets test (5/4; -1/2):
4*(5/4) +2 *(-½) =4
5-1= 4

6*(5/4) -(-½) = 8 → 15/2 +½ =8 → 7.5 +0.5 = 8

Solution (5/4; -½)

30.

**Definitions: Forms of Quadratic Functions**

A quadratic function is a function of degree two. The graph of a *quadratic function* is a parabola.

- The *general form* of a quadratic function is $f(x) = ax^2 + bx + c$ where $a$, $b$, and $c$ are real numbers and $a \neq 0$.
- The standard form of a quadratic function is $f(x) = a(x - h)^2 + k$.
- The vertex $(h, k)$ is located at

$$h = -\frac{b}{2a},\ k = f(h) = f(\frac{-b}{2a}).$$

### How To: Given a quadratic function in general form, find the vertex of the parabola.

1. Identify $a$, $b$, and $c$.
2. Find $h$, the x-coordinate of the vertex, by substituting $a$ and $b$ into $h = -\frac{b}{2a}$.
3. Find $k$, the y-coordinate of the vertex, by evaluating $k = f(h) = f\left(-\frac{b}{2a}\right)$.

31.

Find the vertex of the quadratic function $f(x) = 2x^2 - 6x + 7$. Rewrite the quadratic in standard form (vertex form).

Vertex: -b/2a = - (-6)/2*2 = 6/4 = 1.5
Standard form: a(x-h)^2+k
k= 2(1.5)^2 - 6*1.5 +7 = 2.5

Standard form: 2(x - 1.5)^2 + 2.5

32.

**Given a quadratic function, find the domain and range.**

1. Identify the domain of any quadratic function as all real numbers.
2. Determine whether $a$ is positive or negative. If $a$ is positive, the parabola has a minimum. If $a$ is negative, the parabola has a maximum.
3. Determine the maximum or minimum value of the parabola, $k$.
4. If the parabola has a minimum, the range is given by $f(x) \geq k$, or $[k, \infty)$. If the parabola has a maximum, the range is given by $f(x) \leq k$, or $(-\infty, k]$.

Find the domain and range of $f(x) = -5x^2 + 9x - 1$.

The parabola opens down because a is negative. Vertex -9/2*(-5) = 0.9
f(0.9) = -5 * (0.9)^2 + 9*0.9 -1 = -4.05 +8.1 -1 = 3.05
Domain: R
Range: (-infinity; 3.05]

33.

**Given a quadratic function $f(x)$, find the y- and x-intercepts.**

1. Evaluate $f(0)$ to find the y-intercept.
2. Solve the quadratic equation $f(x) = 0$ to find the x-intercepts.

Find the y- and x-intercepts of the quadratic $f(x) = 3x^2 + 5x - 2$.

Y-intercept: f(0) = -2
X-intercept: 3x^2+5x-2 = 0
(3x-1)(x+2) = 0
3x-1 = 0        x+2 = 0
X = ⅓            x = -2

X- inetrcepts: (⅓; 0) and (-2;0)
34.

Solve the inequality, graph the solution set on a number line and show the solution set in interval notation:

a. $-1 \leq 2x - 5 < 7$
b. $x^2 + 7x + 10 < 0$
c. $-6 < x - 2 < 4$

a. -1 + 5 <= 2x < 7+5 → 4 <=2x<12 —> 2<=x< 6   x is [2; 6)

b. (x + 5)(x+2) < 0 —>
   x+5 < 0 AND x+2 > 0 → x< -5 x> -2 —> no intesection
   x+5 > 0 AND x+2 < 0 → x >-5 x < -2 → x is (-5; -2)

c. -4<x<6 —> (-4; 6)

35.

Solve the inequality and graph the solution set. State the answer in both set builder notation and in interval notation.

$$10 - (2y + 1) \leq -4(3y + 2) - 3$$

10 -2y-1 <= -12y -8 -3
9 -2y <=-12y -11
10y <= -20
Y <= -2 —-> (-infinity; -2]

36.

Solve $x(x + 3)^2(x - 4) < 0.$

(x+3)^2 will always be non-negative

So either x < 0 and then x-4 > 0 → x<0 and x>4 - has no solution OR
x>0 and x-4 < 0 → x >0 and x < 4 → solution 0< x <4 → (0; 4)

37.

Solve: $2x^4 > 3x^3 + 9x^2.$

2x^4 -3x^3 +9x^2 > 0
x^2 (2x^2 - 3x -9) > 0
x^2 will always be non-negative
2x^2 -3x -9 > 0
(2x+3)(x-3) >0

Either both 2x+3 > 0 and x-3 > 0- → x>-3/2 x > 3
OR noth 2x+3 < 0 and x-3 < 0 → x< -3/2 x < 3
(-infinity; -3/2) U (3: + infinity)

38.

Given the function $f(x) = -\frac{1}{2}|4x - 5| + 3$, determine the $x$-values for which the function values are negative.

$-\frac{1}{2}|4x-5| + 3 < 0$
$-\frac{1}{2}|4x-5| < -3$
$|4x-5| > 6$

$4x-5 > 6$ and $4x-5 < -6$
$4x > 11$ and $4x < -1$
$x > 11/4$ and $x < -\frac{1}{4}$

(-infinity; $-\frac{1}{4}$) U (11/4; + infinity)

39.

Solve: $13 - 2|4x - 7| \leq 3$.

$-2|4x-7| <= -10$
$|4x-7| >= 5$
$4x-7 <= -5$ and $4x-7 >= 5$
$x <= \frac{1}{2}$ and $x >= 3$