

1. **Exercise 1.3**

- If  $A \in \mathcal{G}_1$  and is different from empty set and the whole set, then  $A^c$  is closed and does not belong to  $\mathcal{G}_1$ . Thus,  $\mathcal{G}_1$  is not an algebra.
- If  $a = b$  then  $\emptyset \in \mathcal{G}_2$ . If  $b \leq a$ , then  $\mathbb{R} \in \mathcal{G}_2$ . Thus by definition of  $\mathcal{G}_2$ , it is an algebra.
- If  $a = b$  then  $\emptyset \in \mathcal{G}_2$ . If  $b \leq a$ , then  $\mathbb{R} \in \mathcal{G}_2$ . Thus by definition of  $\mathcal{G}_2$ , it is a  $\sigma$ -algebra.

2. **Exercise 1.7**

- $\mathcal{A}$  is a  $\sigma$ -algebra. Thus, by definition it contains  $\{\emptyset, X\}$
- $\mathcal{A}$  is a  $\sigma$ -algebra. By definition it is a family of subsets of  $X$ , thus,  $\mathcal{A} \subset \mathcal{P}(X)$ .

3. **Exercise 1.10**

- $\emptyset \in \mathcal{S}_\alpha \quad \forall \alpha \Rightarrow \emptyset \in \cap_\alpha \mathcal{S}_\alpha$
- If  $A \in \cap_\alpha \mathcal{S}_\alpha$ , then  $A \in \mathcal{S}_\alpha$  and  $A^c \in \mathcal{S}_\alpha \quad \forall \alpha$ , hence,  $A^c \in \cap_\alpha \mathcal{S}_\alpha$
- If  $A_n \in \cap_\alpha \mathcal{S}_\alpha$ , then  $A_n \in \mathcal{S}_\alpha$  and  $\cup_n A_n \in \mathcal{S}_\alpha \quad \forall \alpha$ , hence,  $\cup_n A_n \in \cap_\alpha \mathcal{S}_\alpha$

4. **Exercise 1.22**

- $A \subset B \Rightarrow B = A \cup (B \cap A^c)$ . Since  $A$  and  $B \cap A^c$  are disjoint,  $\mu(B) = \mu(A) + \mu(B \cap A^c)$ . Thus,  $\mu(A) \leq \mu(B)$ .
- $A = (A \cap B^c) \cup (A \cap B)$  and  $B = (B \cap A^c) \cup (B \cap A)$ .  $\mu(A) + \mu(B) = \mu(A \cap B) + \mu(A \cap B) + \mu(A \cap B^c) + \mu(A^c \cap B) = \mu(A \cap B) + \mu(A \cup B)$ . Thus,  $\mu(A) + \mu(B) \geq \mu(A \cup B)$ .

5. **Exercise 1.23**

- $\lambda(\emptyset) = \mu(\emptyset \cap B) = \mu(\emptyset) = 0$ .
- $\lambda(\cup_n \mathcal{A}_n) = \mu((\cup_n \mathcal{A}_n) \cap B) = \mu(\cup_n (\mathcal{A}_n \cap B)) = \sum_n \mu(\mathcal{A}_n \cap B) = \sum_n \lambda(\mathcal{A}_n)$ . ( $\{\mathcal{A}_n\}$  disjoint  $\rightarrow \{\mathcal{A}_n \cap B\}$  disjoint.)

6. **Exercise 1.26**

- $\mu(\cap_n \mathcal{A}_n) = \mu((\cup_n \mathcal{A}_n^c)^c) = \mu(X) - \mu(\cup_n \mathcal{A}_n^c) = \mu(X) - \lim_{n \rightarrow \infty} \mu(\mathcal{A}_n^c) = \mu(X) - \lim_{n \rightarrow \infty} (\mu(X) - \mu(\mathcal{A}_n)) = \lim_{n \rightarrow \infty} \mu(\mathcal{A}_n)$ . By (i).