"DSGE"
Problem Set 1

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1. **Exercise 1**

$$
\begin{aligned}
&\frac{1}{e^{z_t}k_t^{\alpha} - Ae^{z_t}k_t^{\alpha}} = \beta E_t\left\{\frac{\alpha e^{z_{t+1}}k_{t+1}^{\alpha-1}}{e^{z_{t+1}}k_{t+1}^{\alpha} - Ae^{z_{t+1}}k_{t+1}^{\alpha}}\right\}\\
&\frac{1}{e^{z_t}k_t^{\alpha} - Ae^{z_t}k_t^{\alpha}} = \frac{\beta\alpha}{k_{t+1}(1-A)}\\
&\frac{1}{e^{z_t}k_t^{\alpha}(1-A)} = \frac{\beta\alpha}{k_{t+1}(1-A)}\\
&k_{t+1} = Ae^{z_t}k_t^{\alpha},\ \text{where } A = \beta\alpha\ .
\end{aligned}
$$

2. **Exercise 2**

$$
\begin{aligned}
&c_t = (1-\tau)\left[w_t\ell_t + (r_t-\delta)\,k_t\right] + k_t + T_t - k_{t+1}\\
&1/c_t = \beta E_t\left\{1/c_{t+1}\left[(r_{t+1}-\delta)\,(1-\tau)+1\right]\right\}\\
&-\frac{a}{1-\ell_t} = \frac{1}{c_t}w_t(1-\tau)\\
&r_t = \alpha e^{z_t}k_t^{\alpha-1}\ell_t^{1-\alpha}\\
&w_t = (1-\alpha)e^{z_t}k_t^{\alpha}\ell_t^{-\alpha}\\
&\tau\left[w_t\ell_t + (r_t-\delta)\,k_t\right] = T_t
\end{aligned}
$$

3. **Exercise 3**

$$
\begin{aligned}
&c_t = (1-\tau)\left[w_t\ell_t + (r_t-\delta)\,k_t\right] + k_t + T_t - k_{t+1}\\
&c_t^{-\gamma} = \beta E_t\left\{1/c_{t+1}\left[(r_{t+1}-\delta)\,(1-\tau)+1\right]\right\}\\
&-\frac{a}{1-\ell_t} = c_t^{-\gamma}w_t(1-\tau)\\
&r_t = \alpha e^{z_t}k_t^{\alpha-1}\ell_t^{1-\alpha}\\
&w_t = (1-\alpha)e^{z_t}k_t^{\alpha}\ell_t^{-\alpha}\\
&\tau\left[w_t\ell_t + (r_t-\delta)\,k_t\right] = T_t
\end{aligned}
$$

4. **Exercise 4**

$$
\begin{aligned}
&c_t = (1-\tau)\left[w_t \ell_t + (r_t - \delta)\, k_t\right] + k_t + T_t - k_{t+1} \\
&c_t^{-\gamma} = \beta E_t \left\{1/c_{t+1}\left[(r_{t+1} - \delta)(1-\tau) + 1\right]\right\} \\
&-a(1-\ell_t)^{-\xi} = c_t^{-\gamma} w_t (1-\tau) \\
&r_t = \frac{e^{z_t}}{\eta}(\alpha k_t^\eta + (1-\alpha)\ell_t^\eta)^{\frac{1}{\eta}-1} \alpha\eta k_t^{\eta-1} \\
&w_t = \frac{e^{z_t}}{\eta}(\alpha k_t^\eta + (1-\alpha)\ell_t^\eta)^{\frac{1}{\eta}-1} (1-\alpha)\eta \ell_t^{\eta-1} \\
&\tau\left[w_t \ell_t + (r_t - \delta)\, k_t\right] = T_t
\end{aligned}
$$

5. **Exercise 5**

$$
\begin{aligned}
&c_t = (1-\tau)\left[w_t + (r_t - \delta)\, k_t\right] + k_t + T_t - k_{t+1} \\
&c_t^{-\gamma} = \beta E_t \left\{c_{t+1}^{-\gamma}\left[(r_{t+1} - \delta)(1-\tau) + 1\right]\right\} \\
&r_t = \alpha k_t^{\alpha-1}(e^{z_t})^{1-\alpha} \\
&w_t = k_t^\alpha e^{z_t}(1-\alpha)(e^{z_t})^{-\alpha} \\
&\tau\left[w_t + (r_t - \delta)\, k_t\right] = T_t
\end{aligned}
$$

The steady state is given by the variables $\{c, k, w, r, \tau\}$ that solve the following system of equations.

$$
\begin{aligned}
&c = (1-\tau)\left[w + (r - \delta)\, k\right] + T \\
&c^{-\gamma} = \beta E \left\{c^{-\gamma}\left[(r - \delta)(1-\tau) + 1\right]\right\} \\
&r = \alpha k^{\alpha-1}(e^{z})^{1-\alpha} \\
&w = k^\alpha e^{z}(1-\alpha)(e^{z})^{-\alpha} \\
&\tau\left[w + (r - \delta)\, k\right] = T
\end{aligned}
$$

The above described sustem can be analytically solved. The solutions are given by

$$
\begin{aligned}
&r = \frac{1-\beta}{\beta(1-\tau)} + \delta \\
&k = \left(\frac{r}{\alpha}\right)^{\frac{1}{\alpha-1}} \\
&w = k^\alpha(1-\alpha) \\
&c = w + (r-\delta) * k \\
&T = \tau(w + (r-\delta) * k)
\end{aligned}
$$

6. **Exercise 6**

$$
\begin{aligned}
&c_t = (1-\tau)\left[w_t\ell_t + (r_t-\delta)\,k_t\right] + k_t + T_t - k_{t+1} \\
&c_t^{-\gamma} = \beta E_t\left\{c_{t+1}^{-\gamma}\left[(r_{t+1}-\delta)\,(1-\tau)+1\right]\right\} \\
&-a(1-\ell_t)^{-\xi} = c_t^{-\gamma} w_t(1-\tau) \\
&r_t = \alpha k_t^{\alpha-1}(\ell_t e^{z_t})^{1-\alpha} \\
&w_t = k_t^{\alpha} e^{z_t}(1-\alpha)(\ell_t e^{z_t})^{-\alpha} \\
&\tau\left[w_t\ell_t + (r_t-\delta)\,k_t\right] = T_t
\end{aligned}
$$

The steady state is given by the variables $\{c, k, \ell, w, r, \tau, z\}$ that solve the following system of equations.

$$
\begin{aligned}
c &= (1-\tau)\left[w\ell + (r-\delta)\,k\right] + k + T - k \\
1 &= \beta\left\{(r-\delta)\,(1-\tau)+1\right\} \\
-a(1-\ell)^{-\xi} &= c_t^{-\gamma} w(1-\tau) \\
r &= \alpha k^{\alpha-1}(\ell_t e^{z})^{1-\alpha} \\
w &= k^{\alpha} e^{z}(1-\alpha)(\ell e^{z})^{-\alpha} \\
\tau\left[w\ell + (r-\delta)\,k\right] &= T
\end{aligned}
$$