Suggested problems for MATH 31.1 LT2 Review

1. Find the derivative:

   (a) Find $\dfrac{dy}{dx}$ if $x^2y^2 - \cos y = \sin xy$.

   (b) Find $\dfrac{dy}{dx}$ if $x^2e^{y^2} = \tan\left(2^{x-y}\right)$

2. Show that $\dfrac{d}{dx}\left(\tan^{-1}\left(x - \sqrt{1+x^2}\right)\right) = \dfrac{1}{2x^2+2}$

3. Find $\dfrac{d^8}{dx^8}\left(\ln\left(2x-1\right)\right)$.

4. Find the derivative of the given functions.

   (a) $y = \left(\cos^{-1}\left(\sin x^2\right)\right)^2$

   (b) $f(x) = x^2 \sec^{-1}\left(e^{x^3}\right)$

   (c) $g(x) = \dfrac{\log_2\left(\sin^2 x\right)}{\log_2\left(\cos^2 x\right)}$

   (d) $y = \ln\left(3^{2x} + 4^{x^2}\right)$

   (e) $f(x) = \ln\left(\ln\left(\ln(2x-1)\right)\right)$

   (f) $y = \left(\ln x\right)^{\ln x}$

   (g) $y = \sqrt[3]{x}e^{x-2x^2}\sqrt{3x-2}$ (use log diff)

   (h) $y = \left(\sin^{-1} x\right)^{\sin x}$

5. Let a and b be any nonzero constants. Show that if $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ then $y^3 \cdot \dfrac{d^2y}{dx^2}$ is a constant.

6. Find equations of the tangent lines to the graph of

   $$2\left(x^2+y^2\right)^2 = 25\left(x^2-y^2\right)$$

   at the points where the x-coordinate is 3.

7. Use differentials to estimate $\sqrt[4]{(9998)^3}$.

8. Use linear approximation to estimate $\dfrac{200}{100.1} + \sqrt{100.1}$.

9. A window has the shape of a square surmounted by a semicircle. The base of the window is measured as having width 60 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum error possible in computing the area of the window.

10. A spherical tank with radius 2 m. has a coat of paint that is 0.03 cm thick. Use differentials to estimate the amount of the paint.

11. A cone has base radius r and height h.

    (a) Find the rate of change of the volume with respect to the height if the radius is constant.

(b) Find the rate of change of the volume with respect to the radius if the height is constant.

12. The volume of a cube is increasing at a rate of 5 cm^3/min. How fast is the surface area increasing when the length of an edge is 10 cm?

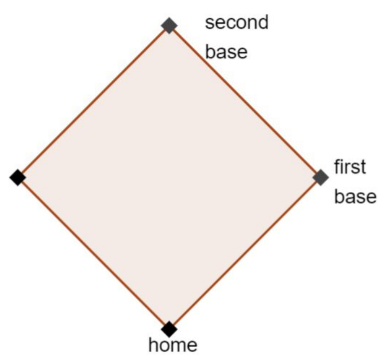
13. Two ants X and Y are at opposite corners A and C, respectively, of square ABCD whose edges measure 12 cm each. X and Y start moving along edges AB and CD, at rates 1 cm/sec and 1/2 cm/sec, respectively. Find the rate at which the distance between them is changing 2 seconds later.

14. A balloon is rising at a constant speed of 2 m/s. A boy is cycling along a straight road at a speed of 5 m/s. When he passes under the balloon, it is 15 m above him. How fast is the distance between the boy and the balloon increasing 3 s later?

15. The angle of elevation of the sun is decreasing at a rate of 0.25 rad/h. How fast is the shadow cast by a 400-m-tall building increasing when the angle of elevation of the sun is $\pi/6$?

16. Two cars are running on two parallel roads in opposite directions at 10 m/sec and 15 m/sec, respectively. If the roads are 75 m apart, how fast is the distance between them increasing four seconds after they pass each other?

17. A baseball diamond is a square with side 26 m. A batter hits the ball and runs toward the first base with speed of 8 m/s. At what rate is his distance from the second base decreasing when he is halfway to the first base?



18. Two ants E and T are inside a rectangular box with dimensions 120 cm × 160 cm × 100 cm. Refer to the figure below. Ant E starts at corner A and walks along the edge AB at the rate of 20 cm/sec, while Ant T starts at corner C and walks along the edge CD at the rate of 15 cm/sec. Find the rate at which the two ants are getting closer to each other when Ant E is halfway between A and B.