CPSC 313 (Winter 2014) L01

University of Calgary
Faculty of Science
Midterm

March 6, 2014.
Time available: 120 minutes.
No books or calculators are permitted (two lettersized sheets of notes are permitted).

Write answers in this booklet only.
Do **not** open this exam until you are told to do so.

Name: ______________________________

Lab section:

☐	☐	☐	☐
T01 Mon 14:00	T03 Mon 17:00	T02 Wed 14:00	T04 Wed 17:00

There are 3 (three) problems in total. There are 34 points (31 points and 3 bonus points) in total. Full answer is **30** points.

For your convenience, the last page (page 14) contains a collection of information.

You may use the blank pages at the end of the exam if you need more space for your answers. Please indicate clearly when your answers are continued on these pages.

Please note that points may be deducted for untidy handwriting.

Problem	Possible score	Score
1	24	
2	7	
3	3 bonus	
Total	31 + 3 bonus	

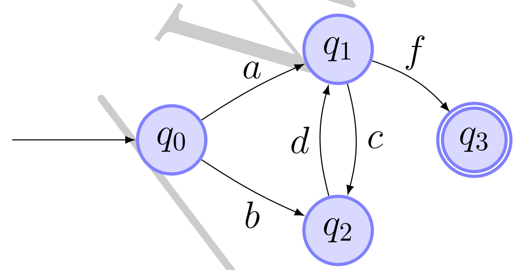
1. Short answers (24 points)

1. Give a regular expression for the language $L = \{w \in \{a, b, c\}^* \mid |w|_a = 1 \text{ and } |w|_b = 1\}$. Briefly justify.

2. Give a grammar for the language $L = \{w \in \{a\}^* \mid |w| \not\equiv 1 \pmod 3\}$. Briefly justify.

3. Give a **DFA** accepting the language $L = \{w \in \{c, d\}^* \mid cd \text{ is } \textbf{not} \text{ a substring of } w\}$. Briefly justify.

4. Give a regular expression for the language accepted by the following NFA. Briefly justify.

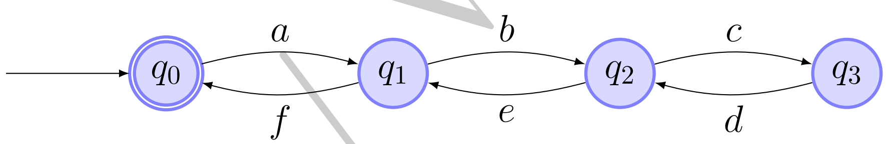


5. Give an **NFA** accepting the regular language $L = \{a^i b^j \mid i \neq 2 \text{ and } i, j \geq 0\}$. Briefly justify.

6. Give a **smallest possible DFA** for the language $L(\boldsymbol{r})$ generated by the following regular expression $\boldsymbol{r} = (\boldsymbol{a}^*\boldsymbol{b} + \boldsymbol{b}^*\boldsymbol{a})^*$. Briefly justify.

7. Give a context-free grammar for the language $L = \{a^i b^j c^k \mid i > j \text{ or } i > k\}$. Briefly justify.

8. Give a regular expression for the language accepted by the following NFA. Briefly justify.



9. Give an infinite regular language L_{reg} and an infinite non-regular language L_{notreg} satisfying that $L_{\text{notreg}} \subseteq L_{\text{reg}}$. Briefly justify.

10. Characterize the language accepted by the grammar containing the following 17 productions.

$$
\begin{array}{lcl}
S & \rightarrow & PXRXTY \\
X & \rightarrow & KNN \\
Y & \rightarrow & NNNN \\
K & \rightarrow & \texttt{9} \mid \texttt{8} \mid \texttt{7} \mid \texttt{6} \mid \texttt{5} \mid \texttt{4} \mid \texttt{3} \mid \texttt{2} \mid \texttt{1} \\
N & \rightarrow & K \mid \texttt{0} \\
P & \rightarrow & \texttt{(} \\
R & \rightarrow & \texttt{)} \\
T & \rightarrow & \texttt{-}
\end{array}
$$

The alphabet Σ is comprised of the following 32 printable ASCII characters,

`!"#$%&'()*+,-./0123456789:;<=>?@`

Briefly justify.

11. Consider the language $L = L(G)$ generated by the grammar G containing the following 6 productions:

$$S \to ASB \mid AB;\ AB \to AAB;\ abb \to ab;\ A \to a;\ B \to b.$$

Give a grammar G' that contains **at most 3** productions, and that accepts the same language as G (i.e. $L(G) = L(G')$). Briefly justify.

12. Give two context-free languages L_1 and L_2 such that $L_1 \cap L_2 = \{a^i b^i c^i d^i e^i \mid i \geq 0\}$. Briefly justify.

2. True or False (7 points)

Answer True or False. No justification required. The languages in questions 1, 2, 3, 8, and 11 are over the alphabet $\Sigma = \{a, b\}$. In question 12, $\boldsymbol{r}$ and $\boldsymbol{s}$ denote regular expressions.

(*Note:* There are 15 questions. One incorrect answer will be ignored.)

	Question	True	False
1	If L is context-free, then so is L^R		
2	If $L = L^*$ and $w \in L$, then $ww \in L$		
3	If L is regular and L' is non-regular, then $L \cap L'$ is non-regular		
4	The smallest DFA accepting a language consisting of exactly 3 strings contains exactly 5 states		
5	The grammar $S \to aSb \mid bSa \mid \lambda$ generates a context-free language		
6	$\{\lambda\}^* = \{\,\}$		
7	$\{\,\}^* = \{\,\}$		
8	If L is regular and $L' \subseteq L$, then L' is also regular		
9	The grammar $S \to SaSaS \mid \lambda$ generates the language $\{w \in \{a\}^* \mid \lvert w\rvert = 2i \text{ for some } i \geq 0\}$		
10	There exists a finite language that is context-free		
11	If L is regular, then so is $L' = \{w \in L \mid \lvert w\rvert \text{ is even}\}$		
12	$\boldsymbol{r}(\boldsymbol{sr})^* = (\boldsymbol{rs})^*\boldsymbol{r}$		
13	All finite languages are context-free		
14	If $L = \{a^i b^i \mid i \geq 0\}$ then $L \setminus L^2 = \{\,\}$		
15	The language $L = \{w \in \{a, b\}^* \mid w = w^R\}$ is non-regular		

3. DFA construction (3 bonus points)

Consider the alphabet $\Sigma = \{0, 1\}$. We can interpret any string $w \in \Sigma^*$ as a non-negative integer written in binary, where we ignore possible leading zeroes. Any string $w \in \{0\}^*$ represents the integer zero. For instance, the string $w_1 = 010011$ represents the integer $16 + 2 + 1 = 19$, $w_2 = 0000101010$ the integer $32 + 8 + 2 = 42$, $w_3 = 00$ the integer 0, and $w_4 = \lambda$ the integer 0. In general, if $w = w_1 w_2 \cdots w_n$, then w represents the integer $\sum_{i=1}^{n} 2^{n-i} w_i$.

We say a string w is *divisible by 3* if the integer w represents is divisible by 3.

Give a **DFA** that accepts exactly the strings in Σ^* that are divisible by 3. Briefly justify your answer.

Note: Full points to correct and justified solutions. No points to partial solutions.

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Theorem 1 (Closure Properties for Regular languages)

Let L_1 and L_2 be regular languages over the same alphabet Σ. Then the following nine languages are regular as well.

1. $\overline{L_1} = \{w \in \Sigma^* \mid w \notin L_1\}$
2. $L_1 \cup L_2$
3. $L_1 \cap L_2$
4. L_1^*
5. $L_1 L_2$
6. $L_1^{\mathrm{R}} = \{w \in \Sigma^* \mid w^R \in L_1\}$
7. $\mathsf{prefix}(L_1) = \{u \in \Sigma^* \mid \exists v \in \Sigma^* \text{ such that } uv \in L_1\}$
8. $\mathsf{suffix}(L_1) = \{v \in \Sigma^* \mid \exists u \in \Sigma^* \text{ such that } uv \in L_1\}$
9. $h(L_1)$, where $h : \Sigma \to \Gamma^*$ is a homomorphism and Γ is an alphabet.

Theorem 2 (Pumping Lemma for Regular Languages)

Let L be a regular language. Then there

$$\textit{exists an integer } m \in \mathbb{Z} \textit{ such that for all } w \in L \text{ with } |w| \geq m,$$

there exist strings x, y, and z, such that $w = x \cdot y \cdot z$ and

1. $|x \cdot y| \leq m$,

2. $|y| \geq 1$, *and*

3. for all $i \geq 0$, $x \cdot y^i \cdot z \in L$.
