CPSC 313 (Fall 2011) L01

University of Calgary

Faculty of Science

Final Examination

December 14, 2011.

Time available: 180 minutes.

No books or calculators are permitted (two 8.5×11 sheets of notes are permitted).

Write answers in this booklet only.

Do not open this exam until you are told to do so.

Name: ____

There are 6 (six) problems in total. There are 31 points (28 points and 3 bonus points) in total. Full answer is 26 points.

The exam contains 18 pages.

You may use the blank pages at the end of the exam if you need more space for your answers. Please indicate clearly when your answers are continued on these pages.

Please note that points may be deducted for untidy handwriting.

Problem	Possible score	Score
1	8	
2	4	
3	4	
4	4	
5	5	
6	3 + 3 bonus	
Total	28 + 3 bonus	

1. Short answers (8 points)

1. Give a **DFA** for the language $L = \{\lambda\}$.

2. Give a PDA for the language $L = \{a^i b^j \mid i \neq j\}$.

3. Describe the language generated by the grammar $S \to aSb$; $S \to \lambda$; $ab \to ba$. Your description should be as simple and succinct as possible.

4. Give a grammar in Chomsky Normal Form for the language $\{a^i b^j c^j \mid i \geq 0, j > 0\}$.

2. Matching parentheses (4 points)

Consider we have two types of parentheses: $\Sigma = \big\{(,),[,]\big\}$, for each type an opening parenthesis and a closing parenthesis. We say a string over Σ is *well-balanced and well-nested* if we can connect every opening parenthesis with a unique later closing parenthesis so that no two connecting lines cross. For instance, the string $w = ()[()[]]([])$ is *well-balanced and well-nested*, as shown here,

$$(\;) \quad [\; (\;) \; [\;] \;] \quad (\; [\;] \;)$$

Let L be the language of all well-balanced and well-nested strings over Σ. Six examples of strings **not** in L are

$$([)] \qquad ([] \qquad)(\qquad ()()[()[)] \qquad [) \qquad []][[[[$$

1. Give a context-free grammar generating L.

2. Using your grammar, show a derivation of the string $()[()]$.

3. Argue why L is not regular. (A complete proof is not required.)

3. Grammars in normal forms (4 points)

We say a grammar $G = (V, T, S, P)$ is on **3NF** (3-normal form) if every rule is on one of the two forms

$$A \to BCD \qquad\qquad A \to a$$

where $a \in \Sigma$ is a terminal symbol and $A, B, C, D \in V$ are variables.

1. Give a grammar G_1 in 3NF that generates the language $L = \{ab^ic^i \mid i \geq 0\}$.

2. Give an example of a context-free language that can **not** be generated by a grammar in 3NF.

3. Give a characterization of all languages L that can be generated by grammars in 3NF. Make your characterization as succinct and precise as possible. Argue that your characterization is correct.

4. Turing machines using finite space (4 points)

The tape cells of Turing machine are labelled $\ldots, -3, -2, -1, 0, 1, 2, \ldots$, with the tape head pointing to cell number 1 in the initial configuation. We say a Turing machine M uses *finite amount of space* if for every input w, there exist two integers ℓ_w and r_w such that the tape head never moves left of cell number ℓ_w, nor right of cell number r_w.

1. Show that if M is Turing machine that uses finite amount of space, then $L(M)$ is recursive. (*Hint:* Consider constructing a Turing machine M' that simulates M and that always halts.)

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5. Language classification (5 points)

Answer **exactly one** of the four possible answers REG, CF, REC, or RE for each of the following languages. Here REG means regular, CF context-free, REC recursive, and RE recursive enumerable. No justification required.

	Question	REG	CF	REC	RE
1	$\{ww^R \mid w \in \{a,b\}^*\}$				
2	$\{ww^Rw \mid w \in \{a,b\}^*\} \setminus \{w^Rww^R \mid w \in \{a,b\}^*\}$				
3	$\{a^i \mid i = n^2$ for some $n \geq 0\}$				
4	$\{a^i \mid i$ is composite$\}$				
5	$\{a^i \mid i$ is odd$\}$				
6	$\{a,b\}^* \setminus \{a^ib^ia^jb^j \mid i,j \geq 0\}$				
7	$\{\langle i,w\rangle \mid$ The i^{th} TM M_i halts on input $w\}$				
8	$\{w \in \{a,b\}^* \mid$ for all $u \in \{a,b\}^6$, u is a substring of $w\}$				
9	$\{\langle w_1,w_2\rangle \mid$ there is a TM that halts on both w_1 and $w_2\}$				
10	$\{\langle M,u\rangle \mid$ TM M halts on some string $w \in \{a,b\}^*$ of which u is a substring				

6. Recursive enumerable (3 points + 3 bonus points)

Let $L_a = \{a^i \mid i \text{ is odd}\}$, and consider the language

$$\mathsf{NotOddAs} \;=\; \big\{ \langle M \rangle \mid L(M) \not\subseteq L_a \;\&\; M \text{ is a TM} \big\}$$

1. Show that the language NotOddAs is recursive enumerable.

2. Show that the language NotOddAs is not recursive by reducing any one of the two languages Member or Halt, to NotOddAs.

   (*Hint:* You can reduce the Halting problem to the NotOddAs problem, Halt $\leq_m$ NotOddAs.)

   (Partial points only to near-complete solutions. No partial points to sketches.)

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