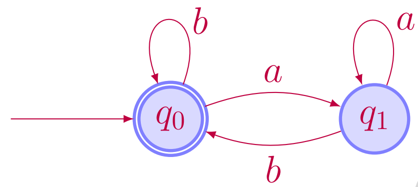
CPSC 313 (Winter 2017) L01

Midterm solution sketches

A 150 minutes exam held on March 2, 2017

1. Problems

1. Give a **DFA** for the language $L(\boldsymbol{r})$ where $\boldsymbol{r} = (\boldsymbol{a}^*\boldsymbol{b})^*$.



2. Give a **right-linear** grammar for the language
$L = \{w \in \{a, b, c\}^* \mid \; w \text{ contains } abc \text{ as a substring}\}$.

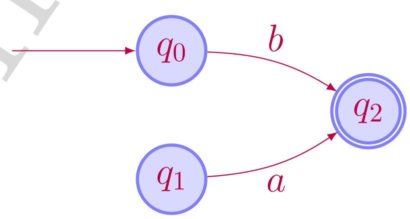
$S \to aS \mid bS \mid cS \mid A; \quad A \to abcD; \quad D \to aD \mid bD \mid cD \mid \lambda$

3. We have seen in class that if L is regular, then so is $\mathsf{suffix}(L)$.

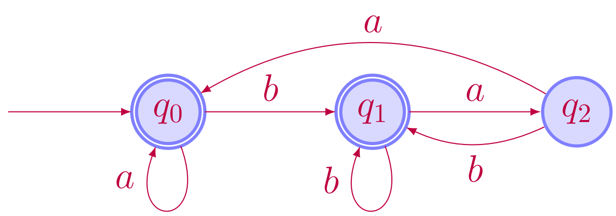
Beeblebrox has created his own "proof" of this fact. But his "proof" is flawed. It contains one critical flaw. Precisely and succinctly identify the critical flaw, explain why it is a flaw, and explain how to fix Beeblebrox's proof.

Proof Given an NFA $N_1 = (Q, q_0, F, \Sigma, \delta)$, we construct a new NFA $N_2 = (Q', q_0', F, \Sigma, \delta')$ as follows. We add a new state $q_0' \notin Q$ to the set of states, $Q' = Q \cup \{q_0'\}$, and we make the new state the new initial state. We add λ-transitions from the new initial state q_0' to every state $q \in Q$. Thus the new transition function δ' contains all the rules in δ plus the new rules $\delta'(q_0', \lambda) = q$ for all $q \in Q$. Then $L(N_2) = \mathsf{suffix}(L_1)$ where $L_1 = L(N_1)$, and $L(N_2)$ is regular since it is accepted by an NFA. $\square$

Let N_1 be the following NFA. Then N_2 will accept the string a which is not a suffix of any string in $L_1 = \{b\}$.



4. Give a **DFA** for the language $L = \{w \in \{a, b\}^* \mid w \text{ does } \textbf{not} \text{ end in } ba\}$.



5. Give a regular expression $\boldsymbol{r}_2$ such that $L(\boldsymbol{r}_2) = \overline{L(\boldsymbol{r}_1)}$ where $\boldsymbol{r}_1 = (\boldsymbol{b}+\boldsymbol{\lambda})(\boldsymbol{ab})^*(\boldsymbol{a}+\boldsymbol{\lambda})$.

   $\boldsymbol{r}_2 = \Sigma^*\Big(\boldsymbol{aa}+\boldsymbol{bb}\Big)\Sigma^*$

6. Given a grammar $G = (V, S, T, P)$, let $\mathsf{more}(G) = (V, S, T, P')$ where $P' = P \cup \{S \mapsto SS\}$. That is, we add the rule $S \mapsto SS$ to the grammar if it is not already there. Give a context-free grammar G_1 for which $(L(G_1))^* \subsetneq L(\mathsf{more}(G_1))$. Demonstrate that the languages are different by giving a string $w \in T^*$ that is in $L(\mathsf{more}(G_1))$ but not in $(L(G_1))^*$.

   Let G be the grammar $S \to aSb \mid \lambda$, and $w = aababb$.

7. We are given an NFA $N_1 = (Q, q_0, F, \Sigma, \delta)$ over the alphabet $\Sigma = \{a, b, c\}$. Consider we remove all edges in N_1 that are labeled by the symbol a, thereby obtaining a new NFA N_2. That is, $N_2 = (Q, q_0, F, \Sigma, \delta')$, where the new transition function δ' contains the rules in δ that do **not** use the symbol a. Give a succinct description of $L(N_2)$.

   $L(N_2) = L(N_1) \cap \{b, c\}^*$

8. Give a context-free grammar for the language $L = \{a^i b^j c^k d^\ell \mid i, j, k, \ell \geq 0, i + j + k = \ell\}$.

   $S \to aSd \mid B; \quad B \to bBd \mid C; \quad C \to cCd \mid \lambda$

9. Sketch a proof that $L = \{uv \mid u \in \{a, b\}^*, v \in \{b, c\}^*, |u| = |v|\}$ is non-regular. You may use the statements on the back of the exam without proof.

   Let $L_1 = L \cap L(\boldsymbol{a}^*\boldsymbol{c}^*)$. Then $L_1 = \{a^i c^i \mid i \geq 0\}$. Let $L_2 = h(L_1)$ with $h(a) = a$ and $h(c) = b$. Then $L_2 = \{a^i b^i \mid i \geq 0\}$.

10. Let $L_1 = \{a^i b^j a^i b^k \mid i, j, k \geq 0\}$ and $L_2 = \{a^\ell b^m \mid \ell, m \geq 0\}$. Compute $L_1 \cap L_2$. Give an expression that is as simple as possible.

    The intersection keeps the strings in L_2 with an even number of a symbols:
    $L_1 \cap L_2 = \{a^{2i} b^j \mid i, j \geq 0\}$. Notice $a^{2\alpha} b^\beta = a^\alpha b^0 a^\alpha b^\beta$.

11. Give two non-regular languages L_1 and L_2 for which $L_1 \cup L_2$ is regular.

    $L_1 = \{a^i b^i \mid i \geq 0\}$ and $L_2 = \overline{L_1}$.

12. The university has implemented a new password policy. A password Φ can contain symbols from the alphabet $\Sigma = \{a, \ldots, z\} \cup \{A, \ldots, Z\} \cup \{0, 1, \ldots, 9\} \cup \{\_, *, \$, \%, \&, !, (,)\}$. The alphabet thus contains the 26 letters in lowercase and uppercase, the ten digits, and eight of the special symbols. A string Φ over Σ is a permissible password if it satisfies the following constraints:

    (a) Contains at least two special symbols

    (b) Starts with a lowercase letter or an uppercase letter

    (c) Is at least 8 characters long

    (d) Contains at least one lowercase and one uppercase letter

    (e) Contains at least two digits

Explain why the language $L = \{\Phi \in \Sigma^* \mid \Phi \text{ is a permissible password}\}$ is regular.

Let $W = \{a, \ldots, z\}$, $U = \{A, \ldots, Z\}$, $D = \{0, 1, \ldots, 9\}$, and $S = \{\_, *, \$, \%, \&, !, (,)\}$, and set $L_a = \Sigma^* S \Sigma^* S \Sigma^*$, $L_b = (W \cup U)\Sigma^*$, $L_c = \Sigma^8 \Sigma^*$, $L_d = \Sigma^* W \Sigma^* \cap \Sigma^* U \Sigma^*$, and $L_e = \Sigma^* D \Sigma^* D \Sigma^*$. Then $L = L_a \cap L_b \cap L_c \cap L_d \cap L_e$.

13. Consider the following regular expressions over the alphabet $\Sigma = \{a, b\}$. For each regular expression $\boldsymbol{r}$ decide whether $L(\boldsymbol{r}) = \Sigma^*$ or not.

	Regular expression	$\{a, b\}^*$	**not** $\{a, b\}^*$
1	$(\boldsymbol{a}^*\boldsymbol{b})^*\boldsymbol{a}^*$	✓	
2	$(\boldsymbol{aa} + \boldsymbol{aaa} + \boldsymbol{bb} + \boldsymbol{bbb})^* + \boldsymbol{a} + \boldsymbol{b}$		✗
3	$\Big((\boldsymbol{a} + \boldsymbol{bb})^+ + (\boldsymbol{aa} + \boldsymbol{b})^+\Big)^*$	✓	
4	$(\boldsymbol{a} + \boldsymbol{b})^* + \boldsymbol{\lambda}$	✓	
5	$(\boldsymbol{a}^+\boldsymbol{b}^+)^* + (\boldsymbol{b}^+\boldsymbol{a}^+)^* + \boldsymbol{a}^* + \boldsymbol{b}^*$		✗
6	$\Big(\boldsymbol{b}(\boldsymbol{a}^*) + \boldsymbol{a}(\boldsymbol{b}^*)\Big)^*(\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{\lambda}$	✓	

3. Bonus questions

1. Give an infinite regular language L satisfying that $L \subseteq \{w \in \{a, b\}^* \mid |w|_a = |w|_b\}$. **NO** justification necessary. It suffices to give the language.

   $L = L((\boldsymbol{ab})^*)$

2. Give a regular language L_1 such that $L_2 = \{w^i \mid w \in L_1 \text{ and } i \geq 0\}$ is **not** regular. **NO** justification necessary. It suffices to give the language.

   $L_1 = L(\boldsymbol{a}^*\boldsymbol{b})$. Notice that then $L' = L_2 \cap L(\boldsymbol{a}^*\boldsymbol{b}\boldsymbol{a}^*\boldsymbol{b}) = \{a^i b a^i b \mid i \geq 0\}$.

2. True or False

Answer True or False. No justification required. The languages are over the alphabet $\Sigma = \{a, b\}$, except in questions 2, 3, 7, and 20, where the alphabet is $\{a, b, c\}$.

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	Question	True	False
1	The grammar $S \to SabS \mid SbaS \mid \lambda$ generates a regular language	✓	
2	$\{a^ib^jc^k \mid i,j,k \geq 0\} \setminus \{c^k \mid k \geq 0\} = \{a^ib^j \mid i,j \geq 0\}$		✗
3	$L = \{ccabc\}$ is a context-free language	✓	
4	If $L = \{uvw \mid u,v,w \in \Sigma^+\}$ then $L^R = \{(v^R)yu \mid u \in \Sigma^+, y, v \in \Sigma\}$	✓	
5	$L^* = L^+ \cup \{\lambda\}$	✓	
6	$(L^+)^* = (L^*)^+$	✓	
7	$\{a^ib^jc^k \mid i \neq j \text{ or } i \neq k\} = \{a^ib^jc^k \mid i \neq j \text{ or } j \neq k\}$	✓	
8	If L is non-regular so is L^2		✗
9	The language $L = \{w \in \{a,b\}^{20} \mid \lvert w\rvert_a = \lvert w\rvert_b\}$ is regular	✓	
10	Let G be the grammar $S \to SaS \mid b \mid \lambda$. Then $L(G) = \{a,b\}^*$		✗
11	The grammar $S \to SaSaS \mid SbSbS \mid \lambda$ generates the language $L_{\text{pal,even}} = \{w \in \{a,b\}^* \mid w = w^R \text{ and } \lvert w\rvert \text{ is even}\}$		✗
12	$\overline{L(\boldsymbol{a}^*)} \cap \overline{L(\boldsymbol{b}^*)} \neq L(\boldsymbol{a}^+\boldsymbol{b}^+)$	✓	
13	If $L = L^R$ then L is not regular		✗
14	If h is a homomorphism and $h(L)$ is regular, so is L		✗
15	Let $L = \{w \in \{a,b\}^* \mid \lvert w\rvert_a \geq 1 \text{ and } \lvert w\rvert_b \geq 1\}$. A minimal DFA for L contains exactly four states	✓	
16	$L = \{\lambda^i \mid i \text{ is prime}\}$ is regular	✓	
17	If L_1 is regular, so is $L_2 = \{v(w^R) \mid v, w \in L_1\}$	✓	
18	If $L_1 \subseteq L(\boldsymbol{a}^*)$ and L_2 is regular, then $L_1 \cap L_2$ is also regular		✗
19	Let L be a regular language and M a minimal DFA for L. If L contains only strings of odd length, then M contains an odd number of states		✗
20	Let $h(a) = a^3$, $h(b) = a^3$, $h(c) = b^3$ and let $L = \{a^ib^ic^j \mid i,j \geq 0\}$. Then $h(L)$ is regular	✓	
21	Let $\boldsymbol{r} = (\boldsymbol{b}^*\boldsymbol{a}\boldsymbol{b}^*)^* + \boldsymbol{\lambda} + \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{abba}$. Then $L(\boldsymbol{r}) = \{a,b\}^*$		✗
22	$\{a^i \mid i \geq 0\}\{b^i \mid i \geq 0\} = \{a^ib^i \mid i \geq 0\}$		✗
23	The grammar $S \to aaaS \mid B;\ B \to BB \mid b \mid \lambda$ is right-linear		✗
24	$L = \{b^i \mid i \text{ is not divisible by } 42\}$ is regular	✓	