CPSC 313 (Winter 2016) L01

University of Calgary
Faculty of Science
Midterm

March 3, 2016.
Time available: 150 minutes.
No books or calculators are permitted (two lettersized sheets of notes are permitted).

Write answers in this booklet only.
Do **not** open this exam until you are told to do so.

Name: ____________________

Lab section:

☐	☐	☐	☐	☐
T01 Mon 14:00	T02 Wed 14:00	T03 Mon 17:00	T04 Wed 17:00	T05 Wed 15:00

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There are 5 (five) problems in total. There are 35 points and 3 bonus points.

For your convenience, the last page (page 16) contains a collection of information.

You may use the blank pages at the end of the exam if you need more space for your answers. Please indicate clearly when your answers are continued on these pages.

Please note that points may be deducted for untidy handwriting.

Problem	Possible score	Score
1	20	
2	4	
3	4	
4	7	
5	3 bonus	
Total	35 + 3 bonus	

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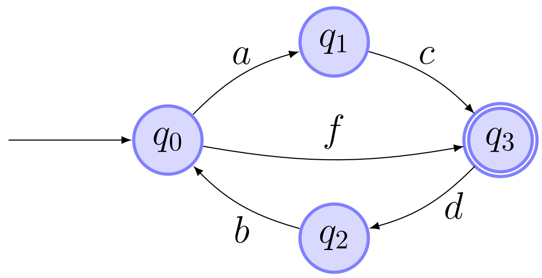
1. Short answers (20 points)

1. Give a regular expression for the language accepted by the following NFA.



Justify your answer.

2. We say a context-free grammar is *symmetric* if whenever $A \to z_1 z_2 \cdots z_m$ is a rule in the grammar, then so is $A \to z_m \cdots z_2 z_1$. Here $z_i \in (V \cup T)$ for all i. Give an example of a language that can **not** be generated by a symmetric grammar. Justify your answer.

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3. Give a DFA M accepting the language $L = \left\{ w \in \{a, b, c\}^* \mid w \text{ ends in } baa \text{ or } caa \right\}$. Justify your answer.

4. Give a right-linear grammar G for the language $L = L(\boldsymbol{r})$ where $\boldsymbol{r} = (\boldsymbol{a}(\boldsymbol{b}^*)\boldsymbol{c})^*$. Justify your answer.

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5. Give a regular expression $\boldsymbol{r}$ for the language $L = \{w \in \{a,b\}^* \mid |w|_a \leq 2 \text{ or } |w|_b \leq 2\}$. Justify your answer.

6. Give a DFA for the complement of the language $L(\boldsymbol{r})$ where $\boldsymbol{r} = (\boldsymbol{a} + \boldsymbol{bb})^*$. Justify your answer.

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7. Give a context-free grammar for the language $L = \{w \in \{a,b\}^* \mid |w|_a = 2|w|_b\}$. That is, a string w is in L if it contains twice as many a symbols as b symbols. Justify your answer.

8. Give a language L_1 such that $L = L_1L_2$ is regular, where $L_2 = \{a^ib^j \mid i \leq j\}$. Justify your answer.

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9. Give a context-free grammar for the language $L = \{a^i b^j c^k \mid i + k = j \text{ and } i, j, k \geq 0\}$. Justify your answer.

10. Give a minimal DFA for the language $L = L(\boldsymbol{r})$ where $\boldsymbol{r} = ((\boldsymbol{a}^*\boldsymbol{b} + \boldsymbol{a}(\boldsymbol{b}^*))^+)(\boldsymbol{a} + \boldsymbol{b})$. Justify your answer.

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2. Non-regular languages (4 points)

Prove that the following language is non-regular

$$L = \left\{ a^{\alpha} b^{\beta} c^{\gamma} d^{\delta} e^{\epsilon} f^{\phi} \mid \alpha \neq \beta \text{ and } \gamma \neq 2\epsilon \text{ and } \delta = \phi, \text{ and } \alpha, \beta, \gamma, \delta, \epsilon, \phi \geq 0 \right\}.$$

You may use the theorems at the back of this exam without proofs.

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3. DFA with a reset button (4 points)

We define a new computational model by equipping a DFA M with a **reset button**. A DFA M_{reset} with a **reset button** works as a DFA, except for the reset button. Consider we are running M_{reset} on some input string $w \in \Sigma^*$. If we press the reset button while being in some state $q_i \in Q$, we are instantaneously being transferred to the initial state q_0. We can press the reset button at most **once** during the entire computation of M_{reset} on w. We can choose when to press the reset button, or not press the button at all, but we can only press it once. Machine M_{reset} accepts a string w if we can end in a final state.

1. Let M be a DFA and $L = L(M)$ the language accepted by M. Let M_{reset} be M equipped with a reset button. Characterize the language L_{reset} accepted by M_{reset}. Explain and justify your answer.

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4. True or False (7 points)

Answer True or False. No justification required.
The languages are over the alphabet $\Sigma = \{a, b\}$, except in questions 1 and 8.

	Question	True	False
1	If $L = \Big\{w \in \{a,b,c\}^* \mid \lvert w\rvert_a = \lvert w\rvert_b \text{ or } \lvert w\rvert_a = \lvert w\rvert_c\Big\}$, then L is context-free		
2	$L^* = (L^+)^*$		
3	The grammar $S \to bSa \mid aSb \mid ab \mid ba$ generates the complement of the language generated by the grammar $S \to aSa \mid bSb \mid a \mid b \mid \lambda$		
4	$(L^+)^+ = L^+ \cdot L^*$		
5	The grammar $S \to aSb \mid aS \mid Sb \mid \lambda$ generates a regular language		
6	If $\lambda \in L$, then $\lambda \in L^+$		
7	$\lambda^2 = \lambda$		
8	$\{a^i b^i c^i \mid i \geq 0\} \cap \{b^i c^i d^i \mid i \geq 0\} = \{b^i c^i \mid i \geq 0\}$		
9	The grammar $S \to aSbSaSb \mid \lambda$ generates the language $\{a^i b^i a^i b^i \mid i \geq 0\}$		
10	The language $L(\boldsymbol{a}^*\boldsymbol{b}^*)$ is context-free		
11	If both L_1 and L_2 are non-regular, then so is $L_1 \cap L_2$		
12	If $\{a^i b^i \mid i \geq 0\} \subseteq L$, then L is non-regular		
13	If $\lambda \in L$ then $L^{13} = L^{17}$		
14	If L is context-free, then so is L^*		

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5. Ambiguities in regular languages (3 bonus points)

Consider a right-linear grammar G. We say that two rules ρ_1 and ρ_2 in G are *ambiguous* if they are on the form

$$\rho_1 = A \to bu \qquad \text{and} \qquad \rho_2 = A \to bv$$

where $A \in V$, $b \in \Sigma$ and $u, v \in T^*(V \cup \{\lambda\})$. That is, the two rules can both be applied on the variable A, and they both start with the same terminal symbol b on the right-hand side.

We say that a grammar G is ambiguous if G contains a pair of rules that are ambiguous, or if G contains a rule of the form $A \to B$ where $A, B \in V$. We say that a grammar is *unambiguous* otherwise.

1. Show that for any right-linear grammar G, there exists another right-linear grammar G' that is unambiguous and that generates the same language.
   (Full bonus points to complete solutions. No points to partial solutions.)

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Theorem 1 (Closure Properties for Regular languages)

Let L, L_1, and L_2 be regular languages over the same alphabet Σ. Then the following ten languages are regular as well.

1. $\overline{L} = \{w \in \Sigma^* \mid w \notin L\}$
2. $L_1 \cup L_2$
3. $L_1 \cap L_2$
4. $L_1 L_2$
5. L^*
6. L^+
7. $L^{\mathrm{R}} = \{w \in \Sigma^* \mid w^R \in L\}$
8. $\mathsf{prefix}(L) = \{u \in \Sigma^* \mid \exists v \in \Sigma^* \text{ such that } uv \in L\}$
9. $\mathsf{suffix}(L) = \{v \in \Sigma^* \mid \exists u \in \Sigma^* \text{ such that } uv \in L\}$
10. $h(L)$, where $h : \Sigma \to \Gamma^*$ is a homomorphism and Γ is an alphabet.

Theorem 2 (Pumping Lemma for Regular Languages)

Let L be a regular language. Then there

$$\textit{exists an integer } m \in \mathbb{Z} \textit{ such that for all } w \in L \text{ with } |w| \geq m,$$

there exist strings x, y, and z, such that $w = x \cdot y \cdot z$ and

1. $|x \cdot y| \leq m$,

2. $|y| \geq 1$, *and*

3. for all $i \geq 0$, $x \cdot y^i \cdot z \in L$.

Theorem 3 (Canonical counting languages) *The following holds*

1. $L_0 = \{a^i \mid i \geq 0\}$ *is regular.*

2. $L_{\mathrm{cnt}} = \{a^i b^i \mid i \geq 0\}$ *is context-free but not regular.*

3. $L_{2\mathrm{cnt}} = \{a^i b^i c^i \mid i \geq 0\}$ *is not context-free.*
