Theorem 1 (Closure Properties for Regular languages)

Let L, L_1, and L_2 be regular languages over the same alphabet Σ. Then the following eleven languages are regular as well.

1. $\overline{L} = \{w \in \Sigma^* \mid w \notin L\}$
2. $L_1 \cup L_2$
3. $L_1 \cap L_2$
4. $L_1 L_2$
5. L^*
6. L^+
7. $L \setminus \{\lambda\}$
8. $L^{\mathrm{R}} = \{w \in \Sigma^* \mid w^R \in L\}$
9. $\mathsf{prefix}(L) = \{u \in \Sigma^* \mid \exists v \in \Sigma^* \text{ such that } uv \in L\}$
10. $\mathsf{suffix}(L) = \{v \in \Sigma^* \mid \exists u \in \Sigma^* \text{ such that } uv \in L\}$
11. $h(L)$, where $h : \Sigma \to \Gamma^*$ is a homomorphism and Γ is an alphabet.

Theorem 2 (Pumping Lemma for Regular Languages)

Let L be a regular language. Then there

$$\textit{exists an integer } m \in \mathbb{Z} \textit{ such that for all } w \in L \text{ with } |w| \geq m,$$

there exist strings x, y, and z, such that $w = x \cdot y \cdot z$ and

1. $|x \cdot y| \leq m$,

2. $|y| \geq 1$, *and*

3. for all $i \geq 0$, $x \cdot y^i \cdot z \in L$.

Theorem 3 (Canonical counting languages) *The following holds*

1. $L_0 = \{a^i \mid i \geq 0\}$ *is regular.*

2. $L_{\text{cnt}} = \{a^i b^i \mid i \geq 0\}$ *is context-free but not regular.*

3. $L_{\text{2cnt}} = \{a^i b^i c^i \mid i \geq 0\}$ *is not context-free.*
