CPSC 313 (Winter 2017) L01

University of Calgary
Faculty of Science
Midterm

March 2, 2017.
Time available: 150 minutes.
No books or calculators are permitted (two lettersized sheets of notes are permitted).

Write answers in this booklet only.
Do **not** open this exam until you are told to do so.

Name: ______________________________

Lab section:

☐	☐	☐	☐	☐	☐
T01	T02	T03	T04	T05	T06
Mon 14	Wed 14	Mon 17	Wed 17	Wed 15	Mon 15

There are 3 (three) problems in total. There are 21 points plus 1 bonus point in total.
Full answer is **21 points**.

For your convenience, the last page (page 18) contains a collection of information.

You may use the blank pages at the end of the exam if you need more space for your answers. Please indicate clearly when your answers are continued on these pages.

Please note that points may be deducted for untidy handwriting.

Problem	Possible score	Score
1	13	
2	8	
3	1 bonus	
Total	21 + 1 bonus	

1. Problems (13 points)

The following contains 13 problems. For each problem, (1) give a solution and (2) argue about your solution. You **must** provide argumentation for why your solution is correct and show how you derived it.

1. Give a **DFA** for the language $L(\boldsymbol{r})$ where $\boldsymbol{r} = (\boldsymbol{a}^*\boldsymbol{b})^*$. (You must argue.)

2. Give a **right-linear** grammar for the language $L = \{w \in \{a, b, c\}^* \mid w \text{ contains } abc \text{ as a substring}\}$. (You must argue.)

3. We have seen in class that if L is regular, then so is $\mathsf{suffix}(L)$.

   Beeblebrox has created his own "proof" of this fact. But his "proof" is flawed. It contains one critical flaw. Precisely and succinctly identify the critical flaw, explain why it is a flaw, and explain how to fix Beeblebrox's proof.

   Proof Given an NFA $N_1 = (Q, q_0, F, \Sigma, \delta)$, we construct a new NFA $N_2 = (Q', q_0', F, \Sigma, \delta')$ as follows. We add a new state $q_0' \notin Q$ to the set of states, $Q' = Q \cup \{q_0'\}$, and we make the new state the new initial state. We add λ-transitions from the new initial state q_0' to every state $q \in Q$. Thus the new transition function δ' contains all the rules in δ plus the new rules $\delta'(q_0', \lambda) = q$ for all $q \in Q$. Then $L(N_2) = \mathsf{suffix}(L_1)$ where $L_1 = L(N_1)$, and $L(N_2)$ is regular since it is accepted by an NFA. □

   (You must argue.)

4. Give a **DFA** for the language $L = \{w \in \{a, b\}^* \mid w \text{ does } \textbf{not} \text{ end in } ba\}$. (You must argue.)

5. Give a regular expression $\boldsymbol{r}_2$ such that $L(\boldsymbol{r}_2) = \overline{L(\boldsymbol{r}_1)}$ where $\boldsymbol{r}_1 = (\boldsymbol{b} + \boldsymbol{\lambda})(\boldsymbol{ab})^*(\boldsymbol{a} + \boldsymbol{\lambda})$. (You must argue.)

6. Given a grammar $G = (V, S, T, P)$, let $\mathsf{more}(G) = (V, S, T, P')$ where $P' = P \cup \{S \mapsto SS\}$. That is, we add the rule $S \mapsto SS$ to the grammar if it is not already there. Give a context-free grammar G_1 for which $(L(G_1))^* \subsetneq L(\mathsf{more}(G_1))$. Demonstrate that the languages are different by giving a string $w \in T^*$ that is in $L(\mathsf{more}(G_1))$ but not in $(L(G_1))^*$. (You must argue.)

7. We are given an NFA $N_1 = (Q, q_0, F, \Sigma, \delta)$ over the alphabet $\Sigma = \{a, b, c\}$. Consider we remove all edges in N_1 that are labeled by the symbol a, thereby obtaining a new NFA N_2. That is, $N_2 = (Q, q_0, F, \Sigma, \delta')$, where the new transition function δ' contains the rules in δ that do **not** use the symbol a. Give a succinct description of $L(N_2)$. (You must argue.)

8. Give a context-free grammar for the language $L = \{a^i b^j c^k d^\ell \mid i, j, k, \ell \geq 0, i + j + k = \ell\}$. (You must argue.)

9. Sketch a proof that $L = \{uv \mid u \in \{a,b\}^*, v \in \{b,c\}^*, |u| = |v|\}$ is non-regular. You may use the statements on the back of the exam without proof (see page 18). (You must argue.)

10. Let $L_1 = \{a^i b^j a^i b^k \mid i, j, k \geq 0\}$ and $L_2 = \{a^\ell b^m \mid \ell, m \geq 0\}$. Compute $L_1 \cap L_2$. Give an expression that is as simple as possible. (You must argue.)

11. Give two non-regular languages L_1 and L_2 for which $L_1 \cup L_2$ is regular. (You must argue.)

12. The university has implemented a new password policy. A password Φ can contain symbols from the alphabet $\Sigma = \{a, \ldots, z\} \cup \{A, \ldots, Z\} \cup \{0, 1, \ldots, 9\} \cup \{\_, *, \$, \%, \&, !, (,)\}$. The alphabet thus contains the 26 letters in lowercase and uppercase, the ten digits, and eight of the special symbols. A string Φ over Σ is a permissible password if it satisfies the following constraints:

    (a) Contains at least two special symbols

    (b) Starts with a lowercase letter or an uppercase letter

    (c) Is at least 8 characters long

    (d) Contains at least one lowercase and one uppercase letter

    (e) Contains at least two digits

    Explain why the language $L = \{\Phi \in \Sigma^* \mid \Phi \text{ is a permissible password}\}$ is regular. (You must argue.)

13. Consider the following regular expressions over the alphabet $\Sigma = \{a, b\}$. For each regular expression $\boldsymbol{r}$ decide whether $L(\boldsymbol{r}) = \Sigma^*$ or not.

    For this one question, no justification is required.

	Regular expression	$\{a, b\}^*$	**not** $\{a, b\}^*$
1	$(\boldsymbol{a}^*\boldsymbol{b})^*\boldsymbol{a}^*$		
2	$(\boldsymbol{aa} + \boldsymbol{aaa} + \boldsymbol{bb} + \boldsymbol{bbb})^* + \boldsymbol{a} + \boldsymbol{b}$		
3	$\Big((\boldsymbol{a} + \boldsymbol{bb})^+ + (\boldsymbol{aa} + \boldsymbol{b})^+\Big)^*$		
4	$(\boldsymbol{a} + \boldsymbol{b})^* + \boldsymbol{\lambda}$		
5	$(\boldsymbol{a}^+\boldsymbol{b}^+)^* + (\boldsymbol{b}^+\boldsymbol{a}^+)^* + \boldsymbol{a}^* + \boldsymbol{b}^*$		
6	$\Big(\boldsymbol{b}(\boldsymbol{a}^*) + \boldsymbol{a}(\boldsymbol{b}^*)\Big)^*(\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{\lambda}$		

2. True or False (8 points)

Answer True or False. No justification required. The languages are over the alphabet $\Sigma = \{a, b\}$, except in questions 2, 3, 7 and 20, where the alphabet is $\{a, b, c\}$.

	Question	True	False
1	The grammar $S \to SabS \mid SbaS \mid \lambda$ generates a regular language		
2	$\{a^i b^j c^k \mid i, j, k \geq 0\} \setminus \{c^k \mid k \geq 0\} = \{a^i b^j \mid i, j \geq 0\}$		
3	$L = \{ccabc\}$ is a context-free language		
4	If $L = \{uvw \mid u, v, w \in \Sigma^+\}$ then $L^R = \{(v^R)yu \mid u \in \Sigma^+, y, v \in \Sigma\}$		
5	$L^* = L^+ \cup \{\lambda\}$		
6	$(L^+)^* = (L^*)^+$		
7	$\{a^i b^j c^k \mid i \neq j \text{ or } i \neq k\} = \{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$		
8	If L is non-regular so is L^2		
9	The language $L = \{w \in \{a, b\}^{20} \mid \lvert w \rvert_a = \lvert w \rvert_b\}$ is regular		
10	Let G be the grammar $S \to SaS \mid b \mid \lambda$. Then $L(G) = \{a, b\}^*$		
11	The grammar $S \to SaSaS \mid SbSbS \mid \lambda$ generates the language $L_{\text{pal,even}} = \{w \in \{a, b\}^* \mid w = w^R \text{ and } \lvert w \rvert \text{ is even}\}$		
12	$\overline{L(\boldsymbol{a}^*)} \cap \overline{L(\boldsymbol{b}^*)} \neq L(\boldsymbol{a}^+\boldsymbol{b}^+)$		

	Question	True	False
13	If $L = L^R$ then L is not regular		
14	If h is a homomorphism and $h(L)$ is regular, so is L		
15	Let $L = \{w \in \{a,b\}^* \mid \lvert w\rvert_a \geq 1 \text{ and } \lvert w\rvert_b \geq 1\}$. A minimal DFA for L contains exactly four states		
16	$L = \{\lambda^i \mid i \text{ is prime}\}$ is regular		
17	If L_1 is regular, so is $L_2 = \{v(w^R) \mid v, w \in L_1\}$		
18	If $L_1 \subseteq L(\boldsymbol{a}^*)$ and L_2 is regular, then $L_1 \cap L_2$ is also regular		
19	Let L be a regular language and M a minimal DFA for L. If L contains only strings of odd length, then M contains an odd number of states		
20	Let $h(a) = a^3$, $h(b) = a^3$, $h(c) = b^3$ and let $L = \{a^i b^i c^j \mid i, j \geq 0\}$. Then $h(L)$ is regular		
21	Let $\boldsymbol{r} = (\boldsymbol{b}^*\boldsymbol{a}\boldsymbol{b}^*)^* + \boldsymbol{\lambda} + \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{abba}$. Then $L(\boldsymbol{r}) = \{a,b\}^*$		
22	$\{a^i \mid i \geq 0\}\{b^i \mid i \geq 0\} = \{a^i b^i \mid i \geq 0\}$		
23	The grammar $S \to aaaS \mid B;\ B \to BB \mid b \mid \lambda$ is right-linear		
24	$L = \{b^i \mid i \text{ is not divisible by } 42\}$ is regular		

3. Bonus questions (1 bonus point)

1. Give an infinite regular language L satisfying that $L \subseteq \{w \in \{a,b\}^* \mid |w|_a = |w|_b\}$.
   NO justification necessary. It suffices to give the language.

   $L =$ ____

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2. Give a regular language L_1 such that $L_2 = \{w^i \mid w \in L_1 \text{ and } i \geq 0\}$ is **not** regular.
   NO justification necessary. It suffices to give the language.

   $L_1 =$ ____

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Theorem 1 (Closure Properties for Regular languages)

Let L, L_1, and L_2 be regular languages over the same alphabet Σ. Then the following eleven languages are regular as well.

1. $\overline{L} = \{w \in \Sigma^* \mid w \notin L\}$
2. $L_1 \cup L_2$
3. $L_1 \cap L_2$
4. $L_1 L_2$
5. L^*
6. L^+
7. $L \setminus \{\lambda\}$
8. $L^{\mathrm{R}} = \{w \in \Sigma^* \mid w^R \in L\}$
9. $\mathsf{prefix}(L) = \{u \in \Sigma^* \mid \exists v \in \Sigma^* \text{ such that } uv \in L\}$
10. $\mathsf{suffix}(L) = \{v \in \Sigma^* \mid \exists u \in \Sigma^* \text{ such that } uv \in L\}$
11. $h(L)$, where $h : \Sigma \to \Gamma^*$ is a homomorphism and Γ is an alphabet.

Theorem 2 (Pumping Lemma for Regular Languages)

Let L be a regular language. Then there

$$\textit{exists an integer } m \in \mathbb{Z} \textit{ such that for all } w \in L \text{ with } |w| \geq m,$$

there exist strings x, y, and z, such that $w = x \cdot y \cdot z$ and

1. $|x \cdot y| \leq m$,

2. $|y| \geq 1$, and

3. for all $i \geq 0$, $x \cdot y^i \cdot z \in L$.

Theorem 3 (Canonical counting languages) *The following holds*

1. $L_0 = \{a^i \mid i \geq 0\}$ is regular.

2. $L_{\text{cnt}} = \{a^i b^i \mid i \geq 0\}$ is context-free but not regular.

3. $L_{2\text{cnt}} = \{a^i b^i c^i \mid i \geq 0\}$ is not context-free.
