CPSC 313 (Winter 2014)

Midterm Sketch Solutions

A two hours exam held on March 6, 2014

1. Short answers

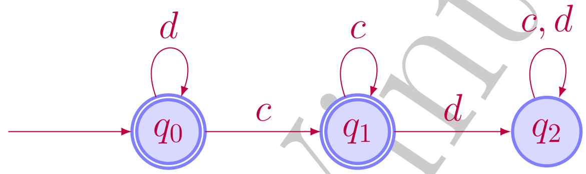
1. Give a regular expression for the language $L = \{w \in \{a, b, c\}^* \mid |w|_a = 1 \text{ and } |w|_b = 1\}$. Briefly justify.

$$\boldsymbol{r = c^* a c^* b c^* + c^* b c^* a c^*}$$

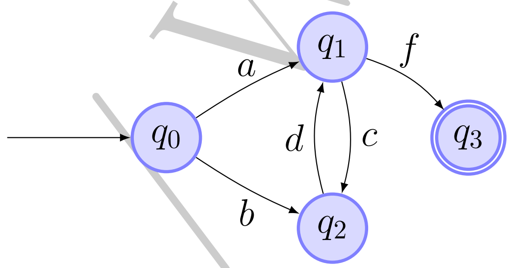
2. Give a grammar for the language $L = \{w \in \{a\}^* \mid |w| \not\equiv 1 \pmod 3\}$. Briefly justify.

$$S \to aaaS \mid aa \mid \lambda$$

3. Give a **DFA** accepting the language $L = \{w \in \{c, d\}^* \mid cd \text{ is } \textbf{not} \text{ a substring of } w\}$. Briefly justify.

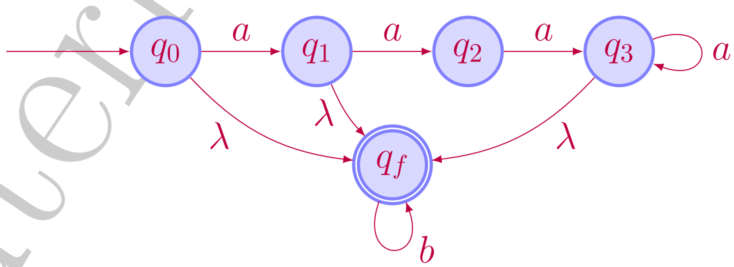


4. Give a regular expression for the language accepted by the following NFA. Briefly justify.

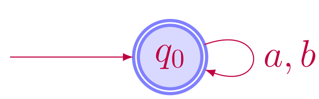


$$\boldsymbol{r = (a + bd)(cd)^* f}$$

5. Give an **NFA** accepting the regular language $L = \{a^i b^j \mid i \neq 2 \text{ and } i, j \geq 0\}$. Briefly justify.



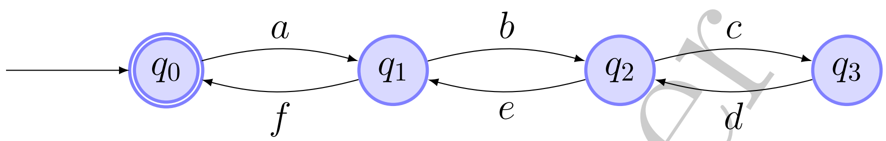
6. Give a **smallest possible DFA** for the language $L(\boldsymbol{r})$ generated by the following regular expression $\boldsymbol{r = (a^* b + b^* a)^*}$. Briefly justify.



7. Give a context-free grammar for the language $L = \{a^i b^j c^k \mid i > j \text{ or } i > k\}$. Briefly justify.

$$\begin{array}{lcl} S & \to & AS_{ab}C \mid AS_{ac} \\ S_{ab} & \to & aS_{ab}b \mid \lambda \\ S_{ac} & \to & aS_{ac}c \mid B \\ A & \to & aA \mid a \\ B & \to & bB \mid \lambda \\ C & \to & cC \mid \lambda \end{array}$$

8. Give a regular expression for the language accepted by the following NFA. Briefly justify.



$$\boldsymbol{r} = (\boldsymbol{a}(\boldsymbol{b}(\boldsymbol{cd})^*\boldsymbol{e})^*\boldsymbol{f})^*$$

9. Give an infinite regular language L_{reg} and an infinite non-regular language L_{notreg} satisfying that $L_{\text{notreg}} \subseteq L_{\text{reg}}$. Briefly justify.

$$\begin{array}{ll} L_{\text{reg}} & = \{a^i b^j \mid \phantom{i = j \text{ and }} i, j \geq 0\} \\ L_{\text{notreg}} & = \{a^i b^j \mid i = j \text{ and } i, j \geq 0\} \end{array}$$

10. Characterize the language accepted by the grammar containing the following 17 productions.

$$\begin{array}{lcl} S & \to & PXRXTY \\ X & \to & KNN \\ Y & \to & NNNN \\ K & \to & \texttt{9} \mid \texttt{8} \mid \texttt{7} \mid \texttt{6} \mid \texttt{5} \mid \texttt{4} \mid \texttt{3} \mid \texttt{2} \mid \texttt{1} \\ N & \to & K \mid \texttt{0} \\ P & \to & \texttt{(} \\ R & \to & \texttt{)} \\ T & \to & \texttt{-} \end{array}$$

The alphabet Σ is comprised of the following 32 printable ASCII characters,

`!"#$%&'()*+,-./0123456789:;<=>?@`

Briefly justify.

North-American phone numbers on the form (ABB)ABB-BBBB, where each A is a nonzero digit, and each B is a digit, for instance (403)123-0987.

11. Consider the language $L = L(G)$ generated by the grammar G containing the following 6 productions:

$$S \to ASB \mid AB; \; AB \to AAB; \; abb \to ab; \; A \to a; \; B \to b.$$

Give a grammar G' that contains **at most 3** productions, and that accepts the same language as G (i.e. $L(G) = L(G')$). Briefly justify.

$$S \to aSb \mid aS \mid ab$$

12. Give two context-free languages L_1 and L_2 such that $L_1 \cap L_2 = \{a^i b^i c^i d^i e^i \mid i \geq 0\}$. Briefly justify.

$$L_1 = \{a^i b^j c^k d^m e^n \mid i = j \text{ and } k = m, \quad i, j, k, m, n \geq 0\}$$
$$L_2 = \{a^i b^j c^k d^m e^n \mid j = k \text{ and } m = n, \quad i, j, k, m, n \geq 0\}$$

2. True or False

Answer True or False. No justification required. The languages in questions 1, 2, 3, 8, and 11 are over the alphabet $\Sigma = \{a, b\}$. In question 12, $\boldsymbol{r}$ and $\boldsymbol{s}$ denote regular expressions.

(*Note:* There are 15 questions. One incorrect answer will be ignored.)

	Question	True	False
1	If L is context-free, then so is L^R	✓	
2	If $L = L^*$ and $w \in L$, then $ww \in L$	✓	
3	If L is regular and L' is non-regular, then $L \cap L'$ is non-regular		✗
4	The smallest DFA accepting a language consisting of exactly 3 strings contains exactly 5 states		✗
5	The grammar $S \rightarrow aSb \mid bSa \mid \lambda$ generates a context-free language	✓	
6	$\{\lambda\}^* = \{\,\}$		✗
7	$\{\,\}^* = \{\,\}$		✗
8	If L is regular and $L' \subseteq L$, then L' is also regular		✗
9	The grammar $S \rightarrow SaSaS \mid \lambda$ generates the language $\{w \in \{a\}^* \mid \lvert w \rvert = 2i \text{ for some } i \geq 0\}$	✓	
10	There exists a finite language that is context-free	✓	
11	If L is regular, then so is $L' = \{w \in L \mid \lvert w \rvert \text{ is even}\}$	✓	
12	$\boldsymbol{r}(\boldsymbol{sr})^* = (\boldsymbol{rs})^*\boldsymbol{r}$	✓	
13	All finite languages are context-free	✓	
14	If $L = \{a^i b^i \mid i \geq 0\}$ then $L \setminus L^2 = \{\,\}$	✓	
15	The language $L = \{w \in \{a, b\}^* \mid w = w^R\}$ is non-regular	✓	

3. DFA construction (optional)

Consider the alphabet $\Sigma = \{0, 1\}$. We can interpret any string $w \in \Sigma^*$ as a non-negative integer written in binary, where we ignore possible leading zeroes. Any string $w \in \{0\}^*$ represents the integer zero. For instance, the string $w_1 = 010011$ represents the integer $16 + 2 + 1 = 19$, $w_2 = 0000101010$ the integer $32 + 8 + 2 = 42$, $w_3 = 00$ the integer 0, and $w_4 = \lambda$ the integer 0. In general, if $w = w_1 w_2 \cdots w_n$, then w represents the integer $\sum_{i=1}^{n} 2^{n-i} w_i$.

We say a string w is *divisible by 3* if the integer w represents is divisible by 3.

Give a **DFA** that accepts exactly the strings in Σ^* that are divisible by 3. Briefly justify your answer.

Note: Full points to correct and justified solutions. No points to partial solutions.