University of Calgary

CPSC 453:

Introduction to Computer Graphics,

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Assignment #3

For
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Q1.

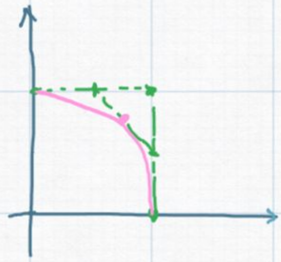
A. $P(u) = (1-u)^2 \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + 2u(1-u) \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + u^2 \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

$(1,0) \qquad (1,1) \qquad (0,1)$

$$= 0.5^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1.0 \times 0.5 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 0.25 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

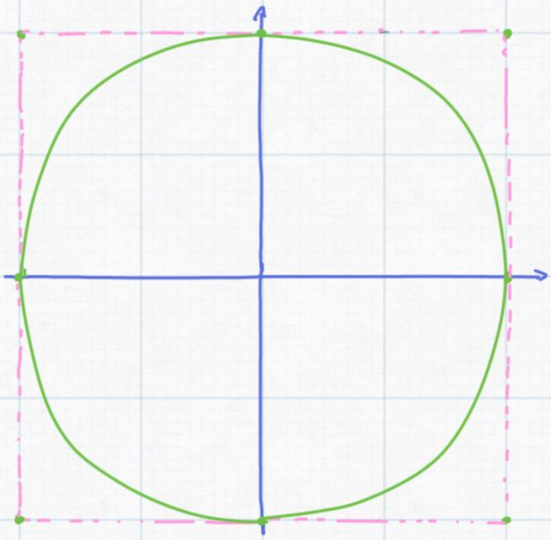
$$= \begin{pmatrix} 0.25 \\ 0 \end{pmatrix} + \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} + \begin{pmatrix} 0 \\ 0.25 \end{pmatrix}$$

$$= \begin{pmatrix} 0.75 \\ 0.75 \end{pmatrix}$$



B.

rounded square



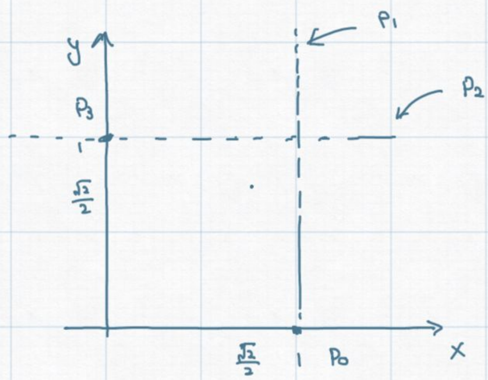
Q1. C

$$P(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u) P_2 + u^3 P_3$$

$$\begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix} = 0.5^3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3 \times 0.5^3 \begin{pmatrix} 1 \\ y \end{pmatrix} + 3 \times 0.5^3 \begin{pmatrix} x \\ 1 \end{pmatrix} + 0.5^3 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\frac{\sqrt{2}}{2} = 0.125 + 0.375 + 0.375x + 0 \quad \Rightarrow \quad x = 0.55228474$$

$$\frac{\sqrt{2}}{2} = 0 + 0.375y + 0.375 + 0.125 \quad \Rightarrow \quad y = 0.55228474$$



Q1. d.

let $u = 0.25$

$$P(u) = (1-u)^3 P_0 + 3(1-u)^2 u P_1 + 3(1-u)u^2 P_2 + u^3 P_3$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = 0.75^3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3 \times 0.75^2 \times 0.25 \begin{pmatrix} 1 \\ 0.55 \end{pmatrix} + 3 \times 0.75 \times 0.25^2 \begin{pmatrix} 0.55 \\ 1 \end{pmatrix} + 0.25^3 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.92109375 \\ 0.38828125 \end{pmatrix}$$

$P_0 = (1, 0)$

$P_3 = (0, 1)$

unite circle : $x^2 + y^2 = 1$

for $(0.92109375, 0.38828125)$

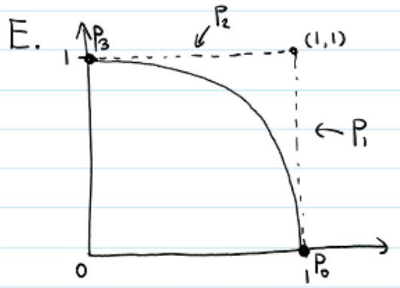
$$0.92109375^2 + 0.38828125^2 = 1$$

P_0 $\quad 1^2 + 0^2 = 1$

P_3 $\quad 0^2 + 1^2 = 1$

So the curve is part of a circular arc.

E.



From 1A we know that the middle point of the curve locates at $\begin{pmatrix} 0.75 \\ 0.75 \end{pmatrix}$

So that, we have the following equation:

$$\begin{pmatrix} 0.75 \\ 0.75 \end{pmatrix} = (1-\tfrac{1}{2})^3 P_0 + 3\cdot\tfrac{1}{2}(1-\tfrac{1}{2})^2 P_1 + 3\cdot(\tfrac{1}{2})^2(1-\tfrac{1}{2})P_2 + (\tfrac{1}{2})^3 P_3$$

$$\begin{pmatrix} 0.75 \\ 0.75 \end{pmatrix} = \frac{1}{8}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{3}{8}\begin{pmatrix} 1 \\ y \end{pmatrix} + \frac{3}{8}\begin{pmatrix} x \\ 1 \end{pmatrix} + \frac{1}{8}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{8} \\ \frac{3}{8}y \end{pmatrix} + \begin{pmatrix} \frac{3}{8}x \\ \frac{3}{8} \end{pmatrix} = \begin{pmatrix} \frac{6}{8} \\ \frac{6}{8} \end{pmatrix} - \begin{pmatrix} \frac{1}{8} \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ \frac{1}{8} \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{8}+\frac{3}{8}x \\ \frac{3}{8}+\frac{3}{8}y \end{pmatrix} = \begin{pmatrix} \frac{5}{8} \\ \frac{5}{8} \end{pmatrix} \Rightarrow$$

to solve x: $\frac{3}{8}+\frac{3}{8}x = \frac{5}{8}$, $\frac{3}{8}x = \frac{2}{8}$, $x = \frac{2}{3}$

to solve y: $\frac{3}{8}+\frac{3}{8}y = \frac{5}{8}$, $\frac{3}{8}y = \frac{2}{8}$, $y = \frac{2}{3}$

So that, four control points of a cubic Bézier that form a curve identical to that of 1A are:

$P_0: \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $P_1: \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix}$ $P_2: \begin{pmatrix} \frac{2}{3} \\ 1 \end{pmatrix}$ $P_3: \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Q2.

A. C^0 continous : 1. 7. 8 . 14

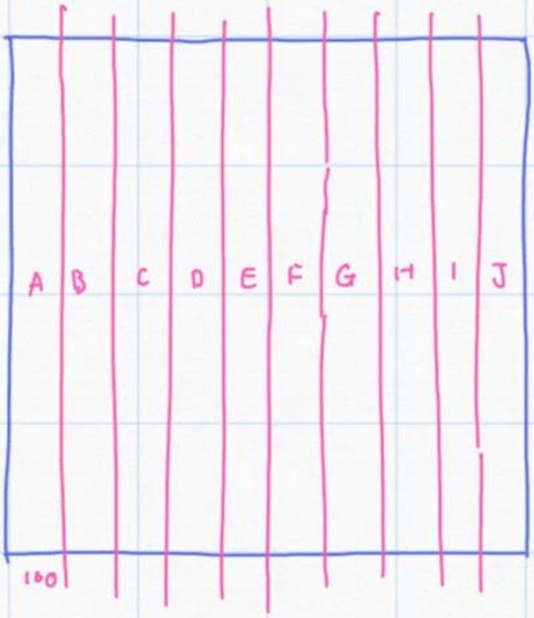
B. G^1 continous : 9. 6. 4. 11. 12 . 3 . 2 . 13

C. C^1 continous : 10, 5,

Q3:

导入

A.



1000 x 1000

3 characters per second

300 pixel per second 60 frames

300 / 60 = 5 pixel

B.

300 / 24 = 12.5 pixel

in order to account for this difference in the programe, we treat frame pre second as a variable.

In the case of three characters pre second, we calculate how many pixels that need to move. by using (3 characters * 100 pixels) / present fps whenever fps got changed