University of Calgary

CPSC 453:

Introduction to Computer Graphics,

Fall 2018

Assignment #1

For
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By
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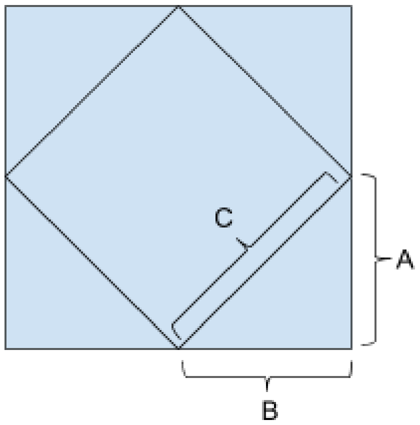
Question 1: Squares & Diamonds

A.

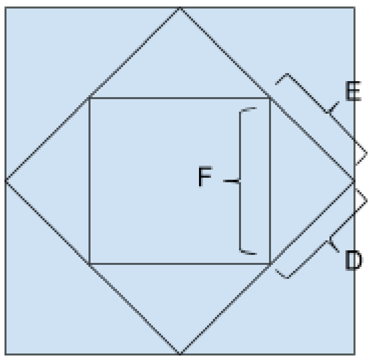


From the question we have known the outermost square has a side length of 1.0. So we can conclude both A and B equal to half of the side which is 0.5. Then we use the Pythagorean theorem to calculate C in which,

$$C^2 = A^2 + B^2$$
$$C = \sqrt{A^2 + B^2}$$
$$C = \sqrt{0.5^2 + 0.5^2}$$
$$C = \frac{\sqrt{2}}{2}$$

So that, the side length of the diamond nested immediately inside is $\frac{\sqrt{2}}{2}$.

B.



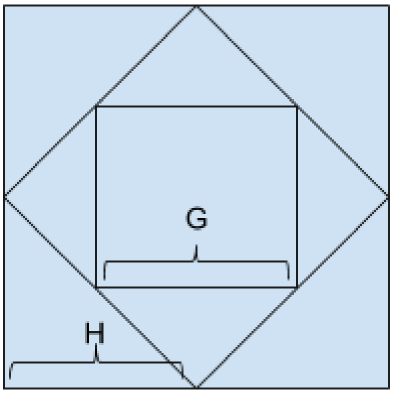
From question 1A we have known the diamond shape has a side length of $\frac{\sqrt{2}}{2}$. So we can conclude both E and D equal to half of the side which is $\frac{\sqrt{2}}{4}$. Then we use the Pythagorean theorem to calculate F in which,

$$F^2 = E^2 + D^2$$
$$F = \sqrt{E^2 + D^2}$$
$$F = \sqrt{(\frac{\sqrt{2}}{4})^2 + (\frac{\sqrt{2}}{4})^2}$$

$$F = 0.5$$

So that, the side length of the square nested immediately inside the diamond is 0.5.

C.



After working on question 1A & 1B, we noticed that the length of side of square in level 2 is ½ of the side of the outermost square in which $H = G$. We can conclude that in each level, the side of the square equals to the side of square from the previous level. In order to find the maximum levels we can have before the smallest square is less than on pixel wide, we create the inequality equation below:

$$\frac{1000}{2^{n-1}} \geq 1 \text{, where n is the number of levels}$$

$$1000 \geq 2^{n-1}$$

From the inequality equation, we can find that the maximum level is 10. We then find the side of the diamond in level 10 in order to make sure that the side of the diamond is larger than 1 as well. After applying Pythagorean theorem we found that the side is larger than 1.

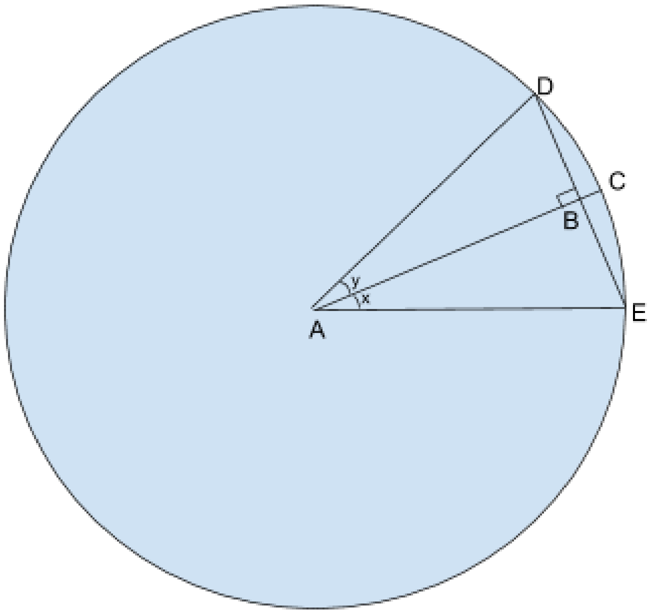
As a result, we can draw maximum of 10 levels in this nested pattern before the smallest square is less than on pixel wide.

Question 2: Parametric Spiral

A. $x(u) = u * sin(u)$

$y(u) = u * cos(u)$, where $0 \le u \le 6\pi$

B.



Assume that point A is the origin of the circle, we draw it's diameter AC. We then rotate the diameter to make an angle and connect DE assuming that BC is 1 pixel. So that, DE is the maximum length of segment that make the drawing no more than one pixel off the true circle. From the question, we know that the radius of the circle is $AD = AC = AE = 1000/2 = 500\ pixels$. By using minimal amount of segments, we assume $BC = 1\ pixel$, then $AB = AC - BC = 500 - 1 = 499\ pixels$. Then we can calculate the angle y in which:

$$cos(y) = \frac{AB}{AD}$$

$$cos(y) = \frac{499}{500}$$

$$y = cos^{-1}(\tfrac{499}{500}) \approx 3.62^{\circ}$$

So that, $\angle DAE = 2 * y = 2 * 3.62^{\circ} \approx 7.25^{\circ}$. Since one segment which makes the drawing no more than one pixel off the true circle, then we need

$$360^{\circ}/7.25^{\circ} \approx 49.7 = 50$$

50 segments minimal in order to make sure that there is no more than one pixel off the true circle.

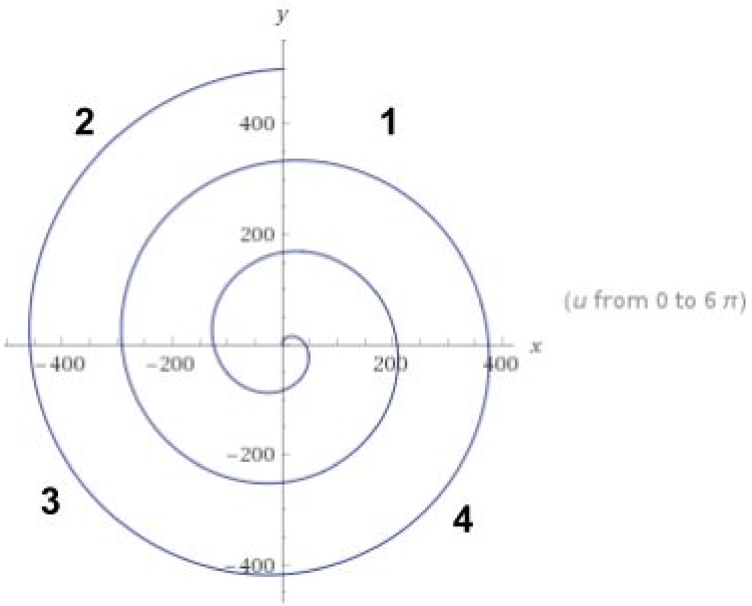
C. In order to fit the equation $x(u) = u * sin(u)$ and $y(u) = u * cos(u)$ where $0 \le u \le 6\pi$ into a $1000 * 1000$ pixel window, we need to calculate the distance from the origin to the y coordinate is 500 pixels when $u = 6\pi$. So that, we have the calculation below:

$$x * 6\pi * cos(6\pi) = 500$$

$$x = 500/(6\pi * cos(6\pi))$$

$$x = 221358/8345$$

After we plotted the equation $x(u) = \frac{221358}{8345} * u * sin(u)$ and $y(u) = \frac{221358}{8345} * u * cos(u)$ where $0 \le u \le 6\pi$ into "**WolframAlpha:** https://m.wolframalpha.com/" we get:



Since the radius of the circle is increasing, in order to have apply the method we used in question 2B, we separate the graph into pieces by using the quadrant. As "u" increases, we take the longest distance from origin to the current coordinate which is the distance between (0, 0) and (x, y). To illustrate, when u travels from 0 to $\frac{1}{2}\pi$ in 1st quadrant, we calculate the x when y = 0 and treat the x as the radius of the circle and calculate the segments. When u travels from $\frac{1}{2}\pi$ to π in 4th quadrant, we calculate the y when x = 0 and treat the y as the radius of the circle and calculate the segments. When u travels from π to $\frac{3}{2}\pi$ in 3rd quadrant, we calculate the x when y = 0 and treat the x as the radius of the circle and calculate the segments. When u travels from $\frac{3}{2}\pi$ to 2π in 2nd quadrant, we calculate the y when x = 0 and treat the y as the radius of the circle and calculate the segments. So on and so forth until we have calculated till the last circle which is y = 500 pixels, then we add them together to get a rough estimate of the line segments needed.

This method has large margin of error starting from the beginning of the graph. As the "u" goes larger, the margin of error will become smaller. For this reason, we will start from $u = 2\pi$, which is after the first cycle has finished.

- when $2\pi \le u \le \frac{5}{2}\pi$, take (x, 0) and $u = \frac{5}{2}\pi$ into equation we get $x \approx 208.3$. after taking x as the radius of the circle into the calculation in question 2B, we get $\angle DAE \approx 11.2^\circ$. then we need

$$90^\circ/11.2^\circ \approx 8.01 = 9$$

9 segments minimal in order to make sure that there is no more than one pixel off the true circle.

- when $\frac{5}{2}\pi \le u \le 3\pi$, take (0, y) and $u = 3\pi$ into equation we get $y =- 250$.

after taking y as the radius of the circle into the calculation in question 2B, we get $\angle DAE \approx 10.25^\circ$. then we need

$$90^\circ/10.25^\circ \approx 8.77 = 9 \text{ segments}$$

- when $3\pi \le u \le \frac{7}{2}\pi$, take (x, 0) and $u = \frac{7}{2}\pi$ into equation we get $x \approx -291.7$. after taking x as the radius of the circle into the calculation in question 2B, we get $\angle DAE \approx 9.49^\circ$. then we need

$$90^\circ/9.49^\circ \approx 9.48 = 10 \text{ segments}$$

- when $\frac{7}{2}\pi \le u \le 4\pi$, take (0, y) and $u = 4\pi$ into equation we get $y \approx 333.3$. after taking y as the radius of the circle into the calculation in question 2B, we get $\angle DAE \approx 8.88^\circ$. then we need

$$90^\circ/8.88^\circ \approx 10.13 = 11 \text{ segments}$$

- when $4\pi \le u \le \frac{9}{2}\pi$, take (x, 0) and $u = \frac{9}{2}\pi$ into equation we get $x = 375$. after taking x as the radius of the circle into the calculation in question 2B, we get $\angle DAE \approx 8.38^\circ$. then we need

$$90^\circ/8.38^\circ \approx 10.75 = 11 \text{ segments}$$

- when $\frac{9}{2}\pi \le u \le 5\pi$, take (0, y) and $u = 5\pi$ into equation we get $y \approx -416.7$. after taking y as the radius of the circle into the calculation in question 2B, we get $\angle DAE \approx 7.94^\circ$. then we need

$$90^\circ/7.94^\circ \approx 11.33 = 12 \text{ segments}$$

- when $5\pi \le u \le \frac{11}{2}\pi$, take (x, 0) and $u = \frac{11}{2}\pi$ into equation we get $x \approx -458$. after taking x as the radius of the circle into the calculation in question 2B, we get $\angle DAE \approx 7.57^\circ$. then we need

$$90^\circ/7.57^\circ \approx 11.89 = 12 \text{ segments}$$

- when $\frac{11}{2}\pi \le u \le 6\pi$, take (0, y) and $u = 6\pi$ into equation we get $y = 500$. after taking y as the radius of the circle into the calculation in question 2B, we get $\angle DAE \approx 7.24^\circ$. then we need

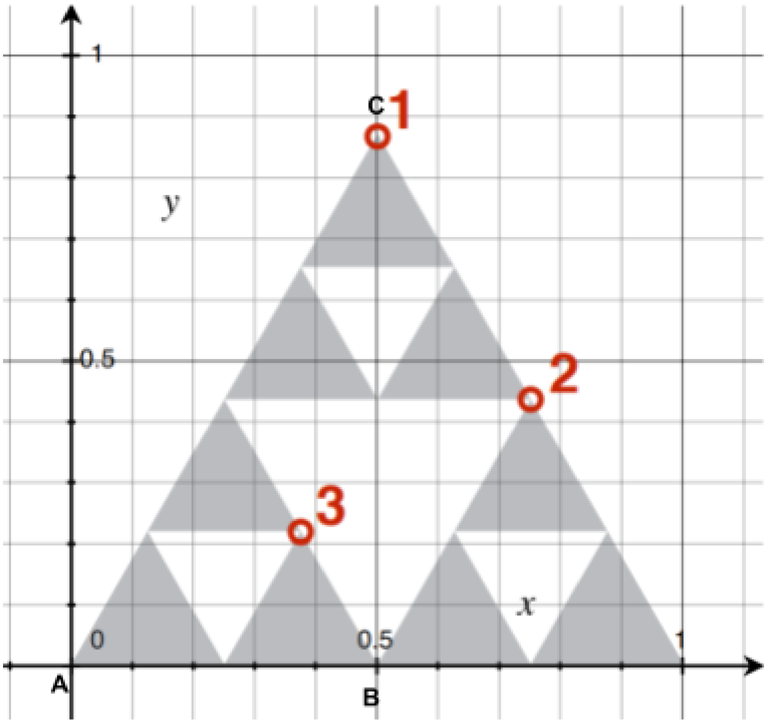
$$90^\circ/7.24^\circ \approx 12.41 = 13 \text{ segments}$$

Then we add all the segments together we get:

$$9 + 9 + 10 + 11 + 11 + 12 + 12 + 13 = 87$$

87 line segments needed to approximate the spiral to an accuracy of one pixel.

Question 3: Sierpinski Triangle & Merge Sponge



A. From the question we can know each small triangle in the biggest Sierpinski triangle is equilateral triangle with the same size. We first calculate the coordinate of vertex 1.

$$sin(\angle CAB) = \frac{CB}{AC}$$
$$sin(60^{\circ}) = \frac{CB}{1}$$
$$CB = sin(60^{\circ})$$
$$CB = \frac{\sqrt{3}}{2}$$

Then we know that the y coordinates for vertex 2 is $\frac{1}{2} * CB = \frac{\sqrt{3}}{4}$ and vertex 3 is $\frac{1}{4} * CB = \frac{\sqrt{3}}{8}$. It is obvious that the x coordinates for vertices 1, 2 and 3 are 0.5, 0.75 and 0.375. As a result, the coordinates for vertices 1, 2 and 3 are (0.5, $\frac{\sqrt{3}}{2}$), (0.75, $\frac{\sqrt{3}}{4}$), and (0.375, $\frac{\sqrt{3}}{8}$).

B. By observing the pattern in each iteration of 2D Menger Sponge, we have found that the 1st iteration begins with a square, the square is separated into 9 subsquares and the central subsquare is removed. In the 2nd iteration, the same procedure is used to the remaining 8 subsquares. If the number of the iteration(s) is x, then we have:

$$number\ of\ squares = 8^x \text{, and}$$
$$8^x \leq 1000000$$
$$x \leq 6.64$$

As a result, we can draw 6 iterations of the 2D Menger sponge before we exceed our maximum capacity.