University of Calgary

CPSC 453:

Introduction to Computer Graphics,

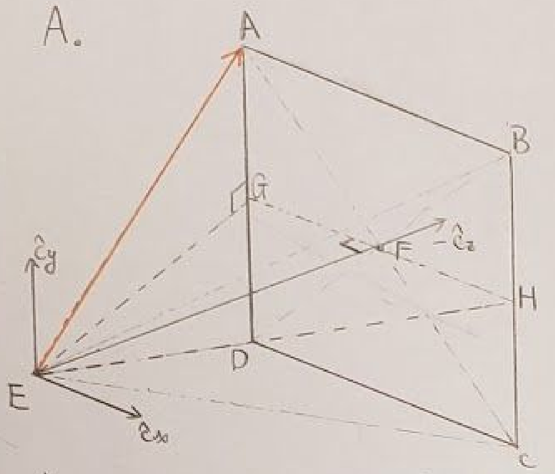
Fall 2018

Assignment #4

For
John Hall, Sonny Chan

Submitted by:
BenKun Chen (ID: 30005337 | Email: benkun.chen@ucalgary.ca)
Heavenel Cerna(ID: 30019914 | Email: Heavenel.cerna@ucalgary.ca)

Question 1: Ray Generation



From the question we know that the square image ABCD is measured 100x100 pixels with a field of view of $\angle GEH = 60°$. So that, we know $GF = FH = AB/2 = 100/2 = 50$ pixels and $\angle GEF = \angle GEH/2 = 60°/2 = 30°$

$$\sin(\angle GEF) = GF/EG$$

$$EG = GF/\sin(\angle GEF) = 50/\sin(30°) = 100 \text{ pixels.}$$

We also know that $AG = GD = AD/2 = 100/2 = 50$ pixels

$$AE^2 = EG^2 + AG^2$$

$$AE = \sqrt{100^2 + 50^2} = \frac{335522}{3001} \text{ pixels.}$$

We know that the coordination for point A is $(-50, 50, -EF)$ from camera reference frame perspective where EF is $\tan(\angle GEF) = GF/EF$

$$EF = GF/\tan(\angle GEF) = 50/\tan(30°) \approx 86.6025 \text{ pixels.}$$

Since $\hat{c} = \frac{\vec{c}}{\|\vec{c}\|}$, the normalized direction vector for A can be calculated as following:

$$\hat{c}_x = \frac{-50}{AE} = -0.4472135955$$

$$\hat{c}_y = \frac{50}{AE} = 0.4472135955$$

$$\hat{c}_z = \frac{-EF}{AE} = -0.7745966693$$

So that, the direction vector after normalization is

$$(-0.4472135955\,\hat{c}_x,\ 0.4472135955\,\hat{c}_y,\ -0.7745966693\,\hat{c}_z)$$

B. It is not possible to crop the image from 1A to form a result that looks as if you had rendered your scene with a 30 degree field of view. If we use 30 degree field of view based on the 60 degree field of view we will get approximately 47 x 47 pixels of cropped image. However, if we rendered our scene with a 30 degree field of view it will be a 100 x 100 pixels image. The difference maybe not noticeable but we will lose pixels if we use 30 degree field of view based on the 60 degree field of view.

Question 2: Ray Intersection

A. We know that $r(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix}$ and the unit sphere has radius of 1 centered at $\begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix}$

$$|O + tD - C|^2 - R^2 = 0$$

$$\left|\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix}\right|^2 - 1^2 = 0$$

$$\left|t\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix}\right|^2 - 1 = 0$$

$$17t^2 - 24t + 9 - 1 = 0$$

$$17t^2 - 24t + 8 = 0$$

$$\begin{cases} t_1 = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} = \dfrac{24 + \sqrt{24^2 - 4 \cdot 17 \cdot 8}}{2 \cdot 17} = 0.8722604193 \\[2ex] t_2 = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a} = \dfrac{24 - \sqrt{24^2 - 4 \cdot 17 \cdot 8}}{2 \cdot 17} = 0.5395042866 \end{cases}$$

So that, the two intersections are

$$\begin{cases} r(t_1) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t_1\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} = 0.8722604193\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.8722604193 \\ -3.489041677 \end{bmatrix} \\[2ex] r(t_2) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t_2\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} = 0.5395042866\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5395042866 \\ -2.158017146 \end{bmatrix} \end{cases}$$

And, the point $\begin{bmatrix} 0 \\ 0.5395042866 \\ -2.158017146 \end{bmatrix}$ is nearest to the camera.

B. We know points $A = (2, -1, -3)$, $B = (-1, 3, -5)$ and $C = (-1, -1, -3)$ so that we calculate

$$\vec{AB} = (-1-2, 3-(-1), -5-(-3)) = (-3, 4, -2)$$

$$\vec{BC} = (-1-(-1), -1-3, -3-(-5)) = (0, -4, 2)$$

Since the vectors $\vec{AB}$ & $\vec{BC}$ lies on the plane and we know the cross product of two vectors will be orthogonal to both of these vectors.

$$\hat{n} = \vec{AB} \times \vec{BC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 4 & -2 \\ 0 & -4 & 2 \end{vmatrix} = \begin{vmatrix} 4 & -2 \\ -4 & 2 \end{vmatrix}\vec{i} - \begin{vmatrix} -3 & -2 \\ 0 & 2 \end{vmatrix}\vec{j} + \begin{vmatrix} -3 & 4 \\ 0 & -4 \end{vmatrix}\vec{k}$$

$$= 0\vec{i} + 6\vec{j} + 12\vec{k}$$

So that, the normal vector of the plane this triangle rests on is $0\vec{i} + 6\vec{j} + 12\vec{k}$.

C. From question 2B we have calculated that the normal vector $\vec{n} = 0\cdot\vec{i} + 6\cdot\vec{j} + 12\vec{k}$

since the equation for the plane is $ax + by + cz + d = 0$, we know $a=0$, $b=6$ & $c=12$. So that

$$0\cdot x + 6\cdot y + 12\cdot z + d = 0$$

$$6y + 12z + d = 0$$

$$-6 - 36 + d = 0 \quad \leftarrow \text{plug in point } (2,-1,-3) \text{ from 2B}$$

$$d = 42$$

So that, the plane equation is $6y + 12z + 42 = 0$

Since the ray's parametric equation is $r(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix}$, we can rewrite the format of

the equation as $\begin{cases} x = 0 + t\cdot 0 = 0 \\ y = 0 + t\cdot 1 = t \\ z = 0 + t\cdot(-4) = -4t \end{cases}$, and we plug them into $6y + 12z + 42 = 0$ and we get:

$$6\cdot t + 12\cdot(-4t) + 42 = 0$$

$$6t - 48t + 42 = 0$$

$$\Rightarrow \quad -42t = -42$$

$$t = 1$$

So that, $\begin{cases} x = 0 \\ y = t = 1 \\ z = -4t = -4\cdot 1 = -4 \end{cases}$, the position of intersection between the above ray and the plane in 2B is $\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix}$.

D. By using the equation $\begin{pmatrix} -d & P_1 - P_0 & P_2 - P_0 \end{pmatrix}\begin{pmatrix} t \\ u \\ v \end{pmatrix} = 0 - P_0$ and $P_0 = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$, $P_1 = \begin{bmatrix} -1 \\ 3 \\ -5 \end{bmatrix}$, and $P_2 = \begin{bmatrix} -1 \\ -1 \\ -3 \end{bmatrix}$.

We have

$$\left(-\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} \begin{bmatrix} -3 \\ 4 \\ -2 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix} \right)\begin{pmatrix} t \\ u \\ v \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$$

$$\left(\begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} \begin{bmatrix} -3 \\ 4 \\ -2 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix} \right)\begin{pmatrix} t \\ u \\ v \end{pmatrix} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{cases} -3u - 3v = -2 & ① \\ -t + 4u = 1 & ② \\ 4t - 2u = 3 & ③ \end{cases}$$

By using $4 \cdot ② + ③$ we get,

$$-\cancel{4t} + 16u + \cancel{4t} - 2u = 4 + 3$$
$$14u = 7$$
$$u = \frac{1}{2}$$

By plug u into ① & ② we get,

$$-3 \cdot \frac{1}{2} - 3v = -2$$
$$-3v = \frac{3}{2} - \frac{4}{2}$$
$$v = -\frac{1}{2} \cdot \left(-\frac{1}{3}\right)$$
$$v = \frac{1}{6}$$

$$-t + 4 \cdot \frac{1}{2} = 1$$
$$-t = 1 - 2$$
$$t = 1$$

So that, $w = 1 - u - v = 1 - \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

As an result, the barycentric coordinates are $w = \frac{1}{3}$, $u = \frac{1}{2}$, $v = \frac{1}{6}$

And $P = \frac{1}{3} \cdot \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix} + \frac{1}{2} \cdot \begin{bmatrix} -1 \\ 3 \\ -5 \end{bmatrix} + \frac{1}{6} \cdot \begin{bmatrix} -1 \\ -1 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix}$ which is the position of intersection between the ray and the triangle.

Question 3: Shading

A. Since we have a surface material that has a purely Lambertian (diffuse), white reflection. To figure out the incident light angle one would see light reflected from the surface at exactly half of the incident intensity we can use the fact that a Lambertian object obeys Lambert's cosine law, which states that the color c of a surface is proportional to the cosine of the angle between the surface normal shown by:

$$c = c_r c_l max(0,\ n \cdot l)\,.$$

We notice that $max(0,\ n \cdot l) \in [0, 1]$. To get half of the incident intensity the following should hold:

$$max(0,\ n \cdot l) = 0.5$$

Which holds if $n \cdot l = cos(60°)$. Therefore the angle needs to be $60°$ measured from the surface normal.

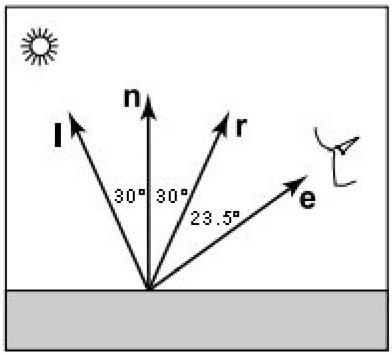
B. To derive the angle, we can use:

$$c = c_l max(0, e \cdot r)^8 .$$

Since r is symmetric to l with regards to the surface normal r is also $30°$ from the surface normal. We know that we get full incident intensity when $e \cdot r = 1$ which geometrically means e and r are lined up. At this case r and e both have the angle $30°$ clockwise from the surface normal. To get half of the incident intensity the following should hold:

$$max(0, e \cdot r)^8 = 0.5$$

Which holds if $(e \cdot r) = cos(23.5°)$ Now that we know that the angle between e and r is $23.5°$. We can find the angle of e by adding the angle between e and r to the angle of r which results to $23.5° + 30° = 53.5°$ clockwise from the surface normal. We also achieve this intensity if we viewed the angle from $30° - 23.5° = 6.5°$ counter-clockwise from the surface normal.



Question 4: Shadows & Reflection

A. The differences between a view ray and a camera ray in terms of how they are treated on our ray tracing system are shown by:

Viewing Ray: $\mathbf{e} + t\mathbf{d}$, $t \in [0,\infty)$
Where:

e: viewpoint
d: view direction

Shadow Ray: $\mathbf{p} + s\mathbf{l}$, $s \in [\epsilon,\infty)$

Where:

$\mathbf{p}$: a point on a surface of an object

$\mathbf{l}$: light source direction

ϵ: some small positive constant

How the viewpoint **e** and view direction **d** of a viewing ray are derived depends on the type of view projection. The point **p** of the shadow ray is the point on the surface of an object that is hit by the viewing ray.

B. The differences between a view ray and a reflected ray in terms of how they are treated on our ray tracing system are shown by:

Viewing Ray: $\mathbf{e} + t\mathbf{d}$, $t \in [0,\infty)$

Where:

$\mathbf{e}$: viewpoint

$\mathbf{d}$: view direction

Reflected Ray: $\mathbf{p} + s\mathbf{r}$, $s \in [\epsilon,\infty)$

Where:

$\mathbf{p}$: a point on a surface of an object

$\mathbf{r} = \mathbf{d} - 2(\mathbf{d}\cdot\mathbf{n})\mathbf{n}$

n: surface normal of an object at point **p**

ϵ: some small positive constant