The Order of Mathematistry

Queering metascience with mathematics

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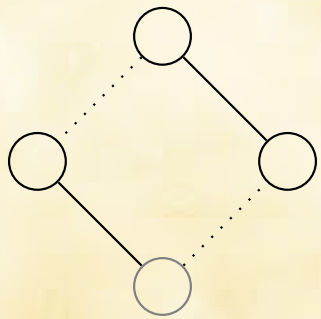
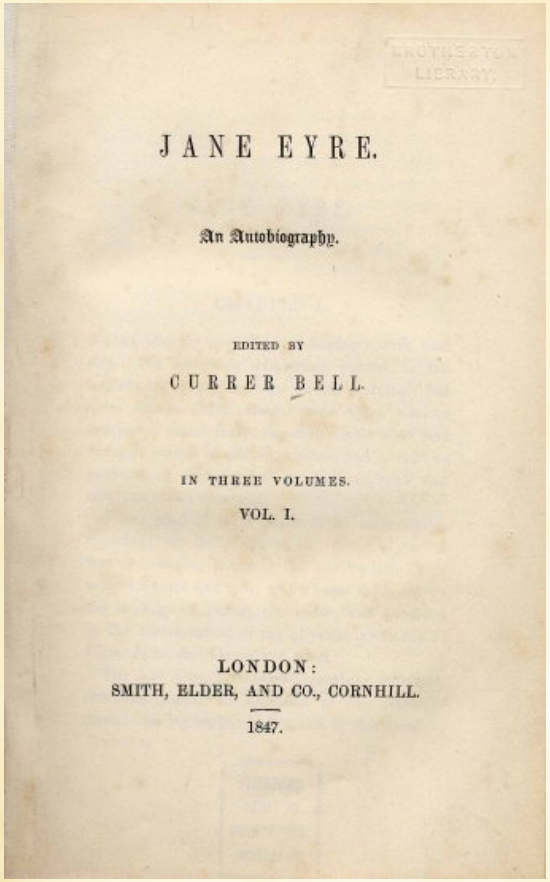


Outline

Queering literature



Jane Eyre, 1847

Charlotte Brontë [2]

Wide Sargasso Sea, 1966

Jean Rhys [8]

Image sources: Wikipedia.

Queering metascience

So let's build an open and reproducible science as a queer reimagining of science and not a small perturbation of the world that is. Such a system will never be perfect.

– Dan Simpson [9]

Preregistration is redundant, at best

The Centre for Open Science defines **preregistration** as specifying the research plan in advance [7].

2019: Preregistration is Hard, and Worthwhile, Nosek *et al.* [6]

2019: Is Preregistration Worthwhile? Szollosi *et al.* [10]

> *The diagonisticity of statistical tests depend entirely on how well statistical models map onto underlying theories, and so improving statistical techniques does little improve theories when the mapping is weak [10].*

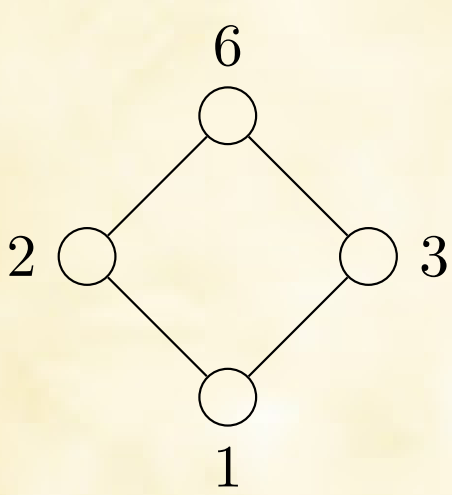
Mathematistry

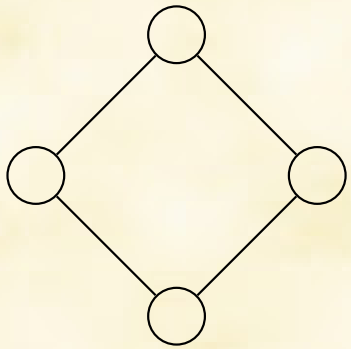
Navarro, a mathematical scientist specialising in psychology, redefines Box's term **mathematistry** [1] to

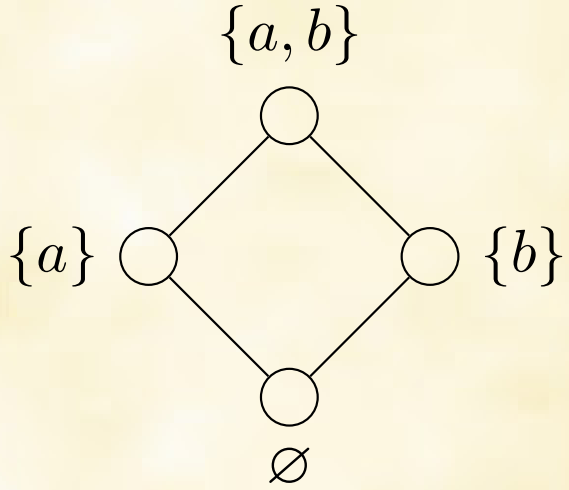
> *'describe using formal tools to define a statistical problem that differs from the scientific one, solving the redefined problem, and declaring the scientific concern addressed' [5].*

We will think of **mathematistry** as the measure of strength of mapping to describe when the mapping is weak [10].

A value judgement is, after all, an order

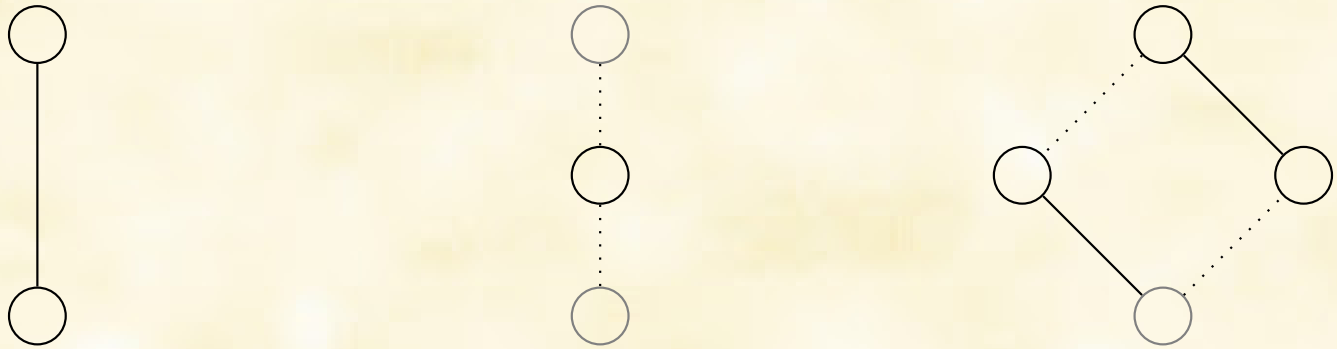






Outline

Heuristics of mathematistry

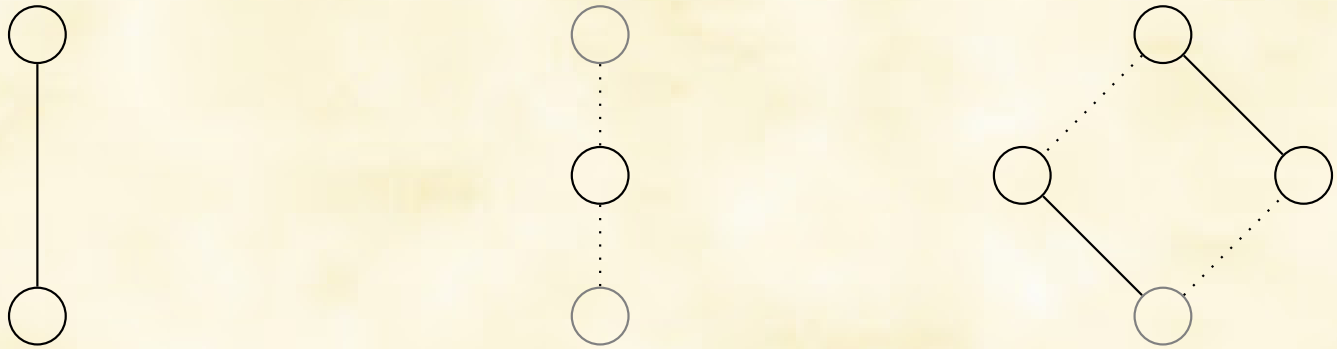


Heuristics of mathematistry

A heuristic can be thought of in terms of a mapping from $C \times M$ to some space of measuring how well a pairing 'furthers the process of scientific discovery' (Devezer *et al., 2019*) [4].

`fpd` := furthers the process of scientific discovery

Heuristics of mathematistry



$$h : C \times M \to \begin{cases} \{0,1\} & \text{if } \texttt{fpd} \text{ or not;} \\ [0,1] & \text{if } \texttt{fpd} \text{ on a spectrum;} \\ H & \text{if } \texttt{fpd} \text{ otherwise.} \end{cases}$$

Heuristics of mathematistry

Let C denote the set of all possible **scientific claims** for which we might provide evidence of, with a scientific method or procedure.

Let M denote the set of all possible **scientific methods** that can be used to provide evidence of scientific claims.

The product $C \times M$ denotes the collection of possible **pairings of claim and methodology**.

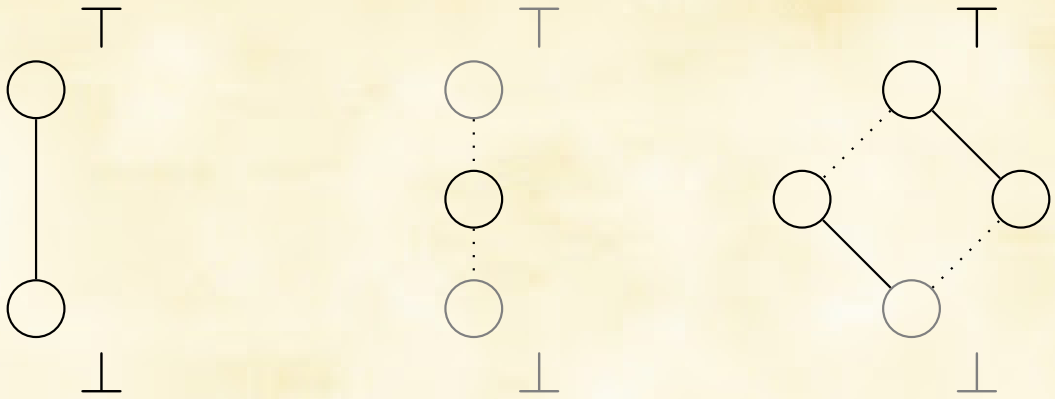
Heuristic of mathematistry

Definition

A **heuristic h of mathematistry** measures the efficacy of a pairing (c, m) of scientific claim $c \in C$, and method $m \in M$ of providing evidence of that claim, measuring how effective method m is at scientifically informing claim c.

We denote $\mathbb{H}$ to be the set of all possible heuristics of mathematistry. For a heuristic h in $\mathbb{H}$, we define the value $h(c, m)$ as the **measure of mathematistry** of pairing (c, m) under heuristic h.

A value judgement on *good enough* [11] science



For a heuristic h to be a member of $\mathbb{H}$, there must exist a scientific method m, and two distinct scientific claims we might reasonably pair m with, c_1 and c_2, such that

$$h(c_1, m) < h(c_2, m)$$

or, conversely, there must exist distinct methods, m_1 and m_2, such that, for a claim c, we have

$$h(c, m_1) < h(c, m_2).$$

A relational operator of mathematistry

Definition

Let $(c_1, m_1) \rightarrow_h (c_2, m_2)$ if and only if $h(c_1, m_1) \leqslant h(c_2, m_2)$ under heuristic h of mathematistry.

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Order

To be considered an order, $\rightarrow_h$ must satisfy three properties [3].

Definition

A binary relation $\leqslant$ on set P is an **order** if, for all $x, y, z \in P$, we have

(i) $x \leqslant x$,

(ii) $x \leqslant y$ and $y \leqslant x$ implies $x = y$,

(iii) $x \leqslant y$ and $y \leqslant z$ imply $x \leqslant z$.

Order

(i) reflexivity $x \leqslant x$,

(ii) antisymmetry $x \leqslant y$ and $y \leqslant x$ implies $x = y$,

(iii) transitivity $x \leqslant y$ and $y \leqslant z$ imply $x \leqslant z$.

When a binary relation satisfies (i) reflexivity and (iii) transitivity, but not (ii) antisymmetry, we say it is a **quasi-order**.

A quasi-order of mathematistry

Definition

When a binary relation satisfies (i) reflexivity and (iii) transitivity, but not (ii) antisymmetry, we say it is a **quasi-order**.

Let $\mathcal{X} \subseteq C \times M$ denote the subset $\mathcal{X}$ of reasonable pairings $C \times M$ of claims and methods.

Lemma

The relation $\to_h$ is a quasi-order on $\mathcal{X}$.

Partitions of mathematistry

Define the equivalence class of mathematistry generated by a reasonable pairing, $(c, m) \in \mathcal{X}$,

$$[[c, m]]_h := \{(x, y) \in \mathcal{X} \mid h(x, y) = h(c, m)\}.$$

We then have reasonable pairings, $\mathcal{X}$, partitioned by mathematistry,

$$\mathfrak{x}_h := \mathcal{X}/_{\to_h}.$$

The Order of Mathematistry

The relation $\rightarrow_h$ is a quasi-order on $\mathcal{X}$, only satisfying reflexivity and transitivity, but not antisymmetry.

It can be shown that $\rightarrow_h$ on $\mathfrak{X}_h$ satisfies reflexivity, antisymmetry, and transitivity.

Theorem

The relation $\to_h$ is an order on $\mathfrak{X}_h$.

Definition

We refer to

$$\langle \mathfrak{X}; \to \rangle_h := \langle \mathcal{X} / \to_h ; \to_h \rangle$$

as the **order of mathematistry** under heurisic $h \in \mathbb{H}$.

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The order of mathematistry

References I

George E. P. Box. 'Science and Statistics'. en. In: *Journal of the American Statistical Association* 71.356 (Dec. 1976), pp. 791–799. ISSN: 0162-1459, 1537-274X. DOI: 10.1080/01621459.1976.10480949.

Charlotte Brontë. *jane eyre*. OUP Oxford, 2000.

B. A. Davey and H. A. Priestley. *Introduction to Lattices and Order*. English. Google-Books-ID: vVVTxeuiyvQC. Cambridge University Press, Apr. 2002. ISBN: 978-0-521-78451-1.

Berna Devezer et al. 'Scientific Discovery in a Model-Centric Framework: Reproducibility, Innovation, and Epistemic Diversity'. en. In: *PLOS ONE* 14.5 (May 2019), e0216125. ISSN: 1932-6203. DOI: 10.1371/journal.pone.0216125.

Danielle Navarro. *Between the devil and the deep blue sea: Tensions between scientific judgement and statistical model selection*. preprint. PsyArXiv, Oct. 2018. DOI: 10.31234/osf.io/39q8y.

Brian A. Nosek et al. 'Preregistration Is Hard, And Worthwhile'. en. In: *Trends in Cognitive Sciences* 23.10 (Oct. 2019), pp. 815–818. ISSN: 1364-6613. DOI: 10.1016/j.tics.2019.07.009.

Centre for Open Science. *Preregistration*. 2020.

Jean Rhys. *Wide sargasso sea*. WW Norton & Company, 1992.

References II

Dan P. Simpson. *What if it's never decorative gourd season? ≪ Statistical Modeling, Causal Inference, and Social Science*. 2019.

Aba Szollosi et al. *Preregistration is redundant, at best*. preprint. PsyArXiv, Oct. 2019. DOI: 10.31234/osf.io/x36pz.

Greg Wilson et al. 'Good enough practices in scientific computing'. English. In: *PLOS Computational Biology* 13.6 (June 2017). Ed. by Francis Ouellette, e1005510. ISSN: 1553-7358. DOI: 10.1371/journal.pcbi.1005510.