Elementary Particle Physics

or

Fundamental Particle Physics

1. Introduction

The terms suggest particles with no internal structure, particles that are not composed of simpler particles.

The terms require clarification as we proceed.

[e.g. $n \rightarrow p + e^- + \tilde{\nu}$

Does this mean the neutron 'is composed of' a proton, an electron and an anti-neutrino? (No.)]

A less equivocal title would be

High Energy Physics

(certainly we need high energies to make progress in investigating the subject)
(and higher and higher ...)

What is the motivation of the subject?

The desire / hope / feeling that all the properties of the Universe may be explained in terms of a few (very few?) basic constituents.

This desire/h/f has been strongly encouraged by experimental findings at various stages of the history of science.

There may also be the hope that we will reach the stage where we may say: "That was a clever way of designing a Universe"

subtle, unexpected....

Einstein: God is subtle but he is not malicious.
(she?

Fundamental Theories of Physics

A. Relativity Space, time, gravitation

B. Particle Physics The basic particles of physics, and the general nature of their interactions (non-gravitational)

C. Quantum Theory (+ Quantum Electrodynamics, QED and Quantum Field Theory) The detailed laws of behaviour of systems (leaving out gravity).

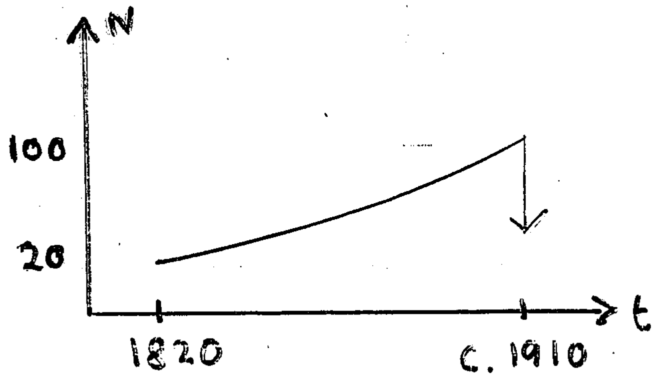
There is, as yet, no quantum theory of gravity [Quantum theory and relativity clash. This is the biggest blemish on today's physics.] When there is gravitation may take its place in B, C above.

How many particles would we accept.
5? 10? 20??
At times historically physicists have hoped to explain everything with very few particles [1928, 2]
At other times the number has been well above 20. [Including today, 38]

1.1 Nineteenth century atomic theory

Dalton c. 1820

Comparatively few atoms, c. 20.
But as the nineteenth century went on, the number grew to around 100.
[Also regularities, periodicities, patterns suggested (at least in retrospect) that there was some deeper structure – a hint of the approach to take in analogous circumstances c. 1965]



[to be continued; for general information only]

1.2 The nuclear atom

For this reason, it was very pleasing when, towards the beginning of the twentieth century, it seemed that the existence and general properties of all the atoms could be explained in terms of a very few elementary particles. (In particular the periodicities, patterns could be explained.)

Electron "Cathode ray"; established as a particle (e/m measured) by Thomson (1897)

Proton Hydrogen nucleus, thus implied by Rutherford (1911) in his nuclear atom.

Photon Einstein (1905) in his theory of photoelectric effect. (Not generally acknowledged till 1923 – Compton effect.)

<u>Neutron</u> Not detected till 1932 (Chadwick). Till then the orthodox view was that ~~an~~ a ~~atom~~ nucleus (A, Z) contains A protons and $A-Z$ electrons [though several people e.g. Rutherford suspected the existence of the neutron]

Today the <u>Heisenberg principle</u> makes it inconceivable that there could be electrons in the nucleus. (Low $\Delta r \rightarrow$ High $\Delta p_r \rightarrow$ High E)

At the time it created great problems for <u>angular momentum</u>.

<u>pre 1932</u> $2A - Z$ particles in nucleus, each with spin $\hbar/2$. If Z is odd, must yield nucleus with spin an <u>odd</u> multiple of $\hbar/2$.

<u>post 1932</u> A particles in nucleus, may be even or odd. Even if Z is odd, nucleus may have zero spin.

The one "good" thing about the pre-1932 model is that it does seem to "explain" β-decay

$$F_7^{16} \longrightarrow O_8^{16} + e^- + \tilde{\nu}_e$$

$$n_0^1 \longrightarrow p_1^1 + e^- + \tilde{\nu}_e$$

If there are electrons in the nucleus, it does seem to explain why we see them coming out!

Lifetime of $n \sim 15$ minutes [exceptionally long because $M_n - (M_p + M_e)$ very small]

Many (candidates ~~for~~ being) elementary particles are unstable with much shorter lifetimes:- 10^{-8}s, 10^{-16}s, 10^{-23}s

Do we (at this stage in the argument) deny they are elementary?

Do we (e.g.) say n is not elementary but "consists of" p, e and $\tilde{\nu}$??

No. At this stage of the argument, we say n is (a good candidate for being) fundamental because it has its own distinct properties:- mass, charge. and various quantum numbers we shall meet. The same goes for the particles of shorter life-times.

[Later we must re-consider.]

2. 1932-50; a few more particles

After 1932, a few more particles became established over the next 15-20 years.

Pre 1932, particles were discovered in the laboratory

Post 1932, those particles discovered experimentally were found in cosmic rays usually using balloons to get above the atmosphere.

Cosmic rays are free (though one may require relatively sophisticated detectors). They contain particles of higher energy than those produced by any particle accelerator ever likely to be constructed on earth.

BUT out of our control.

During this period (as later) there was much interplay between theory and experiment.

e.g ν predicted [empirical] theoretically, then searched for and found much later (1959)

e^+ predicted [high-powered theory] theoretically, then found experimentally by physicists unaware of prediction

μ^-, μ^+ found experimentally, never really explained

2.1 Positrons (and Anti-Matter)

In 1928, Dirac produced a relativistic quantum theory of the electron. . . .

While astoundingly successful in many ways, it appeared to have a major defect. It predicted electrons with negative energy as well as those with positive energy.

In fact the problem stemmed from the well-known non-quantum equation

$$E^2 = p^2c^2 + m^2c^4 \qquad (1)$$

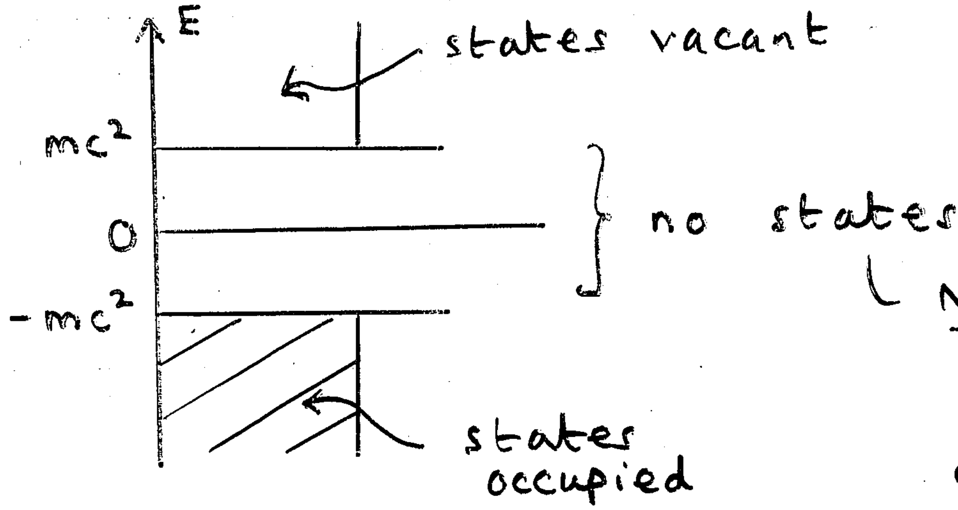
which leads to

$$E = \pm\sqrt{p^2c^2 + m^2c^4} \qquad (2)$$

In quantum theory the problem is exacerbated. One cannot ignore the negative-energy solutions, as electrons with positive energy will automatically make transitions to states with negative energy, if these states are unoccupied

Dirac's solution

The "normal" state of affairs is:



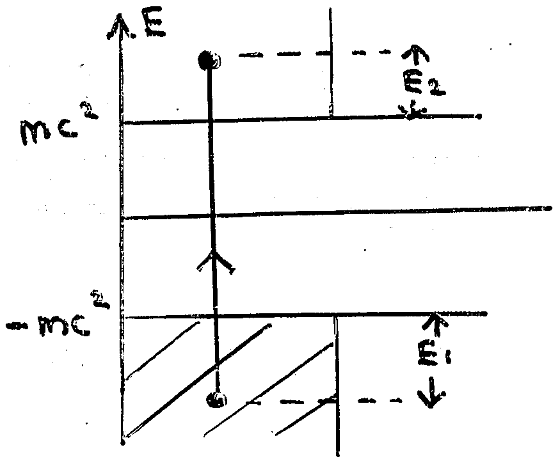
N.B. from (2) $E > mc^2$ or $E < -mc^2$

The exclusion principle prevents any electron losing energy.

Dirac assumed: this "sea" of $-$ve energy electrons is not observed.

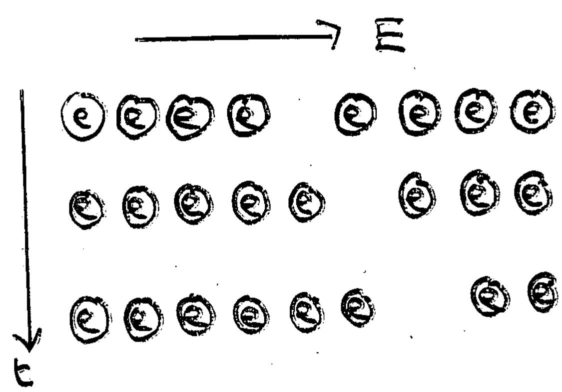
If an electron of $-$ve energy is given energy $> 2mc^2$, it may make a transition to a state of $+$ve energy.

This produces two independent objects, a $+$ve energy electron, and a <u>hole</u> in the sea of $-$ve energy electrons.



Energy of system relative to "normal" state $= 2mc^2 + E_1 + E_2$

Consider "hole"



i.e. hole moves in direction of field as though it has +ve charge.

hole + electron have energy $2mc^2 + E_1 + E_2$

electron has energy $mc^2 + E_2$

$\therefore$ hole " " $mc^2 + E_1$

i.e. hole has +ve mass (m), +ve energy, +ve charge

Hole $\equiv$ positron

Properties exactly those of electron, apart from +ve charge i.e. e^+

[N.B. Argument above a little simple.
Dirac suggested that m(hole) $\neq$ m(electron)
indeed that hole $\equiv$ proton

This would have explained all of
physics in ~~terms~~ of 2 genuinely
different particles - e^- and γ — the least ever!

But it soon became clear that
m(hole) had to equal m(electron)]

1933 Anderson (ignorant of Dirac's theory)
found e^+ in cosmic rays.
X-rays in cosmic rays had caused
pair production : $\gamma \rightarrow e^+ + e^-$
No incident track observed in cloud chamber
2 particles leaving : same mass, opposite
curvatures in mag. field

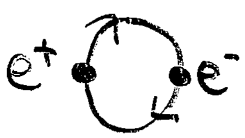
Annihilation
When e^+ and e^- meet, they annihilate each
other

$e^+ + e^- \longrightarrow 2\gamma$

$\geq$ photons produced in order to balance energy and momentum

It is easy to visualise this process in terms of our previous diagrams. The (+ve energy) electron drops back into the hole in the "Dirac sea" of −ve energy electrons, thus removing electron and positron, and leaving radiation.

[N.B. e^- and e^+ will not annihilate immediately, but will circle their common mass (in a Bohr picture) for some time



producing spectrum of positronium]

2.1.1 Anti-Particles and Anti-Matter

Positron is anti-particle of electron

Most particles have anti-particles

[Exceptions of particles known by 1950 − γ, π^0

It is often said: All particles have anti-particles, but some (γ, π^0) are their own anti-particle

Under charge conjugation operator C particles are replaced by anti-particles
$e^- \rightarrow e^+$, $e^+ \rightarrow e^-$, $\pi^0 \rightarrow \pi^0$ etc]

When particle has charge, anti-particle has opposite charge e.g. $\{e^-, e^+\}$, $\{p^+, p^-\}$, $\{\pi^+, \pi^-\}$

Also, quantum numbers such as strangeness, charm have opposite signs for particle and anti-particle.

e.g. K^0 strangeness $= 1$
$\bar{K}^0$ strangeness $= -1$

But this does not cover all cases

Neutron and anti-neutron $n, \bar{n}$

$$\left.\begin{array}{l} n \rightarrow p^+ + e^- + \tilde{\nu} \\ \bar{n} \rightarrow p^- + e^+ + \nu \end{array}\right\}$$

Also relative directions of spin and magnetic moment.

Classically we would not expect a particle with no charge to have a magnetic moment

But $n, \bar{n}$ do

For n, ang. momentum & mag moment are parallel

$\bar{n}$, ang. momentum & mag. moment are anti-parallel

[Eventually we may discuss difference between n and $\bar{n}$ in terms of one being composed of quarks, the other of anti-quarks.]

<u>Neutrino and anti-neutrino</u>

The <u>helicity</u> H of a particle with spin is defined as:

$H = 1$ if direction of spin $\equiv$ direction of motion

$H = -1$ if directions are opposite

It is often said that

For the neutrino, ν $\quad H = -1$

For the anti-neutrino $\bar{\nu}$ $\quad H = 1$

Indeed for a long time this argument was looked on as proof that $m_\nu = 0$

If $m_\nu = 0$, it travels at speed c, and we can never overtake it

However if $m_\nu > 0$, its speed $< c$, and we could overtake it.

Then to us, direction of motion is reversed.

But direction of spin unchanged

[ang. mom $\sim \underset{\sim}{v} \times \underset{\sim}{r}$

$\underset{\sim}{v}$ and $\underset{\sim}{r}$ change sign

$\therefore$ ang. mom does not change sign]

i.e. If we overtake H changed sign

To avoid this happening, let us assume $m_\nu = 0$. [it was said]

But now we know $m_\nu > 0$
see later!

Different types of anti-particles constitute anti-matter

In our Universe, or at least that part of it we experience, matter predominates over anti-matter, and it is not clear why this is the case.
One of the main tasks of cosmology (and Grand Unified Theories or GUTs - see section 10) is to explain this point.

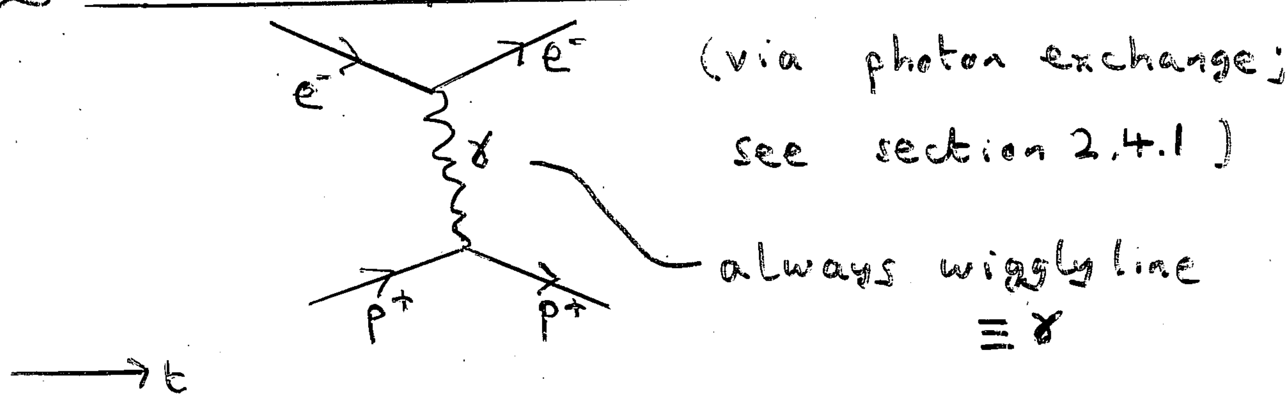
2.1.2 Feynman Diagrams and Anti-Particles

Feynman was one of those who showed how to renormalise quantum electrodynamics (QED) [to remove infinities and so to produce finite values for quantities that agreed with experiment] in the years after 1945.
His technique was to use Feynman diagrams which pictured the various proceses

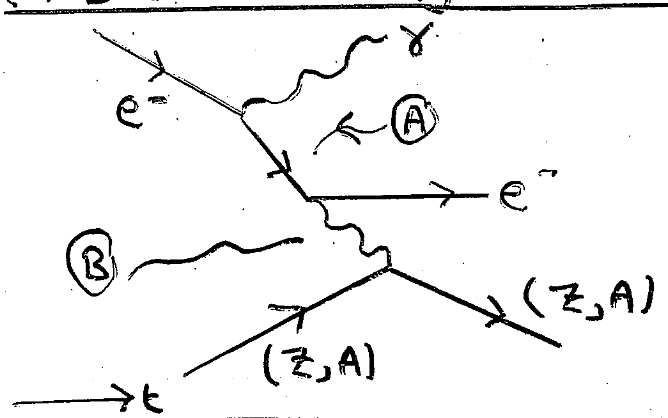
taking place in an experiment, and provided detailed amplitudes ~~for~~ each process.

The technique may also ~~be~~ used merely to provide a picture of a process.

e.g. (1) electron-proton scattering



(2) Bremsstrahlung (Braking radiation)

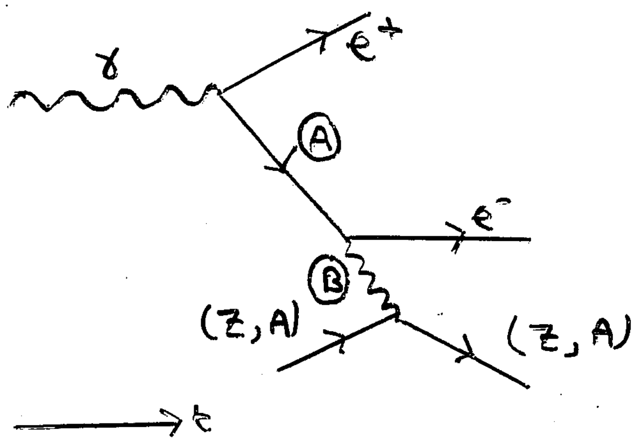


Electric field of nucleus causes e^- to accelerate and decelerate, radiating a photon and losing energy.

At A virtual electron state, since e^- cannot emit a real photon without reducing its rest mass. (Conservation of E and p)

At (B) exchange of virtual photon between e^- and (Z, A) to conserve momentum

(3) Pair production



(A) and (B) as above

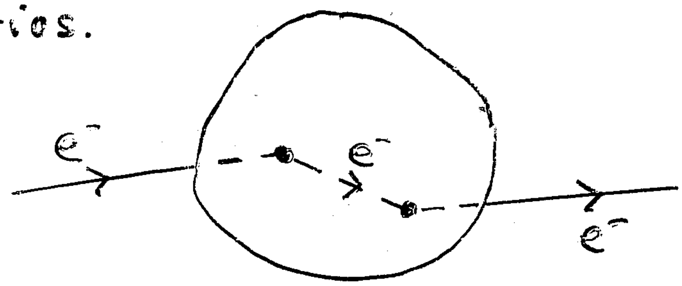
(2) and (3) are related by replacing an incoming e^- by an outgoing e^+
This is a general rule

Another way of expressing the same basic point is that an anti-particle going forwards in time is equivalent to the corresponding particle going backwards in time.

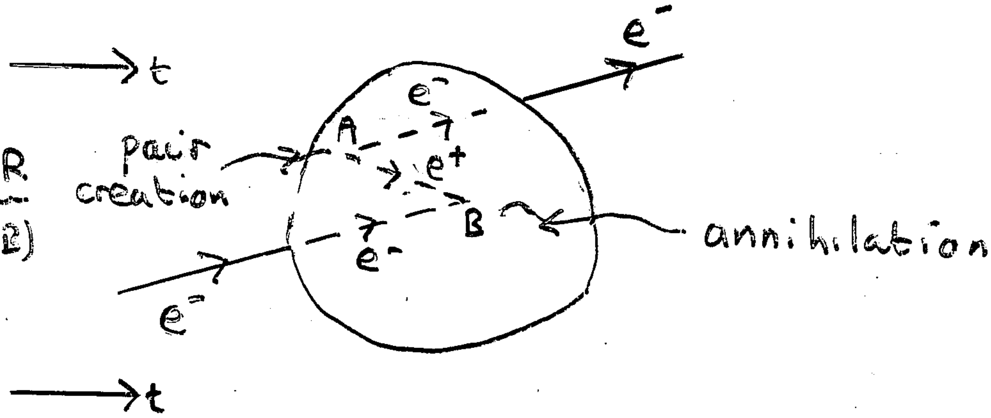
Consider, for example, a double scattering of an electron.
In the following diagrams, what is observed is whatever is outside the closed curves.

Inside the curves are 2 alternative scenarios.

(4A)



OR
(4B)



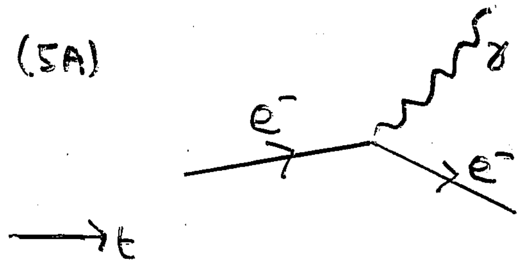
In (4B) between A and B, one has <u>either</u> an e^- going <u>backwards</u> in time, or an e^+ going <u>forwards</u> in time.

All events inside the curve may be regarded as <u>virtual</u> — not energy conserving.

When one calculates the probability of the event, the probabilities for <u>both</u> diagrams must be added up.

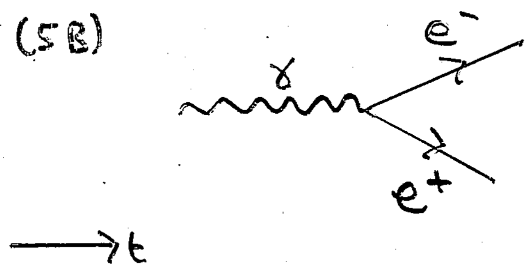
As another example, consider the following three diagrams:

(5A)



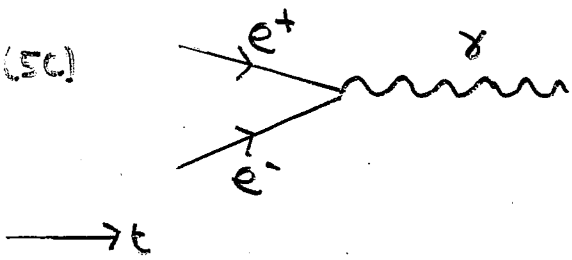
emission of a photon by an electron

(5B)



pair production

(5C)



annihilation

By our rules of handling anti-paticles, all basically the same process.

∴ All processes occur with <u>same strength</u>

<u>2.2 Interactions</u>

Well before 1950, it became clear that there are four fundamental forces/interactions in nature (though not to Einstein who spent much of the last twenty

twenty years of his life (up to 1955) working to produce a "unified field theory" involving only gravitation and electromagnetism. (Gravity is actually the least tractable of the forces.)

The <u>four interactions</u> are

(1) Gravitation
(2) Weak nuclear (or just "weak")
(3) EM
(4) Strong nuclear (or just "strong")

<u>(1) Gravitational interaction</u>

This is exceptionally weak, far too weak to be relevant for us.

Loosely we may say: Grav : Strong $\sim 10^{-38}$

["Loosely" because

(1) Ratios of strengths depend on energies available. (At high enough energy it is predicted that at least the weak, strong, and EM have <u>same</u> strength. See section 10.

(2) There are 2 different ways of "measuring" the strong interaction. See below.]

The grav. interaction is of infinite range. It does of course decay as $1/r^2$ but that is purely geometrical: the "lines of force" at distance r from a point mass are spread over a surface area $\sim r^2$.

The grav. interaction is important cosmologically because of (1) this range, and (2) the fact that all masses are positive, all interactions add up [in contrast to EM].

(2) Weak interaction

The primary role of the weak interaction is to cause β-decay

It has very short range, $\sim 2 \times 10^{-18}$ m

Again loosely (i.e. at energies normally available to us) Weak : Strong $\sim 10^{-14}$

(3) EM

Infinite range

Loosely: EM : Strong $\sim 10^{-3}$

(4) Strong

Binds nucleons (n, p) together to form nuclei. At a more fundamental level, binds quarks together to form n, p etc.

Also causes particle reactions, decays, etc. among strongly-interacting particles; typical time-scale $\sim 10^{-23}$ s.

Attraction short-range $\sim 2 \times 10^{-15}$ m

2.3 Neutrino and anti-neutrino

In β-decay, considered as

$$F_7^{16} \to O_8^{16} + e^- \quad \text{(etc)}$$

2 problems.

(1) A variable amount of energy seemed to disappear.

(2) If A is even, initial and final spin states of nucleus are zero. Electron has spin ½.

Solution (Pauli, 1931)

$$F_7^{16} \to O_8^{16} + e^- + \tilde{\nu}$$ ← anti-neutrino

$\nu, \tilde{\nu}$ have spin ½

[Revision - Coupling of Angular Momenta

Coupling of 2 angular momenta with q. Nos i_1, i_2 yield ang. momentum with q. No I, where
$$I = i_1+i_2,\ i_1+i_2-1, \ldots\ldots |i_1-i_2|$$]

e^- and $\hat{\nu}$ may couple to give spin zero.

Energy considerations tell us that m_ν is exactly zero, or very small.

For a long time it was supposed that m_ν was exactly zero; in this case relativity tells us that ν must travel at c.

However recently theory, experiment, cosmological speculation all suggest it has a mass $\sim 10^{-5}$ MeV/c^2 (in which case it may be stationary with respect to any given observer).

Neutrino extremely unreactive

Charge = 0 ; Mag. moment = 0

unlike n

Really only takes place in reverse of a β-decay process.

$\nu + p \longrightarrow n + e^{+}$

(detection by Cowan and Reines, 1956)

2.4 Pi-mesons, Pions

2.4.1 General remark on field particles

Newton's work assumed "action at a distance"; initially this was much criticised, but the critics stopped when it became clear how well his theory worked.

BUT the critics were right!
We now know all interactions are mediated by field particles (or field quanta or gauge bosons [see section 8])

c.f. tennis players interact by hitting the ball one to another - repulsion.
Throwing a boomerang may be a simple model for attraction

Where does the mass come from?
The particle is virtual; it exists only for

the time it takes to travel from one particle to the other.

Energy is "borrowed" taking advantage of the <u>Heisenberg principle</u>: $\Delta t . \Delta E \sim h$

$\Delta E \sim mc^2$

mass of field quantum

$\Delta t \sim \frac{R}{v}$ ~ range

~ speed of field quantum

To minimise Δt, put $v \sim c$

Then $\Delta t \sim R/c \rightarrow R/c \times mc^2 \sim h$; $Rmc \sim h$

$$\underline{m \sim \frac{h}{Rc}}$$

For <u>gravitational, EM interactions</u>, $R = \infty$

$\therefore$ <u>$m = 0$</u>

<u>EM interaction</u> The field quantum is the <u>photon</u> γ which <u>does</u> have $m = 0$

<u>i.e.</u> 2 (e.g.) electrons interact electromagnetically via <u>exchange</u> of virtual photons.

<u>N.B.</u> When charged particles are accelerated, <u>real</u> photons are produced (using some of kinetic energy of the charged particles

Gravitational interaction

The field particle is called the graviton

Gravitational radiation is so weak that as yet it has not been detected at all (i.e. as analogous to EM radiation) although much effort has been devoted to the pursuit.

It seems unlikely that individual quanta will be detected for a very long time.

2.4.2 Yukawa theory

Yukawa (1935) applied the above to the strong nuclear interaction

$R \sim 2 \times 10^{-15}\,\text{m}$

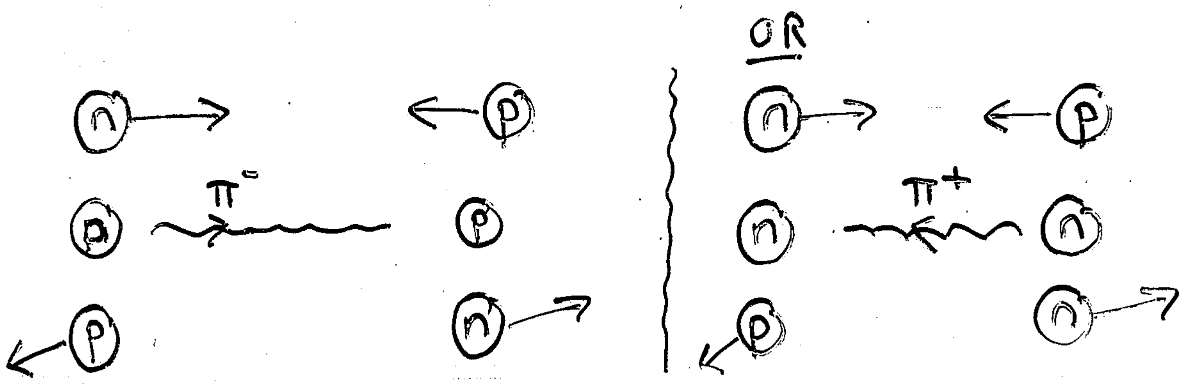
$$\therefore m_\pi \sim \frac{\hbar}{Rc} \sim \frac{10^{-34}\,\text{Js}}{2\times10^{-15}\,\text{m} \times 3\times10^{8}\,\text{ms}^{-1}} \sim 1.5 \times 10^{-28}\,\text{kg}$$

$\sim 150\,M_e$ (obviously very rough)

π-meson, π^+, π^0, π^-

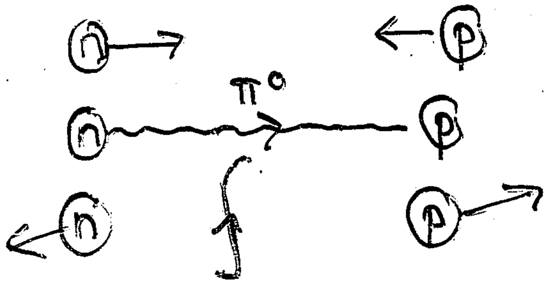
Exchange of π-meson

Small-angle scattering (np)



i.e. (1) charge conserved throughout
(2) particle interchange

But equal probability of



or other direction

i.e. no interchange of particles

(nn) or (pp) only interaction via π^0

History

1936 Anderson and Neddermeyer found particles in cosmic rays $m = 207\ m_e$, close to Yukawa's value.
They were assumed to be Yukawa's particles.

But they weren't !

Yukawa particles should have strong interaction with matter. They did not.

Yukawa particles should have integral spin, symmetric statistics, no magnetic moment.
(Mesons - see next section)

The newly discovered particles were μ^+, μ^- - muons (not mu-mesons)

They were identical to electrons, e^+, e^- spin-$\frac{1}{2}$, anti-symmetric statistics, the same magnetic moment, apart from the mass!

Confusion for a long time because of the war.

Then 1947 $\pi^{\pm}$ - pions or pi-mesons found in cosmic rays by Powell

$$m_{\pi^{\pm}} = 274 \; m_e$$

1950 π^0 found by Moyer

$$m_{\pi^0} = 264 \; m_e$$

Decays of πs and μs

These added to the confusion

$$\left.\begin{aligned} \pi^+ &\rightarrow \mu^+ + \nu \\ \pi^- &\rightarrow \mu^- + \tilde{\nu} \end{aligned}\right\} \quad \tau \sim 2.5 \times 10^{-8}\,s$$

$$\left.\begin{aligned} \pi^0 &\rightarrow 2\gamma \quad (99\%) \\ &\rightarrow e^+ + e^- + \gamma \quad (1\%) \end{aligned}\right\} \quad \tau \sim 10^{-15}\,s$$

$$\left.\begin{aligned} \mu^+ &\rightarrow e^+ + \nu_e + \tilde{\nu}_\mu \\ \mu^- &\rightarrow e^- + \tilde{\nu}_e + \nu_\mu \end{aligned}\right\} \quad \tau \sim 2 \times 10^{-6}\,s$$

[See next section for ν_e, ν_μ etc]

Relativistic wave equation for π-mesons

Start from $\quad E^2 = p^2c^2 + m^2c^4$

[Basic equation of special relativity]

Use associated operators

$E \rightarrow i\hbar\,\partial/\partial t \;; \qquad p \rightarrow -i\hbar\nabla$

This leads to the equation

$$-\hbar^2 \frac{\partial^2\psi}{\partial t^2} = -c^2\hbar^2\nabla^2\psi + m^2c^4\psi$$

or $$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = \frac{m^2c^2}{\hbar^2}\psi$$

This is the <u>Klein - Gordon equation</u>

[or <u>relativistic Schrödinger equation</u>]

It was found by Schrödinger before he obtained his (non-relativistic) time-dependent Schrödinger equation (TDSE).

He abandoned it because it was not good for the hydrogen atom because the electron has spin. (The TDSE was a very good approx. for this problem.)

The Dirac equation was a good relativistic equation for the hydrogen atom (It is first-order in time.)

The Klein-Gordon equation (KGE) is good for particles without spin

If m is put equal to zero, the KGE becomes

$$\nabla^2 \psi = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2}$$

the classical wave equation for photons, the field particles of the EM field.

This has a static solution

$$\psi \sim 1/r$$

corresponding to the electrostatic potential

[Put $\psi = A/r$

N.B. $\nabla^2 \equiv \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d}{dr}\right)$ [No θ, ϕ dependence here]

$\frac{d\psi}{dr} = -\frac{A}{r^2}$; $r^2\left(\frac{d\psi}{dr}\right) = -A$

$\therefore \nabla^2\psi = 0$]

With $m \neq 0$, a static solution of the KGE is

$$\psi \sim \frac{e^{-r/r'}}{r} \quad \text{where} \quad r' = \frac{\hbar}{mc} = R$$

[Put $\psi = -g^2 \frac{e^{-r/r'}}{r}$

$$\frac{d\psi}{dr} = -g^2 \frac{r\left(-\frac{1}{r'}\right)e^{-r/r'} - e^{-r/r'}}{r^2} = g^2 e^{-r/r'}\left(\frac{r}{r'}+1\right)/r^2$$

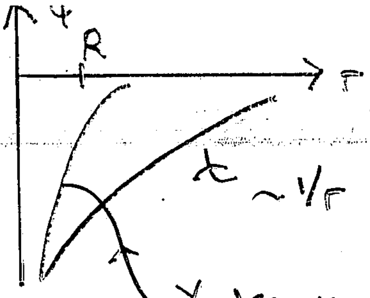
$$d/dr\left(r^2 \frac{d\psi}{dr}\right) = g^2 \frac{d}{dr}\left\{e^{-r/r'}\left(\frac{r}{r'}+1\right)\right\}$$

$$= g^2 e^{-r/r'}\left(\frac{1}{r'} - \frac{r}{(r')^2} - \frac{1}{r'}\right) = g^2\left\{-\frac{r}{(r')^2}\right\}e^{-r/r'}$$

Then $\nabla^2\psi = \left(-g^2 \frac{e^{-r/r'}}{r}\right) \times \frac{1}{(r')^2} = \frac{\psi}{(r')^2} = \frac{m^2c^2}{\hbar^2}\psi$

as required

As expected, this shows the <u>Yukawa potential</u> decays (over and above the "geometrical" ($1/r$) dependence) exponentially with range $\underline{r' = R = \hbar/mc}$



Strengths of the potentials

(EM) $\sim \dfrac{e^2}{4\pi\varepsilon_0\hbar c}$ (the fine structure constant) α $\sim 1/137$

In QED, successive terms are smaller by ratio α i.e. good convergence

(strong) $\sim \dfrac{g^2}{\hbar c} \sim 15$ (empirical) no convergence

weak interaction

The same considerations may apply here

i.e. $m_W \sim \dfrac{\hbar}{R_W c}$

$$R_W \sim R_{strong} \times 10^{-3}$$

$$\therefore M_W \sim M_{strong} \times 10^{3} \sim 2\times10^{5}\, m_e$$

Obviously W much more massive than π

Not found till the 1980s

2.5 Muons and muonic neutrino

This leaves the muon.

Rabi - who ordered that??

Still no understanding of why we require a "repetition" of the electron at higher mass - the generation problem - now much worse!

muonic neutrino ν_μ

There are two distinct neutrinos - ν_e, ν_μ.

Proof When μ decays

$$\mu^- \longrightarrow e^- + \nu_\mu + \bar{\nu}_e$$

the associated neutrino interacts with nucleons producing muons only, not electrons (1962)

$\therefore \nu_\mu$ distinct from ν_e

Also we don't see

$$\mu^- \not\Rightarrow e^- + \gamma$$

or $\mu^- \not\Rightarrow e^- + e^+ + e^-$

which would be allowed if $\nu_e \equiv \nu_\mu$

3. The situation ~ 1950

[though ignoring some "strange particles" detected before 1950 but not at all understood]

Table of particles

	Mass (MeV/c^2)	Electric Charge	Spin	Half life (s)	Principal Decay Scheme
γ photon	0	0	1	stable	
e^- electron	0.5	−1	½	stable	
e^+ positron	0.5	+1	½	stable	
p^+ proton	938	1	½	stable	
p^- anti-proton	938	−1	½	stable	
n neutron	940	0	½	10^3	$\rightarrow p + e^- + \tilde{\nu}$
$\tilde{n}$ anti-neutron	940	0	½	10^3	$\rightarrow p^- + e^+ + \nu$
ν_e electron neutrino	0 (?)	0	½	stable	
$\tilde{\nu}_e$ electron anti-neutrino	0 (?)	0	½	stable	
μ^+ muons	106	1	½	2.2×10^{-6}	$\rightarrow e^+ + \tilde{\nu}_\mu + \nu_e$
μ^- muons	106	−1	½		$\rightarrow e^- + \nu_\mu + \tilde{\nu}_e$
π^+ π-mesons	140	+1	0	2.5×10^{-8}	$\rightarrow \mu^+ + \nu_\mu$
π^0 π-mesons	135	0	0	10^{-16}	$\rightarrow 2\gamma$
π^- π-mesons	140	−1	0	2.5×10^{-8}	$\rightarrow \mu^- + \tilde{\nu}_\mu$
ν_μ muonic neutrino	0(?)	0	½	stable	
$\tilde{\nu}_\mu$ anti muonic neutrino	0(?)	0	½	stable	

N.B (1) Units of mass from $E = mc^2$, $m = E/c^2$

Units MeV/c^2

(2) We could have taken anti-particles for granted and left them out leaving 9 particles

(3) We have not tried to write particles in a "Logical" way, because no obvious periodicity, pattern.

1.2

<u>Discussion</u> We search for "<u>quantum numbers</u>", quantities that remain invariant in interactions i.e. <u>conservation laws</u>

Obviously <u>electric charge</u> is a good quantum number.

Define <u>baryon number</u>, b conserved in all interactions

$$\begin{cases} b = 1 & \text{baryons } (n, p) \\ = -1 & \text{anti-baryons } (\bar{n}, p^-) \\ = 0 & \text{all other particles} \end{cases}$$

<u>e.g</u>

$$n \longrightarrow p + e^- + \tilde{\nu}$$

$$b = 1 \qquad b = 1 + 0 + 0 = 1$$

[baryons decay to baryon of lowest mass which is the p]

$$p + p \longrightarrow p + p + p + \bar{p}$$

$$b = 2 \qquad b = 2$$

This is the route to the production of the anti-proton.

<u>N.B</u> An amount of energy equal to $2 m_p c^2$ is required. But for conservation of baryon no., only $m_p c^2$ might have been required.

$$\text{e.g.} \quad p + p \not\Rightarrow p + p + e^+ + \bar{p} \;]$$

The word <u>baryon</u> means <u>heavy particle</u> - <u>spin 1/2</u>

Define <u>lepton number ℓ</u> conserved in all interactions.

$$\begin{cases} \ell = 1 & \text{leptons} \quad (e^-, \mu^-, \nu_e, \nu_\mu) \\ \ell = -1 & \text{anti-leptons} \quad (e^+, \mu^+, \tilde{\nu}_e, \tilde{\nu}_\mu) \\ \ell = 0 & \text{all other particles} \end{cases}$$

<u>e.g</u>

$$n \rightarrow p + e^- + \tilde{\nu}$$
$$\ell = 0 \qquad \ell = 1 + (-1) = 0$$

$$\mu^+ \rightarrow e^+ + \tilde{\nu}_\mu + \nu_e$$
$$\ell = -1 \qquad \ell = (-1) + (-1) + 1 = -1$$

The word <u>lepton</u> means <u>light particle</u> - <u>spin 1/2</u>

Define <u>muonic number N_μ</u> conserved in all interactions

$$\begin{cases} N_\mu = 1 & \nu_\mu, \mu^- \\ \quad = -1 & \tilde{\nu}_\mu, \mu^+ \\ \quad = 0 & e^-, e^+, \nu_e, \tilde{\nu}_e \\ & \text{and all other particles} \end{cases}$$

<u>e.g</u> $\mu^- \longrightarrow e^- + \nu_\mu + \tilde{\nu}_e$

$N_\mu = 1 \qquad\qquad N_\mu = 1$

$\pi^+ \longrightarrow \mu^+ + \nu_\mu$

$N_\mu = 0 \qquad\qquad N_\mu = (-1) + 1 = 0$

But in the reactions we don't see, N_μ not conserved

<u>e.g</u> $\mu^- \not\longrightarrow e^- + \gamma$

$N_\mu = 1 \qquad\qquad N_\mu = 0$

<u>N.B.</u> (1) We could define Ne

$$\begin{cases} N_e = 1 & \nu_e, e^- \\ \quad = -1 & \tilde{\nu}_e, e^+ \\ \quad = 0 & \text{otherwise} \end{cases}$$

but since $\ell = N_\mu + N_e$, only 2 of these numbers are independent

(2) Do these numbers really help us understand things, or do they just codify what we really know?

Certainly <u>baryon no.</u> (at least) has a significance much beyond what we have said so far.

So far we have been discussing particles, quantities subject to conservation laws.

Now we turn to the opposite.

<u>Mesons and photons - field quanta</u>

$\ell = b = 0$ [i.e. no restrictions on creation/annihilation]

Most important property - spin integral

photon spin = 1

mesons spin = 0

Baryons and leptons are <u>fermions</u>;

½-integral spin, <u>Fermi-Dirac statistics</u>;

obey <u>Pauli exclusion principle</u>

[again restrictions on creation]

mesons and photons are **bosons**;
integral spin, **Bose-Einstein statistics**;
do **not** obey **Pauli exclusion principle**
[as required for field quanta; no
restrictions on creation]

N.B γ, π known as **gauge bosons**
see section 8

Conservation of angular momentum in creation or annihilation of field quantum

e before
e $\xrightarrow{\gamma}$ final

Ang. momentum Q. No. for final system
$= (1+\tfrac{1}{2}) \ldots |1-\tfrac{1}{2}|$
i.e. $\tfrac{3}{2}$ or $\tfrac{1}{2}$
But initial Q. No $= \tfrac{1}{2}$
i.e. we may have **conservation of ang. mom**
[with a π-meson, things are even
easier because π has spin-zero.]
Such a scheme would be **completely impossible** if a field quantum had
half-integral spin.

i.e. we have seen many distinctions between $\left\{\begin{array}{l}\text{leptons} \\ \text{baryons}\end{array}\right\}$ and gauge bosons

[N.B. The word meson means "intermediate mass"]

3.3 Isobaric spin

This may be called isospin or (wrongly) isotopic spin. (Isobars are nuclei with same no. of nucleons, but differing nos. of protons and neutrons; here the concept is extended to similar groups of particles.)

Isospin has nothing to do with a physical spin! It has a mathematical analogy with (quantum-mechanical) spin.

It started with Heisenberg's observation that the nucleon system (n, p) may be regarded as 2 aspects of the same system

$$I = 1/2 \quad \text{with} \left\{\begin{array}{ll} I_z = 1/2 & (p) \\ I_z = -1/2 & (n) \end{array}\right.$$

[analogous to electron $S = 1/2$ $\left\{\begin{array}{l} S_z = 1/2 \\ S_z = -1/2 \end{array}\right.$ (real spin)]

For isospin, z-axis has only a formal significance. (One may write I_3 rather than I_z)

The complete mathematical correspondence between real spin and isospin was developed by Wigner.

Similarly for π-mesons, $I = 1$, multiplicity $= 2I+1 = 3$

$$\begin{cases} I_z = 1 & \pi^+ \\ I_z = 0 & \pi^0 \\ I_z = -1 & \pi^- \end{cases}$$

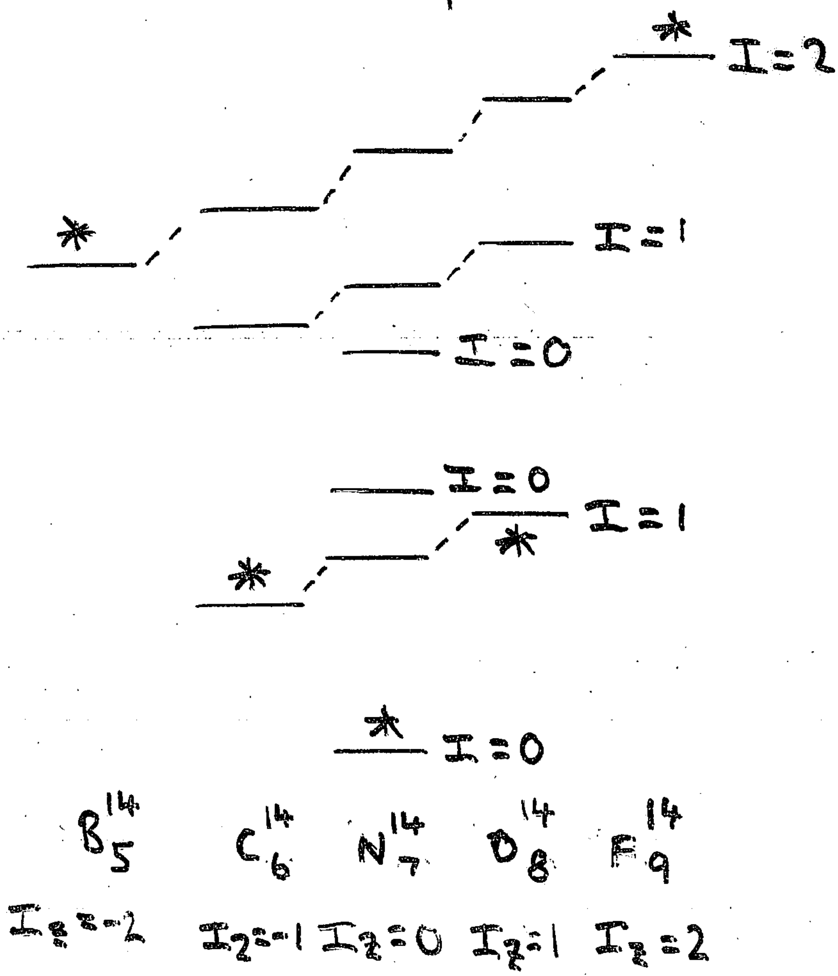
and for other multiplets to come

The idea behind isospin is that, if one could switch off electromagnetism, the 2 states of the nucleon, n and p, would be identical, m_p would equal m_n etc.

We speak of isospin conservation in the strong interaction. Strong interactions depend on I but not I_z. (Of course under EM, this conservation does not hold; EM does distinguish between $I_z = \frac{1}{2}$ and $I_z = -\frac{1}{2}$. But because of the comparative strengths of strong and EM interactions, mass differences between n and p, between $\pi^{\pm}$ and π^{0} small)

Isospin conservation is seen in <u>charge symmetry</u> and <u>charge-independence</u> of nuclear forces, the equivalence of n-p, p-p and n-n forces, once Coulomb effects are subtracted for p-p.

Consider also the low-lying energy levels of the A=14 isobars (Eisberg and Resnick, 2nd edition, p. 633)



<u>N.B.</u> $I_z = \frac{Z-N}{2}$

Each level would have the same energy for each species, if it were not for EM. (As it is, energy increases with Z)

N.B. The ground state masses starred (*) trace out the parabola obtained with the SEMF (semi-empirical mass formula).

This diagram is instructive in an example of conservation of I in a process involving the strong interaction

$$H_1^2 + O_8^{16} \longrightarrow N_7^{14} + He_2^4$$

H_1^2, O_8^{16}, N_7^{14}, He_2^4 all have $Z=N$, $I_Z=0$

In all experimental conditions, H_1^2, O_8^{16} are in their ground states, which have $I=0$.

Provided energy not too high, He_2^4 must also be in its ground state (since first excited state ~ 20 MeV above ground state). This also has $I=0$.

Considerations of energy, angular momentum, parity allow N_7^{14} to be in ground state ($I=0$ from diagram) or first excited state ($I=1$).

Conservation of I in strong interaction means it **must** be in its ground state.

Nucleons N
$$\left\{\begin{array}{lll} p & I_z = 1/2 & Q = 1 \\ n & I_z = -1/2 & Q = 0 \end{array}\right\} \quad B = 1$$

Pions π
$$\left\{\begin{array}{lll} \pi^+ & I_z = 1 & Q = 1 \\ \pi^0 & I_z = 0 & Q = 0 \\ \pi^- & I_z = -1 & Q = -1 \end{array}\right\} \quad B = 0$$

General formula: $\underline{Q = I_z + \frac{B}{2}}$
(so far)

We may also define hypercharge Y

$Y = 2 \times$ average charge over multiplet

$$\left.\begin{array}{ll} \text{For } N, & Y = 1 = B \\ \text{For } \pi, & Y = 0 = B \end{array}\right\} \quad \text{i.e. } \underline{Y = B} \text{ (so far)}$$

[N.B. For $\bar{N}$,
$$\left\{\begin{array}{lll} \bar{p} & I_z = -1/2 & Q = -1 \\ \bar{n} & I_z = +1/2 & Q = 0 \end{array}\right\} \quad B = -1$$
usual rule for antiparticles

$$\left.\begin{array}{l} Q = I_z + B/2 \\ Y = B \end{array}\right\} \text{(as above)}$$
]

So $\left.\begin{array}{l} Q = I_z + B/2 \\ Y = B \end{array}\right\}$ for all families so far

N.B. We are not considering leptons, which do not fit the scheme at all as they do not participate in strong interactions.

Our focus from now on is largely on mesons and baryons, particles which take part in strong interactions - hadrons

4. Strange particles and strangeness

Section 3 has provided a neat classification, but we cheated a little historically.

Before 1950 other particles had been discovered which did not fit naturally into our scheme.

1944 Leprince-Ringuet found particles in cosmic rays with mass different from any known.

1947 Rochester and Butler found 2 tracks in shape of V coming from one point i.e. from a particle of short life which they called a V-particle. (The name did not last)

Soon the synchrotron at Brookhaven produced them in quantity. There were several different types.

They were called strange particles because of a very peculiar property.

Lifetimes $\sim 10^{-8} - 10^{-10}$ s, typical of weak interaction (roughly half-life of $\pi^{\pm}, \mu^{\pm}$, though π^0 decays by EM interaction, $\sim 10^{-16}$ s)

But they appeared in strong interactions, for which time-scale $\sim 10^{-23}$ s

$$\left[\Delta t \sim \frac{\hbar}{m_\pi c^2} \sim \frac{10^{-34}\ \mathrm{Js}}{250 \times 9 \times 10^{-31} \times 9 \times 10^{16}} \sim 10^{-23}\ \mathrm{s} \right]$$

But a fundamental law of Q.M. states that a reaction will take place in either

Why do the strange particles (appear to) break this rule.

Gell-Mann, Nishijima, Pais (Theory)

The two reactions are not identical. Strange particles are produced in pairs by the strong nuclear interaction.

They are then isolated and cannot decay by the strong nuclear reaction, only by the weak nuclear interaction.

Why??

Define a quantum number - Strangeness, S

conserved in strong nuclear interaction

e.g. when pair produced, one had $S=1$, the other $S=-1$

Once separated, they could not decay by the strong nuclear interaction, as strangeness would not be conserved.
But they could decay by the weak nuclear interaction since strangeness is not conserved in the weak nuclear interaction

Some strange particles decay to a proton (+ leptons); they are <u>strange baryons</u> or <u>hyperons</u>

Others decay to <u>leptons</u> – <u>K mesons</u>

Table of strange particles

Particle	Mass (MeV/c^2)	Q	Spin	B	S	Y	Isotopic Multiplet	I_z	Life (s)	Typical Decay
K^+	494	1	0	0	1	1	doublet	$\frac{1}{2}$	10^{-8}	$\nu_\mu + \mu^+$, 2π
K^0	498	0	0	0	1	1		$-\frac{1}{2}$	$\square$	$\square$
Λ^0	1115	0	$\frac{1}{2}$	1	-1	0	singlet	0	10^{-10}	$p + \pi^-$
Σ^+	1189	1	$\frac{1}{2}$	1	-1	0		1	10^{-10}	$p + \pi^0$
Σ^0	1193	0	$\frac{1}{2}$	1	-1	0	triplet	0	10^{-20}	$\Lambda + \gamma$
Σ^-	1197	-1	$\frac{1}{2}$	1	-1	0		-1	10^{-10}	$n + \pi^-$
Ξ^-	1320	-1	$\frac{1}{2}$	1	-2	-1	doublet	$-\frac{1}{2}$	10^{-10}	$\Lambda + \pi^-$
Ξ^0	1311	0	$\frac{1}{2}$	1	-2	-1		$\frac{1}{2}$	10^{-10}	$\Lambda + \pi^0$
Ω^-	1680	-1	$\frac{3}{2}$	1	-3	-2	singlet	0	10^{-10}	$\Xi^0 + \pi^-$

All particles have <u>distinct</u> anti-particles with Q, B, S, Y, I_z of opposite sign.

[<u>N.B.</u> For K_0, $B = Q = 0$.

The quantum numbers for which $\bar{K}^0$ is distinct from K^0, S, Y, I_z relate only to the strong interaction. See later]

Our previous rules are replaced by (or generalised to)

$$\left.\begin{aligned} Q &= I_z + \frac{(B+S)}{2} \\ Y &= B+S \end{aligned}\right\}$$

Some properties of strange particles

K meson doublet – K^+, K^0

$S=1$ $\therefore$ prduced with Λ or Σ or its own anti-particle

e.g. $p+p \longrightarrow \Lambda^0 + K^{\cancel{0}+} + p$

$p+p \longrightarrow \Sigma^+ + K^+ + n$

anti-particle – $K^-, \bar{K}^0$ $\quad S=-1$

e.g. $p+\bar{p} \longrightarrow K^+ + K^-$

Takes part in strong reactions

e.g. $K^- + p \longrightarrow \Sigma^+ + \pi^-$

e.m reactions

e.g. $K^- + p \longrightarrow \Sigma^0 + \gamma$

weak reactions in its decay schemes

Λ^0 hyperon may be regarded as strange neutron

$S=-1$ produced e.g. with K meson

Σ hyperon

$S=-1$ produced e.g. with K meson

Ξ hyperon $S = -2$

can only be produced with 2 paticles of strangeness 1

e.g. $\pi^- + p \rightarrow \Xi^- + K^+ + K^0$

decays via weak interaction into Λ ($S=-1$)

i.e. $\Delta S = 1$ (weak interaction)

general rule $\Delta S = 0, \pm 1$

$\Delta S = \pm 2$ not observed.

Ω^- hyperon Predicted before being found

(SU3; see later) Found 1964

$S = -3$ $\therefore$ production very difficult

e.g. $K^- + p \rightarrow \Omega^- + K^0 + K^+$

$p + \bar{p} \rightarrow \Omega^- + \bar{\Omega}^-$

Decays in 3 stages with $\Delta S = 1$ each step.

4.1 Decay of the K^0, $\bar{K}^0$

The K^0, $\bar{K}^0$ are produced by the strong interaction which conserves strangeness. They thus have specific values of strangeness. (They are eigenfunctions of the strangeness operator)

But they decay via the weak interaction which does <u>not</u> conserve S but <u>does</u> conserve (at least to a very good approximation) CP

C = <u>charge conjugation</u>, replacement of particle by anti-particle

P = <u>parity operator</u>. We are thinking here of <u>intrinsic parities</u> of the fundamental particles

The intrinsic parity of the πs is negative (ER, p. 640). (Also other field particles - other mesons and γ)

<u>By convention</u> the intrinsic parity of the nucleon and other <u>baryons</u> is positive (ER, p. 640)

Intrinsic parity is not defined for <u>leptons</u>

In fact $P(\pi^+) = \pi^-$
$P(\pi^-) = \pi^+$

Now K^0 and $\bar{K}^0$ may decay into $\pi^+ + \pi^-$

But $CP(\pi^+ + \pi^-) = (\pi^+ + \pi^-)$

or $\pi^+ + \pi^-$ is an eigenfunction of CP

with eigenvalue $+1$.

But $$\begin{cases} C(K^0) = \bar{K}^0 \\ C(\bar{K}^0) = K^0 \end{cases}$$

and in fact $$\begin{cases} CP(K^0) = \eta\,\bar{K}^0 \\ CP(\bar{K}^0) = \eta'\,K^0 \end{cases}$$

where η and η' are arbitrary phase factors

which we may define as 1.

So K^0 and $\bar{K}^0$ are <u>not</u> eigenfunctions

of CP and so do not participate in this

form in the decay by weak

interaction.

But consider $$\begin{cases} K_1^0 = \frac{1}{\sqrt{2}}\left(K^0 + \bar{K}^0\right) \\ K_2^0 = \frac{1}{\sqrt{2}}\left(K^0 - \bar{K}^0\right) \end{cases}$$

These <u>are</u> eigenfunctions of CP

$$\begin{cases} CP(K_1^0) = K_1^0 & \text{i.e. } CP = 1 \\ CP(K_2^0) = -K_2^0 & \text{i.e. } CP = -1 \end{cases}$$

Now K_S may decay to 2π ($\pi^0\pi^0$, $\pi^+\pi^-$)

or 3π ($\pi^+\pi^-\pi^0$, $\pi^0\pi^0\pi^0$)

$$\begin{cases} CP\,(2\pi) = +\,2\pi \\ CP\,(3\pi) = -\,3\pi \end{cases} \qquad (*)$$

$$\therefore \begin{cases} K_1^0 \longrightarrow 2\pi \\ K_2^0 \longrightarrow 3\pi \end{cases}$$

* This statement is not <u>precisely</u> true because of angular momentum effects, but it is an exceptionally good approximation.

For the K_2^0 decay, there are 3 particles in the final state, and so the volume occupied in phase space is much smaller. Thus the decay is much slower than that of the K_1^0

$$\begin{cases} K_1^0 \longrightarrow 2\pi & (\tau \sim 10^{-10}s) \\ K_2^0 \longrightarrow 3\pi & (\tau \sim 5\times 10^{-8}s) \end{cases}$$

Regeneration

Note that

$$\begin{cases} K^0 = \frac{1}{\sqrt{2}}\left(K_1^0 + K_2^0\right) \\ \bar{K}^0 = \frac{1}{\sqrt{2}}\left(K_1^0 - K_2^0\right) \end{cases}$$

Now imagine a beam of K^0 is produced.

Let it travel in vacuum for $\sim 10^{-8}$ s.

We are left with K_2^0

Let this interact with matter. For strong interactions we use

$$K_2^0 = \frac{1}{\sqrt{2}}\left(K^0 - \bar{K}^0\right)$$

The beam acts as if it is 50% K^0, 50% $\bar{K}^0$.

The $\bar{K}^0$ is absorbed more strongly than K^0

$\therefore$ We are left with K^0 - regeneration

Strangeness oscillations

This has ignored the mass difference between K_1^0 and K_2^0. Since these particles have different decay modes and lifetimes, their mass must be different. This is a similar (though much smaller) effect to saying that M_p and M_n are different because of their different electromagnetic couplings. Here the couplings are of the weak interaction.

The amplitude of a state which is K_1^0 at $t=0$ is at later times

$$a_1(t) = e^{-iE_1 t/\hbar}\, e^{-t/2\tau_1} \times a_1(0)$$

so that

$$I_1(t) = a_1(t)a_1^*(t) = 1 \times e^{-t/\tau_1}$$

[as required - radioactive decay]

In the rest frame, $E_1 = m_1c^2$

So $a_1(t) = e^{-im_1c^2t/\hbar}\, e^{-t/2\tau_1} \times a_1(0)$

Similarly for a beam which is K_2^0 at $t=0$

$$a_2(t) = e^{-im_2c^2t/\hbar}\, e^{-t/2\tau_2} \times a_2(0)$$

Now consider a beam which is K^0 at $t=0$
So $a_1(0) = a_2(0) = 1/\sqrt{2}$
At later time t, the K^0 intensity will be

$$I(K^0) = \frac{a_1(t)+a_2(t)}{\sqrt{2}} \times \frac{a_1^*(t)+a_2^*(t)}{\sqrt{2}}$$

$$= \tfrac{1}{2}\left\{a_1a_1^* + a_2a_2^* + a_1a_2^* + a_2a_1^*\right\}$$

$$= \tfrac{1}{2}\left\{\tfrac{1}{2}e^{-t/\tau_1} + \tfrac{1}{2}e^{-t/\tau_2} + \tfrac{1}{2}e^{-t/2(1/\tau_1+1/\tau_2)}\, e^{-i(m_1-m_2)c^2t/\hbar}\right.$$

$$\left. + \tfrac{1}{2}e^{-t/2(1/\tau_1+1/\tau_2)}\, e^{i(m_1-m_2)c^2t/\hbar}\right\}$$

Δm in kg

$$= \frac{1}{4}\left\{ e^{-t/\tau_1} + e^{-t/\tau_2} + 2e^{-t/2(1/\tau_1 + 1/\tau_2)} \cos(\Delta m c^2 t/\hbar) \right\}$$

Similarly, for $\bar{K}_0$ we use $\{a_1(t) - a_2(t)\}/\sqrt{2}$

and

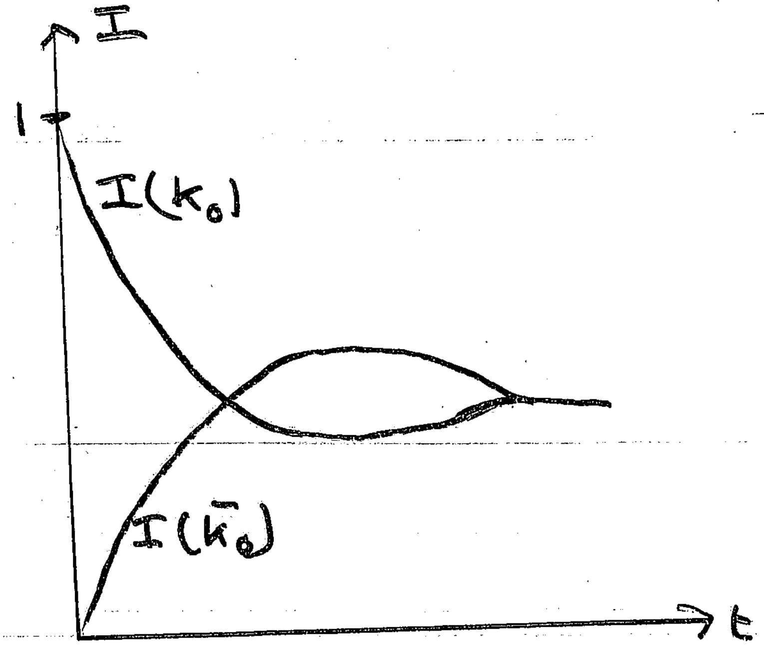
$$I(\bar{K}_0) = \frac{1}{4}\left\{ e^{-t/\tau_1} + e^{-t/\tau_2} - 2e^{-t/2(1/\tau_1 + 1/\tau_2)} \cos(\Delta m c^2 t/\hbar) \right\}$$

(in kg

So K^0 and $\bar{K}^0$ intensities oscillate with frequency $\left(\frac{\Delta m c^2}{\hbar}\right)$

in kg

This may be checked experimentally by the number of interaction events (hyperon yield) as a function of position along the beam.



Gives $\Delta m = 3.52 \times 10^{-12}$ MeV)

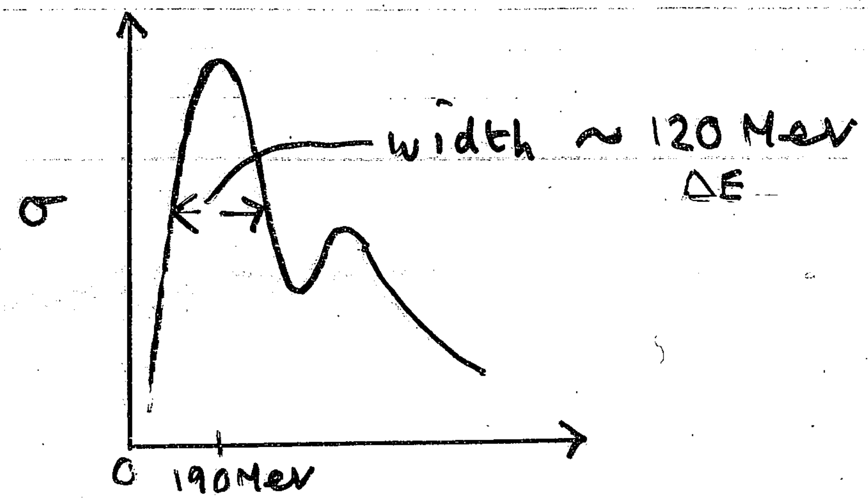
or

$$\frac{\Delta m}{m} = 0.7 \times 10^{-14}$$

5. More particles, and symmetries

In the 1950s and 1960s many more particles, good candidates for being elementary, were detected in accelerators.

1952 Fermi working on the cyclotron at Chicago, measured the cross-section of reaction of π mesons of variable energy bombarding the target.



Resonance at 190 MeV

$$\pi + \underset{\text{nucleon}}{N} \longrightarrow N^*$$

followed by

$$N^* \longrightarrow \pi + N \quad \text{after } 10^{-23}\,s$$

typical time for strong interaction

N.B. $\Delta E \sim \hbar / 10^{-23}s \sim \dfrac{10^{-21}\,MeV\,s}{10^{-23}s} \sim 100\,MeV$

Perkins p.401

Resonant particle

Is it an elementary particle (as we have defined it so far)

Yes! Just as good as n, π^+ etc.
because it has its own well defined quantum numbers

$$M = 1232 \text{ MeV}$$

(N.B. $M_\pi + M_N \sim 1073 \text{ MeV}$

$M_\pi + M_N + 190 \text{ MeV} \sim 1263 \text{ MeV}$

i.e 160 MeV stored in N^*

30 MeV lost as an inelastic component)

$I = 3/2$ (i.e. can exist in $2I+1=4$ states)

Spin $= 3/2$ (π has spin 0
N " " $1/2$

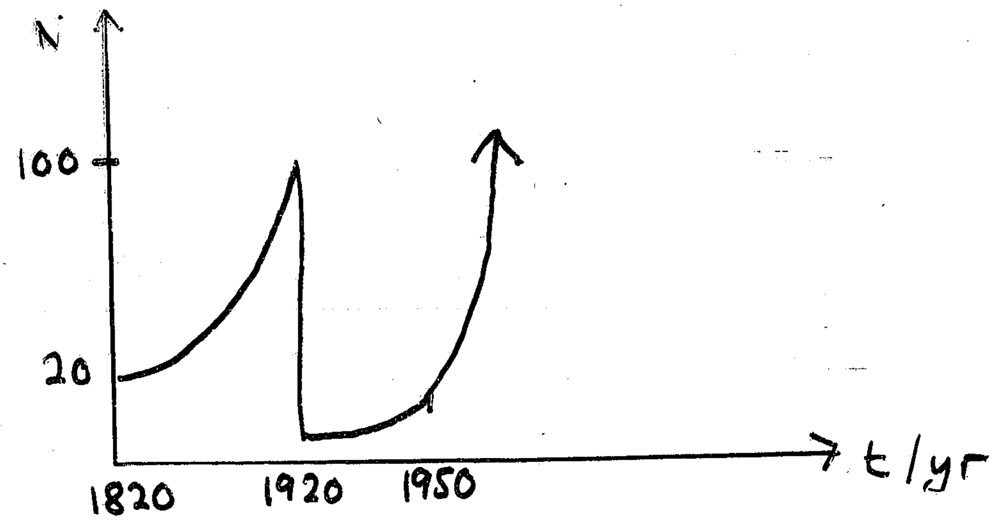
∴ 1 unit of ang. momentum 'absorbed' in collision.

And many many more strongly interacting particles (ER p. 652)

baryons, mesons

(including vector mesons, spin = 1)

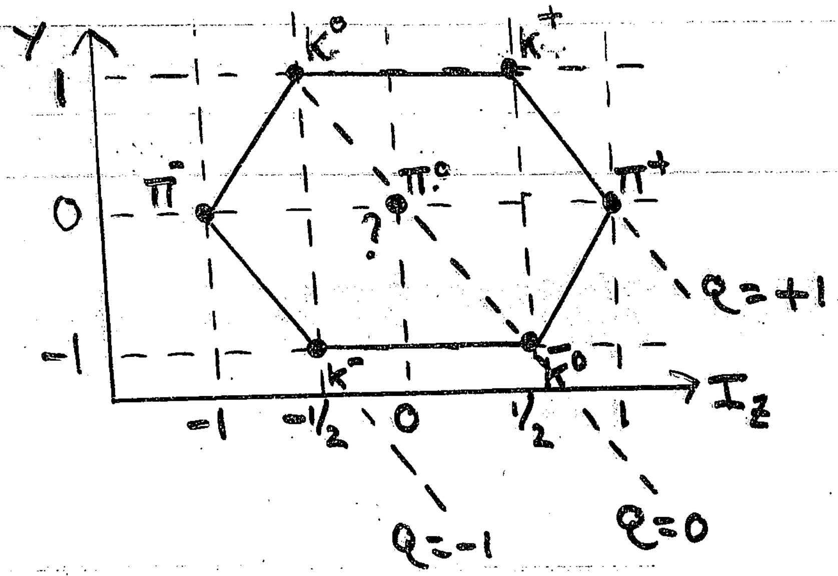
This is a disaster!



The first step forward

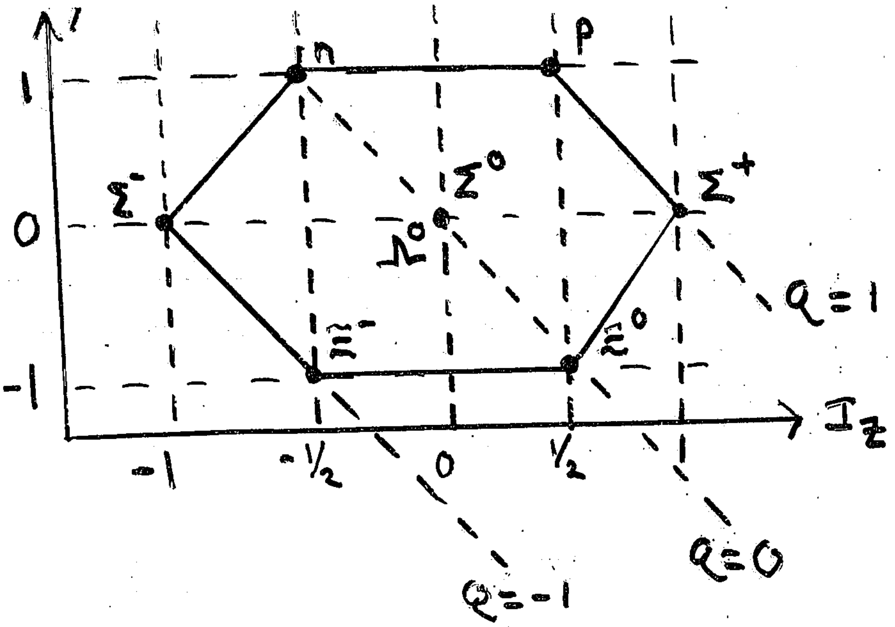
1961 The "Eightfold Way" - Gell-Mann, Ne'eman

These could be regarded as patterns of particles.



0^- parity, spin (mesons)

Actually mathematically we need an extra particle on the $Y=0$ level. There should be an isospin triplet (the πs with $I=1$, $I_z = 1, 0, -1$) but also a singlet with $I = I_z = 0$. This is the η^0 - discovered 1961, mass $\approx$ 550 MeV



$\frac{1}{2}^+$

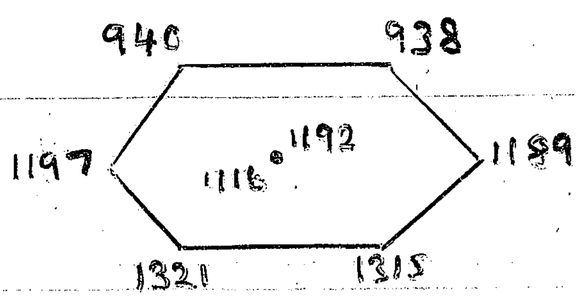
(baryons)

These patterns were actually representations of a particular group from group theory. This is SU(3), special unitary group in 3 dimensions

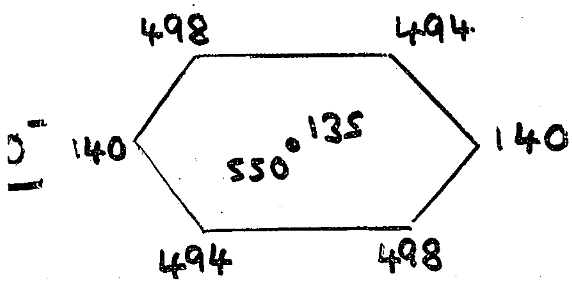
All particles in given octet would have the same mass in the limit of perfect SU(3) symmetry. But this symmetry is broken in 2 ways, along the I_z axis by electromagnetic effects (generally small) and along the Y axis by effects of the strong interaction

masses

$\frac{1}{2}^+$



[quite striking]



The basic representations of $SU(3)$ have 3 members and are called 3 and $\bar{3}$ (see later.)

For mesons it seems that 3 and $\bar{3}$ combine to form a singlet and an octet.

$3 \otimes \bar{3} = 1 \oplus 8.$ already described.

The singlet is the $\eta^{0\prime}$ (mass = 958 MeV) with $I = 0$

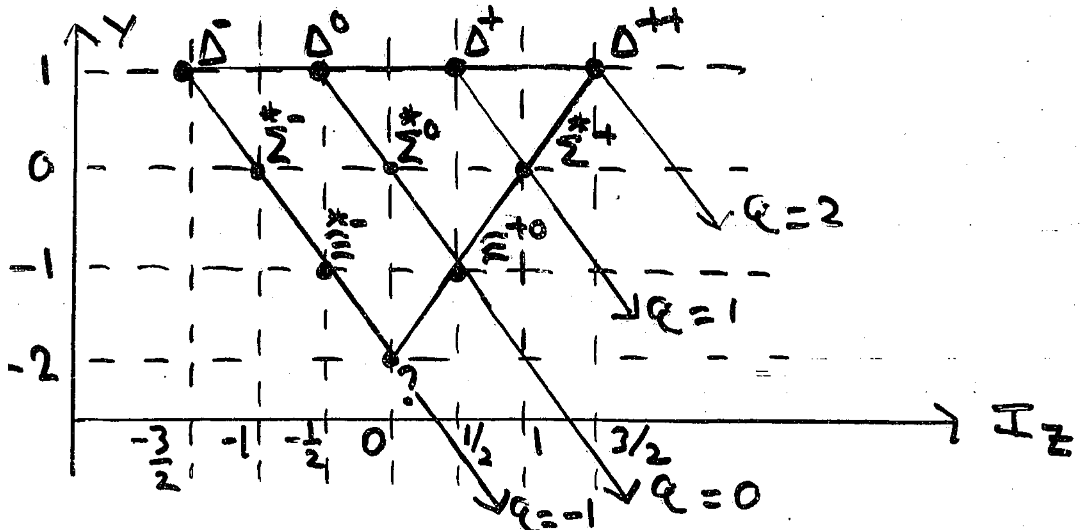
For baryons the combination is as follows

$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$

One octet has been described; others are known with different spins and parities.

The decuplet was very important historically. 9 particles were already known (The Δ was the first resonant particle described above; the Σs here are different from the ones met before.)

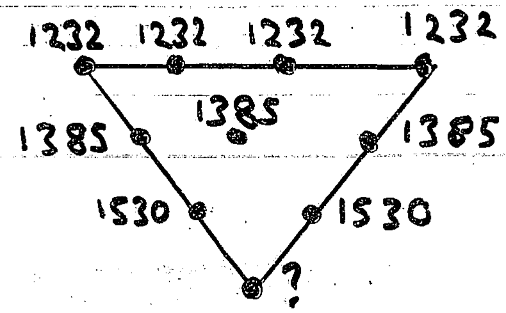
Σ^*, Ξ^*



$\frac{3}{2}^+$

The particle at the bottom was not known.
It would have $S = Y - B = -2-1 = -3$.
As such it could not decay via strong interaction, but by the weak interaction.

masses



$$M_\Sigma - M_\Delta = 153 \text{ MeV}$$

$$M_\Xi - M_\Delta = 145 \text{ MeV}$$

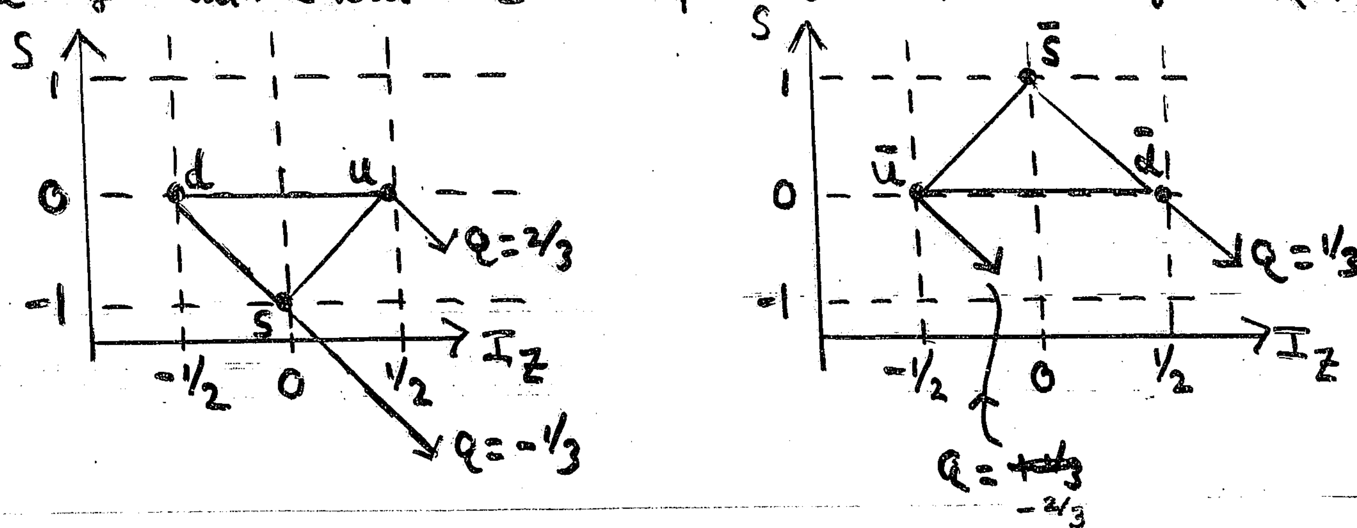
Prediction for mass of Ω^- 1530 + 150 MeV
= 1680 MeV (Gell-Mann, Ne'eman)

1963 Ω^- discovered at Brookhaven (CERN
Mass = 1679 MeV. Strangeness = -3 etc.

The SU(3) of Gell-Mann and Ne'eman was broadly equivalent to the Periodic Table of Mendeleev. Both were exceptionally important. Both showed very meaningful patterns. The patterns however required an explanation. For the Periodic Table it was the nuclear atom. For SU(3), what was it?

6. Quarks. The basic idea

1964. Gell-Mann and Zweig independently realised that one could go backwards to the fundamental 3 representation of SU(3).



quarks   anti-quarks

[These were Gell-Mann's terms
Zweig called them 'jacks']

<u>quantum numbers</u> u, d, s quark <u>flavours</u>

	$\underline{u}$	$\underline{d}$	$\underline{s}$
charge Q	$2/3$	$-1/3$	$-1/3$
baryon no. B	$1/3$	$1/3$	$1/3$
strangeness S	0	0	-1
isospin I	$1/2$	$1/2$	0
I_z	$1/2$	$-1/2$	0

<u>N.B.</u> In Gell-Mann-Nishijima relation

$$Q = I_z + \frac{B+S}{2}$$

For u, d
$$Q = \begin{matrix} 1/2 \\ -1/2 \end{matrix} + \frac{1/3}{2} = \begin{matrix} 1/2 \\ -1/2 \end{matrix} + \frac{1}{6} = \begin{matrix} 2/3 \\ -1/3 \end{matrix}$$

For s
$$Q = 0 + \frac{(1/3)-1}{2} = -1/3$$

<u>Also</u> $Y = B + S = 1/3$ (u, d)

$= -2/3$ (s)

[<u>N.B.</u> Y = twice the average value of charge over multiplet]

Fractional charges!

Fractional baryon numbers!

Initially Gell-Mann and Zweig's idea was treated with contempt. Zweig was never able to publish his paper.

<u>Spin and parity of quarks</u>

Each quark has spin-$\frac{1}{2}$.

All three quarks must have the same parity, taken as even.

<u>Construction of baryons and mesons</u>

To obtain agreement with experiment, it must be said that q, $\bar{q}$ are combined as

$$qqq \rightarrow \text{baryons}$$
$$q\bar{q} \rightarrow \text{mesons}$$

<u>baryons</u> $qqq \rightarrow B = 1$

p and n use <u>only</u> u and d quarks.

[<u>N.B.</u> The u and d form the <u>first generation</u>; the s is in the <u>second generation</u>; there are other quarks yet to come.

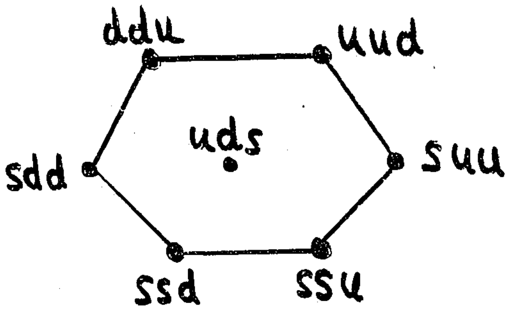
This is another warning of the <u>generation problem</u>. The first generation apparently provides all the particles we actually need, but the basic pattern is repeated in higher generations.]

$$\left.\begin{array}{l} uud \rightarrow p \\ udd \rightarrow n \end{array}\right\} \text{ i.e. differ by 1 quark}$$

Not all combinations of quarks appear in the basic octet. [In outline, group theory tells us at least 2 quarks must be different. There are two possible combinations of uds]

<u>i.e.</u>

	$\underline{S}$	$\underline{Q}$		
u u u	0	2	–	
u u d		1	p	<u>basic baryon octet</u>
u d d		0	n	
d d d		−1	–	
u u s	−1	1	Σ^+	
u d s		0	$\Sigma^0\ \Lambda^0$	
d d s		−1	Σ^-	
u s s	−2	0	Ξ^0	
d s s		−1	Ξ^-	
s s s	−3	−1	–	



As we move to right, d replaced by u

As we move in ↘ direction, d replaced by s

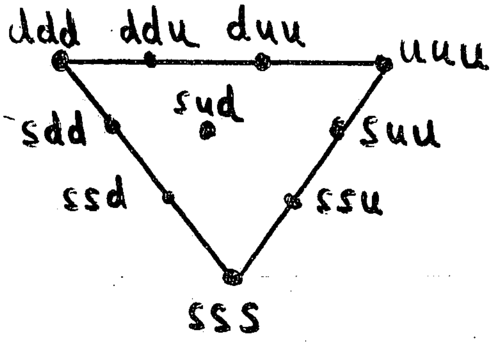
" " " " ↙ " , u " " s

[Broadly, as we move downwards, no. of s quarks increases.]

For baryon decuplet, all possible combination of u, d, s allowed

	S	q		
u u u	0	2	Δ^{++}	
u u d		1	Δ^{+}	
u d d		0	Δ^{0}	
d d d		−1	Δ^{-}	
u u s	−1	1	Σ^{*+}	baryon decuplet
u d s		0	Σ^{*0}	
d d s		−1	Σ^{*-}	
u s s	−2	0	Ξ^{*0}	
d s s		−1	Ξ^{*-}	
s s s	−3	−1	Ω^{-}	

(Only one combination of uds)



(same quark relations as for previous case)

Quark masses

This suggests isospin mass differences may be attributed to a small $(m_u - m_d)$, the larger differences as we move down the tables to a larger $(m_s - {m_u \atop m_d})$.

In particular, it suggests that $m_s - m_{u,d}$ might be around 150 MeV (from prediction of Ω^- and so on).

The 'standard effective masses' accepted today are

$m_u \sim 5$
$m_d \sim 10$ all in MeV
$m_s \sim 180$

The above criterion is obeyed.

But clearly quark masses do not just add up. $2m_u + m_d \neq M_p$!

In the p. the quarks are not just added together, but there is a large contribution from energy of interaction. Good agreement may be obtained from detailed models involving details of potential well, spin-orbit, spin-spin interactions etc. These 'accepted' quark models are then the basis of the values for the masses of large numbers of baryons (and mesons).

Quarks have <u>spin $\frac{1}{2}$</u>

$\therefore$ By the usual laws of adding together angular momenta, baryons may have spin $\frac{3}{2}$ or $\frac{1}{2}$

$\left\{ \begin{array}{l} \text{octet has spin } \frac{1}{2} \\ \text{decuplet " " } \frac{3}{2}. \end{array} \right.$

<u>mesons</u>

Mesons produced from $q\bar{q}$

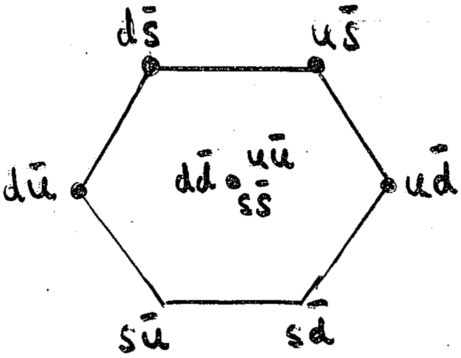
This gives $B=0$ as required

The result is that, while baryons actually appear in families of 8 or 10, mesons appear in families of 9 (breaking the direct correspondence via the 'Eightfold Way).

Spin may be 0 or 1

Parity is $-$ due to relationship between q and $\bar{q}$

<u>nonet 0^-</u>



	$\underline{S}$	Q	
$u\bar{s}$	1	1	K^+
$d\bar{s}$		0	K^0
$u\bar{d}$	0	1	π^+
$u\bar{u}$		0	π^0, η^0, η'^0
$d\bar{d}$		0	
$s\bar{s}$		0	
$d\bar{u}$		-1	π^-
$s\bar{u}$	-1	-1	K^-
$s\bar{d}$		0	$\bar{K}^0$

The nonet of <u>vector mesons</u> 1^- are broadly analogous. Analogous to the Ks there are K^{*+}, K^{*0}, K^{*-}, $\overline{K^{*0}}$ (mass 890 MeV)

Analogous to the πs are ρ^+, ρ^0, ρ^- (mass 770 MeV)

Analogous to η^0 and η'^0 are ω^0 (mass 770 MeV) and ϕ^0 (mass 1020 MeV).

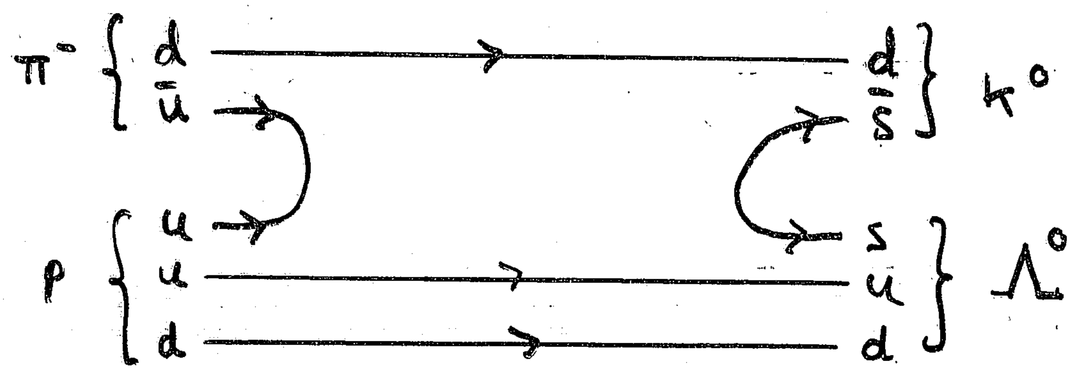
The discovery of these particles, in particular that of the ϕ in 1963, added to the acceptance of the quark hypothesis.

Quarks in Feynman diagrams

Feynman diagrams may be shown clearly and meaningfully in terms of quarks.

e.g. (a) $\pi^- + p \rightarrow \Lambda^0 + K^0$

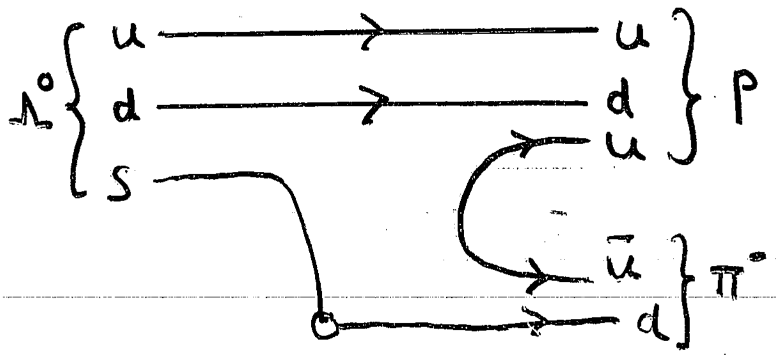
(strangeness conservation in production of strange particles via strong interaction)



An $s\bar{s}$ pair is produced; a $u\bar{u}$ pair is annihilated.

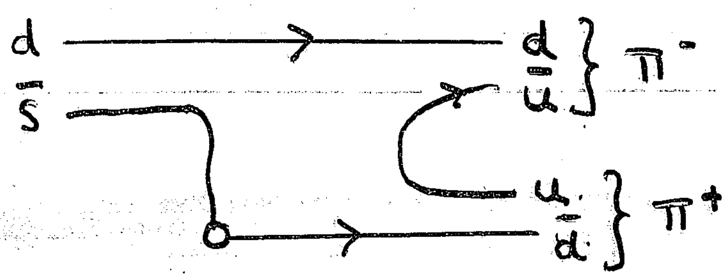
(b) $\Lambda^0 \rightarrow p + \pi^-$

(strangeness non-conservation in weak decay. [weak decay represented as a circle; will be treated more completely later].)



(c) $K^0 \rightarrow \pi^+ + \pi^-$

(strangeness non-conservation in weak decay)



<u>Families with orbital angular momentum</u>

So far the families we have considered have non-zero spin (in some cases) but zero orbital angular momentum.

Many families are now known which have orbital angular momentum; the higher the orbital angular momentum of the quarks, the higher the spin of the hadrons, and the higher their mass.

[N.B. hadrons - particles that take part in the strong interaction - baryons and mesons]

Meson families known with spin 2, 3, 4 ...

Baryon families known with spin $5/2$, $7/2$ .... $15/2$.

- overall many hundreds of hadrons!

6.1

Basic questions

This has taken us a long way, but we have not addressed fundamental questions

(1) Do quarks exist? If so, why do we not observe them?

(2) Why do they only combine as qqq, $q\bar{q}$?

Much work has been done on searching ~~for~~ free quarks. (They should be readily observable because of non-integral charge.)

(1) Searches for pre-existing quarks in moon rock, crushed oyster shells, deep-sea sludge etc. Max. possible concentration $< 10^{-26}$ quarks / nucleon.

(2) Millikan oil-drop experiments: Fairbank et al, 1977 used niobium balls at low temperature; claimed evidence for fractional charges $\pm 1/3$; also claims that Millikan's data showed similar effects. Effects not confirmed subsequently. Max. concentration $< 10^{-21}$ quarks / nucleon.

(3) Cosmic rays / accelerator laboratories McCusker claimed to detect quarks in cosmic rays, but not confirmed or believed.

It was thought that quarks might be 'mathematical artefacts'.

6.2

Partons

However corroborating evidence for the existence or 'existence' of quarks came from experiment.

Initially it might be suggested that the non-zero size ($\sim 10^{-15}$ m) of hadrons (as distinct from the pointlike nature of leptons) might indicate 'quarks in motion'. (quarks are considered as pointlike.)

In the 1960s, a two-mile long machine was built at SLAC, Stanford, California accelerating electrons to 20 GeV

At these energies, electrons can resolve structures $< 10^{-15}$ m, and should be able to probe the structure (if any) of a proton.

The results were broadly analogous to Rutherford scattering. If the charge on the proton were spread uniformly throughout it, scattering would be unexciting. However this was not the case; it appeared that the electric charge was concentrated on three localised regions, and when the electron came close to one of these, there would be violent deflections.

Similar experiments using neutrino beams were performed at CERN.

Overall and in detail the results of these experiments could be explained accurately by the parton model of Feynman (1969).

In this model, the proton was composed of three partons, with spin-½ which behaved in these experiments as effectively free particles, which were pointlike.

However not all the energy and momentum of the proton was in these partons. Around half was in particles without electrical charge called <u>gluons</u>. It was suspected that at this level, the gluons would be carriers of the strong nuclear interaction, the interaction that binds quarks together.

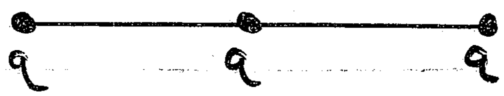
Quarks carry a new property <u>'colour'</u>, analogous for the strong interaction to charge for the EM interaction. (see later.)

<u>Problem</u>: In the nucleus quarks appear to be very light. Also quarks appear to be totally free!

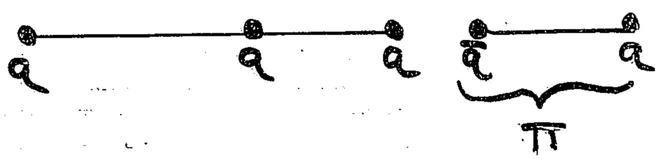
If this were the case, they should be easily ejected from the proton, but in no cases were quarks ~~or~~ gluons produced - just pions and other well-known particles.

Quarks act as if they are free inside a proton ("bag of sweets"), but free quarks are never found.
This is the problem of <u>quark confinement</u>.
For quarks the proton is a perfect prison.
Inside they are almost totally free - <u>asymptotic freedom</u>, but they cannot escape.

One model is a string model



If one applies energy to "pull off" a quark it does not work. A $q\bar{q}$ pair is formed at the point of fracture, giving a p and a π.



These facts may be explained by the theory of <u>quantum chromodynamics</u> <u>QCD</u> which is the analogy for the strong or colour force of <u>QED</u> for the electromagnetic force.

The relationship between the (old-style) strong interaction and the colour force

c.f. van der Waals forces — "forces between molecules"
But ~~the~~ van der Waals forces are certainly not fundamental. They are a remnant of the fundamental electrical forces between atoms.

Similarly the strong nuclear interaction (binding nucleons) is a remnant of the more fundamental colour force between quarks.

7. Quarks and leptons in the standard model

(For some time it will just be quarks.)

7.1 Colour

There is a remaining problem with the quark model.

Consider Δ^{++} (uuu), Δ^{-} (ddd), Ω^{-} (sss)

These have 3 quarks of the same flavour ~~colour~~. They also have spin $= 3/2$, so all quarks are spinning in the same direction (i.e. have same quantum no m_s)

But quarks have spin $1/2$, so should obey Pauli. Yet the above suggests they are in flagrant disobedience of Pauli!

Solution (Greenberg, 1964). Quarks has an additional property called colour, similar in many ways to charge, but it occurs in three varieties - red, yellow or blue colour charges.

Quarks have positive colour charges - red, yellow or blue.

Antiquarks have negative colour charges.

Thus, for quarks we have s_R, s_Y, s_B, u_R, u_Y, u_B, d_R, d_Y, d_B.

For anti-quarks, we have $\bar{s}_{\bar{R}}, \bar{s}_{\bar{Y}}, \bar{s}_{\bar{B}}$ etc

Observed particles must be white : baryons must consist of R, Y, B quarks ; mesons of $R\bar{R}$, $Y\bar{Y}$, $B\bar{B}$; anti-baryons of $\bar{R}, \bar{Y}, \bar{B}$

Thus the Δ^{++}, Δ^{-}, Ω^{-} consist of three different quarks, and Pauli is obeyed.

Proof of colour idea

The colour idea is extremely expensive in terms of new elementary particles. But there is experimental evidence for it. (Review it up till 1972)

Annihilation of e^-, e^+ at high energies may yield μ^-, μ^+ or $q\bar{q}$, which in turn will yield nuclear particles. The probability of the latter is proportional to $(\text{charge})^2$ for each variety of quark.

<u>production rate of nuclear particles</u>

production rate of μ^-, μ^+

$$= (2/3)^2 + (-1/3)^2 + (1/3)^2 = 2/3 \text{ (no colour)}$$

$$= \{ \qquad \} \times 3 = 2 \text{ (if colour idea correct)}$$

<u>xperiment</u> (Rome 1970), definitely $> 2/3$

(Stanford 1972), ~ 2

i.e. colour idea seems to be correct

[Details adjusted later, for higher energies, more flavours of quark]

.2 Charm and the charmed quark

The theory of the unification of the weak and EM interactions (see section B) led one to write lepton and quark pairs

$$Q_1 = \begin{pmatrix} u \\ d \end{pmatrix} \qquad L_1 = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} s \\ \uparrow \\ ? \end{pmatrix} \qquad L_2 = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$$

In his original quark paper, Gell-Mann postulated a fourth quark — but no experimental evidence, no hadrons containing this quark.

1970 Glashow, Iliopoulos, Maiana (GIM) : postulation of quark with charm, charmed quark, c

C, Charm (large) is a new quantum number analogous to strangeness (conserved in strong interaction).

The argument was from symmetry; it gives us $Q_2 = \begin{pmatrix} s \\ c \end{pmatrix}$ (small, small), but more deeply from weak-EM unification. (Section 8.)

quark quantum numbers (up to c)

	d	u	s	c
Charge	$-1/3$	$2/3$	$-1/3$	$2/3$
Isospin	$1/2$	$1/2$	0	0
I_z	$-1/2$	$1/2$	0	0
B	$1/3$	$1/3$	$1/3$	$1/3$
S	0	0	-1	0
C	0	0	0	1

we must generalise our previous relation (Gell-Mann - Nishijima) to

$$\left.\begin{aligned} Q &= I_z + \frac{B+S+C}{2} \\ Y &= B+S+C \end{aligned}\right\}$$

$\Big($For c, $I_z + \dfrac{B+S+C}{2} = 0 + \dfrac{4/3}{2} = 2/3 \equiv Q$

$B+S+C = 4/3 = 2 \times (\text{av. charge}) = Y \Big)$

<u>Why had no charmed particles been discovered?</u>

Presumably because the mass of c was too high. (To conserve charm a c <u>and</u> a $\bar{c}$ would need to be created.)

But for GIM theory to work, ~~mass could~~ not be too high - within energy range of machines of early 1970s.

[Masses of quarks as stated today (not of course measured directly)

$\underline{u}$	5 MeV/c^2
$\underline{d}$	10 "
$\underline{s}$	180 "
$\underline{c}$	1600 "]

Experimental note

In the late 1960s there was a great change in the type of experiments favoured by experimental particle physicists.

In the 1950s, 1960s, the aim was to produce new particles by firing protons at matter. During the 1960s the data could be used to analyse / confirm SU3 and later the quark model.
Particles with charm were too massive to be produced.

In the late 1960s emphasis switched to experiments using leptons (such as the experiments indicating partons - section 6.2).

The great development was the storage ring where beams of electrons and positrons were stored and then collided head - on.

N.B. This arrangement is difficult experimentally but highly efficient from an energy point of view.

Total momentum in the collision is zero, so all the energy is available for creation of particles.

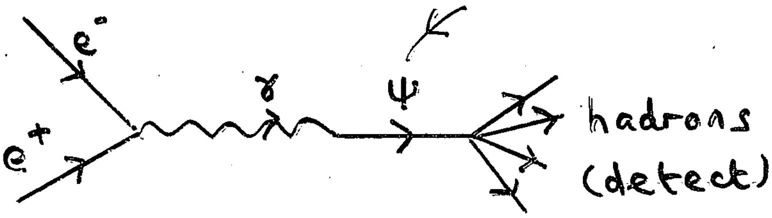
[compare a stationary target experiment where, in the lab. frame, there is considerable total momentum, thus considerable final K.E. Thus a considerable fraction of the initial KE (which is so expensive) is wasted].

November 1974 (The November revolution)

Discovery of J/ψ ($c\bar{c}$)

The first machine with enough energy to produce $c\bar{c}$ was the storage ring 'SPEAR' at Stanford, California (using a 2-mile electron accelerator).

The energy of 10 GeV (10,000 MeV) in each beam yields 20 GeV in the centre of mass frame. (In contrast, to get the same energy in the centre of mass frame of a stationary target experiment would require a laboratory energy of about 4×10^5 GeV.)

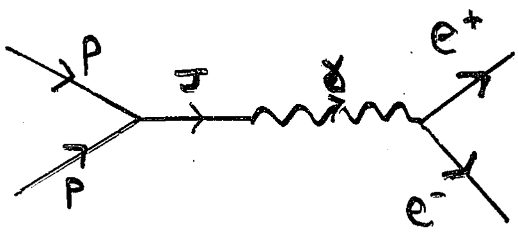


<u>Discovery of ψ by Richter</u>

The same particle was simultaneously discovered by Ting at M.I.T. and called the J
(Richter and Ting shared the Nobel Prize for Physics, 1976)

<u>Ting</u> essentially did Richter's experiment in reverse. He looked at collisions of protons and nuclei. He was not interested in the ~99% of cases which ended up as pions.
He <u>was</u> interested in the rare events where an e^+ and e^- are produced. By measuring the

energies of the pair, they could see if the e^+ and e^- preferred to emerge frequently with a particular value for energy sum. This would imply a meson had been produced, which had subsequently decayed into the e^+ and e^-.



The J/ψ is the $c\bar{c}$ (charmonium - in analogy with e^+e^- - positronium)

$c\bar{c}$ is a vector meson - spin 1, parity odd, $\hat{C}$ cap [charge conjugation] negative [i.e. eigenfunction of C with eigenvalue -1] hidden charm (as distinct from naked charm

Because it has spin 1, it can decay electromagnetically, via a virtual photon (as above).

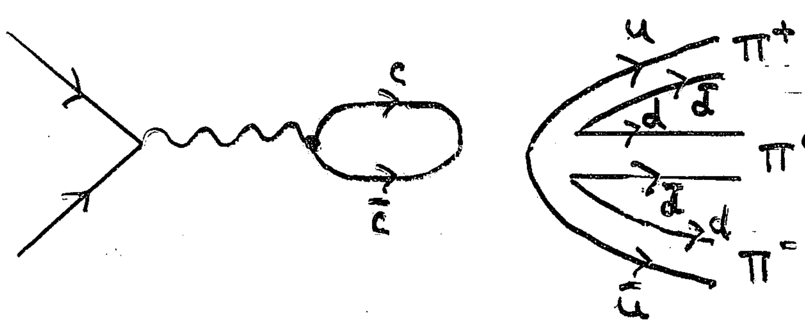
Ting found a massive narrow peak at 3095 MeV.

Its narrowness implies the state is very stable. (Its width is ~ 0.06 MeV/c^2 compared to ~ 100 MeV/c^2 for a typical resonant particle. So lifetime ~ 10^{-20} s.)

Why?

Apart from mass, ψ could readily decay into two mesons, one with a c, one with a $\bar{c}$. But even the least massive such mesons (D meson and $\bar{D}$ meson) are too massive i.e. $M_D + M_{\bar{D}} > M_{\psi/J}$.

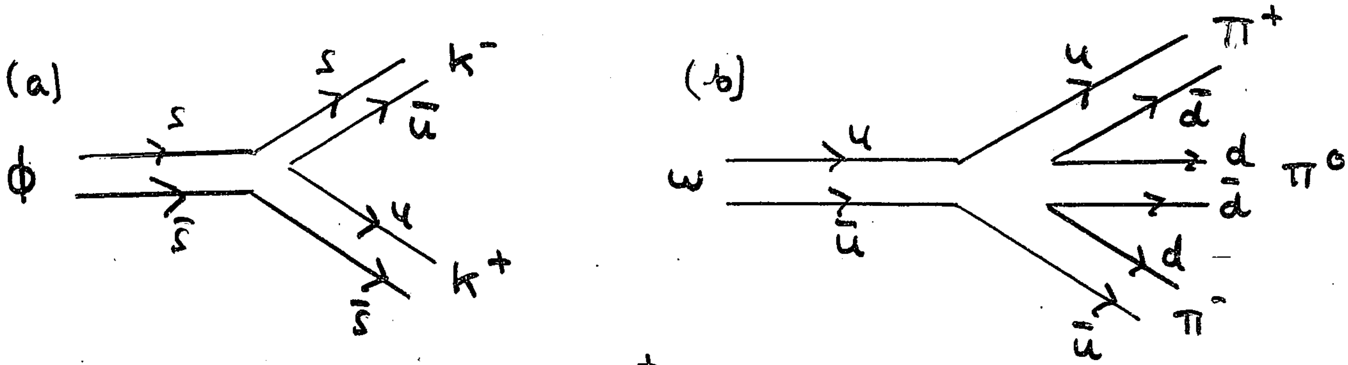
Decays that are allowed energetically are Zweig forbidden. (e.g. $\pi^+ + \pi^0 + \pi^-$)

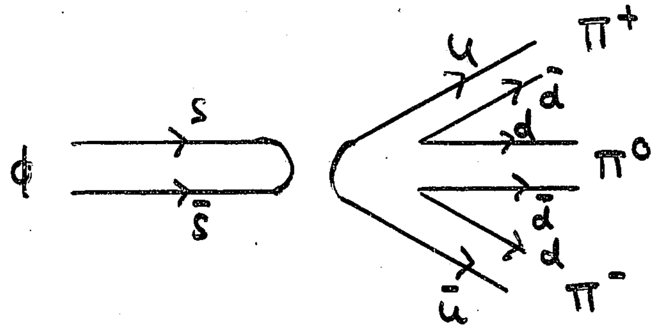


This is because there are unconnected lines in the decay $c\bar{c} \rightarrow u\bar{u}$. Thus the narrowness of the J/ψ peak indicated a new quantum number, C, was involved.

(At a deeper level, we can relate the Zweig rule to gluon exchange.)

[The Zweig rule also comes in for the decay of the ϕ, a vector meson, $s\bar{s}$



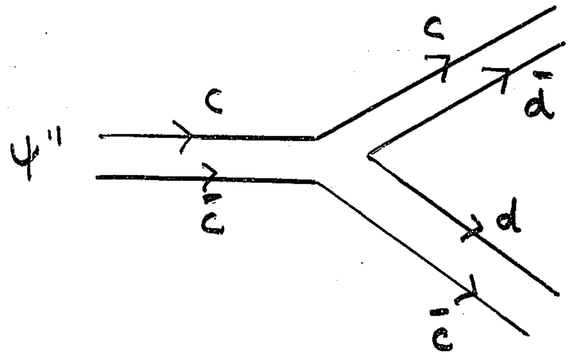


The decay of ϕ to $\pi^+\pi^0\pi^-$ has an energy balance of 600 MeV, while that to K^+K^- has a balance of only 24 MeV, but the latter is dominant because of the Zweig rule.

In contrast, the decay of ω to π^+, π^0, π^- is not hindered.]

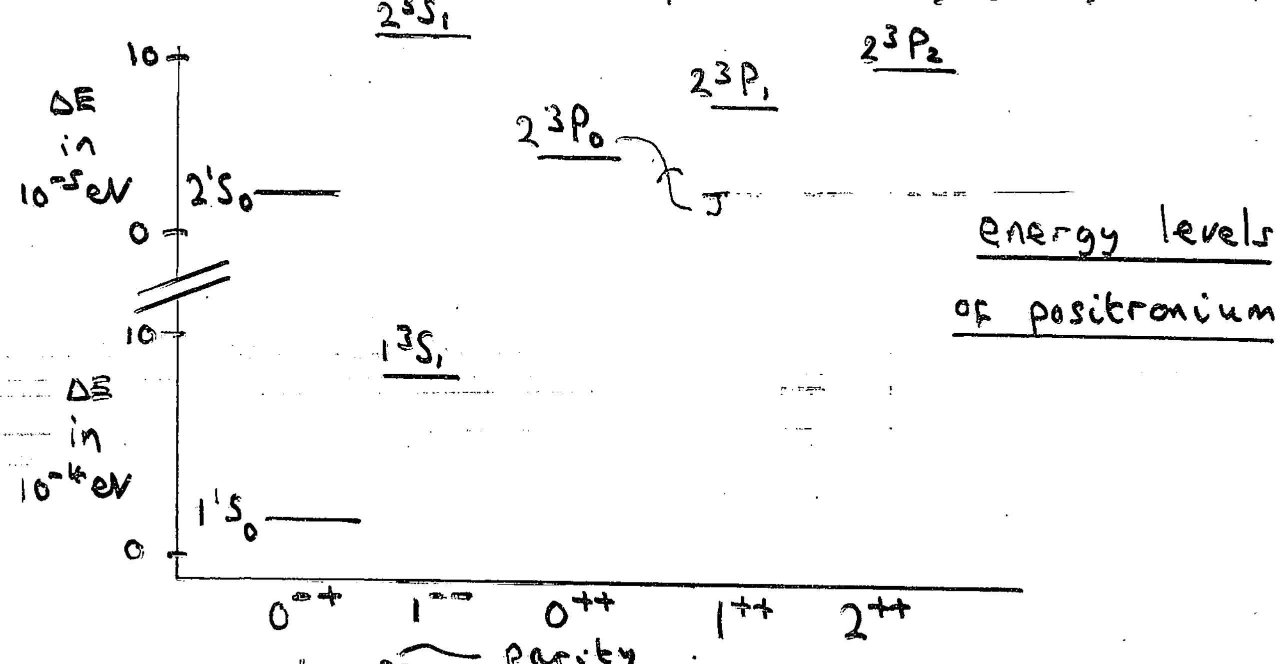
An excited state of $c\bar{c}$, called ψ', was discovered with a mass of 3685 MeV/c^2 at SPEAR in

1975, and subsequently another, ψ'', at 3767 MeV/c^2. The ψ'' is massive enough to decay into $D + \bar{D}$, so it has a short lifetime and a large mass width.



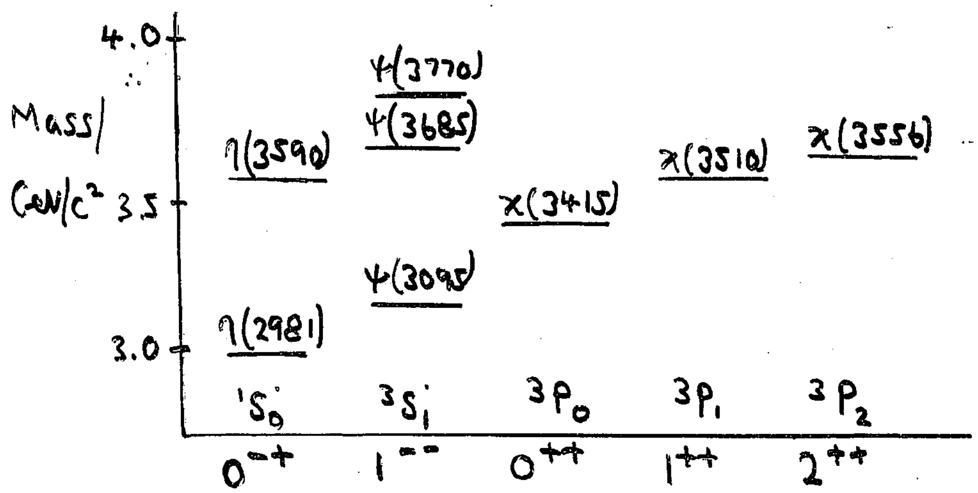
(not Zweig forbidden)

Many other charmonium states have been discovered. There is a strong analogy between the states of $c\bar{c}$ and those of positronium, e^+e^-.



J^{PC}: P — parity, C — charge conjugation quantum number, J — spin

[triplet states ~ ortho-positronium
singlet " - para-positronium]



energy levels of charmonium

N.B. $P = (-1)^{\ell+1}$
$C = (-1)^{\ell+s}$

[N.B The $\eta(2981)$ is a singlet state, reached by radiative decay from $\psi(3685)$]

Note close analogy between e^+e^- and $c\bar{c}$ systems

Particles with naked charm

$c\bar{c}$ does not have charm

The lightest mesons with charm will be $c\bar{d}$, $c\bar{u}$, $\bar{c}d$, $\bar{c}u$, mass ~ 1850 MeV

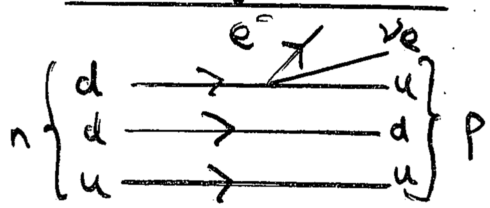
The J/ψ is too light to produce 2 such mesons.

It was discovered that when e^- and e^+ collide with energy $\gtrsim$ 4 GeV, probability of annihilation increases.

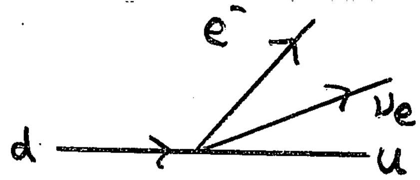
This suggests that charmed mesons were emerging in pairs.
To test this, note that when a charmed meson decays, it should produce a strange meson in its decay products.

[Transmutation

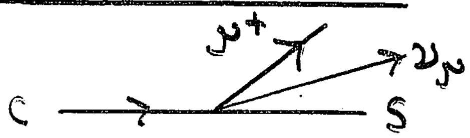
first generation



or

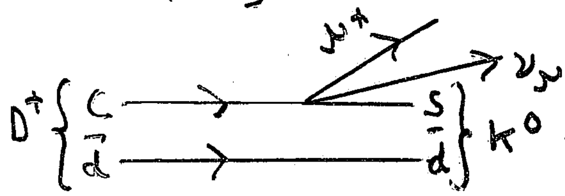


second generation



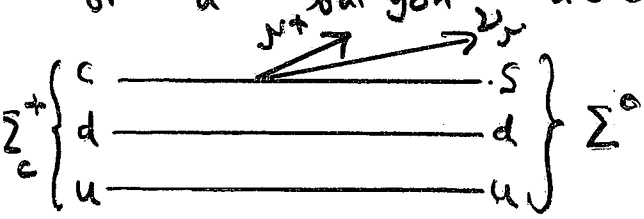
[N.B. μ second generation]

This may be the essence of a meson decay



detected

or a baryon decay



This confirms that the strange and charmed quarks form the second generation.]

N.B. $D^+ \equiv c\bar{d}$, $D^0 \equiv c\bar{u}$, $F^+ \equiv c\bar{s}$ etc.

$\Lambda_c \equiv udc$ (just as Λ but c replacing s)

[But $q = -1/3$ for s, $+2/3$ for c

$\therefore$ charge of $\Lambda = 0$

" " $\Lambda_c = 1$ etc.]

7.3 Further leptons and quarks

Much of the point in establishing c had been to complete the 2 generations of leptons and quarks

BUT 1975. Perl (SLAC) established the evidence for a particle analogous to e^- and μ^- - the τ^- (tau)

Mass around 1784 MeV/c^2. ['heavy lepton']

It has anti-particle τ^+, and associated ν_τ and $\tilde{\nu}_\tau$

The tau has a variety of decays because of its high mass

e.g. $\tau^- \rightarrow e^- + \tilde{\nu}_e + \nu_\tau$

$\tau^- \rightarrow \mu^- + \tilde{\nu}_\mu + \nu_\tau$

$\tau^- \rightarrow \pi^- + \nu_\tau$

Because of these many modes of decay, lifetime quite short $\sim 10^{-13}$ s

[conservation laws:

previously we had either

$\left\{\begin{array}{ll} \text{conservation of } \ell \\ \text{and } \quad " \quad\quad " \; N_\mu \end{array}\right\}$

or

$\left\{\begin{array}{ll} \text{conservation of } N_e \\ \text{and } \quad " \quad\quad " \; N_\mu \end{array}\right\}$

By analogy (and inspection of decays of τ) we now have either

conservation of ℓ, N_μ, N_τ

or " " N_e, N_μ, N_τ

with $N_\tau = 1$ (τ, ν_τ)

$= -1$ $(\tau^+, \tilde{\nu}_\tau)$

$= 0$ (other particles)]

Since there was now a third generation of leptons, it was expected there should be a third generation of quarks $\binom{b}{t}$ (for bottom and top; or beauty and truth)

1977 Lederman at Fermilab near Chicago searched debris from p-p collisions (analogous to Ting discovering the J/ψ) and found a massive resonance, which was very narrow - Υ (upsilon) with higher states - Υ', Υ''. These are all $^3S_1$ states of $b\bar{b}$. The energies are 9.46, 10.02 and 10.35 GeV/c^2. The energy level diagram was analogous to those of $c\bar{c}$ and positronium.

The widths of these levels was narrow e.g. that of Υ about the same as that of ψ, about 0.06 MeV/c^2.

A fourth state Υ''' at 10.57 GeV/c^2 has a much broader level, as it has the energy to decay into a $B\bar{B}$ pair of mesons, where $B^+ \equiv \bar{b}u$, $B^0 \equiv \bar{b}d$. The B meson has a mass of 5.27 GeV/c^2. (It may be said to have 'naked bottom' as distinct from Υ which has 'hidden bottom'.)

1978 There was now a machine, the DORIS at Hamburg, colliding e^+ and e^- with enough energy to produce an Υ (as Richter discovered the J/ψ).

We deduce that the mass of b is around 4.5 GeV.

Top quark

It was now confidently expected that the t quark existed and would soon manifest itself.

Yet it took till 1995!

This is because the mass of t is around 180 GeV. (To achieve this the LEP (Large Electron Positron Collider) at CERN had to be upgraded to LEP2.)

Summary of quarks

'Bottom' and 'Top' may be regarded as quantum numbers, analogous to S and C. Each of these quantum nos. is conserved in the strong and EM interactions, but may change by one in the weak interaction. This means that in the strong or EM interactions, the no. of quarks − no. of antiquarks of each flavour remains constant. In the weak interaction, a quark may change flavour, the preferred sequence being $t \to b \to c \to s \to \begin{matrix} u \\ d \end{matrix}$

e.g. $D^0 \to K^- + \pi^+$

or $c\bar{u} \to s\bar{u} + u\bar{d}$

$\Delta C = \Delta S = -1$

Quark quantum numbers

	d	u	s	c	b	t
charge	−1/3	2/3	−1/3	2/3	−1/3	2/3
isospin I	1/2	1/2	0	0	0	0
I_z	−1/2	1/2	0	0	0	0
baryon no. B	1/3	1/3	1/3	1/3	1/3	1/3
strangeness S	0	0	−1	0	0	0
charm C	0	0	0	1	0	0
bottom B	0	0	0	0	−1	0
top T	0	0	0	0	0	1

distinguish from baryon no.

Gell-Mann – Nishijima formula becomes

$$Q = I_z + \frac{B + S + C + B + T}{2}$$

Review of the 'proof' for colour. (p. 84)

Experiments at DESY (Hamburg) have enough energy to excite pairs of u, d, s, c, b.

Factor should equal $3\{(2/3)^2 + (1/3)^2 + (2/3)^2 + (1/3)^2 + (1/3)^2\}$

$$= 3 \times \frac{11}{9} = \frac{11}{3} \sim 3.7$$

Experimental factor $\sim 3.9 \pm 0.3$

Do we expect more and more generations?

It seems there are just 3.

(See decay of Z^0 in section 8)

7.4 General features of the colour interaction

The analogy between the energy levels of e^+e^-, ψ and Υ shows they are all pointlike fermion-fermion pairs with an analogous potential $\sim (1/r)$.

$\therefore$ From Klein-Gordon equation (p. 32) the gluon must be massless like the photon.

However a detailed fitting of the charmonium levels $\rightarrow$

$$V_c = -\frac{k_1}{r} + k_2 r$$

The first term is Coulomb-like term due to exchange of massless gluons, which are emitted and absorbed by colour charge.

The second term is the crucial new feature of the colour force.

It is small for small distances $\rightarrow$ asymptotic freedom. (So energy levels of $c\bar{c}$, $b\bar{b}$ largely determined by first term.) This is what makes the parton model work; when the quarks are close together, they act as almost free non-relativistic particles.

As r gets large, this second term gets very large. This confines quarks and gluons to be parts of hadrons, not free – quark (and gluon) confinement

Hence colour is never observed directly.

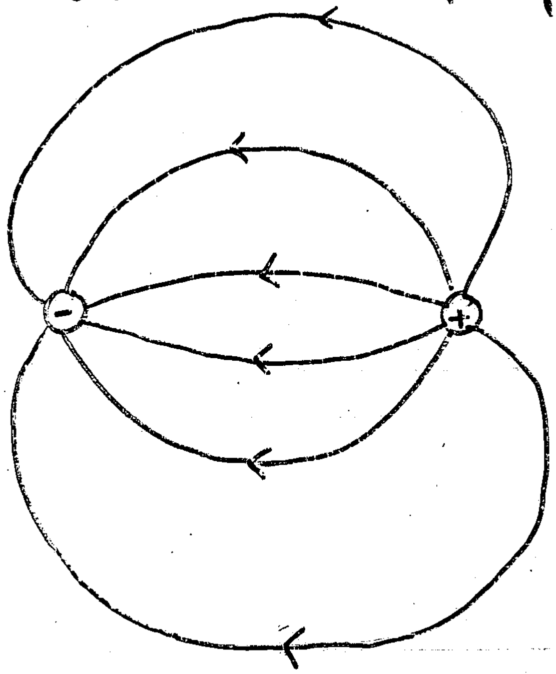
This implies a major difference between QED and QCD.

The photons do not themselves carry electric charge.
Gluons do carry colour charge (and also anti-colour)

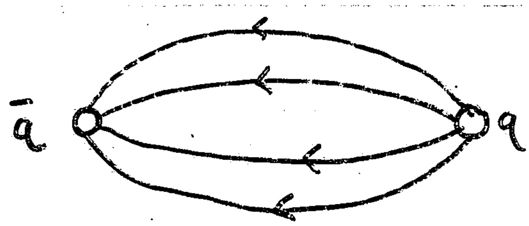
Consider trying to separate a quark from a proton. As the quark moves away from the other 2, the gluon field increases, and also its associated energy. It thus becomes very likely that the gluon will break into $q\bar{q}$. The new quark reconstitutes the proton; the new anti-quark combines with the separating quark to give a meson.

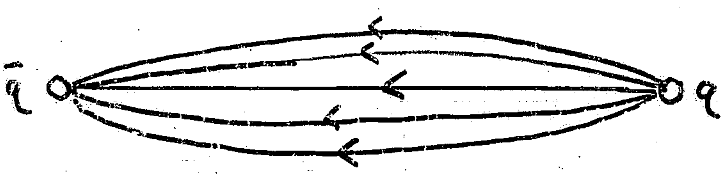
All the energy we provide hoping to pull off particles <u>with colour</u> goes into providing the masses of colourless particles.

Difference in electric lines of force and colour lines of force:



electric lines of force between +ve and −ve charges; because photons do not carry charge, there is no interaction between the lines of force

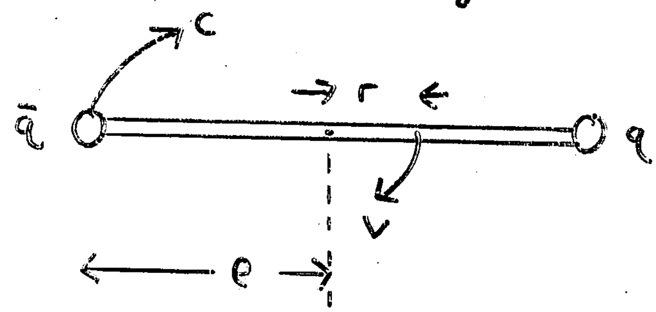




colour lines of force between quark and anti-quark; the gluon-gluon force pulls these together; as separation increases, the interaction energy increases and lines get closer together.

This is aleady a good picture of a meson. However the colour force lines will in general form a rotating tube of force

This model enables us to make some order of magnitude comparisons between mass and angular momentum.



Assume the interaction energy is high enough that the mass of the quarks may be neglected.

Now $k_2 dr$ is the rest mass energy of a length dr, so its relativistic energy is $k_2 dr / \sqrt{1 - v^2/c^2}$.

$\therefore$ Total mass M of the system is given by

$$Mc^2 = 2 \int_0^{\rho} \frac{k_2 dr}{\sqrt{1 - v^2/c^2}}$$

At a distance r from the centre of the tube, $v = cr/\rho$

$$\therefore Mc^2 = 2k_2 \int_0^{\rho} \frac{dr}{\sqrt{1 - r^2/\rho^2}} = 2k_2 \frac{1}{(1/\rho)} \int_0^{\rho} \frac{dr}{\sqrt{\rho^2 - r^2}}$$

$$= 2k_2\rho \left[\sin^{-1} r/\rho\right]_0^{\rho} = 2k_2\rho \cdot \frac{\pi}{2} = \underline{\pi k_2 \rho}$$

The angular momentum of the mass of length dr at distance r from the centre is

$$\frac{vr\,(k_2\,dr)}{c^2\sqrt{1-v^2/c^2}} \qquad \left(\text{Energy} = k_2\,dr \quad \therefore \text{Mass} = k_2\,dr/c^2\right)$$

$\therefore$ Angular momentum of the tube in units of $\hbar$ is

$$J = \frac{1}{\hbar}\,2\int_0^{e}\frac{vr\,k_2\,dr}{c^2\sqrt{1-v^2/c^2}} = \frac{2k_2}{\hbar c^2}\int_0^{e}\frac{vr\,dr}{\sqrt{1-r^2/e^2}}$$

$$= \frac{2k_2}{\hbar c e}\int_0^{e}\frac{r^2\,dr}{\sqrt{1-r^2/e^2}} \qquad (\text{using } v = cr/e)$$

Put $I = \displaystyle\int_0^{e}\frac{r^2\,dr}{\sqrt{1-r^2/e^2}} = e\int_0^{e}\frac{r^2\,dr}{\sqrt{e^2-r^2}}$

$$= e\int_0^{e}\frac{r^2-e^2+e^2}{\sqrt{e^2-r^2}}\,dr = e\left[-\int_0^{e}(e^2-r^2)^{1/2}\,dr + \int_0^{e}e^2(e^2-r^2)^{-1/2}\,dr\right]$$

$$= e\left[-I_1 + I_2\right]$$

$I_2 = e^2\,\pi/2$ as before

In $I_1 = \displaystyle\int_0^{e}(e^2-r^2)^{1/2}\,dr$ put $r = e\sin\theta$, $dr = e\cos\theta$

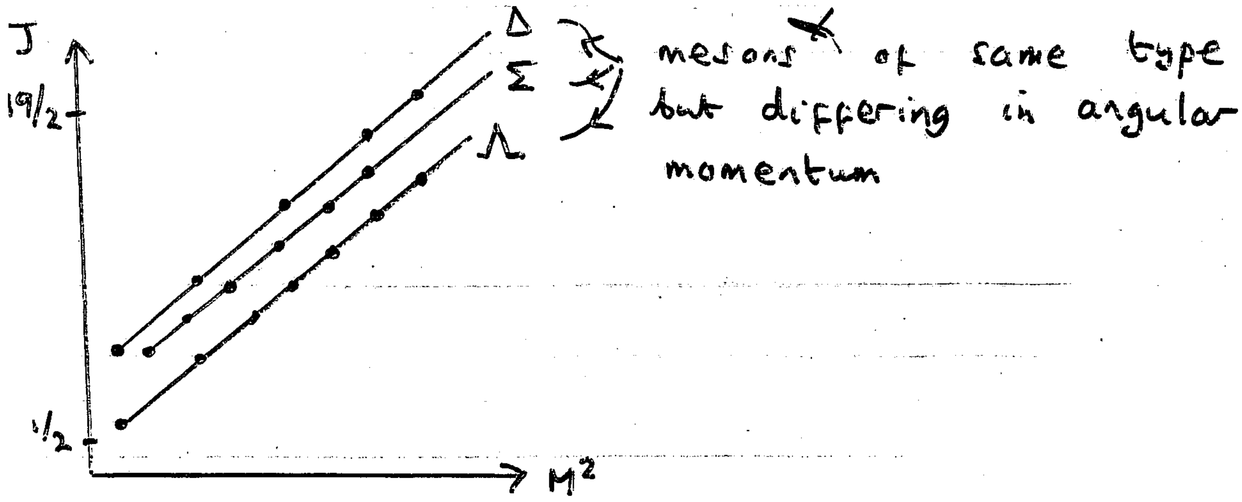
$$I_1 = e^2\int_0^{\pi/2}\cos^2\theta = e^2\int_0^{\pi/2}\frac{1+\cos 2\theta}{2}\,d\theta = e^2\left[\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right]_0^{\pi/2} = e^2\pi/4$$

$$I = e\left[-I_1+I_2\right] = e^3\pi/4$$

$$\therefore J = \frac{2k_2}{\hbar c e}\times e^3\frac{\pi}{4} = \frac{\pi k_2 e^2}{2\hbar c}$$

But $\rho = \frac{Mc^2}{\pi k_2} \Rightarrow J = \frac{\pi k_2}{2\hbar c} \times \left(\frac{Mc^2}{\pi k_2}\right)^2 = \underline{\frac{(Mc^2)^2}{2\pi k_2 \hbar c}}$

The model is simplistic, but the prediction $J \sim M^2$ agrees with experiment for baryons as well as mesons.



Gradient $\simeq 0.9\,(\text{GeV})^{-2}$

According to the model, $\frac{dJ}{d(Mc^2)^2} = (2\pi k_2 \hbar c)^{-1}$

$\Rightarrow k_2 = 1\,\text{GeV}/F$ the Fermi, 10^{-15} m

or $\underline{k_2 = 10^{15}\ \text{GeV m}^{-1}}$

This is a reasonable answer, since the proton has a rest mass energy ~ 1 GeV and a radius ~ 1 F

So at a typical hadron distance of 1F, the confinement energy of a quark $\sim$ 1GeV

(100 times nuclear binding energies)

7.5 Gauge theories

QCD is a gauge theory.

So are QED, general relativity, weak + EM (see section 8.)

i.e all fundamental interactions in nature are described by gauge theories.

Gauge theories have their origin in gauge invariance in classical electromagnetism.

Gauge invariance implies that only differences of electric potential can have physical significance not actual values.

Wigner showed the connection between gauge invariance and conservation of charge.

He supposes charge is not conserved, and a charge creating and destroying device exists at potential V, which creates Q, requiring energy W

Then the charge and device are moved to a

place where the potential is V', with $V' < V$.
The charge and device gain energy $Q(V-V')$ in this transfer.
At this point, the device is used to destroy the charge, regaining the energy W used in its creation. (Gauge invariance tells us that regaining W is independent of the value of the potential.)
Now the chargeless device is moved to its original location without doing any work.

Net gain of energy $= Q(V-V')$

i.e. if gauge invariance and non-conseration of charge are assumed, energy conseration is violated.

The indefiniteness of V is a <u>global gauge symmetry</u>. Changing the value of V everywhere has no physical effect. The <u>global</u> symmetry assures <u>global</u> charge conservation; the total charge in the universe is constant.

|| Can the global symmetry be converted into a local gauge symmetry, assuring local charge conservation?

This is what Maxwell did in 1868.

He had 4 equations of electromagnetism. Leaving out details ($4\pi\varepsilon_0$ etc) they are

$$\nabla \cdot \underset{\sim}{E} = \rho, \quad \nabla \times \underset{\sim}{E} = -\frac{\partial B}{\partial t}, \quad \nabla \cdot \underset{\sim}{B} = 0, \quad \nabla \times \underset{\sim}{B} = \underset{\sim}{j}$$

Taking the divergence of the last gave

$$\nabla \cdot (\nabla \times \underset{\sim}{B}) = \nabla \cdot \underset{\sim}{j}$$

But the divergence of a curl is zero

So we should have $\nabla \cdot \underset{\sim}{j} = 0$ *

But continuity of charge $\Rightarrow \nabla \cdot \underset{\sim}{j} = -\frac{\partial \rho}{\partial t}$ *

These equations are in conflict, so Maxwell modified Ampère's Law to be

$$\nabla \times \underset{\sim}{B} = \underset{\sim}{j} + \partial \underset{\sim}{E} / \partial t$$

$$\Rightarrow \nabla \cdot (\nabla \times \underset{\sim}{B}) = \nabla \cdot \underset{\sim}{j} + \partial/\partial t \; \nabla \cdot \underset{\sim}{E}$$

$$0 = \nabla \cdot \underset{\sim}{j} + \frac{\partial \rho}{\partial t}$$ which is correct

This ensures the <u>local conservation of charge</u>, since the continuity equation says no charge can be created or destroyed <u>in an arbitrarily small volume</u>.

We may clarify the situation by dealing with <u>vector and scalar potentials</u> $\underset{\sim}{A}$ and V. We have

$$\underset{\sim}{B} = \nabla \times \underset{\sim}{A}; \qquad \underset{\sim}{E} = -\nabla V - \frac{\partial \underset{\sim}{A}}{\partial t}$$

<u>Gauge invariance</u> says that $\underset{\sim}{A}$ and V are not unique for given physical fields $\underset{\sim}{E}$ and $\underset{\sim}{B}$.

<u>Gauge transformations</u> on $\underset{\sim}{A}$ and V leave $\underset{\sim}{E}$ and $\underset{\sim}{B}$ unaltered. (The associated invariance of the Maxwell equations is <u>gauge invariance</u>.)

<u>e.g.</u>
$$\begin{cases} V \longrightarrow V' = V - \dfrac{\partial \chi}{\partial t} \\ \underset{\sim}{A} \longrightarrow \underset{\sim}{A}' = \underset{\sim}{A} + \nabla \chi \end{cases}$$

arbitrary

Then $$\underset{\sim}{E}' = -\nabla V + \nabla\left(\frac{\partial \chi}{\partial t}\right) - \frac{\partial \underset{\sim}{A}}{\partial t} - \frac{\partial (\nabla \chi)}{\partial t}$$

$$= -\nabla V - \frac{\partial \underset{\sim}{A}}{\partial t} = \underset{\sim}{E}$$

$$\underset{\sim}{B}' = \nabla \times \underset{\sim}{A} + \underbrace{\nabla \times (\nabla \chi)}_{0} = \nabla \times \underset{\sim}{A} = \underset{\sim}{B}$$

i.e. $\underset{\sim}{E}$ and $\underset{\sim}{B}$ unchanged in this gauge transformation.

The global symmetry of the electric field and global charge conservation have been converted into a local symmetry with local charge conservation by addition of a new field, $\underset{\sim}{B}$.

Now V may be made different at any point, not just changed everywhere at once, by introducing a corresponding change in $\underset{\sim}{A}$.

The above process may actually be reversed. Local invariance requires a relation between V and $\underset{\sim}{A}$, and hence between $\underset{\sim}{E}$ and $\underset{\sim}{B}$.

With the aid of Lorentz invariance, Maxwell's equations may be derived from the local symmetry requirement.

(This is broadly how gauge theories are obtained; a global symmetry is changed into a local one by addition of one or more new fields, and from the resulting relations, the field equations are obtained.)

General relativity follows the same pattern. The global space-time coordinate transformations of special relativity are turned into local ones by addition of a field, gravity. The result is the gauge theory of general relativity.

Electromagnetic gauge invariance in quantum theory

Analogous to the indeterminacy of the absolute value of V is the fact that the absolute phase of a wave-function cannot be measured.

A physical observable is, for example

$$\bar{O} = \int_{-\infty}^{\infty} \psi^*(x,t)\, \hat{O}\, \psi(x,t)\, dx$$

the expectation value of $\hat{O}$.

This is invariant under a global phase transformation

$$\psi(x,t) \longrightarrow \psi'(x,t) = e^{i\theta}\psi(x,t)$$

This is a global phase transformation, because θ is a scalar, independent of x and t.

For local phase invariance, we would require

$$\psi(x,t) \rightarrow \psi'(x,t) = e^{i\theta(x,t)}\psi(x,t)$$

for any $\theta(x,t)$

However, if $\psi(x,t)$ satisfies the free particle Schrödinger equation,

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)$$

then

$$\frac{\partial \psi'(x,t)}{\partial t} = \frac{\partial}{\partial t}\left\{ e^{i\theta(x,t)}\psi(x,t)\right\}$$

$$= i e^{i\theta(x,t)} \frac{\partial}{\partial t}(x,t) + i\frac{\partial \theta(x,t)}{\partial t} e^{i\theta(x,t)}\psi(x,t)$$

(A)

$$
\begin{aligned}
\frac{\partial^2}{\partial x^2}\psi'(x,t) &= \frac{\partial^2}{\partial x^2}\left\{ e^{i\theta(x,t)}\,\psi(x,t)\right\} \\
&= \frac{\partial}{\partial x}\left\{ i\,\frac{\partial\theta(x,t)}{\partial x}\,e^{i\theta(x,t)}\,\psi(x,t) + e^{i\theta(x,t)}\,\frac{\partial\psi(x,t)}{\partial x}\right\} \\
&= i\,\frac{\partial^2\theta(x,t)}{\partial x^2}\,e^{i\theta(x,t)}\,\psi(x,t) - \left(\frac{\partial\theta}{\partial x}\right)^2 e^{i\theta(x,t)}\,\psi(x,t) \\
&\quad + i\,\frac{\partial\theta(x,t)}{\partial x}\,e^{i\theta(x,t)}\,\frac{\partial\psi(x,t)}{\partial t} + i\,\frac{\partial\theta(x,t)}{\partial x}\,e^{i\theta(x,t)}\,\frac{\partial\psi(x,t)}{\partial x} \\
&\quad + \underbrace{e^{i\theta(x,t)}\,\frac{\partial^2\psi(x,t)}{\partial x^2}}_{B}\Big\}
\end{aligned}
$$

It is clear that, if there were just terms A and B, the free-particle Schrödinger equation would be obeyed by ψ', but the other terms, involving differentials of $\theta(x,t)$ do not allow this.

How can local phase invariance be obtained? We may introduce a new field to provide compensating local changes.
The appropriate Schrödinger equation will no longer be force free and will not describe a free particle.
The invariance will be manifested in the inability to distinguish whether a particle

motion is due to local phase change or the new field of force.

The compensating field is just the electromagnetic field. In the phase transformation, if

$\theta = q\chi(x,t)$ then $\psi(x,t) \rightarrow \psi'(x,t)$

q: charge, χ: arbitrary function

$$= e^{iq\chi(x,t)}\psi(x,t)$$

Since the EM field is now included, it is necessary to make the same correlated gauge transformation on $\underset{\sim}{A}$ and V as in the classical case.

If gauge and phase transformations are made simultaneously, the Schrödinger equation will be invariant to the changes, and is then <u>gauge invariant</u>.

The equation may be obtained from the free-particle equation by substituting

$$\nabla \rightarrow \nabla - iq\underset{\sim}{A}/\hbar; \qquad \frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} + \frac{iqV}{\hbar}$$

So the equation is

$$\left[\frac{1}{2m}\left(-i\hbar\nabla - q\underset{\sim}{A}\right)^2 + qV\right]\psi(x,y,z,t) = i\hbar\frac{\partial\psi(x,y,z,t)}{\partial t}$$

A very similar substitution of derivatives serves to insert the compensating fields in other gauge theories. (Exactly the same substitution works for the Klein-Gordon and Dirac equations.)

To summarise setting up a gauge theory,
(1) a global gauge symmetry or invariance must be found, expressed by a transformation
(2) the global symmetry is converted to a local symmetry by changing the transformation so that it depends on space and time and contains something equivalent to a charge
(3) the local transformation is compensated by adding new fields that may be put into the field-free wave equations by a suitable substitution of derivatives.

<u>QED</u> follows the lines above.
$\underset{\sim}{A}$ becomes the wave function of the photon. When an electron emits a photon, the phase of its wave-function changes.

When the photon is reabsorbed by the same or a different electron. there is a compensating phase change.
The phase changes are correlated, and the overall symmetry is maintained because the electrons are indistinguishable.

Yang-Mills theory (1954)

QED was used as a guide by Yang and Mills in trying to develop a theory of the strong interaction. (The theory was ultimately unsuccessful, but instructive.)

They tried to get a local symmetry out of the global symmetry of isaspin invariance for the strong interaction.

The global symmetry is that, in the absence of EM, changing $\left.\begin{matrix} n \to p \\ p \to n \end{matrix}\right\}$ would leave the universe unchanged. It may be expressed as a phase transformation, but the wave function must have 2 components.

$$\begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \rightarrow \begin{pmatrix} \psi_p' \\ \psi_n' \end{pmatrix}$$

The transformation acts on both wave-function components, and so correlates numbers of protons and neutrons.
A 2x2 matrix is required, rather than a simple phase angle θ.

This is an important difference making the Yang-Mills theory <u>non-Abelian</u> while QED is <u>Abelian</u>.
An <u>Abelian transformation</u> is <u>commutative</u>.
If 2 transformations are made in succession, the order is irrelevant
<u>e.g.</u> In QED, successive phase shifts can be made without regard to order.

<u>Non-Abelian transformations</u> are <u>not</u> commutative.
The Yang-Mills theory is non-Abelian because 2 isospin rotations usually lead to

different final numbers of protons and neutrons, depending on the order in which they are performed.

(From now on, all subsequent gauge theories will be non-Abelian. For QCD this has important physical consequences.)

We now turn the global symmetry into a local one, by inserting a "charge" and making the transformation depend on space and time. (The "charge" is a coupling constant.) Fields are introduced to compensate for the equivalent of a local phase change.

For the fields, 2×2 matrices are involved, and more than 1 compensating field is required.

The symmetry group of isospin is SU(2) and its simplest representations are 2 and $\bar{2}$. The no. of compensating fields =

$2 \otimes \bar{2} = 1 \oplus 3$.

The singlet field is as in QED just $\underset{\sim}{A}$ which is the wave-function of the photon.

The triplet of fields are also massless, but, unlike the photon, carry isospin, and so must have charges +1, 0 and -1.

This is the important distinction between Abelian and non-Abelian transformations.
In the Abelian case, e.g. QED, the carrier of the field, γ, does not possess the source of the field, charge.
In the non-Abelian case, e.g. Yang-Mills, the carrier does have the source of the field, isospin.

Charged massless fields or particles would have been detected.
$\therefore$ They do not exist.
$\therefore$ Yang-Mills fails.
[QCD and the electroweak (EM-weak) theory are both non-Abelian.]

7.6 Quantum Chromodynamics QCD

In QCD, the SU(2) of isospin of the Yang-Mills theory is replaced by the SU(3) of colour

N.B. The SU(3) of flavour (section 5) was a broken symmetry because the masses of u, d and s are not the same.

But the SU(3) of colour is exact, because all 3 colours are equivalent.

The global symmetry of colour is that if all rs became ys, all ys became bs, all bs became rs, all hadrons would still be colourless.

This symmetry may be expressed as a transformation, but 3-component wave-functions are needed, and so 3×3 matrices.

To make the global symmetry a local one, the same prescription is used as for Yang-Mills. The transformation is altered to include a coupling constant, and to make it a function of x and t.

This transformation by itself would change the colour of one quark, and hence create a coloured hadron.

∴ Compensating fields – gauge fields – must be added by substituting derivatives as above. Now 3×3 matrices are involved.

Since the simplest representations of $SU(3)$ are 3 and $\bar{3}$ (the 3 colours and anticolours respectively) we expect $3 \otimes \bar{3} = 1 \oplus 8$ gauge fields.

The octet are the gluons. Each possesses a colour and an anti-colour

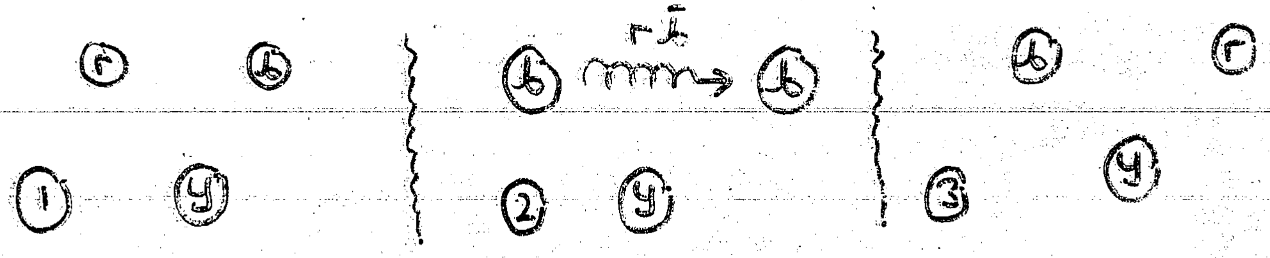
The 8 are $r\bar{y}$, $r\bar{b}$, $y\bar{r}$, $y\bar{b}$, $b\bar{r}$, $b\bar{y}$, $\frac{1}{\sqrt{2}}(r\bar{r} - y\bar{y})$ and $1/\sqrt{6}\,(r\bar{r} + y\bar{y} - 2b\bar{b})$

[One may write down different permutations of r, y, b for the last 2, but there will be no physical difference.]

The other combination of $r\bar{r}$, $y\bar{y}$ and $b\bar{b}$ is $\frac{1}{\sqrt{3}}(r\bar{r} + y\bar{y} + b\bar{b})$. This has no colour and is the singlet.

<u>The gluons and local colour symmetry</u>

e.g.



In a baryon (rby), ① the r quark emits a $r\bar{b}$ gluon and becomes b.

② The b absorbs the $r\bar{b}$ gluon and becomes r.

③ The baryon remains colourless.

So colours of quarks may be changed differently at any point of space-time, and the gluon field restores the symmetry.

[Gluons convert a <u>global</u> symmetry into a <u>local</u> one; they are able to do this because they possess colour.]

N.B Like the γ, the gluons have spin-1

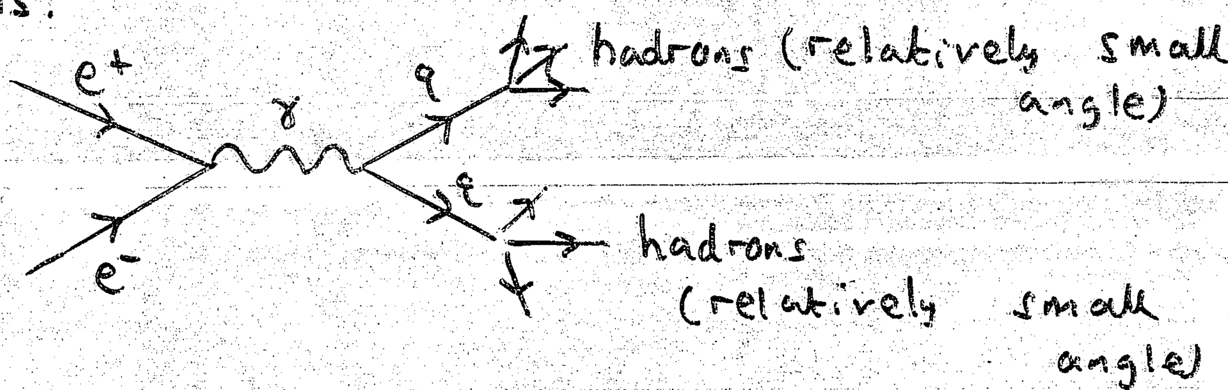
<u>Glueballs</u> Gluons should cluster for the same reason as quarks do, because they

carry colour. So we will not see free gluons any more than free quarks, but must look for indirect evidence.

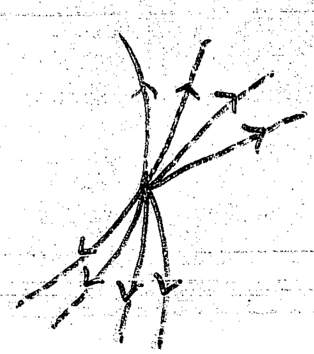
Clusters of gluons or <u>glueballs</u> are being sought, and some candidates have been found.

<u>Two-jets and three-jets</u>

The PETRA (Hamburg) e^+e^- colliding beam accelerator has provided direct evidence for gluons.



In rest frame of $q\bar{q}$

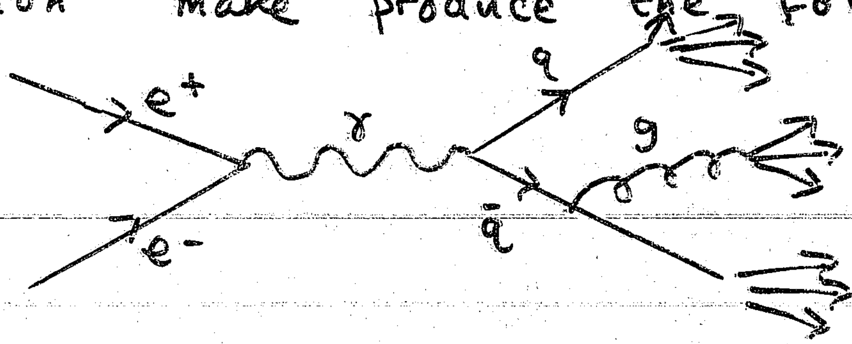


particle trajectories curved because they are in a magnetic field

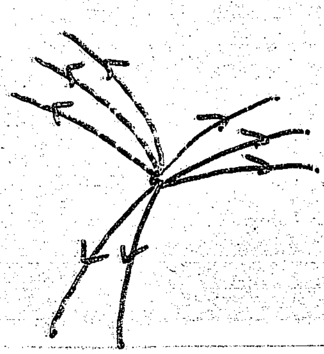
This is a <u>2-jet event</u>

As energy increased, gluons are produced widening beams.

As it is increased further, an energetic gluon make produce the following diagram:



In rest frame of $q\bar{q}$



3-jet event: fairly direct evidence of gluon

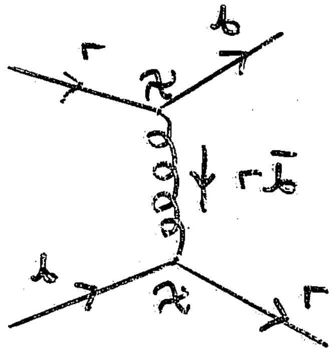
Formation of baryons and mesons

We need to discuss the probabilities for various couplings between quarks due to gluons. They depend on the colour charge, χ.

For an antiquark the colour charge is $-\chi$ (just as all Q. nos. change sign for anti-particles).

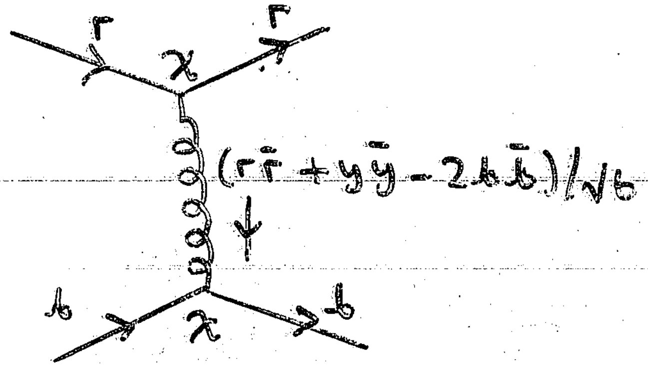
There are only 6 genuinely different cases. (All other exchanges just involve a permutation of colours.)

(a)



colour charge product = x^2

(b)



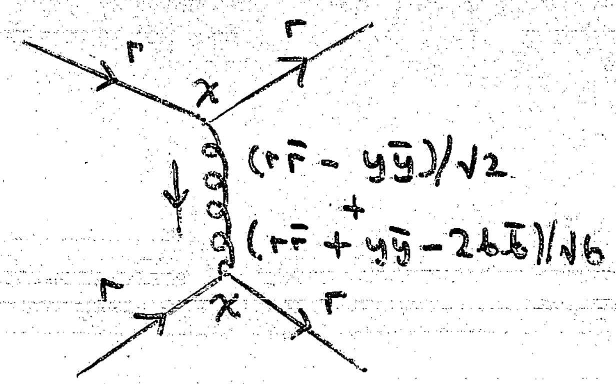
colour charge product = $-x^2/3$

At upper vertex, $r \to r$, so the part of the gluon eigenfunction involving $r\bar{r}$ contributes i.e. $1/\sqrt{6}$ of whole eigenfunction $\Rightarrow$ $x/\sqrt{6}$ as contribution to coupling.

At lower vertex, $b \to b$, and the $b\bar{b}$ part of the gluon eigenfunction contributes $-2x/\sqrt{6}$ to coupling.

Total colour charge product $= \left(x/\sqrt{6}\right)\left(-\frac{2x}{\sqrt{6}}\right) = -x^2/3$

(c)



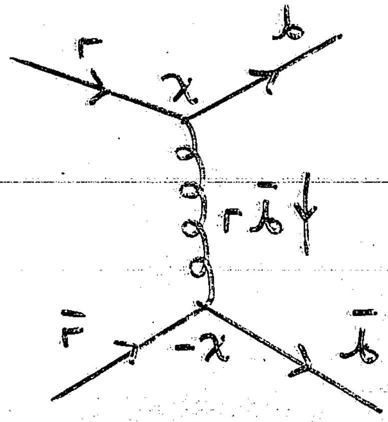
colour charge product = $2x^2/3$

Both colour nonchanging gluons can contribute.
At upper vertex, $r\bar{r}$ part of first contributes $x/\sqrt{2}$ and of second $x/\sqrt{6}$, and also at the lower vertex.

Total colour charge product $= \frac{x^2}{2} + \frac{x^2}{6} = 2x^2/3$

The other 3 diagrams involve the exchange of the same gluons as the first 3. The colour charge products are reversed, since 1 vertex involves antiquarks and has $-x$ rather than x

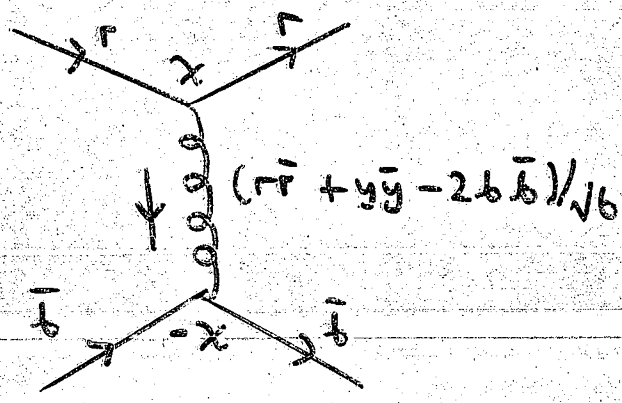
(d)



colour charge product

$= -x^2$

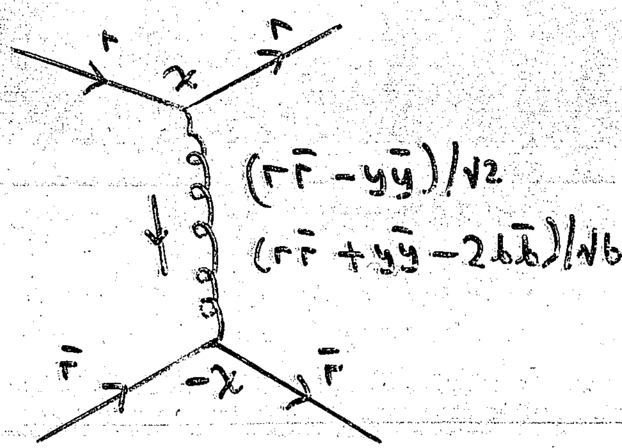
(e)



colour charge product

$= x^2/3$

(f)



colour charge product

$= -2x^2/3$

(1) Binding of baryon

The baryon will have a totally antisymmetric colour eigenfunction for its 3 quarks:

$[(rb - br)y + (by - yb)r + (yr - ry)b]/\sqrt{6}$

[antisymmetric under interchange of any 2 colour labels]

Consider first the interaction of the r and the b.

use $\dfrac{rb - br}{\sqrt{6}}$

Since this is an initial and final state eigenfunction interaction strength involves the square i.e. $(rb - br)^2/6 = 1/6\,(rbrb + brbr - rbbr - br\,\cancel{br}\,rb)$

The first two terms represent $rb \to rb$ and correspond to (b)

The last two terms represent $rb \to br$ and correspond to (a)

$\therefore$ Interaction strength $= \frac{1}{6}\left[\left\{2 \times \left(-\frac{x^2}{3}\right)\right\} - \left\{2 \times x^2\right\}\right]$

$= 1/6\; x^2\left(-\frac{2}{3} - 2\right) = -\frac{4}{9}x^2$

But there are equal contributions from interaction of b and y, and of r and y

$\therefore$ Total coupling $= \underline{-\frac{4}{3}x^2}$

For Coulomb potential, coupling of $-e^2 \to$ potential between e^+ and e^- $= -\dfrac{e^2}{4\pi\varepsilon_0 r}$

coupling of $-\dfrac{4x^2}{3} \longrightarrow$ potential $= -\dfrac{4x^2}{3r}$

The minus sign in both cases denotes <u>binding</u>

(2) Binding of meson

The meson is a boson and so has a totally symmetric colour part to its eigenfunction:

$$(r\bar{r} + y\bar{y} + b\bar{b})/\sqrt{3}$$

The term $r\bar{r} \to r\bar{r}$ contributes $(1/\sqrt{3})^2(-2x^2/3)$ from (f) $= -2x^2/9$

One must add in equal contributions for

$y\bar{y} \to y\bar{y}$, $b\bar{b} \to b\bar{b}$

Total so far $-2x^2/3$

Also $r\bar{r} \to b\bar{b}$, $r\bar{r} \to y\bar{y}$, $y\bar{y} \to r\bar{r}$, $y\bar{y} \to b\bar{b}$,
$b\bar{b} \to r\bar{r}$, $b\bar{b} \to y\bar{y}$ each contribute $\left(\frac{1}{\sqrt{3}}\right)^2(-x^2)$
from (d) $\to$ total of $6\times\left(-\frac{x^2}{3}\right) = -2x^2$

Net coupling strength $= -2x^2 - \frac{2x^2}{3} = -\frac{8x^2}{3}$

Potential $= \underline{-\frac{8x^2}{3r}}$ (i.e binding)

(3) Quark-antiquark pair with colour e.g. $r\bar{b}$

We cannot use colour swapping gluons, since $r \to \bar{b}$ is not possible.

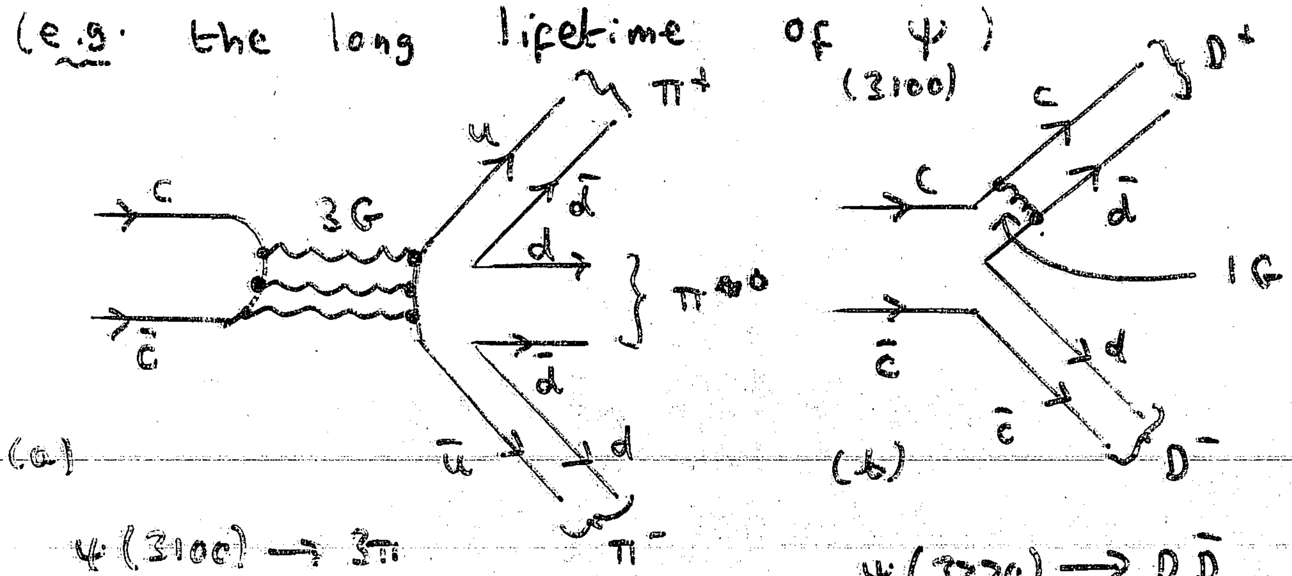
$\therefore$ Only a non-colour-changing gluon can be involved, only, in fact $(r\bar{r} + y\bar{y} - 2b\bar{b})/\sqrt{6}$

(e) $\to$ colour charge product $= +x^2/3$

Potential $= \underline{+\frac{x^2}{3r}}$ (non-binding)

Interpretation of the Zweig rule

(e.g. the long lifetime of $\psi_{(3100)}$)



$\psi(3100) \longrightarrow 3\pi$

$\Gamma = 0.063\,MeV$

$\psi(3770) \longrightarrow D\bar{D}$

$\Gamma = 25\,MeV$

In (a), all states are colour singlets, so at least 2 gluons must be exchanged.

In fact, because ψ has C (charge conjugation number) equal to -1, an odd number of gluons must be exchanged, and the simplest possibility is 3-gluon exchange. (This is very unlikely).

The decay of $\psi(3770)$ to D^+D^- requires only single gluon exchange, and so is not inhibited.

8. Electroweak Theory

(Unification of the weak and EM interactions)

8.1 General Discussion

Return to Yang - Mills; this produced 4 gauge fields, all corresponding to massless particles.

~~One could be the γ~~

The other 3 had charges 1, 0, -1; such charged massless particles would have been detected.

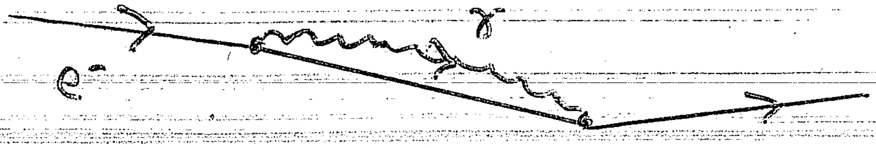
Could theory give the gauge fields mass? very difficult

(1) A mechanism is required.

(2) A gauge boson with mass would violate gauge invariance. We need this not just to have a gauge theory, but to have a <u>renormalisable</u> theory.

QED and renormalisation

There is a basic problem with QED: emission of virtual photons.



$\Delta E \sim \hbar / \Delta t$

As $\Delta t \to 0$, $\Delta E \to \infty$ and the effective mass of the electron becomes infinite.

The problem was resolved for QED around 1948 by Feynman, Tomonaga, Schwinger (Nobel Prize 1965)

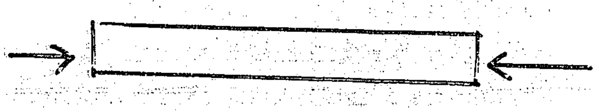
This is done by <u>renormalisation</u>, so that infinities cancel, and the resulting theory is in excellent agreement with experiment. (Gauge invariance is required.)

<u>Spontaneous symmetry breaking</u>

A process was produced which gave the gauge bosons mass as well as preserving gauge invariance - <u>spontaneous symmetry breaking</u>.

<u>compare</u>

<u>(i) rod under axial pressure</u>



all equations are symmetric under rotations about the axis, <u>BUT</u> as pressure increases, it will buckle in an unpredictable direction

(2) Ferromagnet

Above the Curie temperature T_c, there is rotational symmetry (all directions equivalent). Below T_c, spins line up → ferromagnetism, in an unpredictable direction.

In both cases one cannot predict which non symmetric final state will be chosen, but all have a lower energy than the symmetric state.

At a critical point, spontaneous symmetry breaking occurs.

1964 Higgs was able to show how to generate mass using a spontaneously broken local symmetry.

He used a local phase transformation as discussed for QED. In group theory terms, the group is $U(1)$ - unitary group in 1 dimension.

The local field transformation is compensated by a vector field. Higgs obtained from the spontaneous symmetry breaking two scalar fields, one with mass and one without.

Then a suitable gauge transformation has a miraculous result – the massless scalar field disappears, and the vector field gets mass.

brief history of the electroweak gauge theory

1961 **Glashow** set up the general form of the theory, but could not give the gauge bosons mass.

1967, 1968 **Weinberg, Salam** applied the Higgs mechanism to give the gauge bosons mass, and produced a consistent theory.

1971 **t' Hooft** showed (to great surprise) that the theory was renormalisable (which was essential).

1979 Glashow, Salam, Weinberg – **Nobel Prize**

8.2 Sketch of the Theory

Yang-Mills was an attempt to make a *local* symmetry out of the global SU(2) symmetry of isospin.

Since isospin is a property of the strong interaction, what has this theory to say for the weak interaction?

Define <u>weak isospin</u>, $\underline{I_w}$

$$I_{wz} = 1/2 \text{ for } \nu, \quad -1/2 \text{ for } e^-$$

Then $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$ is a weak isospin doublet

analogous to $\begin{pmatrix} p \\ n \end{pmatrix}$.

[<u>N.B.</u> Each generation may be considered separately i.e. $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$, $\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$.]

In fact we write $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ because only a specific helicity of neutrinos is involved — "left-handed".

We do not require $(\nu_e)_R$ but we <u>do</u> require $(e^-)_R$. Thus we introduce EM, which does not violate parity, and so treats $(e_-)_L$ and $(e_-)_R$ equivalently.

The EM part of the theory requires a local phase symmetry with $U(1)$ transformations.

The Yang-Mills part requires a local phase symmetry with $SU(2)$ transformations.

So the overall theory is $U(1) \times SU(2)$.

[N.B. (1) Just as isospin symmetry for n and p implies a physical equivalence - the strong interaction acts equivalently on n and p; weak isospin symmetry for ν_e and e^- implies a physical fact:

The processes

$$\nu_e \longleftrightarrow e^- + W^+$$

and

$$e^- \longleftrightarrow \nu_e + W^-$$

have the same strength. ν_e and e^- are in this sense equivalent under the weak interaction.

(2) For these lepton doublets, we may write $Q = I_z + \frac{Y}{2}$ as in Gell-Mann - Nishijima relation for hadrons.

However Y must be -1; it is not as given for hadrons $B+S+C+B+T$.

We could say, for leptons, $Y = -\ell$]

lepton number

To compensate for the local changes, 4 gauge fields are required. They may be called B (from the $U(1)$ transformation) and W_1, W_2 and W_3 (for the $SU(2)$ transformation).

B and W_3 have no electric charge

The photon γ is $\quad \gamma = B\cos\theta_w + W_3 \sin\theta_w$

(where θ_w is the Weinberg angle (or weak mixing angle) which is, from experiment, given by $\sin^2\theta_w \sim 1/5$, $\theta \sim 26°$)

The combination of B and W_3 orthogonal to γ is

$$Z^0 = W_3 \cos\theta_w - B\sin\theta_w$$

W_1 and W_2 carry electric charges. The states of definite charge are:

$$\left.\begin{aligned} W^+ &= W_1 + iW_2 \\ W^- &= W_1 - iW_2 \end{aligned}\right\}$$

W^+, W^- and Z^0 are the carriers of the weak force.

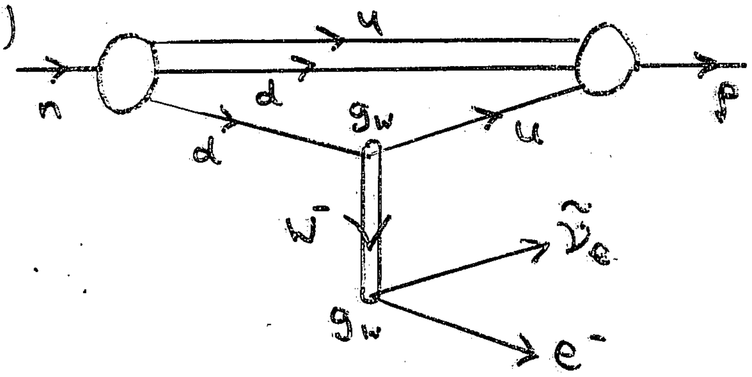
They may be given mass via spontaneous symmetry breaking with 4 Higgs scalar fields, 2 charged, 2 not.

The two charged Higgs particles give the W^+ and W^- mass; one of the neutral Higgs particles gives the Z^0 mass.
These three particles disappear with a suitable gauge transformation.

The other neutral Higgs remains as a real particle. It may be written ϕ^0.
One of its properties is different from any other particle.
In the vacuum state, it has a non-zero expectation value; this value is not invariant under a gauge transformation - spontaneously broken gauge invariance. The actual interactions are still gauge invariant.
(N.B. It costs energy to make the Higgs disappear from the vacuum.)

Weak interaction diagrams

(a)

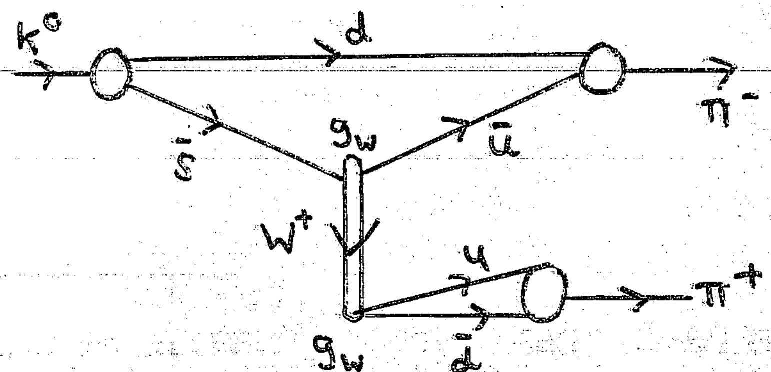


$$n \rightarrow p + e^- + \tilde{\nu}_e$$

or

$$\left.\begin{array}{l} n \rightarrow p + W^- \\ W^- \rightarrow e^- + \tilde{\nu}_e \end{array}\right\}$$

(b)

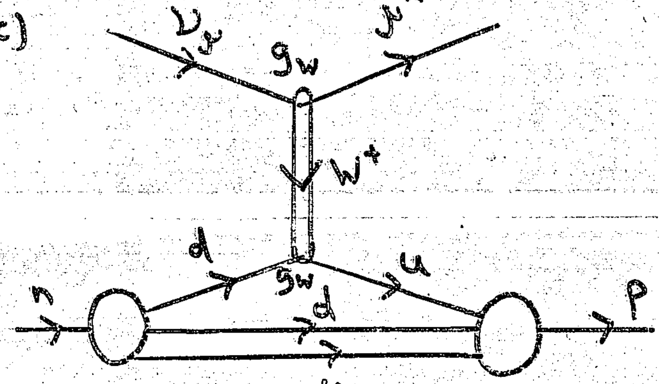


$$K^0 \rightarrow \pi^+ + \pi^-$$

or

$$\left.\begin{array}{l} K^0 \rightarrow \pi^- + W^+ \\ W^+ \rightarrow \pi^+ \end{array}\right\}$$

(c)

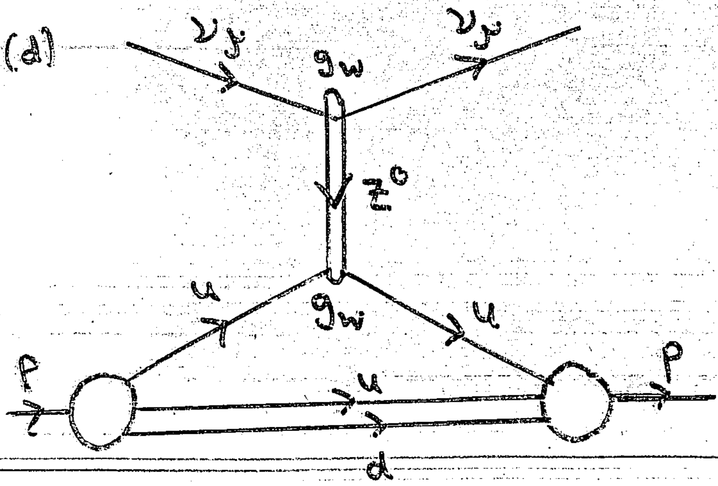


$$\nu_\mu + n \rightarrow \mu^- + p$$

or

$$\left.\begin{array}{l} \nu_\mu \rightarrow W^+ + \mu^- \\ n + W^+ \rightarrow p \end{array}\right\}$$

(d)



$$\nu_\mu + p \rightarrow \nu_\mu + p$$

or

$$\left.\begin{array}{l} \nu_\mu \rightarrow \nu_\mu + Z_0 \\ p + Z_0 \rightarrow p \end{array}\right\}$$

<u>N.B.</u> Quark changes as well as lepton changes are involved. <u>Quarks and leptons have equal strengths of weak interactions.</u>

(a) - (c) are charged current processes using the expected W^+, W^- particles.

(d) is a neutral current process using the Z^0 which was an unexpected result of the theory.

Such a process was discovered at CERN in 1973. The key is a charged particle suddenly moving off from rest when neutrinos are fired at it, with no accompanying phenomena.

Quark mixing

1963 Cabibbo found that if the decay constant for a purely leptonic process like

$$\mu^- \rightarrow e^- + \tilde{\nu}_e + \nu_\mu$$

is β, then that for a non-strangeness changing process like

$$\pi^- \rightarrow \mu^- + \tilde{\nu}_\mu$$

is $\beta \cos \theta_c$, and that for one in which $\Delta S = 1$ like

$$K^- \rightarrow \mu^- + \hat{\nu}_\mu$$

is $\beta \sin \theta_c$

where θ_c, the Cabibbo angle is about 0.23 radians.

Then the ratio of $\Delta S = 1$ to $\Delta S = 0$ decays $\sim \tan^2 \theta_c \sim 0.06$.

At the quark level this means that the s quark does not couple to W^+ as strongly as the u quark.

For <u>weak</u> isospin doublet, we must use, not $\begin{pmatrix} u \\ d \end{pmatrix}_L$ but $\begin{pmatrix} u \\ d_c \end{pmatrix}_L$ with

$$d_c = d \cos \theta_c + s \sin \theta_c .$$

This then gives the correct Cabibbo couplings for $\Delta S = 0$, $\Delta S = 1$.

For the Z^0 though, there is a problem. If the Z^0 interacts with d_c, there is a prediction of $s \longrightarrow d$ (strangeness changing) neutral current processes, known experimentally not to happen for ordinary weak interactions.

<u>GIM mechanism</u> (See section 7.2)

They proposed a new c quark, and a new <u>weak</u> isospin doublet $\begin{pmatrix} c \\ s_c \end{pmatrix}_L$ with

$$s_c = s \cos \theta_c - d \sin \theta_c \qquad \text{(orthogonal to } d_c\text{)}$$

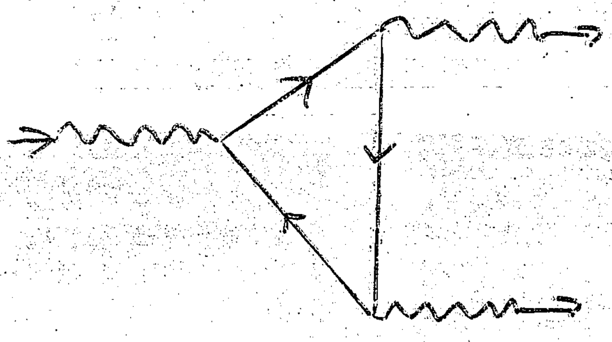
This cancels the strangeness-changing neutral current. (The "charm" wards off the evil.)

[N.B. When third quark doublet found, weak isospin doublet being $\begin{pmatrix} t \\ b_c \end{pmatrix}_L$, a more complicated mechanism ensures no flavour-changing neutral current process.]

Triangle anomaly or Adler-Bell-Jackiw anomaly

There is independent evidence for quark-lepton symmetry.

A triangle anomaly graph would give infinite contributions to decay rates unless the sum of charges of all fermions in each generation adds to zero.



Lepton doublet has total charge -1

Quark doublet has total charge $\underset{\text{no. of colours}}{3}(2/3 - 1/3) = 1$

8.3 Discovery of $W^\pm$ and Z^0

The electroweak theory predicts

$$M_W = \left(\frac{e^2\sqrt{2}}{8\beta \sin^2\theta_W}\right)^{1/2} = \frac{38.5}{\sin\theta_W}\frac{\text{GeV}}{c^2} = M_{Z^0}\cos\theta_W$$

weak interaction parameter

So $M_W \simeq 82\ GeV/c^2$; $M_{Z^0} \simeq 92\ GeV/c^2$

CERN had just built a super proton synchrotron (SPS) accelerating protons to several hundred GeV (1979).

Then came the idea of creating anti-protons in a lab. next to the SPS, accumulating them till huge quantities were available, and injecting them into the SPS.

The SPS is a circle of 4 miles circumference; protons travel one way, anti-protons the other. The difficulty was storing the p^-s (van der Meer).

1982 Collisions of p^+ and p^-. Usually $\rightarrow$ quarks/antiquarks $\rightarrow$ showers of particles, one along line of p^+, one that of p^-. Occasionally, one quark, one antiquark annihilate $\rightarrow$ radiant energy, W and Z as well as photons at these energies.

W and Z decay in $\sim 10^{-23}s$ into $e^-, e^+, \nu, \bar{\nu}$ which can shoot out perpendicular to the beam of the p^+, p^-.

9 examples by 1983 (Masses as above.)

1984 Rubbia (organiser), van der Meer Nobel Prize

Z^0 lifetime - quark/lepton generations

The Z^0 may decay into <u>any</u> pair of objects that has less mass than itself, including, obviously*, all pairs of neutrinos.
The more types of decay, the shorter the lifetime.
Measurement and calculation $\rightarrow$ $\underline{\underline{3}}$ types of neutrino i.e. 3 generations of quarks/leptons.

[* Could there be neutrinos with mass $\geq 45\ \text{GeV}/c^2$??]

8.4 The Standard Model

The main weakness of the standard model is that the Higgs has not been found, and it is difficult to estimate its mass.

The reaction
$$Z^0 \rightarrow \phi^0 + \nu + \hat{\nu}$$
is not observed, so $M_\phi \gtrsim 60\ \text{GeV}/c^2$

LEP-II gave some indication of the ϕ^0 in the reaction $e^+ + e^- \rightarrow \phi^0 + Z^0$

We wait for conclusive evidence in the LHC (<u>large hadron collider</u>) which has max energy $8\ \text{TeV} = 8 \times 10^{12}\ \text{eV}$ per beam

Will produce ϕ if $M_\phi \lesssim 1\,\text{TeV}$ by

$$p + p \rightarrow \phi + \underset{\text{any particle allowed by conservation laws}}{X}$$

Standard model

Name	Symbol	Mass (MeV/c^2)	Charge	N_c (no. of colour components)	
spin 1, gauge bosons					
photon	γ	0	0	1	$U(1)$
vector bosons for weak force	Z^0	91,188	0	1	$SU(2)$
	W^+	80,280	1	1	
	W^-	80,280	-1	1	
gluon	A_s	0^*	0	8	$SU(3)$
spin 0, Higgs	ϕ or H^0	$>60,000$	0	1	
spin ½, quarks					
I up	u	5^*	$2/3$	3	
I down	d	10^*	$-1/3$	3	
II charm	c	1600^*	$2/3$	3	
II strange	s	180^*	$-1/3$	3	
III top	t	$180,000^*$	$2/3$	3	
III bottom	b	$4,500^*$	$-1/3$	3	
spin ½, leptons					
I e-neutrino	ν_e	~ 0	0	1	
I electron	e^-	0.510999	-1	1	
II μ-neutrino	ν_μ	~ 0	0	1	
II muon	μ^-	105.6584	-1	1	
III τ-neutrino	ν_τ	~ 0	0	1	
III tau	τ	1777	-1	1	
spin 2, graviton	g	0	0	1	

* is colour force ignored

Total N = 38

9. C, P, T symmetries

pseudovectors

A vector should change sign under reflection; a scalar should not.

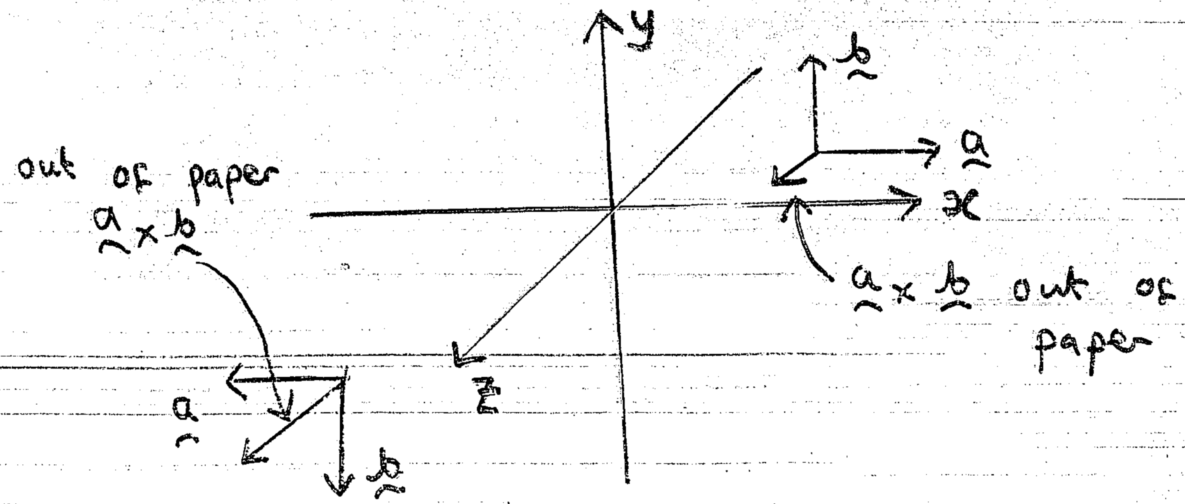
In elementary work, one says the vector product $\underset{\sim}{a} \times \underset{\sim}{b}$ is a vector, and the triple scalar product, $\underset{\sim}{a} \cdot (\underset{\sim}{b} \times \underset{\sim}{c})$ is a scalar

Yet if $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ change sign on reflection, $\underset{\sim}{a} \times \underset{\sim}{b}$ will not change sign, and $\underset{\sim}{a} \cdot (\underset{\sim}{b} \times \underset{\sim}{c})$ will change sign.

$\underset{\sim}{a}$, $\underset{\sim}{b}$, $\underset{\sim}{c}$ are [normal] polar vectors

$\underset{\sim}{a} \times \underset{\sim}{b}$ is a pseudovector or axial vector

$\underset{\sim}{a} \cdot (\underset{\sim}{b} \times \underset{\sim}{c})$ is a pseudoscalar



N.B. (1) Parity operations should really be reflections in xy xz and yz planes ($x \to -x$, $y \to -y$, $z \to -z$) but in practice we may use single reflection, provided it is a suitable one

(2) Axial vectors have directions which are not directly physical, but more matters of definition

e.g. $\underset{\sim}{L} = m\, \underset{\sim}{V} \times \underset{\sim}{r}$

(The direction of $\underset{\sim}{L}$ is not a physical given.)

e.g. $\underset{\sim}{V} = \underset{\sim}{\omega} \times \underset{\sim}{r}$ ~ polar vector (changes)

polar vector (changes)   pseudovector (does not change)

$\underset{\sim}{F} = m\{\underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})\}$

polar

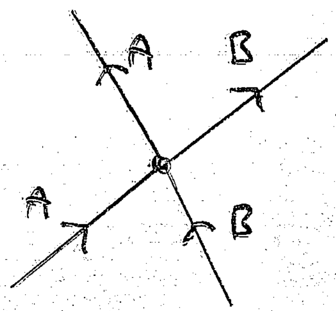
Parity P

The parity operator may be regarded as a reflection operator.

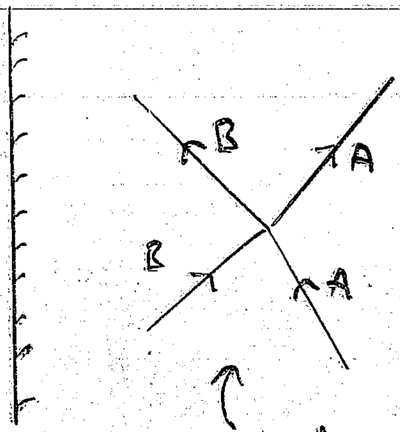
Conservation of parity implies that, if a physical event is reflected, the resultant event is a proper physical event.

Till 1955, it was assumed that parity <u>was</u> conserved.
It <u>is</u> conserved in all interactions except the weak.
(We may worry about left-hand-rules, right-hand-rules in EM!)

<u>e.g.</u> (1)



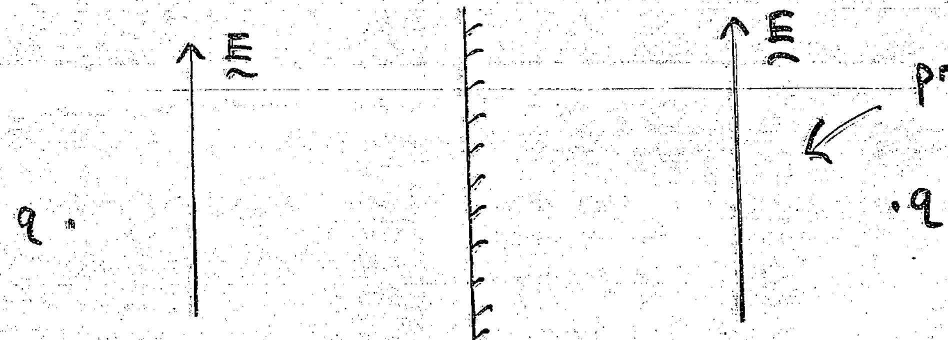
actual event



reflected event

proper physical event

(2)

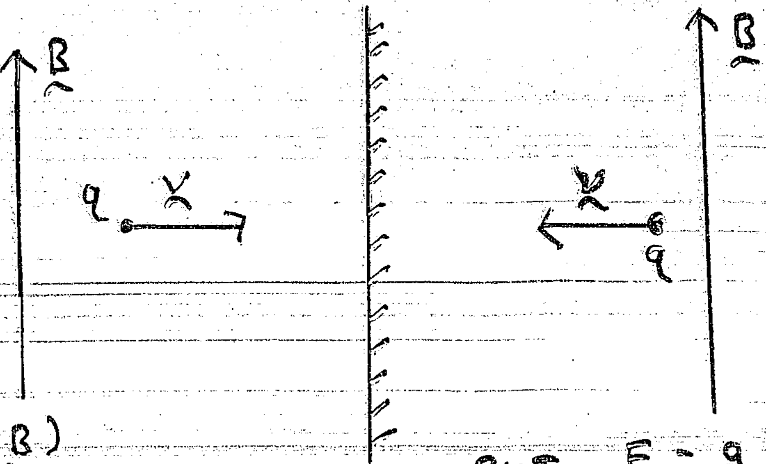


proper physical event

$\underset{\sim}{F} = q\underset{\sim}{E}$ in direction of $\underset{\sim}{E}$

$\underset{\sim}{F} = q\underset{\sim}{E}$ in direction of $\underset{\sim}{E}$

(3)
(naive)



reflection → $\underset{\sim}{F}$ out of paper

$\underset{\sim}{F} = q(\underset{\sim}{v} \times \underset{\sim}{B})$ out of paper

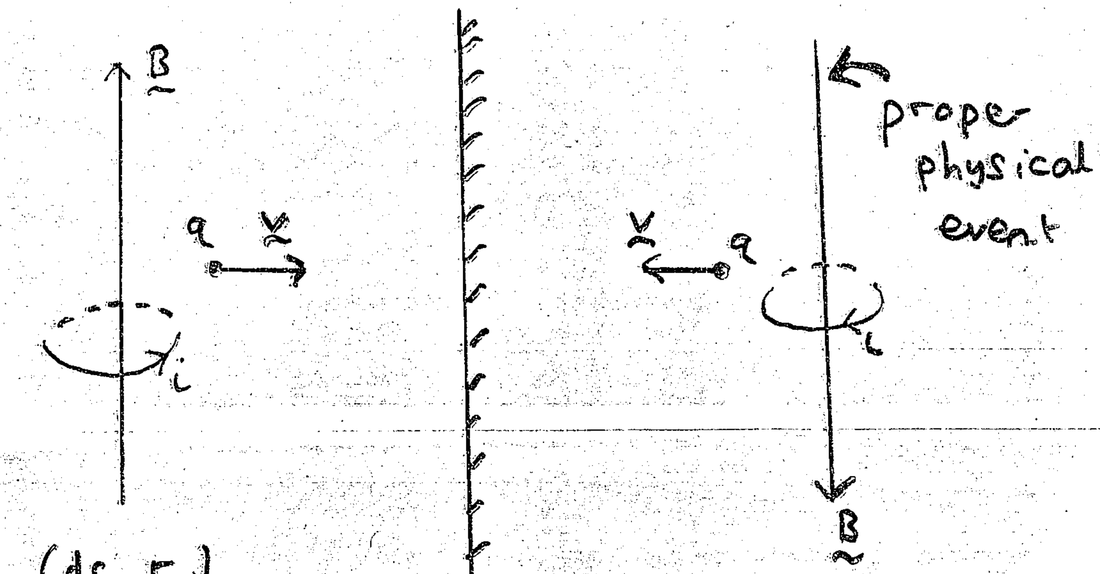
<u>BUT</u> $\underset{\sim}{F} = q(\underset{\sim}{v} \times \underset{\sim}{B})$ into paper

The reflection (as we have performed it) does <u>not</u> lead to a proper physical event.

The problem is that $\underset{\sim}{B}$ is a pseudovector and so behaves in the opposite way to a polar vector on reflection.

Return to something more genuinely physical i.e. current in circuit

(4)



$$d\underset{\sim}{B} = \frac{i}{4\pi r^3}(d\underset{\sim}{s} \times \underset{\sim}{r})$$

force out of paper

force out of paper

Parity of wave-functions, particles etc.

Define $\hat{P}\,\psi(x,y,z) = \psi(-x,-y,-z)$

$$\hat{P}^2\,\psi(x,y,z) = \psi(x,y,z)$$

i.e. $\psi(x,y,z)$ is an eigenfunction of $\hat{P}^2$

~~If $\psi(x,y,z)$ is an eigenfunction of $\hat{P}$~~

$$\hat{P}\,\psi(x,y,z) = P\,\psi(x,y,z)$$

$$\hat{P}^2\,\psi(x,y,z) = P^2\,\psi(x,y,z)$$

$$\therefore P = \pm 1$$

Elementary particles have even or odd parity
Conservation of parity implies parity should not change during reactions.

Charge conjugation C

Conservation of C implies that replacing all particles by anti-particles in a physical event yields a proper physical event.

Time reversal T

Conservation of T implies that filming a physical event and showing it backwards yields a proper physical event.

CPT There is a rigorous theorem of quantum field theory (Pauli-Lüders, Bell) that physics is invariant under CPT

Non-conservation of parity

π^0 - mesons have -ve parity

But K^0 - mesons decay to 2π (even parity) and also to 3π (odd parity) (weak interaction)

(For some time people called the K^0 two names θ meson and τ meson, and hoped to show that $m_\theta \neq m_\tau$, but this proved not to be the case)

1956 Yang and Lee suggested parity not conserved in the weak interaction. (Nobel Prize 1957)

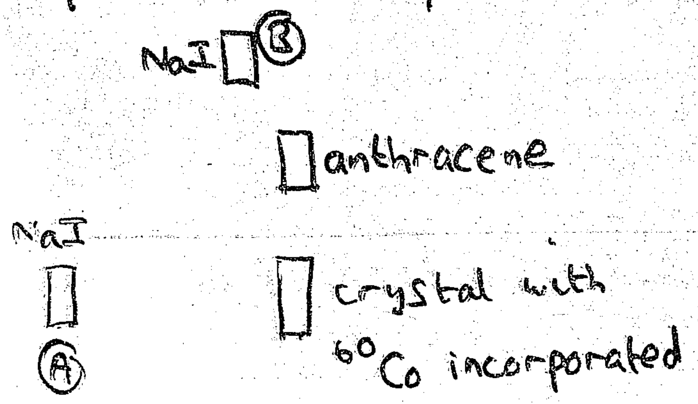
1956 Wu et al - experiment

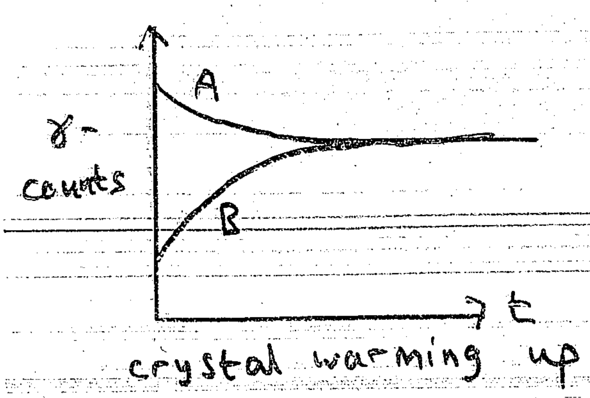
They studied the decay

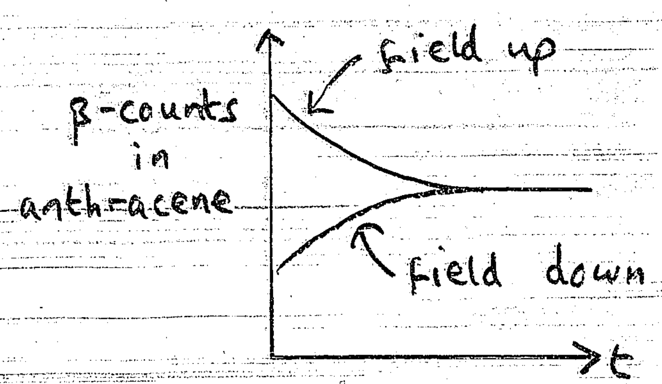
$$Co^{27}_{60} \longrightarrow Ni^{27}_{60} + \beta^- + \tilde{\nu}$$

The nuclei were polarised by being incorporated in a crystal of cerium magnesium nitrate, which has a strong internal field from electrons to nuclear spins.

The crystal is cooled by adiabatic demagnetisation to 0.01K so electron spins and hence nuclear spins are polarised.



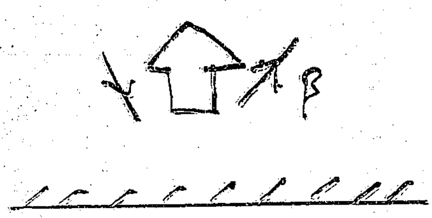




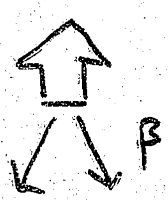
i.e. if spin polarised as 

the electron tends to come off as

But spin is angular momentum, a pseudo-vector so reflection gives



<u>not</u> a physical event



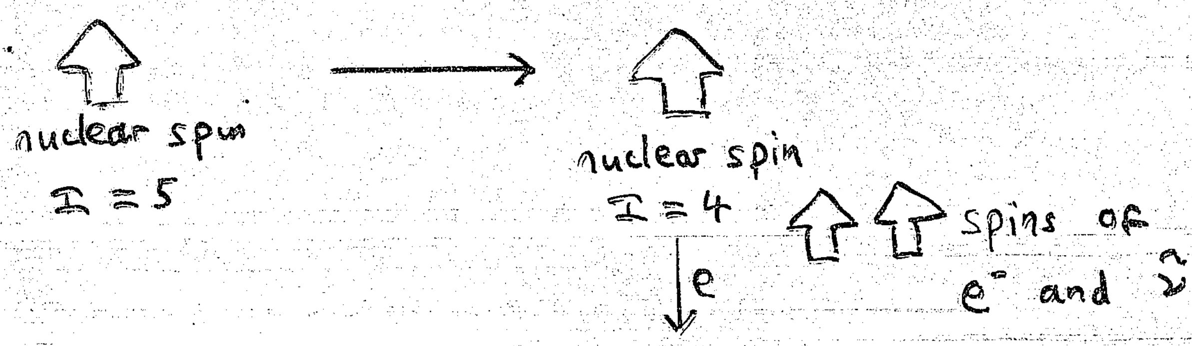
physical event

Physics <u>not</u> invariant to reflection

i.e. Parity <u>not</u> conserved.

The electrons produced are polarised

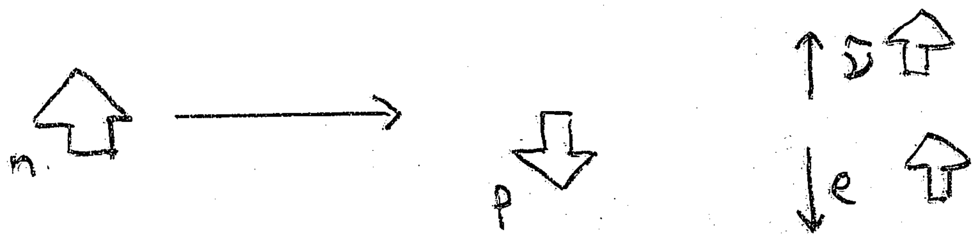
i.e.



correlation of momentum and spin directions of electron itself contradicts conservation of parity since one is a polar vector, the other a pseudovector.

The weak interaction also fails to be invariant under <u>charge conjugation</u>

<u>e.g.</u> decay of n



(spin of n and p must have z-component differing by 1 from theory of β-decay)

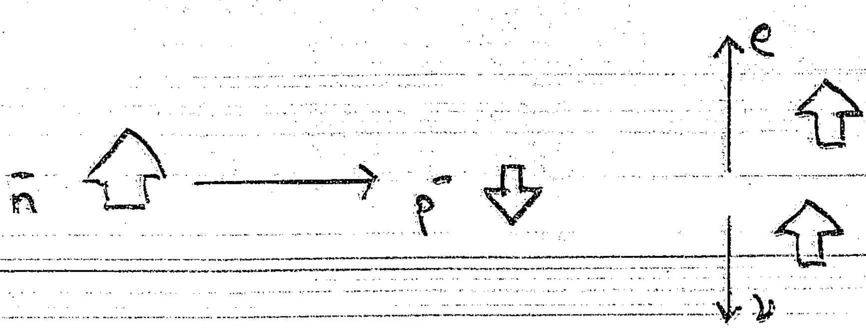
<u>C operator</u> →

<u>not</u> what is observed in nature

(1) e^+ and ν should be oppositely directed to first case

(2) helicity of ν wrong

<u>CP operator</u> →



<u>as in nature</u>

Indeed CP is conserved at the 99% level in the weak interaction.

But 1964 Cronin, Fitch et al:

Decay of K^0 at stage at which all K_1^0 all decayed, only K_2^0 left, which should have decayed into 3π (section 4)

0.1% decayed into 2π (violating conservation of CP). (Cronin, Fitch - Nobel Prize 1980)

If CPT invariance is to be maintained (which seems essential), it seems this decay may not be invariant under T.

(N.B. This effect may be a result of the mixing of quarks (Cabibbo) as in section 8. Thus the effect would not be present in leptonic processes.)

10. Beyond the Standard Model

The Standard Model is extremely successful within its self-imposed limitations

The chief next aim of physicists is to unify the electroweak interaction with the strong (colour) interaction.

(In addition they would like to understand the 3 generations, and also be able to calculate all the various constants that appear in the theory.)

Grand Unification Theories (GUTs)

The differences between EM and weak interactions is caused by spontaneous symmetry breaking. At high energy, full symmetry should apply.

Let us examine the behaviour with energy of the coupling constants.

For EM, don't use e corresponding to γ (after symmetry breaking) but the coupling g' for B of the $U(1)$ transformation before symmetry breaking.

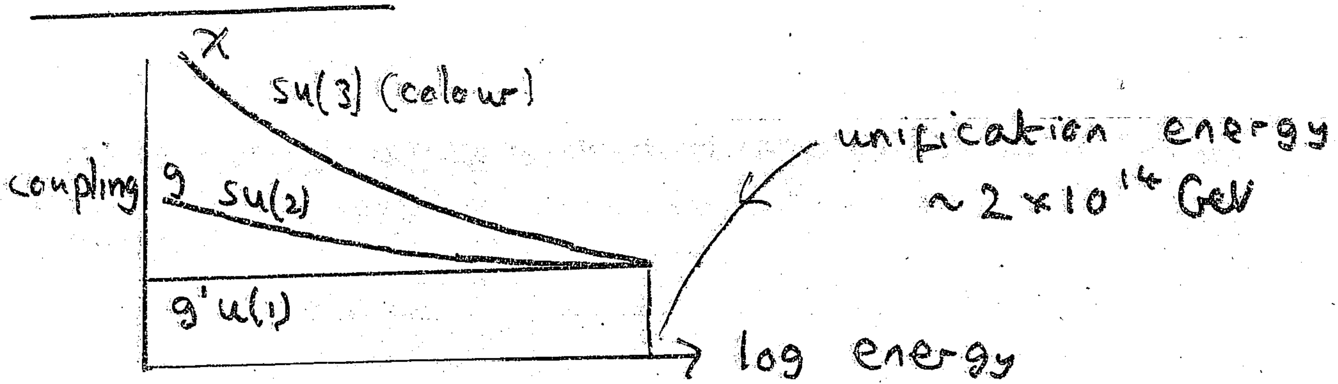
$g' = e/\cos\theta_W$ and θ_W increases slowly with energy, so g' increases slowly.

For the weak interaction, instead of using g_W appropriate to $W^\pm$, Z^0 *after* symmetry breaking, use g for W_1, W_2, W_3 of $SU(2)$ *before* symmetry breaking

$$g = g_W \times 2\sqrt{2}$$

In fact g decreases as energy increases.

For the strong interaction, the appropriate coupling also decreases as energy increases, and at an energy of $\underline{\sim 2 \times 10^{14}\ \text{GeV}}$, <u>the three come together.</u>



[N.B. 10^{14} GeV seems an impossible energy for us to create on earth; we are led to consider its occurrence soon after the Big Bang, i.e. led into cosmological speculation.]

Much effort has been spent developing GUTs unifying the $SU(3)$ of the strong colour interaction, the $SU(2)$ of the weak interaction, and $U(1)$ of EM. (These 3 would result from spontaneous symmetry breaking of the GUT.)

e.g.
1974 Georgi and Glashow $SU(5)$

This theory contains $5 \otimes \bar{5} = 1 \oplus 24$ gauge bosons. (The singlet is not of interest.)

Of the 24, 8 are the gluons, 4 are γ, $W^\pm$, Z^0.

The other 12 are X and Y bosons, which are called leptoquarks.

There are antiparticles $\bar{X}$ and $\bar{Y}$.

$X, Y, \bar{X}, \bar{Y}$ come in 3 colours $\rightarrow$ 12.

Just as $\bar{d}_{\bar{b}} + g(b\bar{y}) \rightarrow \bar{d}_{\bar{r}}$ (section 7)

and $\nu_e + W^- \rightarrow e^-$ (section 8)

now $\bar{d}_{\bar{r}} + X \rightarrow e^-$

charge $= +1/3$ charge $= -4/3$

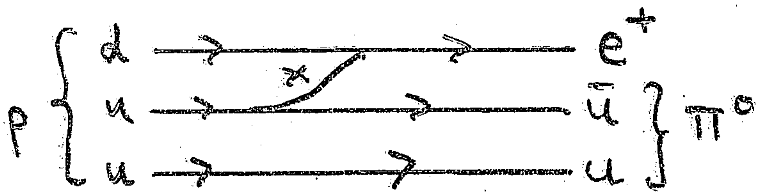
(The $X, Y, \bar{X}, \bar{Y}$s carry charges $+4/3$, $-4/3$, $1/3$ or $-1/3$)

The X will also turn quark to antiquark

$$\text{i.e.} \quad u \longrightarrow X + \bar{u}$$

$$\text{charge} \quad 2/3 \qquad 4/3 \qquad -2/3$$

Put together these processes will cause <u>proton decay</u> i.e. <u>non-conservation of baryon number</u>.



i.e. $p \to e^+ + \pi^0$

If $m_X \sim 10^{15}$ GeV, it can transmit forces only over a distance of $\sim 10^{-31}$ m.

This is so small that proton decay has a very long lifetime $\sim 10^{30}$ years

(Lifetime of universe $\sim 10^{12}$ years.)

Tests are being carried out on large masses of material underground (to avoid spurious effects due to cosmic rays)

Experiments suggest lifetime of proton $> 10^{32}$ years !!

N.B. This theory, like any theory linking quarks and leptons, will also predict non-conservation of lepton number.

The families in SU(5) are five-membered and ten-membered, specifically

left-handed

$$
\begin{array}{l}
\bar{d}_{\bar{R}} \\
\bar{d}_{\bar{B}} \\
\bar{d}_{\bar{Y}} \\
\begin{bmatrix} e^- \\ \nu_e \end{bmatrix}
\end{array}
\quad \text{and} \quad
\begin{array}{l}
\begin{bmatrix} d_R \\ d_B \\ d_Y \\ u_R \\ u_B \\ u_Y \end{bmatrix} \\
\bar{u}_{\bar{R}} \\
\bar{u}_{\bar{B}} \\
\bar{u}_{\bar{Y}} \\
e^+
\end{array}
$$

right-handed

$$
\begin{array}{l}
d_R \\
d_B \\
d_Y \\
\begin{bmatrix} e^+ \\ \tilde{\nu}_e \end{bmatrix}
\end{array}
\quad \text{and} \quad
\begin{array}{l}
\begin{bmatrix} \bar{d}_{\bar{R}} \\ \bar{d}_{\bar{B}} \\ \bar{d}_{\bar{Y}} \\ \bar{u}_{\bar{R}} \\ \bar{u}_{\bar{B}} \\ \bar{u}_{\bar{Y}} \end{bmatrix} \\
u_R \\
u_B \\
u_Y \\
e^-
\end{array}
$$

The brackets contain pairs connected by the SU(2) weak interaction

e.g. $e^- \leftrightarrow \nu_e + W^+$

$d \leftrightarrow u + W^-$

N.B All particles (of first generation) appear in left-handed and right-handed categories apart from ν_e and $\tilde{\nu}_e$.

[we defined ν_e and $\tilde{\nu}_e$ as having definite handedness (definite helicity). This could only strictly be ~~true~~ if they have zero mass; otherwise Lorentz transformations may be made from ν to $\tilde{\nu}$ (and back).

Hence since ν and $\tilde{\nu}$ have a non-zero mass, this theory must be adjusted.

If the neutrino has a mass, it may be identical to the $\tilde{\nu}$, and may be called a Majorana neutrino (as distinct from a Dirac neutrino).
This obviously again leads to non-conservation of lepton number.

The concept of the Majorana neutrino could lead to neutrinoless double β-decay. For the decay 161a

$$2n \rightarrow 2p + 2e^- + 2\nu_e$$

may, if $\nu_e \equiv \tilde{\nu}_e$, be written as

$$2n \rightarrow 2p + 2e^-$$

but there is no experimental evidence for this.

<u>neutrino oscillations</u>

Over the last ~20 years, it has become clear that 1/3 - 1/2 of the electronic neutrinos emitted by the sun are detected at earth.

[At both 'ends' of the experiment, there are major difficulties - the theory of neutrino production in the sun, and the detection of neutrinos on earth.

However the understanding of both

has steadily improved, and now it is certain that there are large discrepancies.]

GUTS allow for the possibility that neutrino flavour states may mix along the lines of the Cabibbo quark mixing. Then neutrinos will oscillate between different flavours. (This can only happen if their masses > 0 to give time for the process according to special relativity.)

Recent experiments at the <u>Sudbury Neutrino Observatory</u> in Canada, which detect <u>all</u> flavours of ν indicate that the total flux is as predicted but ~2/3 ν_e have changed flavour before they reach earth.
They suggest
$m(\nu_e) < 2.2eV$; $m(\nu_\mu) < 170keV$; $m(\nu_\tau) < 15.5MeV$]

There are nice features of this type of theory.

(1) It can predict the Weinberg angle, θ_W, well

(2) It predicts that the sum of charges of any of the 5-membered or 10-membered families must be zero. This relates the charge of the electron to those of the quarks, and hence that of the proton. It gives the quark and lepton doublets (in each generation) and gives $Q(\nu_e) - Q(e^-) = Q(u) - Q(d)$

Other types of GUTs

(1) The use of preons, supposed to be constituents of quarks and leptons.

(2) Supersymmetry In this theory, every boson has a fermion partner, and vice versa:- squarks, sleptons, photinos, gluinos... (None found!)

(3) Supergravity In this theory gravity does the symmetry breaking, and so may perhaps be unified in single theory with the other 3 interactions.

N.B overall, GUTs have not made successful predictions.

11. Particle Physics and Cosmology

Modern aspects of our understanding of particle physics are integrated into our understanding of the early history of the Universe.

GUTs predict (section 10) that the natural forces show uniformity at high energies which is obscured at lower energies.

Thus the hot early universe may demonstrate aspects of physics which have since disappeared.

Aspects of today's experience which we may hope to explain in ~~terms~~ of models of the early universe:

(1) Abundance of H relative to He
(2) Density of matter relative to photons
(3) The presence of matter and no anti-matter.

Important dates in the history of the universe

Time/s	Temp/K	Typical Energy/GeV	
10^{-43}	10^{32}	10^{19}	Gravity strong. Quantum gravity theory required (not yet available).
10^{-37}	$>10^{29}$	10^{16}	Strong, EM and weak forces united.
10^{-33}	10^{27}	10^{14}	Processes caused by leptoquarks start to freeze. Predominance of matter over antimatter probably established.
10^{-9}	10^{15}	10^{2}	Production of W bosons starts to freeze. Weak interaction weakens relative to EM. (Max. energies of today's accelerators.)
10^{-2}	10^{13}	1	Colour forces acting on quarks and gluons cluster into 'white' hadrons. Colour forces hidden. Protons & neutrons appear.
100	10^{9}	10^{-4} ($=\frac{1}{10}$ MeV)	Nucleosynthesis – helium and deuterium created
10^{6} yrs	10^{3}	10^{-7} ($= {}^{1}/_{10}$ eV)	Photons decouple from matter. Background radiation originates (optical and EM astronomy cannot see back beyond this epoch
10^{10} yrs	3	10^{-9}	Today. Galaxies and life exist. Background radiation. Stars provide local hotspots
$\lesssim 10^{32}$ yrs			All matter erodes away or big crunch – followed by new big bang?

① ultra-hot 10^{32}K

Quantum gravity strong.
General theory of relativity: space-time and gravitational forces profoundly related, so it is not clear what 'time' means when gravitational forces are strong.

Thus the 'start of the universe' ill-defined.
We cannot really argue further back than the time we call 10^{-43}s

② hot 10^{29}K

Energies so great that particles like $X, \bar{X}$ created as easily as light quarks, etc.
Strong, weak and EM interactions have equal strength; quarks and leptons transmute back and forth.

③ warm; down to 10^{15}K

As cooling takes place, strong, then EM and weak interactions freeze out and go their separate ways. (Try to create lower end in lab - symmetry between weak and EM.)

④ cool; down to 10^9K

Colour disappears. The "strong interaction" becomes that which binds nuclei.

5 <u>cold</u> The different interactions and different families of particles, behave in different ways → rich structure:- galaxies, stars, crystals, people.... (The <u>opposite</u> to 1)

Light elements and neutrinos

The heavy elements had their nuclei formed inside stars. There light nuclei fuse together producing heavy nuclei and releasing energy.

How were the light elements, H and He formed?

They were produced ~ 3 minutes after the big bang. The temperature was ~ 10^9 K

<u>Nucleosynthesis</u> occurred very rapidly from the abundant neutrons and protons. The temperature balance is critical. It must be cool enough that there is no <u>photodisintegration</u> - ripping apart of deuterium atoms by hot photons. It must be hot enough that two deuterons may collide violently, overcoming their electromagnetic repulsion.

All ns form d; all deuterium forms He-4.

$n + p \rightarrow d + \gamma$;

$\gamma + d \not\rightarrow n + p$ (no photodisintegration)

$d + d \rightarrow He^4$

[Small amounts of He^3 and Li^7 were also synthesised, but production of heavier elements was prevented because there are no stable isotopes with atomic mass 5 or 8.]

Excess ps remain and form H. So the ratio of He to H today gives the n to p ratio at nucleosynthesis. (Can be compared with models of the universe, in particular with how fast it was expanding - gives independent evidence about the number of quark/lepton generations, $\leqslant 4$.)

<u>Quarkosynthesis and the Preponderance of Matter</u>

Quarks were produced in the original fireball, where radiation energy was converted into quark <u>and an equal number of antiquarks</u>.

How was our present excess of quarks (and so matter) generated?

GUTs contain interactions that do not preserve <u>net quark number</u> (= no. of quarks − no. of antiquarks)

Up to $t \sim 10^{-32}$ s, X and $\bar{X}$ were produced copiously <u>in equal amounts</u>.

As the Universe cooled, these particles decayed

$$X \longrightarrow q + q \quad \text{or} \quad \bar{q} + \bar{l}$$
$$\bar{X} \longrightarrow \bar{q} + \bar{q} \quad \text{or} \quad q + l$$

In fact say

$$X \longrightarrow a(qq) \quad + (1-a)(\bar{q}\bar{l})$$
$$\bar{X} \longrightarrow b(\bar{q}\bar{q}) \quad + (1-b)(ql)$$

[a, $1-a$, b, $1-b$ are fractions of occurrences]

If $a = b$ then $N_q = N_{\bar{q}}$, $N_l = N_{\bar{l}}$

matter-antimatter symmetry

But in fact $a > b$, so $N_q > N_{\bar{q}}$, $N_l > N_{\bar{l}}$

excess of matter.

The minority $\bar{q}, \bar{l}$ are annihilated by q, l
The excess q, l form the material from which our present Universe was built.

The background radiation today is the remnant of the photons produced in the annihilation, and is a measure of the number of antiquarks and antileptons.

But today ratio of baryons to photons $= \frac{N_B}{N_\gamma} \sim 10^{-9 \pm 1}$

i.e. Following decay of $X, \bar{X}$,

no. of particles : no. of anti-particles $\sim \frac{1}{1-10^{-9}}$

Say $N_P = n$, $N_A = n(1-10^{-9})$

Then $n(1-10^{-9})$ pairs annihilate $\Rightarrow$ n photons

$10^{-9} n$ particles remain

Neutrinos and the Future of the Universe

If the average density of the Universe is sufficiently low, the Universe will expand for ever.

GUTs predict matter will decay with half-life 10^{30} years. All matter will erode away and the Universe will end up as expanding cold radiation.

If the average density is greater than a critical value, gravitation will cause the expansion of the Universe to halt, and the Universe to collapse to a Big Crunch, followed (?) by a Big Bang...... ad infinitum (?)

It is suspected that the first alternative is correct.
Even so there is much mass in the universe that is not understod - cannot be found - <u>missing mass</u>, <u>"dark mass"</u>, <u>"dark energy"</u>

There should be 10^9 neutrinos today for every baryon, since at the beginning of the universe, neutrinos were produced as copiously as photons.

Thus if neutrinos have ever a small mass, they may help to solve this problem.