

Quantum Mechanics in symmetry language

Houri Ziaeeepour

Max Planck Institut für Extraterrestrische Physik (MPE),
Giessenbachstraße, Postfach 1312, 85741 Garching, Germany ¹
houriziaeeepour@gmail.com

Abstract

We consider symmetry as a foundational concept in quantum mechanics and rewrite quantum mechanics and measurement axioms in this description. We argue that issues related to measurements and physical reality of states can be better understood in this view. In particular, the abstract concept of symmetry provides a basis-independent definition for observables. Moreover, we show that the apparent projection/collapse of the state as the final step of measurement or decoherence is the result of breaking of symmetries. This phenomenon is comparable with a phase transition due to spontaneous symmetry breaking. Additionally, we demonstrate that the property of state space as a vector space representing symmetries is more fundamental than being an abstract Hilbert, and L^2 integrability of state space can be obtained from conditions imposed on it to be a representation of a symmetry group and general properties of probability distributions.

1 Introduction

More than one century has passed since the discovery of quantum mechanics. During this period every aspect of its predictions - specially the most weird ones such as nonlocality, superposition and entanglement - has been tested thousands of times, and up to precision of measurements, so far no deviation has been found. Nowadays quantum effects are not mere subjects of abstract interest for physicists, but make the backbone of 21st century technology, from electronic communication and devices to drug design. However, despite being so far the most successful theory in the history of science, principles of quantum mechanics are still considered not be well understood. According to Wikipedia there are at least 14 well known, more or less distinct, interpretations of quantum mechanics and many more less-known ones, see e.g. [1, 2] for few examples. Various tests of completeness of quantum mechanics against hidden variable hypothesis [3] such as Einstein-Podolsky-Rosen (EPR) experiments and similar apparent paradoxes [4], and what is generally called *Bell inequalities* [5], after the original work by J.S. Bell [6], are proposed and experimentally implemented to verify this model. The original aim of Bell inequalities were providing a quantitative mean for testing quantum mechanics in EPR gedanken experiment setup that involves entangled particles. Evidently, such experiments are not anymore just thought exercises. They have been carried out using various techniques, even at long distances [7], and make the backbone of quantum computing technology. More recently, a new no-go theorem has been proposed which provides a mean to test hidden variables and incompleteness hypothesis with non-entangled particles [8] (from here on PBR). The first test of contextuality - Kochen-Specker theorem [9] - and independence on initial conditions [10] with non-entangled particles conclusively confirms predictions of quantum mechanics and their deviation from classical systems.

The most remarkable and mysterious property of quantum mechanics is its nonlocality which is the origin of the violation of Bell-inequalities, Hardy's paradox, the PBR no-go theorem², and the inherent inconsistency of quantum mechanics with classical gravity [11, 12, 13]. Another controversial issue is the nature of a quantum state and whether it is a *real* physical entity or it simply presents *the state of information* that an observer has about a physical system. Here *reality* means the complete representation, i.e. no hidden observable is averaged out. In a classical view this implicitly means that a state should be *measurable* for each instance of a system. In classical physics, the state of a system is defined by the list of values $\{x\}$

¹Present address: 14, allée Jean Rostand, 91000, Evry, France

²This theorem assumes that copies of a system can be prepared independently. Properly speaking this violates nonlocality, but can be considered as a good approximation if correlation between systems is much smaller than measurements noise which is taken into account in the construction and proof of the theorem [8].

measured or predicted for a minimal number of observables $\{X\}$ that characterize the system completely, i.e. any other observable - physical property - \mathcal{O} accessible to an observer will be a single-valued function of these observables $o = \mathcal{O}(\{x\})$:

$$o \neq o' \implies [\{x\} = \mathcal{O}^{-1}(o)] \neq [\{x'\} = \mathcal{O}^{-1}(o')] \quad (1)$$

It is important to emphasize on the single valuedness because when multiple values are possible, it is assumed that some characterizing quantities are missed i.e. averaged out³. Classical statistical mechanics assumes that it is not possible to measure the complete set of characterizing observables $\{x_c\}$ of a system. The impact of averaging out hidden quantities is the random behaviour of accessible characteristics $\{x\}$ with a probability distribution $P(\{x\})$, see e.g. [5]. This probability function determines *the state of information* of the observer about the system. Because any physical property by definition depends uniquely on $\{x_c\}$, the exclusion relation (1) is not necessarily satisfied when some variables are averaged out. The no-go theorem of PBR proves that in quantum mechanics the state satisfies no-overlap condition (1), and if quantum mechanics is a complete description of nature, the quantum state of a system is one of its physical properties. However, in their conclusions PBR consider the collapse/projection of the state after a measurement as problematic for a physical property which should have *a reality of its own* [8]. This brings us to the well known measurement problems of quantum mechanics, namely the apparently nonunitary collapse of the state. Although the concept of decoherence by an environment (see e.g. [15, 16, 17] for recent reviews) provides a better solution than the collapse, it is not completely flawless and devoid of criticisms, see e.g. [18] for a review of opinions on this subject.

Apart from numerous alternative interpretations of quantum mechanics, to better understand the origin and foundation of its principles a number of authors have tried to construct quantum theory from a set of axioms that give it a structure very close to statistics. For instance, in a frequentist/measurement approach, Hardy [19] constructs both classical statistical mechanics and quantum theory based on 5 axioms from which 4 are satisfied by both theories and the last one only by quantum mechanics. More recently a similar model inspired by quantum information theory with only 3 axioms is suggested [20]. In both models the complex Hilbert space of quantum states is projected to a real vector space which presents probabilities for outcomes of measurements. This line of investigation seems to be an attempt of *quantum pessimists* who try to present quantum mechanics as a simple extension of statistical physics.

Criticisms against quantum mechanics can be essentially summarized in the following questions [15, 16, 17]:

Meaning of a quantum state: Similar to classical statistical physics, a quantum mechanical state defines a probability for measurement outcomes. But why does quantum mechanics seem to be a complete description of nature, i.e. violates Bell inequalities ? In contrast to classical statistical mechanics, why does not the measurement of the probability of each outcome determine the state completely, i.e. decomposition coefficients are in general complex, create interference, and their norm square rather than themselves corresponds to probability ?

Measurement problems: After a measurement, why does the measured quantities behave classically, i.e. immediate measurements afterward give the same outcome ? Collapse, entanglement, decoherence ? What is really measured, i.e. how is a pointer basis selected ?

In this letter we show that the issues and questions summarized in the previous paragraphs can be understood if we interpret the relation between a system and an observer or environment as a symmetry and the process of measurement as breaking of symmetries. This interpretation does not modify the established principles of quantum mechanics and the process of measurement [22, 21]. For this reason, we prefer to call it a new *description* or *language* for presenting quantum mechanics rather than a new interpretation or construction. Evidently, the concept of symmetry is not new for quantum mechanics and quantum information, but in our knowledge it has not been used as a fundamental principle for the foundation of quantum mechanics. For

³In presence of chaos in a classical system the noise from environment plays the role of *hidden variable* and determines which branch is taken by the system, see e.g. [14]

instance, in algebraic quantum mechanics [22, 23] symmetries of state space and their relation with their classical analogues are extensively used to demonstrate the appearance of superselections by decoherence. However, symmetries are not considered as a foundational necessity even when issues such as measurement problems are addressed [24, 25]. The aim of present work is to introduce a description of quantum mechanics axioms based on symmetries.

In Sec. 2 we reformulate axioms of quantum mechanics based on symmetries. We discuss their content and consequences for properties of state space in Sec. 3 and for how state space of composite systems is related to states of their components in Sec. 4. Measurement and related subjects are discussed in Sec. 5. In Sec. 6 we obtain the Born rule for probabilities. Issues related to decoherence and classicality are discussed in Sec. 7. Finally, main conclusions of this work are recapitulated in Sec. 8 through their explanations for issues mentioned here.

2 Quantum mechanics postulates in symmetry language

In this section we present our axioms for quantum mechanics and compare them with axioms à la Dirac [21] and von Neumann [22] which from now on we call the *standard quantum mechanics*:

- i. A quantum system is defined by its symmetries. Its state is a vector belonging to a projective vector space called *state space* representing symmetry groups of the system. The set of independent observables is isomorphic to subspace of commuting elements of the space of self-adjoint (Hermitian) operators. This subspace generates the maximal abelian subalgebra of the symmetry group of the system.
- ii. The state space of a composite system is homomorphic to the direct product of state spaces of its components. In the special case of non-interacting (separable) components, this homomorphism becomes an isomorphism.
- iii. Evolution of a system is unitary and ruled by conservation laws imposed by its symmetries.
- iv. Decomposition coefficients of a state to eigen vectors⁴ of an observable presents the symmetry/degeneracy of the system with respect to its environment according to that observable. Its measurement is by definition the operation of breaking this symmetry/degeneracy. The outcome of the measurement is the eigen value of the eigen state to which the symmetry is broken. This spontaneous symmetry breaking reduces the state space to subspace generated by other independent observables.
- v. A probability independent of measurement details is associated to eigen values of an observable as the outcome of a measurement.

These axioms are very similar to their analogue in the standard quantum mechanics except that we do not assume a Hilbert space. We demonstrate this property of state spaces using axioms (i) and (v) in Sec. 6. In addition we emphasize on the presence of symmetries in any quantum system. Furthermore, these postulates introduce a definition for measurement operation as symmetry breaking, independent of how it is performed i.e. by a designed apparatus or by interaction with an arbitrary system or environment. It may look like a projection/collapse - a jump - as it is the case in first-order phase transitions or be continuous through entanglement with environment and decoherence. Additionally, we will show that the abstract concept of symmetry provides a basis-independent mean for describing observables. It can explain the problem of selected basis and ambiguity of observed quantities raised in the literature [22, 23].

In standard quantum mechanics, measurement axioms attribute a probability to projection/collapse/decoherence to i^{th} eigen vector of a state $|\psi\rangle$ equal to $tr(\rho P_i)$ where ρ is the density operator associated to the state $|\psi\rangle$

⁴To be more precise we should use the term *rays* in place of vectors because vectors differing by a constant are equivalent. Thus, we assume that state are normalized.

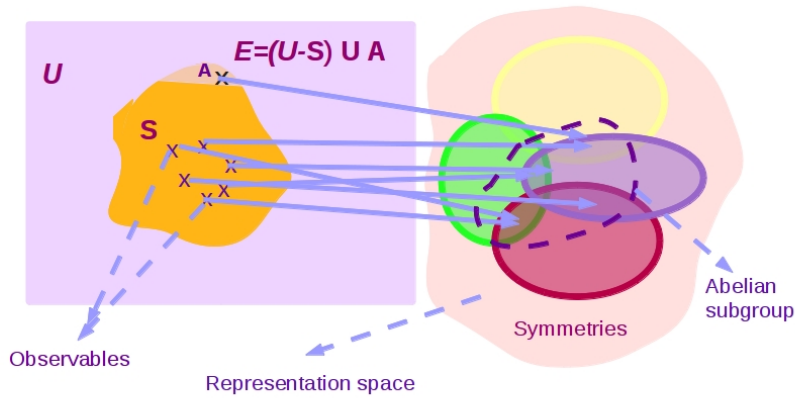


Figure 1: Abstraction of systems by their symmetries. The region A indicates the shared/entangled information between system and environment (or observer) usually called apparatus.

and P_i is the projection operator on i^{th} eigen vector. Here we will obtain this relation from properties of probability distribution functions and their application to composite systems described under axiom (ii).

Symmetry can be considered as the generalization of equality and similarity concepts which are the basis of our logical and physical understanding of the Universe. The operation of measurement is nothing else than determining similarity between a characteristic of object with ones belonging to a reference. The statement that a system is characterized by n observables means that its similarity or difference from other systems in the same category is determined by similarity or difference of n categories of *things/characteristics* associated to the system. It is clear that without the concept of similarity (symmetry) the previous sentence could not be even made. Therefore, symmetry is our *only* tool for abstraction of the Universe. A question that arises here is whether it is necessary to assume a division of the Universe to *system* and *observer* or *environment*. In a logical view the concept of similarity makes sense only if there are at least two objects to be compared. In a mathematical view and in the framework of set theory, which we assume to be applicable to all objects including the Universe, only an empty set does not have subsets. In this sense any indivisible set either is a subset of larger set or is isomorphic to an empty set, therefore it is meaningless to discuss about its properties. Consequently, a nontrivial Universe must have at least two non-empty nontrivial subsets, which can be called *system* and *observer* or *environment*. This argument shows that the necessity of having an observer/environment is not because without it quantum mechanics becomes meaningless but for assuring that Universe is not trivial. Fig 1 shows a schematic description of this description. Note that system and observer/environment must overlap i.e. interact, otherwise they cannot exchange information and each of them can be considered as a separate universe. We call the overlap subset an *apparatus*.

In the following sections we describe in more details these axioms and their consequences for better understanding enigmatic issues in quantum mechanics summarized in the Introduction.

3 State space and symmetries

Symmetries are inherent to physical laws. Examples are Poincaré symmetry of spacetime, gauge symmetries of fundamental interactions, global leptonic and flavor symmetries, etc. These symmetries are intrinsic and always preserved unless broken explicitly by specific interactions. In standard quantum mechanics a Hilbert space is introduced as an abstract object. Nonetheless, it is always related to the symmetries of the system. In symmetry description, the emphasis is on symmetries and they are considered as a backbone for the definition of physical systems. According to axiom (i) a physical system which is assumed to be a subset

of a universal set - the Universe - is abstracted by its symmetries and their representations which define its state space. Symmetries of physical systems are dominantly (if not always) are Lie groups. Therefore, for the sake of simplicity through this work we restrict our arguments to them.

Vector spaces representing symmetry groups are generated by eigen vectors of their commuting subgroup, which according to axiom (i) defines independent observables. Evidently the decomposition of an arbitrary state vector depends on the selected basis. However, this does not affect the fact that the new basis continues to correspond to eigen vectors of commuting elements of symmetries and represents independent observables. Therefore, in symmetry description observables have an identity independent of the basis of state space. For this reason in symmetry view the question: *what basis is selected in a measurement* seems irrelevant. For example, spin of particles are related to $SO(3) \cong SU(2)$ symmetry which its abelian subalgebra is one dimensional. Because this is the rotation group in a 3-dim space, the generator of abelian subalgebra is usually associated to one of the frame axes. Nonetheless, any other vector can be equally considered as the generator of abelian subalgebra. What matters is the concept of commuting subgroup not its basis-dependent representation.

Fundamental symmetries usually impose constraints on related observable. Nonetheless, they can also induce symmetries because sometimes nature is inherently degenerate. This fact can be shown in the following example: Consider as system a π^0 particle decaying to two photons with opposite spins and the rest of the Universe as environment/observer. If the preparation is made such that a magnetic field is present, the interaction of the field with charged quark-antiquark of π^0 defines a preferred direction for opposite spins of oppositely propagating photons. This breaks spherical symmetry of pion's environment at least partially, and gives the observer some information about the spin of e.g. the photon emitted to right hand side with respect to the magnetic field direction. On the other hand, if the preparation is performed such that no magnetic field or another distinguishable difference between directions are produced, the direction of spins of the photons at production are completely arbitrary i.e. symmetric. Thus, we expect that the state of the system has equal coefficients when it is decomposed to eigen states. The only restriction on their direction is the conservation of angular momentum which forces them to be opposite to each other. Furthermore, the symmetry/degeneracy of eigen vectors is independent of other properties and symmetries of the system, notably geometry and distance between particles and is global.

If eigen states are transformed by application of a group member, the state of the system changes too:

$$|\psi\rangle = \sum_{\alpha} a_{\alpha} |\alpha\rangle, \quad |\alpha\rangle \rightarrow |\alpha'\rangle = U|\alpha\rangle, \quad U \in G \quad \implies \quad |\psi\rangle \rightarrow |\psi'\rangle = \sum_{\alpha} a_{\alpha} |\alpha'\rangle \quad (2)$$

where G is the symmetry group. Evidently the operation performed in (2) does not correspond to a basis transformation which leaves $|\psi\rangle$ unchanged but not decomposition coefficients $a_{\alpha} \rightarrow a_{\alpha'}$. The new base does not necessarily correspond to eigen vectors of the abelian subalgebra, and consequently coefficients $a_{\alpha'}$ do not have the same physical interpretation. This is the source of the confusion and what is called preferred basis selection by measurements. In fact the measurement of the spin of photons in the example above in absence of an external magnetic field or other external indicators results to 50% probability for each spin eigen value in the direction chosen by the observer/apparatus, whatever this direction would be. Therefore, decomposition coefficients a_{α} define the true physical state of the system when the basis corresponds to eigen vectors of the maximal abelian subalgebra. For instance, in the case of spin, one of the generators of the $SU(2)$ group, usually called σ_x , σ_y , and σ_z , generates the abelian subalgebra. What is measured is this generator and it does not make any physical difference to call it the component x , y , z or anything else, and others are not independent observables. This is the physical manifestation of rotational symmetry in an active manner. Once the direction is fixed by symmetry breaking, then the only freedom is selection of a frame. This behaviour is consistent with contextuality of quantum mechanics and the above example shows that it arises through irreversible process of symmetry breaking.

In classical statistical mechanics if the direction of the angular momentum cannot be known from preparation procedure/initial conditions, the system has a rotational $SO(3) \cong SU(2)$ symmetry which is reflected in the equal probability for the projection of its angular momentum on the three reference directions. Thus, in

analogy with quantum mechanics we can define a configuration (state) space consisting of states $|S_{\perp}, S_z, S\rangle_C$. The state of the system can be described as (subscript c means classical):

$$|\psi\rangle_c = \sum_i f_i |i\rangle_c \quad i = \{S_{\perp}, S_z, S\} \quad (3)$$

One can even *quantize/discretize* the value of coefficients by considering a *resolution* ΔS for projections which presents the smallest detectable difference between two states. It is natural to identify coefficients f_i as the probability for state i . Thus, they must be semi-positive $f_i \geq 0$. The spherical symmetry is reflected in the invariance of coefficients f_i 's under permutation of axis indices x, y, z and under a rotation of the reference frame that projects f_i 's to themselves. Because projections S_{\perp}, S_z can independently take any values as long as $S^2 = S_{\perp}^2 + S_z^2$, the symmetry represented by the configuration/state space is in fact $T \otimes T \cong U(1) \otimes U(1)$. Evidently, the spherical symmetry is present, but it plays a passive role, namely it does not impose any constraint on the value of S_{\perp} and S_z except the above relation. This is the main difference between the role of symmetries in classical and quantum mechanics. In contrast to classical mechanics, in quantum systems symmetries have an operative role and induce a symmetry which reflects the history and relation of a system with its environment. In classical physics initial conditions play a comparable but not equal role.

Once an observable is *measured* (we define this operation in Sec. 5), according to axiom (iv) the symmetry or degeneracy between its eigen vectors breaks. Thus, what we called state symmetries are ephemeral and related to history, initial conditions, and/or preparation of systems, and can evolve. They may be total or partial. In the former case permutation of eigen states does not change the state, otherwise the symmetry is partial, meaning that the system's history or environment somehow discriminate between eigen states but cannot single out one of them. In another word if the coefficients of decomposition of the state to eigen vectors are all equal the symmetry is total, otherwise it is partial. We should remind that the symmetry of eigen states is by definition associated and inseparable from symmetries of the system. Consequently, their breaking is comparable and analogue to spontaneous symmetry breaking in which the symmetry is broken in states but not in the Lagrangian.

According to axiom (i), state space is a vector space representing the symmetry group of the system, and observables are self-adjoint linear operators acting on this space. By definition for every group element $g \in G$ there is an inverse element $g^{-1} \in G$ | $gg^{-1} = g^{-1}g = I$, and only in vector spaces defined over complex numbers \mathbb{C} operators with non-zero determinant are always invertible. Moreover, properties of the system are by definition conserved under application of G . As observables are self-adjoint operators, representation of symmetry group by state space must be unitary to preserve the norm and should be invertible. These requirements enforce the necessity of definition of state space over the field of complex numbers. Evidently many groups are unitarily represented by vector spaces defined over real numbers, but in general a complex field is necessary.⁵ In practice in standard quantum mechanics procedures similar to what is described here are used to define the state space for a given system, but the role of symmetries is not considered as a foundational aspect of quantum systems.

In classical statistical mechanics a system with n observables has a configuration space isomorphic to $\bigotimes^n U(1) \cong \mathbb{R}^n$, but its states are defined as points in \mathbb{R}^n rather than a vector and probabilities make a $n - 1$ simplex in \mathbb{R}^n [17]. More importantly, internal symmetry between observables is applied passively. For instance, rotational symmetry allows to transform coordinates, but does not apply any restriction on observables because states do not present a representation of this symmetry which needs a vector space. It is noteworthy to remind that in general symmetry groups are non-abelian and their maximal abelian algebra presenting independent observables has a smaller number of generators than the whole group. Therefore, quantum systems have less independent observables than their classical analogue. This is another manifestation of the active role of symmetries in quantum mechanics which imposes more constraints on quantum systems than in their classical analogue.

⁵Representations over \mathbb{C} are isomorphic to real representations with double dimensions which are not minimal and can be considered as alternative definition of a complex representation. They are analogue to presenting complex numbers by their real and imaginary components.

4 Composite systems, interactions and environment

According to axiom (ii) the ensemble of two or more subsystems can be considered as a system with a symmetry group homomorphic to direct product of symmetries of its components. Deviation of this homomorphism from a trivial isomorphism presents the strength of interaction between subsystems. This axiom is particularly important for understanding measurement problems and decoherence of quantum systems. In this regard, symmetries provide a natural criteria for division of the Universe to subsystems/components according to their different symmetries. Examples of such decomposition can be found in particle physics where particles are grouped by their symmetries which in turn define their interactions. For instance, hadrons and gluons can be considered as a subsystem with $SU(3)$ color symmetry, and leptons and electroweak bosons as another subsystem with $SU(2) \otimes U(1)$ symmetry. Evidently these systems overlap each other because in addition to $SU(3)$ quarks have also $SU(2) \otimes U(1)$ symmetry. In the example of a decaying π^0 , the system is composed of three subsystems each with $SU(2) \cong SO(3)$ symmetry. Their ensemble has $SU(2) \oplus \mathbb{Z}_2 \in \bigotimes^2 SU(2)$ symmetry which corresponds to the global symmetry under rotation and permutation symmetry of photons. Evidently this division is valid only when photons are sufficiently far from their production place and can be considered as non-interacting.

From the close relation of fundamental interactions and symmetry groups, and from universality of gravitational interaction - for which we do not yet have a quantum description - we conclude that the decomposition of the Universe to subsystems is not orthogonal, i.e. it is not possible to decompose the Universe to two quantum subsystems S_1 and S_2 with state spaces satisfying properties imposed by axioms (i) and (ii) such that $S_1 \cap S_2 = \emptyset$. Therefore, the Universe is an ensemble of intertwined vector spaces rather than a separable closed set. This may mean that the Universe is topologically open and for any system $S \in U$ there is a system S' such that $S \in S' \in U$. A direct consequence of this observation is the fact that *there is no isolated system in the Universe* and division to system/apparatus/environment is an approximation because these entities overlap each others. In another word, classical locality and separability of the world are only approximately valid and the Universe is inherently nonlocal, composite but inseparable. This observation answers the criticism of decoherence as the origin of classical behaviour of macroscopic systems, based on the fact that it only approximately suppress interferences. Classicality and locality of macroscopic world are simply approximations and consequences of a coarse-grained presentation of an otherwise quantic Universe.

Because classicality is just an approximation, the Universe as a whole must be considered as a quantum system, and there should exist a state space representing global symmetries of the Universe. They must be related to symmetries of its components by a homomorphism similar to (14) below. If the projection is trivial, the Universe will have a single state meaning that it globally looks classical. If it is roughly isomorphic, its state is simply the direct multiplication of state spaces of its components. If it is neither trivial nor isomorphic, but isomorphic to a small nontrivial symmetry, the Universe may have global structures. Observations of Cosmic Microwave Background (CMB) [27] do not show any signature of a nontrivial topology up to their precision, and anisotropies seem to have a continuous spectrum up to modes $k \rightarrow 0$. This is topologically consistent with an open ball. Nonetheless, the Universe may be in a nontrivial rotational ($SO(3) \cong SU(2)$) state, see Sec. 7.1 for more details.

5 Measurement

A measurement operation in classical physics consists of using a tool to determine the value of a quantity - an observable or the state of a complex system, e.g. the occurrence of head or tail when a macroscopic coin is thrown. By definition this operation results in one value or state for the system. In classical mechanics if the dynamics of the system is known, evolution of the observable from its initial value, determined by the measurement, is ruled by dynamical equations. In classical statistical mechanics exact equations for all degrees of freedom are not available and because of summation over inaccessible degrees, only the probability of outcomes in measurements can be estimated. They are usually assumed to be independent

of the measurement operation and evolve unitarily after every measurement. As we mentioned in the previous section, the state of a classical system, and thereby the value of observables that characterize its state, are uniquely associated to a point in an \mathbb{R}^n space where n is the number of observables and their probabilities belong to a $n - 1$ simplex in this space. These points are independent, therefore it is natural that a measurement gives a single value for each observable as outcome. By contrast, the state of a quantum system is a vector usually described with respect to a basis. Since the discovery of quantum mechanics the issue of which basis is chosen to present the state in a measurement, or in general in interaction between two systems, has been discussed by physicists and philosophers. Despite various proposed explanations, specially through introduction of interaction/decoherence as the selector of pointer basis, this issue is still considered to be not well understood, see e.g. [16, 18] for a review of criticisms.

In the symmetry description, measured quantities presents the abelian subalgebra of the symmetry group of the system. This provides a basis-independent notation of independent observables and corresponding measured quantities. According to postulate (iv) a measurement by definition breaks what we called symmetry of state in Sec. 3. In addition, this process reduces the state space to the subspace presenting the abelian subspace of the measured observable. For this reason we can say that after a measurement the symmetry is *fixed*. For instance, in the example of entangled photons in the previous section, the measurement of the spin of one of photons, fixes the direction of spin, and spontaneously breaks the spherical symmetry of space. This fact is reflected in the state space which becomes trivial and stays unchanged in future measurements of the spin as long as the measurement procedure is exactly repeated. A measurement along another direction - for instance a measurement in which the direction of electric current in apparatus is different - will change the state because the corresponding operator does not commute with the operator in the first measurement. This is consistent with the claim of symmetry breaking in the first measurement. Because it has determined the state of the system, if the symmetry was not broken, the application of a member of symmetry group would project the state space to itself. This phenomenon is similar to fixing the gauge in models with gauge symmetry. Once the gauge is fixed, the formulation is only valid in the assigned gauge.

Comparing the symmetry fixing with collapse or projection postulate in standard quantum mechanics, the latter is a mean to reduce the state to a point in configuration space in analogy with classical systems. Thus, it does not provide any physical explanation why this should happen. By contrast, breaking of symmetries has a definitive physical, mathematical, and operational meaning. Evidently neither the projection description nor symmetry breaking provides an explicit explanation how this process occurs. The reason is simply the fact that it depends on the details of the system and apparatus/environment. Even in decoherence proposition for solving measurement issues, there is no detailed description of how the density matrix of environment becomes diagonal. This is in fact one of the criticisms raised about decoherence [16]. On the other hand, symmetry breaking is a process which occurs in both macroscopic and microscopic systems in various circumstances independent of details. The presence of scaling relations in phase transitions which are associated to spontaneous symmetry breaking is the evidence that this process is independent of detailed properties of systems and their environment. Algebraic approach to decoherence [23, 24] demonstrates that this process leads to superselections and divide state space to independent blocks/representations of symmetries. This action is by definition discontinuous, similar to a first order phase transition, and looks like a jump. Thus a projection can be considered as an idealized mathematical description of this operation.

Many examples of decoherence of quantum systems can be found in the literature. Nonetheless, the following simple example shows the relation of this phenomenon to interactions, symmetries, and their breaking. Consider the state of a system in an arbitrary basis:

$$|\psi\rangle = \sum_{\alpha} a_{\alpha} |\alpha\rangle \quad (4)$$

To measure an observable, the system must weakly interact with an apparatus because due to no cloning theorem, see e.g. [28], its state cannot be simply compared with a system with known state (a reference). According to discussions in Sec. 4 interactions are related to symmetries, and assuming a weak interaction we expect it only slightly modifies the state of the system, namely symmetries are not modified and the

basis $|\alpha\rangle$ can be used to describe the modification of the state due to interaction:

$$|\psi'\rangle = \mathcal{S}|\psi\rangle = \sum_{\alpha} a'_{\alpha} |\alpha\rangle \quad (5)$$

Because state space is assumed to represent symmetry groups, application of \mathcal{S} corresponds to an isomorphism of the representation to itself, and in general a new set of expansion coefficients $a'_{\alpha} \neq a_{\alpha}$ is obtained. A designed measurement apparatus by definition must be capable to distinguish between different states. Therefore, in the basis consisting of eigen vectors of \mathcal{S} , there must be a significant difference between eigen values that preferentially selects some states with respect to others. Moreover, during a measurement the interaction is continuous or can be considered to be repeated a large number of times before a stable number appears on a counter. For example, if the spin of a charged particle is measured by coupling it to an external magnetic field, at lowest perturbative order $\mathcal{S} \propto \alpha_e \epsilon_{\mu} \gamma^{\mu}$ where ϵ_{μ} is photons polarization vector, γ^{μ} is the Dirac matrix, and α_e is the electromagnetic (fine-structure) coupling constant. The fixed polarization of the field breaks rotational symmetry, and selects a preferred direction for the Dirac matrix which in turn determines the preferred basis for \mathcal{S} . Moreover, continuous exchange of soft photons of a magnetic field with the charged particle is equivalent to:

$$|\psi'\rangle = \mathcal{S}^n |\psi\rangle = \sum_{\alpha} [\mathcal{S}^n]_{\alpha\alpha} a_{\alpha} |\alpha\rangle, \quad [\mathcal{S}^n]_{\alpha\alpha} \Big|_{n \rightarrow \infty} \rightarrow 0 \quad (6)$$

where we have assumed that matrix \mathcal{S} is normalized to its element with largest amplitude. The final result of this operation is the projection of the state to one of the pointer states. Clearly, this breaks the symmetry between pointer states and according to axiom (iv) only at this point the operation of measurement may be considered to be accomplished. This simple example shows how the interaction between system and apparatus or any two systems selects the pointer basis.

Modern literature about foundation of quantum mechanics usually presents the measurement as entanglement between the system and apparatus such that the quantum nature of both be involved explicitly [16]:

$$\sum_{\alpha} s_{\alpha} |\alpha\rangle \otimes |a_0\rangle \rightarrow \sum_{\alpha} s_{\alpha} |\alpha, a_{\alpha}\rangle \quad (7)$$

where $|a_0\rangle$ is the initial state of the apparatus and $|\alpha\rangle$ an arbitrary basis for the system. The problem of this approach is that in general $|\alpha, a_{\alpha}\rangle$ states are not orthogonal to each others, and a probabilistic interpretation for outcomes is not possible. Usually it is assumed that this regress breaks when the apparatus becomes macroscopic. But the fact that at each step the apparatus get bigger does not solve the conceptual problem that one of states must be selected among many other possibilities. In decoherence explanation this chain is broken by introduction of an environment which its state is assumed to be inaccessible to observer:

$$\sum_{\alpha} s_{\alpha} |\alpha\rangle \otimes |a_0\rangle \otimes |e_0\rangle \rightarrow \sum_{\alpha} s_{\alpha} |\alpha, a_{\alpha}\rangle \otimes |e_0\rangle \rightarrow \sum_{\alpha} s_{\alpha} |\alpha, a_{\alpha}, e_{\alpha}\rangle \quad (8)$$

where $|e_0\rangle$ is the initial state of environment. Because the state of environment is not accessible, it must be traced out. Moreover, assuming that it includes a very large number of degrees of freedom, a process similar to (5) eliminates non-diagonal elements. In fact, in the framework of algebraic approach to quantum mechanics it is demonstrated that when $n \rightarrow \infty$ number of freedom degrees are traced out, a *superselection* [23, 24, 17] occurs and density matrix of the combined system-apparatus (or more generally two systems) becomes approximately diagonal in a basis which is chosen by the interaction between apparatus and environment - system-environment interaction are usually neglected, see also Sec. 7 for more details. However, the final step in the process of measurement must be projected or collapsed from the entangled states to one of them. Therefore, although decoherence clarifies why most systems in the nature seem to have preferred pointer basis, it does not explicitly solve the collapse issue [29, 16].

5.1 Influence of environment and decoherence on measurements in symmetry description

Symmetry description helps to understand the last step of decoherence and its projection. According to axiom (i) a system is abstracted by its symmetries and its state space represents them. Notably, symmetry or degeneracy between states of a pointer basis is determined by initial conditions/history/environment of the system which due to intrinsic nonlocality and inseparability of quantum world influence its state, see also Sec. 6.1. Therefore, symmetry of states can never be complete, simply because Universe is not completely symmetric, homogeneous or isotropic. Consequently, the concept of decoherence and breaking of symmetries is inherent to this description. Assuming entanglement between the three components system-apparatus-environment (8), they can be considered as a single indivisible quantum system, and according to axiom (ii) their state space represents the symmetry of the composite system. The environment includes everything in the Universe except the system (and apparatus if it is present). According to the argument about the openness of the Universe, the environment has a large number of degrees of freedom, thus a very large symmetry. On the other hand, a large symmetry is equivalent to trivial symmetry, i.e. any single state can be considered as the representative of the system. For instance, circle one point on a circle is a representative of all points points, because there is no difference between them. In a mathematical view this is equivalent to tracing out environment states which reduces the state space to a trivial representation presented by a single state in of the pointer basis. The standard description of decoherence does not clarify how physically the tracing occurs. By contrast, symmetry description provides a logical and physical explanation.

Assuming that environment interacts with the system only through the apparatus - described by the intermediate state in (8) - if the apparatus is a reliable measuring tool, the states of composite apparatus-environment system must have a complete symmetry/degeneracy. In this case, the effect of tracing them out is similar to unobserved degrees of freedom in classical system. They behave like a statistical ensemble and induces a probability for the reduction to a single pointer state [24] for the ensemble which depends only on the system. The reduction of the state space is a direct consequence of being the representation of a symmetry group before the establishment of entanglement. Moreover, tracing operation on which the standard description of decoherence relies to describe the apparent projection to one of the entangled states is nonunitary operation and does not preserve total probability.

Because the operation described in (6) breaks the symmetry, a measurement can be defined as breaking of what we called symmetry of information/states. This breaks the active presentation of the symmetry by the state space which is made trivial by measurement, but leaves its passive effect i.e. the possibility of changing the basis untouched, as explained for EPR experiment earlier. In fact, before the first measurement in what concerns the value of independent observables, the environment is globally symmetric, i.e. they affect it in a same way. Once the observer/apparatus/another system breaks this symmetry by performing the entanglement (6), the global symmetry is broken and cannot be restored automatically. For this reason the similarity of environment effect to a statistical ensemble is just only on analogy, and in contrast to a classical system, a quantum system preserves its state after the first measurement.

A question arises here: Should a quantum universe have a large number of freedom degrees such that a large chunk of it plays the role of environment and makes the measurement of smaller subsystems meaningful? Here we argue that this is not necessary, and although decoherence helps to understand the classical world more easily, it is not a necessary condition. Consider a universe with two particles/subsystems as its content. We call one of them the *observer* and its state $|\psi_1\rangle$, and the other one the *system* and its state $|\psi_2\rangle$. By definition no other substructure/subsystem exists. In this case, the only observable of this universe is the comparison between states $|\psi_1\rangle$ and $|\psi_2\rangle$ and the symmetry group they present is \mathbb{Z}_2 . The only reference/apparatus that the observer has is its own state. We assume the following operation as interaction : $H = \alpha|\psi_1\rangle\langle\psi_1|$ for an arbitrary α , which for simplicity can be considered to be 1. If the observer applies H to itself, its state does not change. It can also define a state $|\bar{\psi}_1\rangle$ which the application of H to it gives $|\odot\rangle$, i.e. $\langle\bar{\psi}_1|\psi_1\rangle = 0$. The space generated by $|\psi_1\rangle, |\bar{\psi}_1\rangle$ is a representation of \mathbb{Z}_2 . Evidently any other basis can be chosen for this space, but the observer cannot be aware of it because the interaction defined by H compares

other states with its own and the result (projection) would be the same. In particular, the application of H to $|\psi_2\rangle = \beta|\psi_1\rangle + \bar{\beta}|\bar{\psi}_1\rangle$ leads to the measurement of $|\beta|$ that determines what is the probability of similarity of the states of two subsystems. There is however a caveat here. To be able to measure β , the observer must distinguish between $|\psi_1\rangle$ and $\beta|\psi_1\rangle$ or determine the average outcome $|\langle\psi_1|H|\psi_2\rangle|^2$. However, because the state space is projective, $|\psi_1\rangle$ and $\beta|\psi_1\rangle$ are indistinguishable. The second operation is also in practice impossible, because due to interaction, after the first measurement - encounter - the states of the two systems would be either equal or opposite and stay as such for ever. Therefore, it would not be possible to repeat the comparison operation unless there are multiple universes of the same type and an external observer who performs the averaging between outcomes. In conclusion, the only thing that the observer can verify is the complete similarity or opposition between itself and the other system. This is consistent with \mathbb{Z}_2 as the symmetry of this universe and confirms the fact that the concept of randomness is meaningful only when an experiment/measurement can be performed on copies prepared independently and under the same initial conditions. In this case randomness is the evidence of a degeneracy/symmetry in nature. This fact applies to both classical and quantum systems. On the other hand, when in classical statistical physics just probability distribution is associated to possible states, in quantum systems states present much more than just probabilities.

5.2 Modification of state space by measurements

The introduction of state symmetry and its breaking by measurement, interaction or decoherence shows why the state space of a system changes afterward. According to postulate (i) and arguments in Sec. 3, the state space of a system represents its global symmetries with respect to the rest of the Universe and laws of physics. Therefore, their spontaneous breaking automatically causes a definitive modification of the state space. We know many examples of such phenomena in physics. For example, symmetry breaking due to formation of a condensate in Higgs sector gives mass to SM particles. In turn, massive bosons break $SU(2) \times U(1)$ symmetry and changes the state space of SM particles. In electrodynamics gauge symmetry of massless photons forbids their parallel polarization in vacuum. On the other hand, if photons had mass, they could not have gauge symmetry but had a parallel polarization and their state space was different. Similar to these examples a measurement operation which according to its definition in axiom (iv) breaks the symmetry can be considered as a phase transition from initial mixed state (in statistical sense) of system and apparatus to their ordered - projected/collapsed - state. In this interpretation interaction operators similar to \mathcal{S} in (6) (more precisely the coupling constant it contains) can be considered as a disorder parameter for this transition.

The definitive modification of the state space after a measurement is usually considered to put doubt in the status of the state as a physical property of the system. As mentioned in the Introduction, even in [8] where the reality of state in the framework of their definition and PBR no-go theorem is proved, the authors consider it as counter-intuitive. The description of quantum mechanics in symmetry language shows that the concept of reality of the state space and not being measurable or invariant are not mutually exclusive. In fact, even in macroscopic world we can find many examples of this sort. For instance, consider topological properties of a Riemannian surface. The knowledge about local geometry do not provide any information about global topology. Another example is the functionality of body parts of an organism. Their functionality is complete and meaningful only in the context of the organism as a whole. If they are cut, they do not have any more the same properties. An abstract mathematical example is a set of objects. If no subset is defined, all members have exactly the same relation with respect to the only existing structure i.e. the set as a whole. They are all members, nothing differentiate them, and the set is invariant under their permutations. If we define two subsets, the nature of the set and its members is untouched, but they acquire a difference and additional structure because they can belong to one or other subset, and the permutation symmetry partially breaks. The common attribute of these examples is their global nature. The state of a system presents the complete information that laws of physics provide about it. Not only a measurement changes the system or its relation with environment, but also a single value that it provides by definition cannot represent the complete state that includes all information about the state.

The issue of reality mentioned above seems more philosophical than physical. A better example of what is acquired by describing quantum mechanics in symmetry scheme is the entanglement and its nonlocality which can occur in composite systems. Depending on the type of interaction between subsystems some of their properties - symmetries - may be entangled where others stay independent. In other words, systems can be separable with respect to some quantities but not others, and none of quantities has any preference with respect to others. In particular, in contrast to classical physics spacetime does not have any special role. In addition to nonlocality, another conclusion of this observation is that systems are only approximately isolated and independent. In the previous section we arrived to this conclusion from universality of gravity. This conclusion may be other way around, i.e. the indivisibility of the Universe reflects itself as a universal and attractive force that we call gravity. If true, this would be a hint to an inherent relation between quantum mechanics and gravity [12].

6 Fundamental randomness and probability

Axiom (v) is very similar to its analogue in standard quantum mechanics but it does not specify how the probability is determined because we obtain it in this section from axioms and some general properties of probability distributions. The introduction of symmetries in the foundation of quantum mechanics helps to better understand the concept of fundamental randomness in this theory and its conceptual difference with statistical mechanics. In classical physics “*There is a rule for everything !*”. Therefore, the randomness of complex systems and phenomena is assumed to be due to inaccessibility of complete degrees of freedom to measurements, i.e. *hidden variables* which are averaged over in measurements with limited resources and resolution. Since the discovery of quantum mechanics and observation of randomness in outcome of measurements - unsharp outcomes - such as atomic absorption and emission of electromagnetic waves [31], physicists have been searching for *hidden variables* [3], coarse-graining of spacetime [2], and many other means to establish (hidden) rules and explain the randomness of quantum systems in the same line of reasoning as classical physics. However, Bell inequalities [6, 5] and other proposed quantitative tests of quantum mechanics and difference of its predictions from classical statistical mechanics and their experimental verifications [32] have demonstrated the success of quantum mechanics. Therefore, we must accept randomness as an inherent reality of nature. This means that there are situations in which nature is intrinsically degenerate/symmetric and outcomes of repetition of the same process are inherently random. As discussed earlier, symmetries are generated by their abelian subalgebra. Thus, it is logical to associate probability to these states and not their superposition which present a degenerate configuration of nature.

Following axiom (i), the state of a system contains all obtainable information about it, including probabilities associated to pointer states consisting of eigen states of independent observables. Because any vector belonging to the state space can be decomposed to pointer basis, it is logical to conclude that the expansion coefficients must be related to the probability that the system be in the corresponding eigen state when the symmetry/degeneracy is broken. Giving the fact that probabilities must be positive real numbers, the square of the absolute value of coefficients $|a_i|^2$ is the most natural candidate. Because state space is projective, without loss of generality we may assume that they are orthonormal⁶, thus:

$$\sum_i^n |a_i|^2 = 1 \quad (9)$$

where n is the dimension of the state space representing symmetries of the system and can be ∞ or even innumerable. Equation (9) is a proper relation for $|a_i|^2$ as probabilities. However, apriori any positive real function $f(|a_i|^2)$ is an equally valid candidate. In this case normalization of the probability leads to:

$$\sum_i^n f(|a_i|^2) = 1 \quad (10)$$

⁶Orthogonality is a necessary condition for non-degenerate eigen vectors.

Consider two copies of the same system prepared independently but in the same manner. According to postulate ii the state space of the composite system made from these subsystems is:

$$|\psi\rangle = \frac{1}{2}(|\psi_1\rangle \otimes |\psi_2\rangle + |\psi_2\rangle \otimes |\psi_1\rangle) = \sum_i a_i^2 |i\rangle \otimes |i\rangle + \sum_{i \neq j} a_i a_j (|i\rangle \otimes |j\rangle + |j\rangle \otimes |i\rangle) \quad (11)$$

Symmetrization over two subsystems means that due to similar preparation conditions, they are indistinguishable. From this expression we conclude that the probability for subsystems to be in (i, j) state is $f(|a_i a_j|^2)$. On the other hand, because these subsystems are prepared independently, the mutual probability for the systems to be in (i, j) state is $f(|a_i|^2) f(|a_j|^2)$. Therefore:

$$f(|a_i|^2) f(|a_j|^2) = f(|a_i|^2 |a_j|^2) \quad (12)$$

It has been proved [19] that the only function with such property is a positive power-law⁷:

$$f(x) = x^\beta, \quad \beta > 0 \quad (13)$$

Considering this property along with (9) and (10), it is straightforward to see that only $\beta = 1$ can satisfy all these relations. Therefore, $f(|a_i|^2) = |a_i|^2$ is the only possible expression for the probability of pointer states. When they are normalized coefficients $|a_i|^2$ form a $n - 1$ simplex similar to classical systems. Note that we did not impose any condition on the dimension of the configuration space. When $n \rightarrow \infty$, equations (10) and (13) impose L^2 integrability condition on the state space. Therefore, it must be a Hilbert space. This completes the proof that axioms presented in Sec. 2 leads to standard quantum mechanics, and from this point there is no difference between the latter and the construction of quantum mechanics according to axioms (i) to (v).

6.1 Separability

Expectation value of an observable \mathcal{A} on a state with density ρ is $E[\mathcal{A}] = \text{tr}(\rho \mathcal{A})$. Density operators by definition have the common property of $\text{tr}(\rho) = \text{tr}(\rho^2) = 1$. It is easy to see that density operators are invariant under a unitary transformation and thereby under symmetry group of a system. For composite systems a priori one should be able to determine partial traces over one or multiple components. However, considering axiom (ii), meaningfulness of such an operation is guaranteed only when the state (Hilbert) space of the composite system is isomorphic to direct product of state spaces of components, i.e. when they do not interact or their interaction is negligible. Entanglement arises when the state space of a composite system is not isomorphic to direct product of state spaces of its components. This is usually due to some interaction between components at the time of their preparation or later which has reduced their symmetry. In this case, the entangled components cannot be any more considered as *components* and are *inseparable*. It is well known that in such systems $\text{tr}_i(\rho) \neq 1$ where the index i means that states of the i^{th} component are traced out. Such systems are usually called *mixed*. Composite systems for which $\text{tr}_i(\rho) = 1 \forall i$ are called pure. When the state of the composite system is presented by:

$$\mathcal{H} : \mathcal{H}_1 \times \mathcal{H}_2 \times \dots \times \mathcal{H}_n \rightarrow \mathcal{H}_1 \times \mathcal{H}_2 \times \dots \times \mathcal{H}_n \quad (14)$$

where \mathcal{H} is not an isomorphism, it is not guaranteed that a partial trace preserve symmetry representation condition which axiom (i) imposes on \mathcal{H} as the state space of a quantum system, although it preserves the Hilbert space property of the space. For instance, consider two particles of spin up or down. If they are not entangled, their state space represents $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ symmetry. When their states are entangled e.g. $(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}$, the symmetry is broken to $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ which mixes particles identification with their spin

⁷More exactly, in [19] this theorem is proved for the case in which the function f is applied to positive integer numbers. Isomorphy of integer and fractional numbers extends (12) to them. Because fractional numbers are a dense subset of real numbers, the power-law form must be the unique solution for all real numbers as long as we assume f is a smooth function with at most countable number of discontinuities.

(if particles are different, the symmetry reduces to \mathbb{Z}_2). It is why the result of partial tracing is equal to $1/2$ rather than 1, like if we had summed over half of the states. This means that entangled particles are indivisible, thus partial tracing is a meaningless operation. Evidently, partial tracing preserves the Hilbert space property of the states. Therefore, this shows that axiom (i) is more fundamental and restrictive than Hilbert space condition. A well known corollary of this observation is the fact that quantum systems keep a memory of their interactions. In another word, they are global in both space and time which is a consequence of axioms (i) and (ii). In the example above the complete symmetry group (after fixing the direction of spin axis) is $\mathbb{R}^4 \otimes (\mathbb{Z}_2 \oplus \mathbb{Z}_2)$ where the first group presents spacetime translation symmetry. Particles are separable with respect to spacetime, but inseparable with respect to the second symmetry.

7 Dynamics, decoherence and classicality

Axiom (iii) about the evolution of a quantum system is essentially the same as its analogue in standard quantum mechanics. The evolution is unitary, i.e. preserve states norm and is ruled by conservation laws imposed by symmetries. Because probabilities and interference coefficients determine the degree of symmetry or degeneracy of the state, they must evolve unitarily. Although the evolution of quantum states preserve the norm of the state i.e. the total probability, strictly speaking it does not satisfy the condition of unitary evolution in classical statistical mechanics. In classical systems there is no interference between possible outcomes because they are associated to independent points of \mathbb{R}^n space. In quantum mechanics unitarity evolution is assumed to be broken by measurements because in contrast to classical statistical physics immediate repetitions of a measurement operation on a system give the same outcome, i.e. after the first measurement the probability of the outcome jumps to 1 from whatever value it was. The best example of such behaviour is the Zeno effect [40]. Moreover, partial tracing of composite systems (which is a non-unitary operation) do not preserve probability, in contrast to similar case in classical physics. According to axiom (iv) by definition the process of measurement breaks the symmetry of states to a trivial representation and accordingly, the state space is reduced. Symmetries cannot be restored automatically and it is perfectly logical to see no signature of them in following measurements. The contribution of interference is also understandable because in contrast to states of a classical system that do not correspond to a symmetry representation, state space of a quantum system is a vector space representing symmetries and for the reasons discussed in Sec. 3 must be defined on complex numbers. Note that in standard quantum mechanics an abstract complex Hilbert space is postulated without any physical reason. In symmetry description these properties are necessary for representing symmetries. Therefore, related effects such as contribution of off-diagonal terms in probabilities and interference, if a phenomenon or measurement depends on noncommuting observables or commuting observables with different eigen states, are consequences of active influence of symmetries on the behaviour and evolution of quantum systems.

We argued earlier that discontinuous behaviour of quantum systems after a measurement/decoherence is similar to a first order phase transformation due to spontaneous symmetry breaking. This point may be criticized because phase transitions in classical systems are usually the macroscopic manifestation of micro-physics, i.e. quantum effects and the inferred argument seems circular. However, it is not difficult to find purely macroscopic classical examples with discontinuity. For instance, when a freely propagating particle enters a box and is trapped, its configuration space suddenly becomes reduced to the box. This is analogue to quantum mechanics of a particle for which translation symmetry is broken by measurement of its position. The size of the box for the classical particle is analogue to precision of measurement of a quantum particle. Once the classical particle is trapped or equivalently its position is measured, further measurements give the same position for up to resolution of measurement i.e. the size of the box.

7.1 Decoherence and classical behaviour

We discussed decoherence in the framework of measurement in Sec. 5. In general, when a quantum system seems to behave classically it is called to be decohered. Examples of such systems are practically every

macroscopic object in the Universe. In Copenhagen interpretation an observer is necessary to give an objective meaning to physical world by measuring and consequently collapsing its state to a designated pointer state. In more modern descriptions of quantum mechanics classicality and the lack of interference between states arise from interaction with a large environment which is not studied/measured or accessible. Nonetheless, there is nothing special about environment except that it is not practical to follow/measure its huge number of freedom degrees. As inferred before, measurement procedure relies on decoherence to explain the selection of pointer basis of systems by interaction with the environment. More precisely if the interaction Hamiltonian is \mathcal{H}_{int} and system and environment states are respectively $|\psi\rangle$ and $|\phi\rangle$, the evolution of these states from initial time t_0 for time t in interaction picture is [17]:

$$e^{-i\mathcal{H}_{int}(t-t_0)}|\psi\rangle \otimes |\phi(t_0)\rangle \xrightarrow{t \gg t_0} |\alpha\rangle \otimes |\phi(t)\rangle \quad (15)$$

where $|\alpha\rangle$ is one of the states of a pointer basis selected by the interaction. Here it is assumed that \mathcal{H}_{int} depends on the observable α and dynamical time of interaction with environment is considered to be much shorter than its interaction with apparatus used by an observer. For these reasons, very shortly after the initial time t_0 the system seems behaving classically. Although decoherence by interaction with environment provides a natural and well established mean, both theoretically and experimentally, for explaining the existence of a classical world a number of unclear issues still persist. They can be summarized as the followings [18, 16]:

- a. Interference is only partially removed by decoherence.
- b. Decoherence needs open systems, thus cannot be applied to the Universe as a whole.
- c. How does the separation of system and environment influence the state to which the system decohere ?
- d. Decoherence in relativistic quantum field theory.

To these criticisms we must add the ambiguity of pointer basis that we discussed in Sec. 5. In addition, we somehow addressed system-environment separation in symmetry view and argued that separability is always an approximation. This means that symmetries are always approximations or more precisely idealizations, and the presence of anisotropies at all scales in the Universe and nonlocality of quantum systems makes the decoherence a natural and omnipresent phenomenon. Therefore, here we concentrate on the rest of raised issues and discuss how symmetry description may help to understand them.

The answer to (a) is trivial. Classicality is an approximation in a universe which is intrinsically quantic. This question made sense in the early days after discovery of quantum mechanics, but nowadays with application of quantum mechanics in every aspect of micro-physics and verification of its role in the formation of the Universe through inflation, see e.g. [33] for a review and [27] for latest observations, and probably dark energy [35, 36, 37], there is no doubt that we live in a quantum universe, and classicality is just a good approximative perception of the local Universe.

As for (b), considering the Universe globally, evidently there is no environment for its decoherence. Nonetheless, the example of a very simple universe in Sec. 5.2 showed that there is no inconsistency in such a system. According to postulate (iv) a measurement has to break the symmetry related to the observable globally. This reduces the representation to a trivial one presented by one of the pointer states. However, in such systems the result of a measurement cannot have a probabilistic interpretation unless many copies of them are available. This is in contradiction with the assumption of a single copy of the Universe. Moreover, as we discussed in Sec. 4, a quantum universe is an intertwined ensemble of its components, and for this reason its global states are simply union of the states of its components. Furthermore, inflation in the early Universe and expansion at present make the Universe effectively an open system because the influence of processes occurring outside the past and possibly future horizons on everything inside becomes very small and negligible. Therefore, for all practical applications the Universe behaves like an open system. It rests

however one question: Does the Universe globally represent a symmetry non-trivially, i.e. is a homomorphism similar to (14) consisting of all entities in the Universe nontrivial? As explained before, this does not impose any inconsistency and if we can observe the corresponding observable, we find its value but cannot associate a probability to it - this is the infamous *cosmic variance*. An example of a plausible nontrivial symmetry is a rotating universe. In fact, recent observations of the CMB anisotropies by Planck satellite shows hemispherical anisotropies [38]. However, it is too soon to be certain about their origin and possible association to a globally nontrivial representation of rotation symmetry.

Regarding (c), apriori it seems that the definition of system-environment influence outcomes. This is certainly a fact. For instance, in the example of a decaying pion in Sec. 3, considering a magnetic field, even a weak one, as part of the system completely changes the definition of the system from first place, and thereby symmetries that define the state space, possible outcomes, etc. The situation would be different if we change the environment. According to axiom (i) a system is defined by its symmetries. Therefore, characteristics of the environment do not matter for the system as long as they do not change its symmetries.

Finally, (d) refers to the fact that relativistic quantum field theories do not admit a superselection [24], and thereby algebraic description cannot explain decoherence. Although apriori in the framework of field theory state space and algebra of observables in such systems are infinite dimensional, in practice for perturbative systems far from interaction zone, particles can be considered as isolated and usual rules of decoherence of quantum mechanics can be applied to them, see e.g. [17] for detailed discussion and examples of decoherence of relativistic particles. Moreover, a recent proposition for describing the vacuum of field theories as a superposition of coherent states with asymptotically zero amplitude [37] provides a natural environment for decoherence of quantum fields. As for non-perturbative systems, they are usually in highly correlated/entangled states and no classical description for them exists. Examples of such systems are Bose-Einstein Condensate (BEC) of atoms, condensate of Cooper pair of electrons in superconductors, other quantum exotic phenomena at low temperature such as quantum Hall effect, anions and topological materials, neutron stars, quark-gluon plasma in early Universe and ion collisions, etc. The strong interaction or quantum correlation in these systems usually provide a natural pointer basis, and due to extreme physical conditions for their formation, despite their theoretical infinite dimension they are very sensitive to environment and decoherence [39]. A notable common property of all these systems is the role of symmetries in their formation. Strong correlation between components/particles breaks the symmetry/degeneracy of a large number of states to very few ones, for instance to states where spins of all bosonic particles are in the same direction. In this case, the symmetry of individual spins $\bigotimes^{n \rightarrow \infty} \mathbb{Z}_2$ breaks to \mathbb{Z}_2 . Thus, during measurement of their direction particles behave as a single entity and the rules of decoherence of finite systems can be applied to them.

8 Summary

In this work we introduced symmetry as a fundamental concept with a central place in foundation of quantum mechanics, rewrote its axioms based on this idea, and showed their equivalence to standard quantum mechanics which is intensively verified by experiments. Our description replaces some of abstract entities such a Hilbert space with symmetry related objects which have better and more understandable physical interpretations. We also interpreted the decomposition of states to pointer states as an induced symmetry that we called state symmetry. It is associated to the history and environment of the system. In this sense the state of a system has a physical reality and presents the complete achievable knowledge about it. Furthermore, we used this new description to discuss some of foundational issues still under discussion in the literature. We summarize conclusions of this work through their answers to these questions:

Meaning of a quantum state: We discussed the fundamental and active role of symmetries in the definition of quantum states and argued that they are the extension of the concept of similarity which is essential for logical perception and interpretation of the Universe. Quantum states provide the complete obtainable information about systems and are much more than simple probabilistic

distribution of measurement outcomes which classical statistical states represent. In addition, state space as a representation of symmetries must be defined on complex numbers and the presence of interference between states is the manifestation of active role of symmetries in characterization of quantum systems.

Measurement problems: We showed that observables generate the abelian subalgebra of the symmetry group of the system. This provides a basis-independent meaning for the observables. What is called preferred pointer states are defined/chosen by interaction between components of a composite system or with environment and their associated symmetries. In this regard, the process of decoherence by measurement leads to complete breaking of symmetries and is unitary but irreversible.

References

- [1] O. Diego, “Toward the unification of the postulates of Quantum Mechanics”, [arXiv:0801.2839], G. Bertoldi, A.E. Faraggi & M. Matone, “Equivalence Principle, Higher Dimensional Moebius Group and the Hidden Antisymmetric Tensor of Quantum Mechanics”, [hep-th/9909201].
- [2] J.B. Hartle, *Phys. Rev. D* **44**, 1991 3173 [gr-qc/9210004].
- [3] D. Bohm, “A Suggested Interpretation of the Quantum Theory in Terms of Hidden Variables, I and II”, *Phys. Rev.* **85**, 1952 166.
- [4] L. Hardy, “Quantum mechanics, local realistic theories, and Lorentz-invariant realistic theories”, *Phys. Rev. Lett.* **68**, 1992 2981, L. Hardy, “Nonlocality for two particles without inequalities for almost all entangled states”, *Phys. Rev. Lett.* **71**, 1993 1665.
- [5] J.S. Bell, “Speakable and unspeakable in quantum mechanics”, Cambridge University Press, Cambridge, UK (1987).
- [6] J.S. Bell, “On the Einstein-Podolsky-Rosen paradox”, *Physics* **1**, 1964 195, J.E. Clauser, M.A. Horn, A. Shimony, R.A. Holt, “Proposed Experiment to Test Local Hidden-Variable Theories”, *Phys. Rev. Lett.* **23**, 1969 880.
- [7] J. Yan, “Quantum teleportation and entanglement distribution over 100-kilometre free-space channels”, *Nature* **488**, 2012 185, X.S. Ma, *et al.*, “Quantum teleportation over 143 kilometres using active feed-forward”, *Nature* **489**, 2012 269.
- [8] M.F. Pusey, J. Barrett & T. Rudolph, “On the reality of the quantum state”, *NAP* **8**, 2012 476 [arXiv:1111.3328].
- [9] S. Kochen & E.P. Specker, “The problem of hidden variables in quantum mechanics”, *J. Math. & Mech.* **17**, 1967 59.
- [10] V. D’Ambrosio, I. Herbauts, E. Amselem, E. Nagali, M. Bourennane, F. Sciarrino, A. Cabello, “Experimental implementation of a Kochen-Specker set of quantum tests”, *PRX* **3**, 2013 011012 [arXiv:1209.1836].
- [11] Eppley K., & Hannah E., “The necessity of quantizing the gravitational field”, *Found. Phys.* **7**, 1977 51.
- [12] H. Ziaepour, “And what if gravity is intrinsically quantic?”, *J. Phys. Conf. Series* **174**, 2009 012027 [arXiv:0901.4634].
- [13] A. Almheiri, D. Marolf, J. Polchinski, J. Sully, “Black holes: complementarity or firewalls?”, *J. High Ener. Phys.* **02**, 2013 062[arXiv:1207.3123].

- [14] S. Strogatz, “Nonlinear Dynamics And Chaos”, Westview Press, (2001).
- [15] W.H. Zurek, “Decoherence, einselection, and the quantum origins of the classical”, *Rev. Mod. Phys.* **75**, 2003 715 [quant-ph/0105127], W.H. Zurek, “Decoherence and the transition from quantum to classical – REVISITED”, *Physics Today* **44**, 1991 36[quant-ph/0306072].
- [16] M. Schlosshauer quant-ph/0312059, M. Schlosshauer, & A. Fine, in “Quantum Mechanics at the Crossroads: New Perspectives from History, Philosophy and Physics”, Eds. J. Evans & A.S. Thorndike, Springer (2006).
- [17] E. Joos, H.D. Zeh, C. Kiefer, D. Giulini, J. Kupsch, and I.O. Stamatescu, “Decoherence and the Appearance of a Classical World in Quantum Theory”, 2nd Ed. (2003), Springer, Germany.
- [18] D. Wallace, “Decoherence and its role in the modern measurement problem”, *Phil.Trans.Roy.Soc.Lond. A* **370**, 2012 4576 [arXiv:1111.2187].
- [19] L. Hardy, “Quantum Theory From Five Reasonable Axioms”, [quant-ph/0101012].
- [20] B. Dakić & C. Brukner, “Deep Beauty: Understanding the Quantum World through Mathematical Innovation”, Ed. H. Halvorson, Cambridge University Press, (2011) 365 [arXiv:0911.0695].
- [21] P.A.M. Dirac, “The Principles of Quantum Mechanics”, Oxford University Press (1958).
- [22] “Mathematical Foundation of Quantum Theory”, Princeton University Press, (1955).
- [23] J.M. Jauch, “Foundation of Quantum Mechanics”, Addison-Wesley (1968).
- [24] N.P. Landsman, “Observation and superselection in quantum mechanics”, [hep-th/9411173].
- [25] V.P. Belavkin, “On the Dynamical Solution of Quantum Measurement Problem”, [quant-ph/0512187].
- [26] L. O’Raifeartaigh, “Group Structure of Gauge Theories”, Cambridge Univ. Press, (1986), W. Ziller, “Lie Groups, Representations Theory and Symmetric Spaces”, Univ. Pennsylvania, (2010).
- [27] Planck Collaboration “Planck 2013 results. XXII. Constraints on inflation”, [arXiv:1303.5082], “Planck 2013 results. XXVI. Background geometry and topology of the Universe”, [arXiv:1303.5086].
- [28] W. Wootters, W. Zurek, Wojciech, “A Single Quantum Cannot be Cloned”, *Nature* **299**, 1982 802, D. Dieks, “Communication by EPR devices”, *PLA* **92**, 1982 271.
- [29] O. Pessoa Jr., “Can the decoherence approach help to solve the measurement problem”, *Synthese* **113**, 1997 323.
- [30] L. de Broglie, in Solvay (1928), D. Bohm, “Proof that Probability Density Approaches —2 in Causal Interpretation of Quantum Theory”, *Phys. Rev.* **89**, 1953 458.
- [31] N. Bohr, H.A. Kramers, and J.C. Slater, “Über die Quantentheorie der Strahlung” *Zeitschr. f. Physik* **24** (1924) 69, “Quantum Theory of Radiation”, *Phil. Mag.* **47** (1924) 785.
- [32] M.A. Rowe, D. Kielpinski, V. Meyer, C.A. Sackett, W.M. Itano, C. Monroe & D.J. Wineland, “Experimental violation of a Bell’s inequality with efficient detection”, *Nature* **409**, 2001 791, D. Salart, A. Baas, J.A.W. van Houwelingen, N. Gisin, & H. Zbinden, “Spacelike Separation in a Bell Test Assuming Gravitationally Induced Collapses.”, *Phys. Rev. Lett.* **100**, 2008 220404, M. Ansmann, *et al.*, “Violation of Bell’s inequality in Josephson phase qubits”, *Nature* **461**, 2009 504506.
- [33] A. Linde, “Inflation and String Cosmology”, *Prog. Theor. Phys. Suppl.* **163**, 2006 295 [hep-th/0503195].
- [34] E.J. Copeland, M. Sami, & S. Tsujikawa, “Dynamics of dark energy”, [hep-th/0603057].

- [35] H. Ziaeeepour, “Cosmological Condensation of scalar fields”, *Phys. Rev. D* **81**, 2010 103526 [arXiv:1003.2996].
- [36] G.E. Volovik, “Superfluid Universe”, [arXiv:1004.0597].
- [37] H. Ziaeeepour, “Issues with vacuum energy as the origin of dark energy”, *Mod. Phys. Lett. A* **27**, 2012 1250154 [arXiv:1205.3304].
- [38] Planck Collaboration, “Planck 2013 results. XXIII. Isotropy and statistics of the CMB”, [arXiv:1303.5083].
- [39] M. Schlosshauser, “Experimental motivation and empirical consistency in minimal no-collapse quantum mechanics”, *Annals Phys.* **321**, 2006 112 [quant-ph/0603202].
- [40] E.C.G. Sudarshan, B. Misra, “The Zenos paradox in quantum theory”, *J. Math. Phys.* **18**, 1977 756.