On defining the undefinable: jets

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On defining the undefinable: jets†

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1. Introduction

Since this is a meeting concerned primarily with jets at LEP and HERA, I will begin with a brief review of how we view jets [1, 2] arising in $p\bar{p}$ collisions, illustrated schematically in figure 1. The colliding hadrons (in their centre-of-mass system) are composed of many partons (quarks and gluons in QCD). Two such partons can undergo a large angle ('hard') scattering generating two (or more) final-state partons at large momentum transverse to the initial beam direction (large P_T). As a final step the partons that now find themselves isolated in momentum space tend to radiate more partons and eventually to evolve into the readily apparent 'sprays' of hadrons that we label jets as indicated in the figure. For later reference it is important to note that the full final state contains another component consisting of primarily low P_T particles as suggested by the small arrows in figure 1. In general this 'underlying' event will itself have two components. One component involves the initial- and final-state interactions of (largely radiation from) the partons that participate in the hard scattering and is correlated with the hard scattering. The other component involves the interactions of the 'spectator' partons and is essentially uncorrelated with the hard scattering.

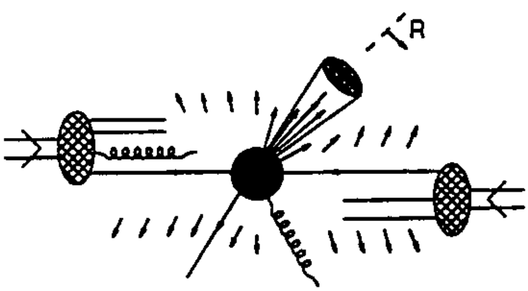

Figure 1. Hard-scattering process.

The factorized [3] form of the inclusive jet cross section has the schematic structure

$$\begin{aligned}\sigma = \sum_{i,j,n} \int \mathrm{d}x_i \int \mathrm{d}x_j \int \mathrm{d}z \\ \times G_{i/\mathrm{p}}(x_i, \mu_{\mathrm{CO}}) G_{j/\bar{\mathrm{p}}}(x_j, \mu_{\mathrm{CO}}) \\ \times \hat{\sigma}_{ij\to n}(\hat{s}, \text{angles}; \mu_{\mathrm{UV}}, \mu_{\mathrm{CO}}) \\ \times \mathcal{S}_n(E_{\mathrm{T}}, \Omega_{\mathrm{J}}; R).\end{aligned} \tag{1}$$

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In this expression the Gs represent the distributions of the partons i, j inside the proton or anti-proton as a function of the longitudinal momentum fraction x (and characterized by a factorization/collinear cutoff scale μ_{CO}) . The parton cross section $\hat{\sigma}$ describes the process $i + j \to n$ partons and is a function of both the kinematic variables and the factorization and renormalization scales, μ_{CO} and μ_{UV}. Finally the jet 'projection' function, $\mathcal{S}_n$, contains the information about how subsets of the n partons are combined to form a jet, which is characterized by a transverse energy E_{T} and direction Ω_{J}. This function depends on the precise way in which the jet is defined (see later) as schematically indicated by the dependence on the 'jet radius' R. Non-perturbative effects associated with the 'fragmentation' of partons into hadrons, with the jet now constructed from the hadrons, could also be included in $\mathcal{S}$. It is important to recognize that, even though the presence of jets in the final state is qualitatively obvious in the data, there is no fundamental, quantitatively unique definition of a jet. The constraints arising from the conservation of energy–momentum, colour, etc, guarantee that

- a jet is NOT the remnant of a single parton;
- a jet is NOT the result of a single parton shower;
- a jet is NOT uniquely defined;
- the properties of a jet will, in general, depend on how the jet is defined.

Thus the jet definition itself will introduce a form of *trigger bias*.

The presence of this trigger bias is one of the features that suggests that it would be very informative to study and compare jet samples obtained by employing the *same* jet definition to different physical processes. For example, the physical situation described earlier for $\mathrm{p\bar{p}}$ collisions is quite different from that which obtains in $\mathrm{e^+e^-}$ annihilations. In the latter case there is no 'underlying' event! The interplay of the jet definition with this 'underlying' event can then be studied by comparing two such samples, one from $\mathrm{p\bar{p}}$ collisions and one from $\mathrm{e^+e^-}$ annihilations. Let us pursue this possibility further by considering in more detail how jets can and are being defined.

2. Jet definitions (What is a jet?)

As we have just described, the essential point to remember about jets, which has not been widely appreciated due to the remarkable success of lowest-order perturbation theory, is that a jet is *not* actually well defined. While jets can certainly be loosely associated with single partons, quarks or gluons, which are initially isolated in momentum space just after they are scattered or produced, the real experimental situation is not that simple, especially in hadron–hadron collisions. The case of $\mathrm{e^+e^-}$ collisions is rendered simpler by the absence of an underlying event that includes the 'beam jets'. Simple invariant mass cuts (e.g. $M_{\mathrm{jet}}^2/Q^2 < y_{\mathrm{cut}}$) serve to define multi-jet events, at least for small jet numbers (in the simplest and dominant case of two jets, the selection is especially easy—half of the event = a jet). In the collision of two hadrons the situation is much more complex. Even in the absence of a hard-scattering process, a collision involving only soft exchanges typically produces many particles, mostly of low P_{T} but populating all of the available phase space in η (the underlying event). When a hard scattering does occur the final state is characterized by collimated sprays of large P_{T} hadrons, the jets. But of all the other hadrons produced in the event, which are to be associated with the jets? Since the underlying partons are not colour singlets while the produced hadrons are, a single parton cannot evolve into a jet of hadrons in isolation. This fragmentation process must involve the collective (and coherent!) action of several partons (possibly spanning a large region in momentum space). Hence, except for the largest P_T hadrons, there can be no unique, *a priori*, experimental definition of which hadrons are to be associated with which jet (if any). The interesting feature of the theoretical jet calculations at order α_S^3 that are now being carried out [4, 5], is that this ambiguity also appears in the perturbative calculation. The jets can now include either one *or* two of the possible three partons in the final state. Further, the two partons associated with a single jet may or may not be easily associated in a Feynman diagram sense, e.g. as a gluon radiated from a quark in the final state. One of the partons included in the jet may, in fact, be more usually thought of as initial-state radiation.

The jet definition employed in practice in hadron–hadron collisions can be simply characterized in terms of the energy measured inside a cone in η–ϕ space, where $\eta = -\ln\tan\theta/2$ and ϕ is the usual azimuthal angle around the beam direction. The quantity used is, in fact, the 'transverse energy'. This is effectively the transverse momentum of the jet treated as a scalar rather than a two-vector (hence energy rather than momentum). The transverse energy can also be defined directly in terms of the energy, $E_T = E\sin\theta$. The attractive feature of this quantity is its invariance under longitudinal Lorentz boosts. Recall that the parton centre of mass is, in general, moving in the $p\bar{p}$ system. Thought of in terms of calorimeter cells, i, inside a cone defined by

$$\Delta R_i \equiv \sqrt{(\eta_i - \eta_J)^2 + (\phi_i - \phi_J)^2} \leqslant R \tag{2}$$

we define the transverse energy of the jet E_T as

$$E_T = \sum_{i\in\text{cone}} E_{T,i}. \tag{3}$$

The energy-weighted direction of the jet is given by

$$\eta_J = \frac{1}{E_{T,J}} \sum_{i\in\text{cone}} E_{T,i}\eta_i \tag{4}$$

and

$$\phi_J = \frac{1}{E_{T,J}} \sum_{i\in\text{cone}} E_{T,i}\phi_i. \tag{5}$$

This procedure implies some number of iterations of the jet defining process in equation (2) until the quantities defined in equations (3), (4) and (5) are stable with the jet cone remaining fixed. This is the definition used in the theoretical calculations described here [4]. The precise definitions used by the various experiments differ to a greater or lesser extent from this form [6]. The CDF group use a definition [7] that is similar but not identical to this one. In particular, the CDF definition performs the sum over the cells in the cone before doing the projection on the transverse direction as in

$$E_{T,\text{CDF}} = \left(\sum_{i\in\text{cone}} E_i\right)\sin\theta_J \tag{6}$$

where

$$\sin\theta_J = \frac{p_{T,J}}{P_J}$$
$$p_{T,J} = \sqrt{\left(\sum_{i\in\text{cone}} p_{x,i}\right)^2 + \left(\sum_{i\in\text{cone}} p_{y,i}\right)^2}$$
$$p_J = \sqrt{p_{T,J}^2 + \sum_{i\in\text{cone}} (p_{z,i})^2}. \tag{7}$$

The primary difference from the definition described earlier concerns information about the invariant mass of the jet and is not expected to be numerically important for high E_T jets. On the other hand, the differences with the definition utilized by UA2, which involves summing contiguous calorimeter elements with $E_{T,i} > 0.4$ GeV, are clearly relevant numerically. The differences with the way in which jets have been defined in e^+e^- collisions are more fundamental.

One of the more interesting activities carried out at the Snowmass Workshop last July was to define a 'standard' jet definition, called the Snowmass Accord [8]. The point was not necessarily to select an optimum jet definition, as this depends on the specific application, but rather to formulate a jet definition that satisfies reasonable criteria and can be used by all experimentalists and theorists to generate at least one sample of jets or jet cross sections that can be meaningfully compared between different groups. Suggested criteria are:

- easy to implement in all experiments;
- easy to implement in theoretical calculations;
- yields reliable (finite) results at all orders in perturbation theory.

The jet definition outlined in the Snowmass Accord is as described in equations (3) through (5).

The central thesis of this talk is that a particularly interesting application of this standard definition would be to generate a sample of jets from e^+e^- events (and deep inelastic lepton–proton scattering events) that can be compared to those from hadron collisions. As discussed later the systematic issues associated with jets in hadron collisions, especially those due to the underlying event, are quite different from those in the e^+e^- case. It will be an informative test of our understanding of these issues to make this comparison.

We should emphasize that the dependence on the jet definition is not only an obstacle for comparing different experiments but also for comparing experiment to theory. Precise comparisons will be possible only if the detailed dependence on the jet definition exhibited by the data is well reproduced by the theory. This is an essential limitation of the Born level calculation, which has only one parton per jet and thus exhibits no structure to the jet at all. In Monte-Carlo simulations [9] the finite size of a jet arises from the subsequent showering of this parton plus the final non-perturbative fragmentation into hadrons. While of differing reliability, both of these features contribute to the theoretical uncertainty. The inclusion of exact higher-order corrections helps to reduce this uncertainty in the jet definition dependence. At order α_S^3 [4] jets consist of a linear combination of both one and two partons and thus have a finite size. This is illustrated in figure 2 where the inclusive cross section for a single jet is plotted against the cone size R (all of the curves of theoretical calculations displayed in this talk use the jet definition of [4], which is essentially identical to the Snowmass Accord definition), and where HMRS(B) parton distribution functions [10] have been used in the calculation. While the order α_S^2 (Born) result exhibits its *independence* of R, the order α_S^3 cross section clearly depends on R. The general form of this dependence is approximately

$$\frac{d\sigma}{dE_T\,d\eta} \simeq a + b\ln R + cR^2 \tag{8}$$

where we recognize the second term as arising from the collinear divergence characteristic of gauge theories. The curve in the figure is well described by a value $b \simeq 0.15$ nb GeV^{-1}, which can be understood as $\sim 2 \times \alpha_S \times \sigma_{\rm Born}$ where the number ~ 2 is the result of the detailed calculation [4] and is consistent with the naive expectation of order 1. Preliminary data from CDF [11] indicate a somewhat more rapid variation with R, corresponding to somewhat broader jets, than that exhibited by the perturbative calculations. This is in at least qualitative agreement with the expectation that non-perturbative fragmentation effects, not included in the theoretical calculations, will tend to broaden the jets. It will be informative to study this issue in detail.

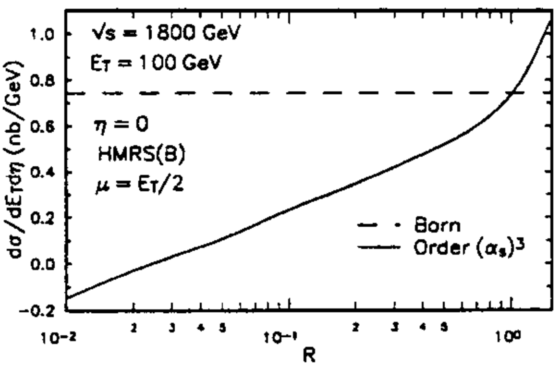

Figure 2. Inclusive jet cross section against cone size R.

In order to emphasize the differences between jets in hadron–hadron collisions and in e^+e^- annihilation, let us turn briefly to the question of the role of the underlying event in determining the dependence on the jet definition.

3. Jet structure and the underlying event

Let us consider the dependence on the jet definition, here characterized by the parameter R, arising from the expected structure of an *ideal* jet in a hadron collision event. This is illustrated (in schematic form) in figure 3 where the E_T in a jet (arbitrary units) is plotted against ΔR (recall equation (2)). (This is the one-dimensional projection of what is actually a two-dimensional structure.) The shaded region represents the E_T within the cone defined by R.

Clearly, for *any* specific choice of jet size R, there will probably be energy that falls outside of the cone that might correctly be associated with the jet. At the same time some of the energy within the cone is actually the result of the underlying, uncorrelated soft event (recall figure 1). One might, for example, characterize this extra contribution to the jet by extending the 'pedestals' that exist far from the centre of the jet back under the jet. Thus in the naive view, illustrated in figure 4, one has to correct for both 'splash out' (the correlated energy outside of the cone, expected to be of order 2 GeV for a 100 GeV jet) and 'splash in' (the uncorrelated energy inside the cone, expected to be of order 1 GeV for $R \sim 0.7$). UA1 has reported [12] that they observe approximately 1.3 GeV of E_T per unit area in R^2 in the pedestals of the jets, i.e. well away from direction of the jets, for jets with $E_T \geqslant 15$ GeV.

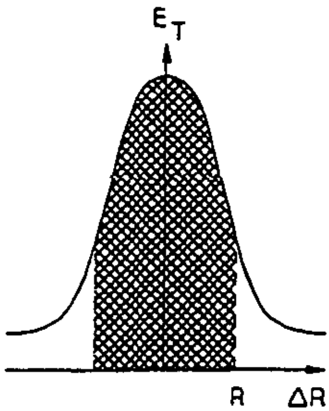

Figure 3. Idealized jet.

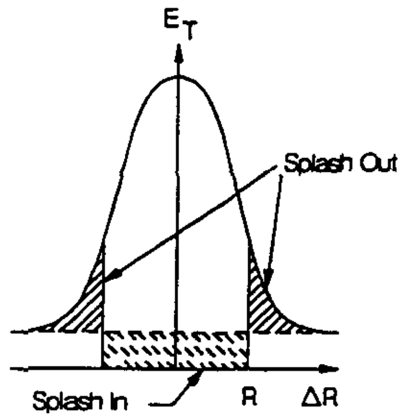

Figure 4. Naive picture of jet with 'pedestals'.

However, to be comparable with the higher-order calculation, the corrections should be both more sophisticated and, it turns out, *smaller*. In fact the higher-order calculation already correctly includes (to first non-trivial order in perturbation theory) the effect of the correlated energy that falls outside the cone. It is also accounts for the fact that a large fraction (perhaps more than half) of the energy in the pedestals is in fact correlated, corresponding to soft but correlated perturbative bremsstrahlung in the hard event [13]. This feature helps to explain the observed [12] correlation between the general level of E_T in an event and the presence of a hard-scattering process. In minimum bias events, with apparently no hard scattering involved, the level of observed E_T is more like 0.5 GeV per unit R^2. Thus the required sophisticated corrections should correspond only to non-perturbative 'splash out' (the effects of non-perturbative fragmentation) and truly uncorrelated 'splash in' and should be quite small (of order 1 GeV in total) as suggested in figure 5.

It should in fact be possible to confirm the validity of these ideas and the relative smallness of the required corrections by studying in detail the structure within jets in both the theoretical results and in the experimental numbers. An example quantity is the energy distribution within a jet as illustrated in figure 6. The jet sample is defined by the outer cone radius R and a total jet transverse energy E_T. We consider the fraction of E_T that falls inside the inner cone defined by r. Preliminary theoretical results are illustrated in figure 7 corresponding to $E_T = 100$ and 400 GeV (for $\sqrt{s} =$ 1800 GeV), $\eta = 0$ and $R = 1.0$. Note the expected result that the larger E_T jet is narrower. In such a plot the Born result would correspond to a constant value of

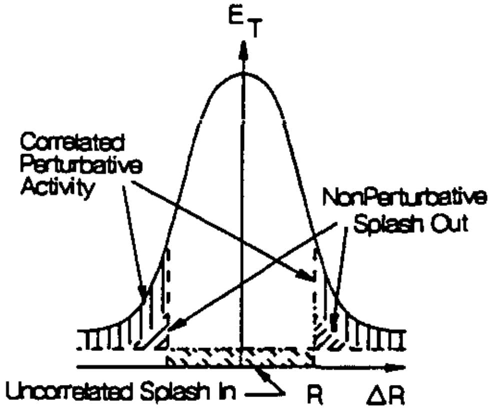

Figure 5. Sophisticated picture of jet with 'pedestals'.

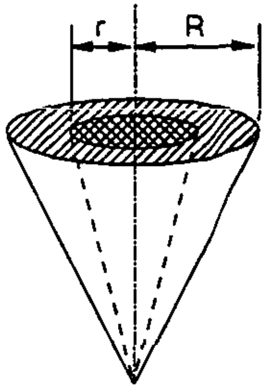

Figure 6. Schematic picture of a cone within a cone.

1 at all $r > 0$ while the higher-order calculation exhibits a non-trivial distribution, although still with a (double) logarithmic singularity for $r \to 0$. This is just the sort of study that would be very informative to perform also for jets defined in the same way in e^+e^- events, where the underlying event structure is dramatically different, i.e. essentially absent.

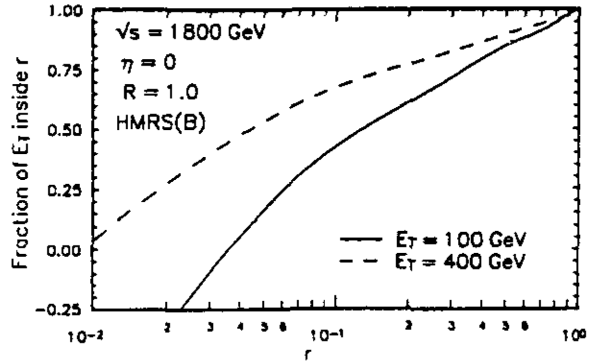

Figure 7. E_T distribution inside a jet.

The only published data directly relevant to this quantity come from UA1 [12] (although data from CDF [14] will appear soon) and these are compared with the preliminary theoretical results in figure 8. The experimentally observed jets are clearly broader (a smaller fraction of E_T inside $r = 0.2$) than those suggested by the perturbative calculation at small E_T. As noted earlier, this is to be intuitively expected from the fact that non-perturbative fragmentation will make the distribution broader rather than narrower. Furthermore, the difference between theory and experiment is consistent with vanishing as an inverse power of E_T as expected for non-perturbative effects.

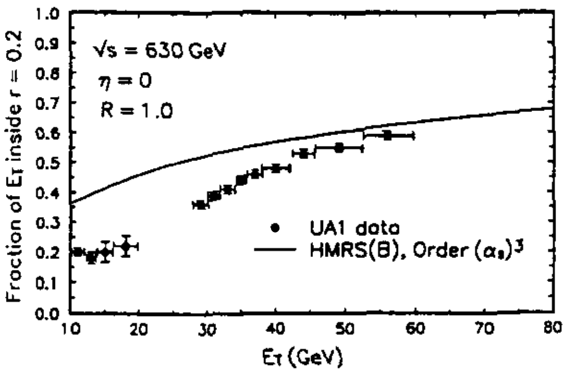

Figure 8. Fraction of E_T inside a cone, $r = 0.2$, against E_T. Data from [12].

4. Conclusion

The analysis of the inclusive jet cross section in hadron–hadron collisions, including contributions to order α_S^3, has now achieved a high level of sophistication and success. The quality of the present comparison to data is illustrated in figure 9 where the data are preliminary numbers from CDF [11] corresponding to an average over $0.1 < |\eta| < 0.7$ with $R = 0.7$. Only the (vertical) statistical uncertainties in the data are indicated. The calculated curve corresponds to the HMRS(B) distribution functions [10]. It is important to note that there are no fitted parameters involved here. Everything is determined from the theory and independent measurements. Figure 10 shows the same data but now divided by the theoretical result for the HMRS(B) distribution functions. The HMRS(E) result is also shown as an indication of the theoretical uncertainty due to the uncertainty in the parton distribution functions. Ignoring for the moment the overall experimental systematic uncertainty, we see that such jet data are just at the threshold of resolving the differences in the distribution functions.

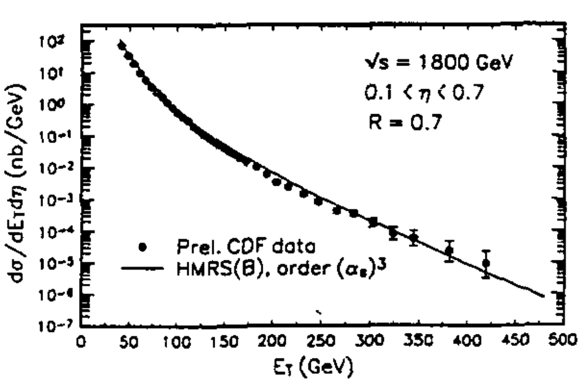

Figure 9. Jet cross section compared to data [11].

An important part of the explanation for this excellent agreement of theory with data is that the trigger bias effects inherent in the necessity of defining a jet is also reflected in the theoretical result. To push this field further will require even more detailed studies of such systematic effects. The point of this paper is to emphasize that a particularly useful approach will be the comparison of equivalent (i.e. with the same jet definition) jet samples from the full range of possible processes, $p\bar{p}$ collisions,

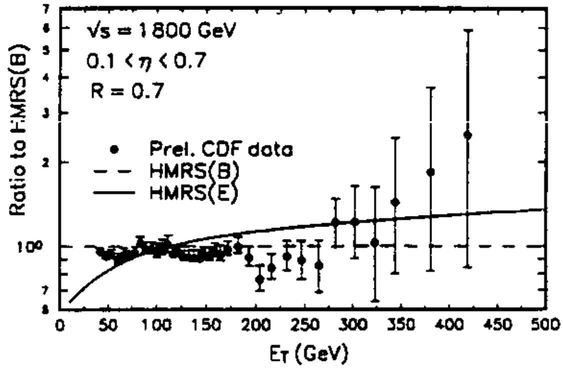

Figure 10. Ratio of jet cross sections to HMRS(B) result.

e^+e^- annihilations and deep inelastic lepton–proton collisions. I encourage both experimentalists and theorists to perform this exercise using the Snowmass Accord jet definition.

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