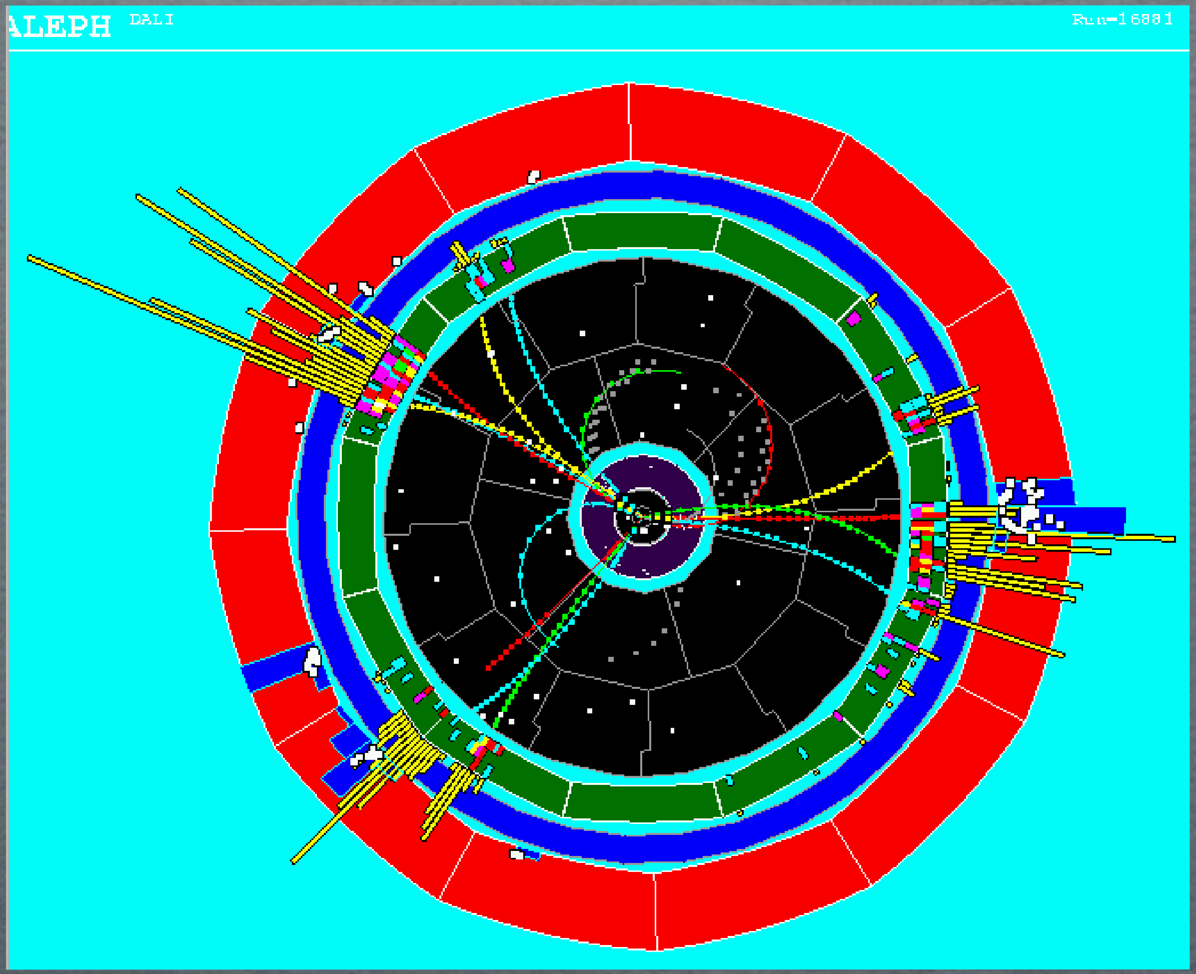
Jet algorithms



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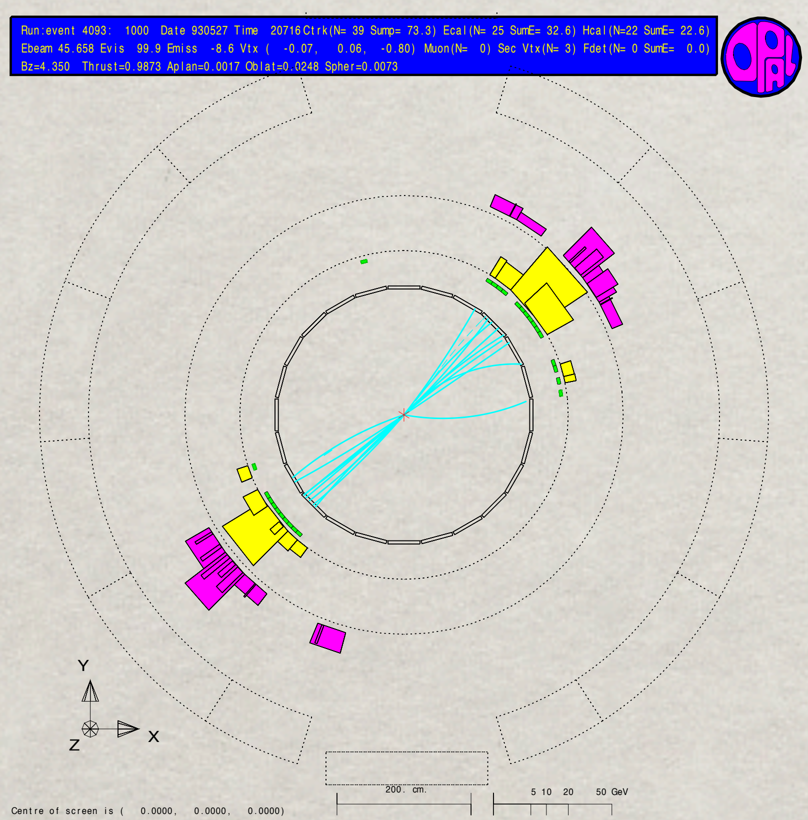
Outline

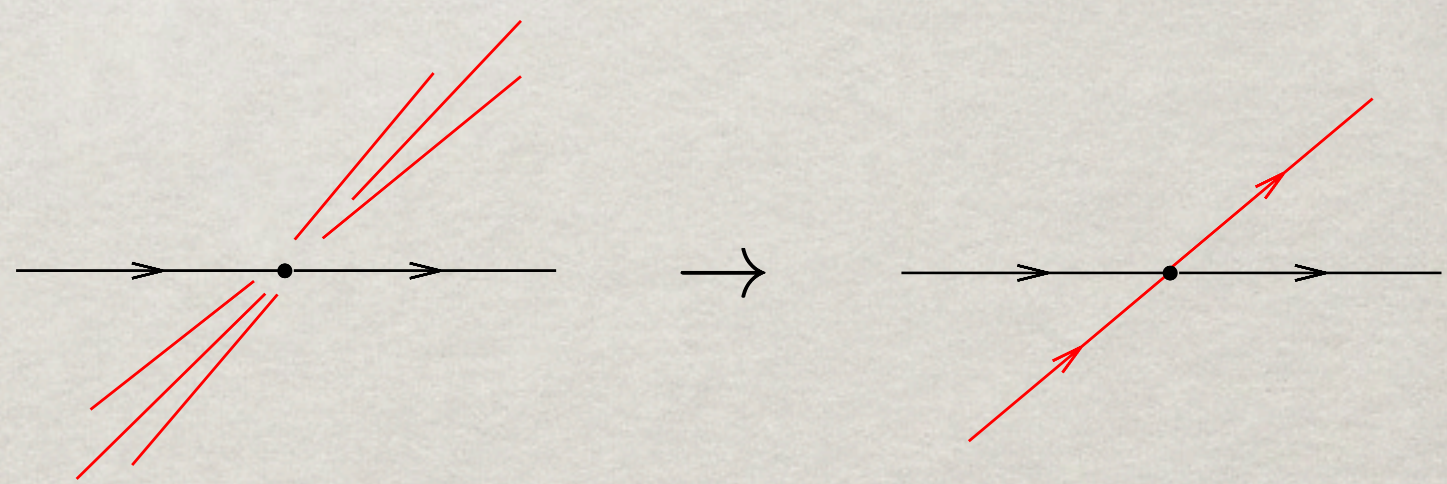
- Lecture 1: jet algorithms and QCD
  - Cone and sequential jet algorithms
  - Computing jet observables in PT QCD
- Lecture 2: using jets at the LHC
  - Hadronisation and underlying event effects
  - Performance of jets for reconstructing resonances
  - Fat jet techniques for boosted object decays
- Most material in these lectures has been taken from

  G.P. Salam "Towards Jetography" 0906.1833 [hep-ph]

Why jets?

- Quarks and gluons, the basic objects entering the QCD Lagrangian, are never observed into detectors
- QCD final states involve highly collimated sprays of energetic hadrons, a.k.a. jets

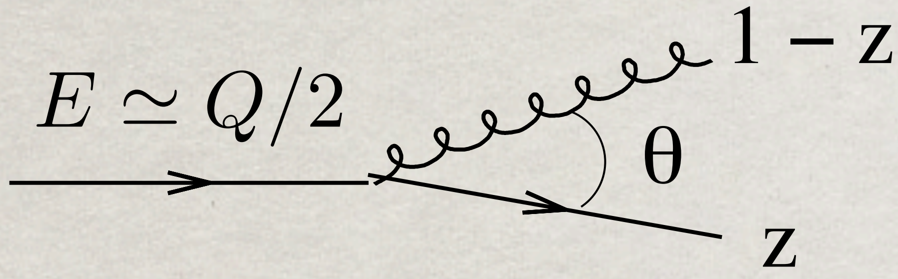




- Jets are the footprints of partons (quarks and gluons) in the detector $\Rightarrow$ ideally 1 jet $\sim$ 1 parton

Jets and QCD radiation

- Accelerated quarks always radiate gluons



$$E_q \simeq z\, E$$

$$E_g \simeq (1 - z)\, E$$

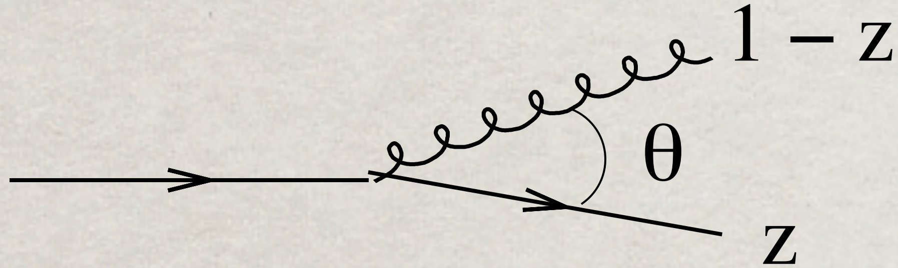
$[C_F = 4/3,\; C_A = 3,\; T_R = 1/2]$

$$dP_{q \to qg} = C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} dz \frac{1+z^2}{1-z}$$

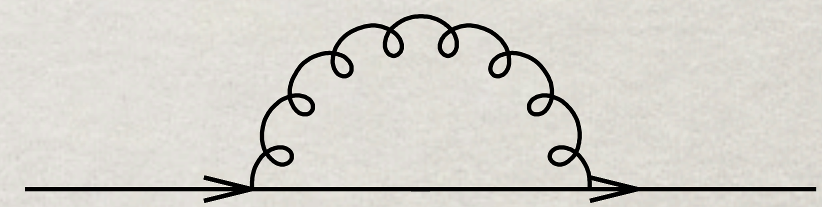
- The structure of jets reflects the properties of underlying QCD radiation
  - High probability of emission of soft ($z \to 1$) and collinear ($\theta \to 0$) gluons
  - Extra hard gluon emission $\propto \alpha_s(Q)$
- Asymptotic freedom: $\alpha_s(Q) \to 0$ for $Q \to \infty \Rightarrow$ the higher the energy the more collimated the jets

Infrared safety

- Gluon emission probability gives infinity if integrated in the soft and collinear phase space regions



$+\infty$



$-\infty$

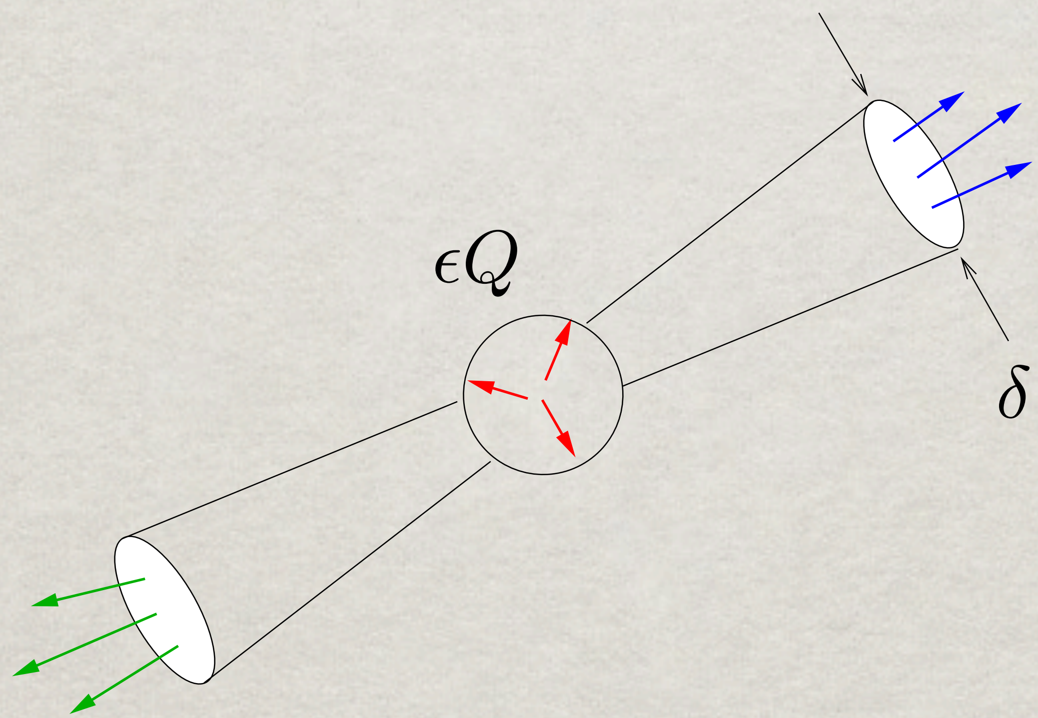
- The same (negative) infinity occurs in virtual corrections

- If jet definitions are insensitive to soft and collinear gluons, real and virtual infinities cancel in jet cross sections ⇒ jet cross sections are computable in perturbative QCD using the language of quarks and gluons

Sterman-Weinberg jets

- Sterman-Weinberg jets: cones of opening angle δ containing all but a fraction ϵ of the total energy in the event Q

[Sterman Weinberg PRL 39 (1977) 1436]



- SW jets are collinear and infrared safe $\Leftrightarrow$ probability of having two SW jets (two-jet rate) can be computed in QCD

$$R_2(\delta, \epsilon) = 1 - 4\, C_F \frac{\alpha_s}{\pi} \ln \delta \ln \epsilon + \ldots$$

Snowmass accord

- SW procedure is not easily generalisable to jets in hadron collisions (no meaning of the total energy, position of cones, multi-jet events with overlapping cones)

- In early '90s the "Snowmass accord" was elaborated, stating desirable properties of jet algorithms

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in theoretical calculation;
3. Defined at any order of perturbation theory
4. Yields finite cross sections at any order in perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronisation

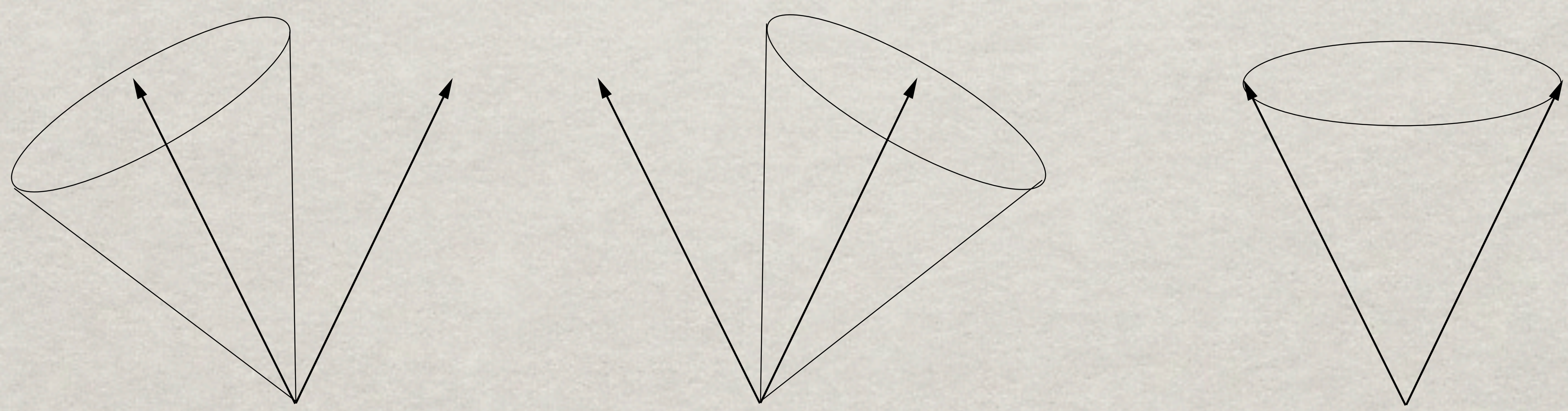
[3] J. E. Huth et al., "Toward a standardization of jet definitions", FNAL-C-90-249-E

Cone algorithms

- Till recently, the blessed algorithms in hadronic collisions were cone algorithms
- A stable cone is a set of particles p_i that are within a circle or radius R in the y-ϕ plane around their centre of mass p_J

$$\Delta R_{iJ}^2 = (y_i - y_J)^2 + (\phi_i - \phi_J)^2 < R^2$$

- Examples of stable cones

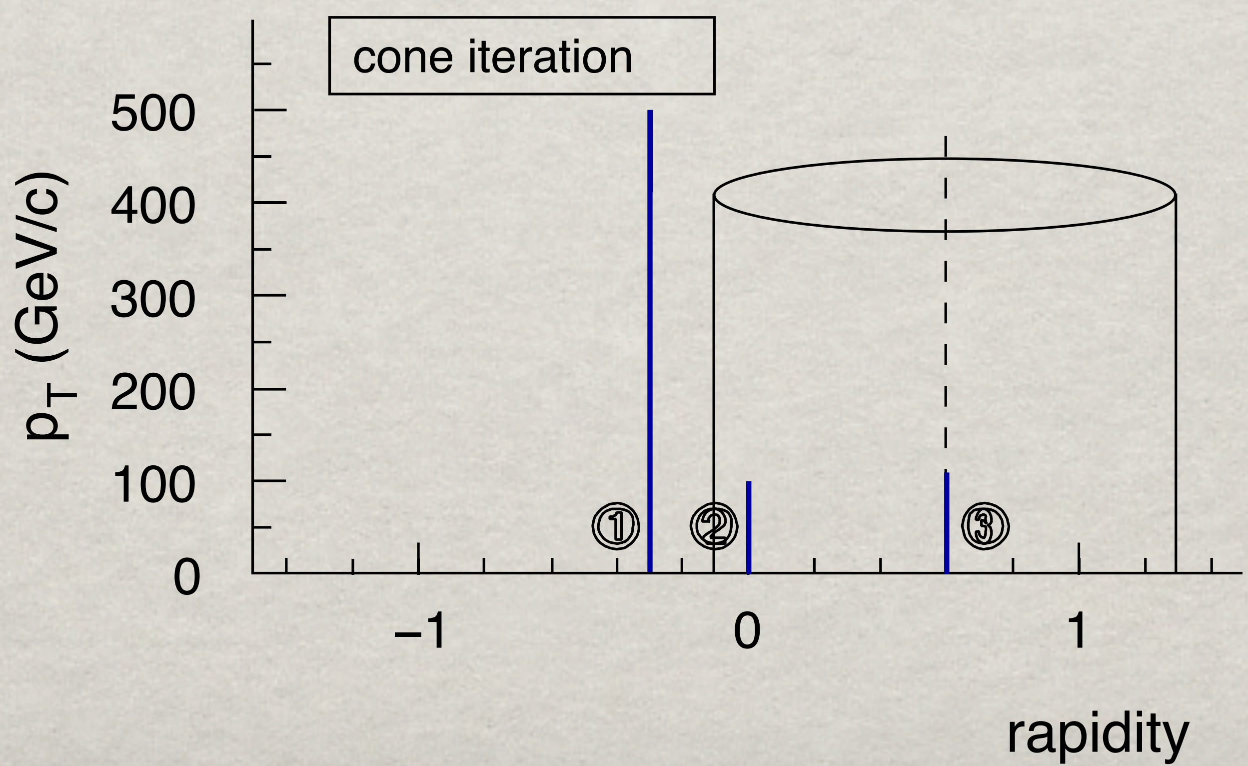


Seeded cone algorithms

- Seeded cone algorithms are attempts to find stable cones via an iterative-cone (IC) procedure
  - One starts from a seed particle p_i (not necessarily a particle momentum) and considers the set of all particles p_j such that

$$\Delta R_{ij} < R$$

  - If the cone is stable the procedure stops, otherwise the cone centre p_J is taken as a new seed until one reaches a stable cone



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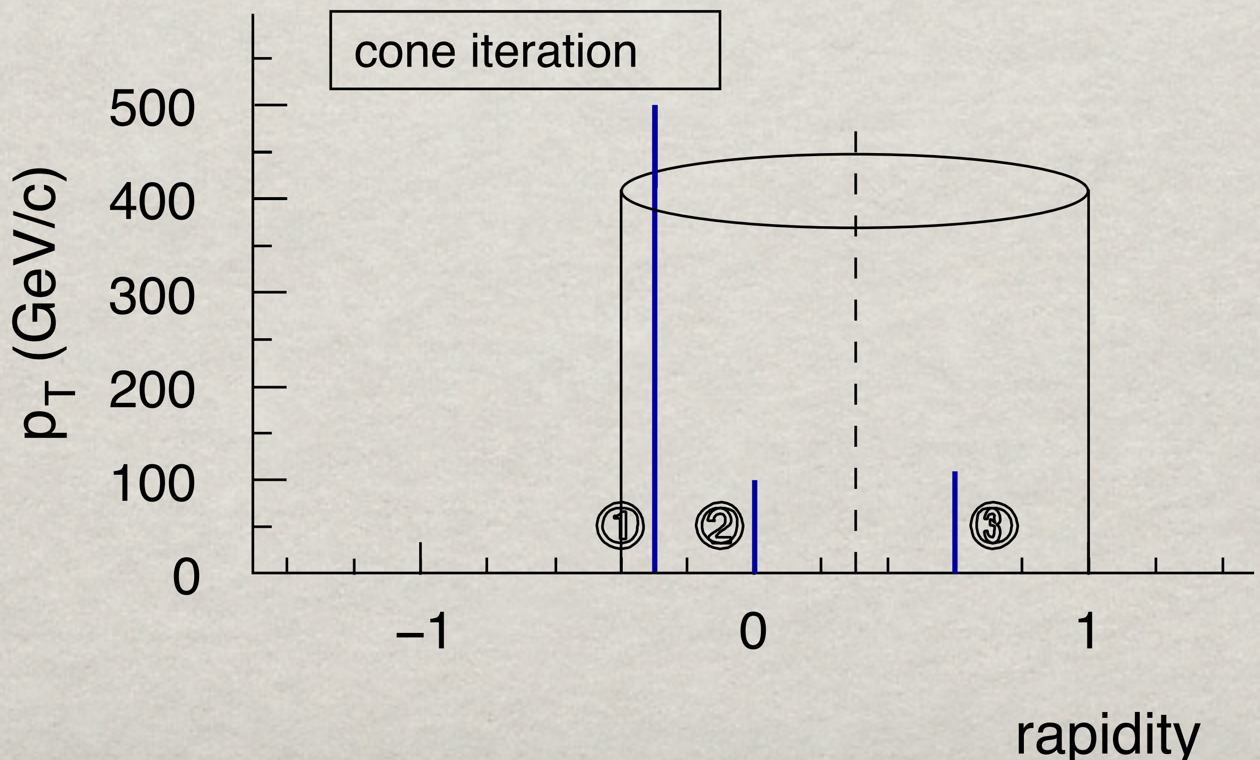

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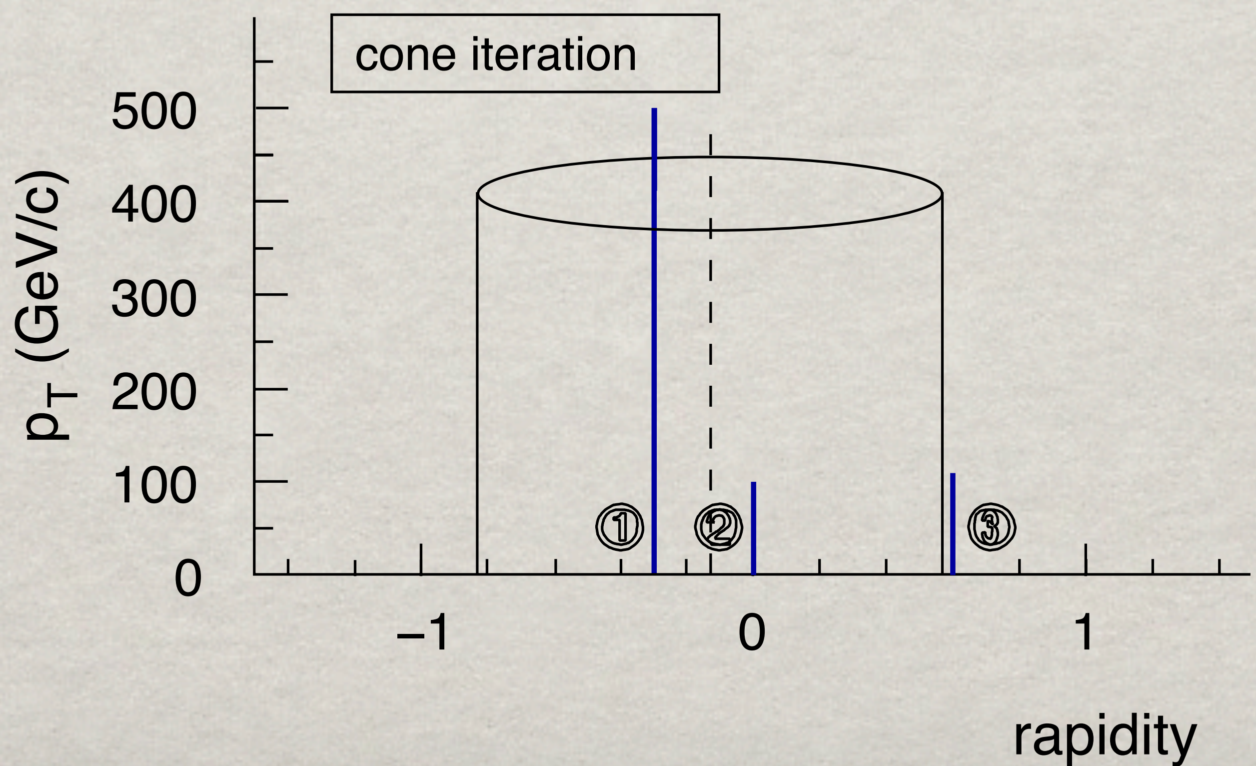

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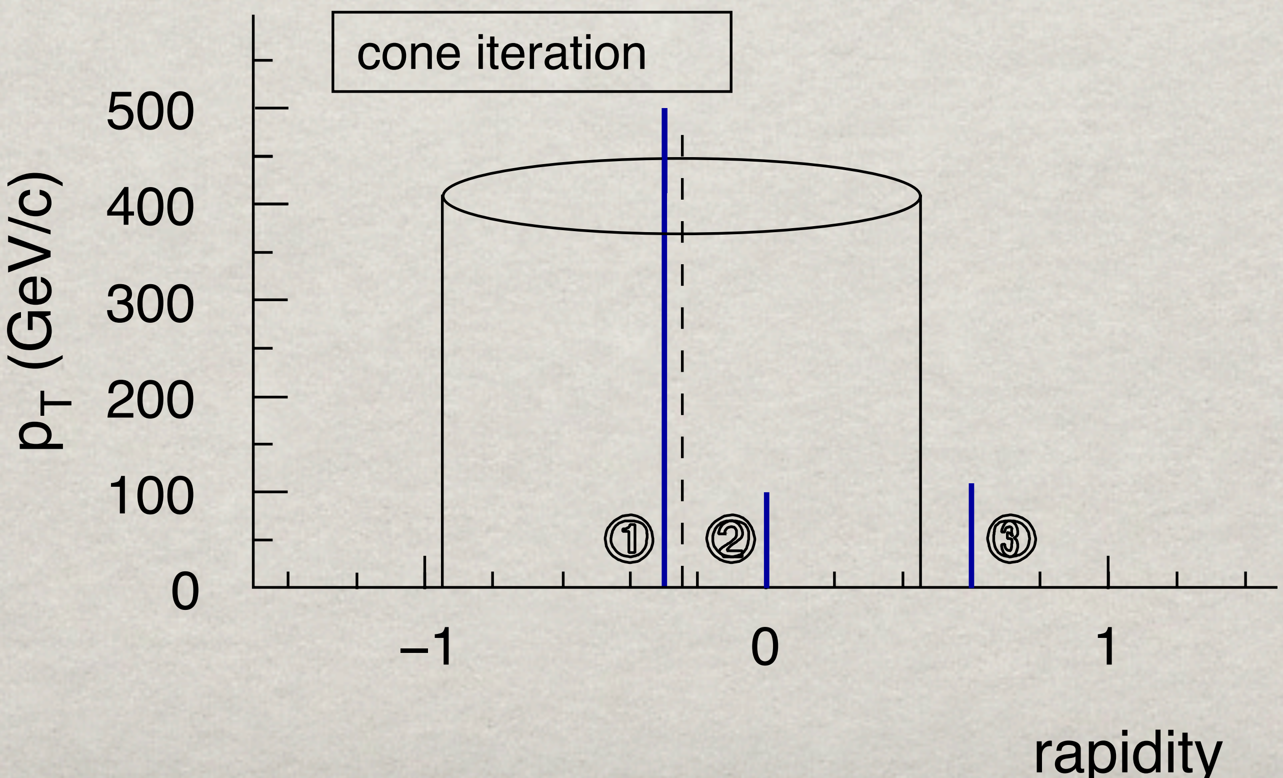
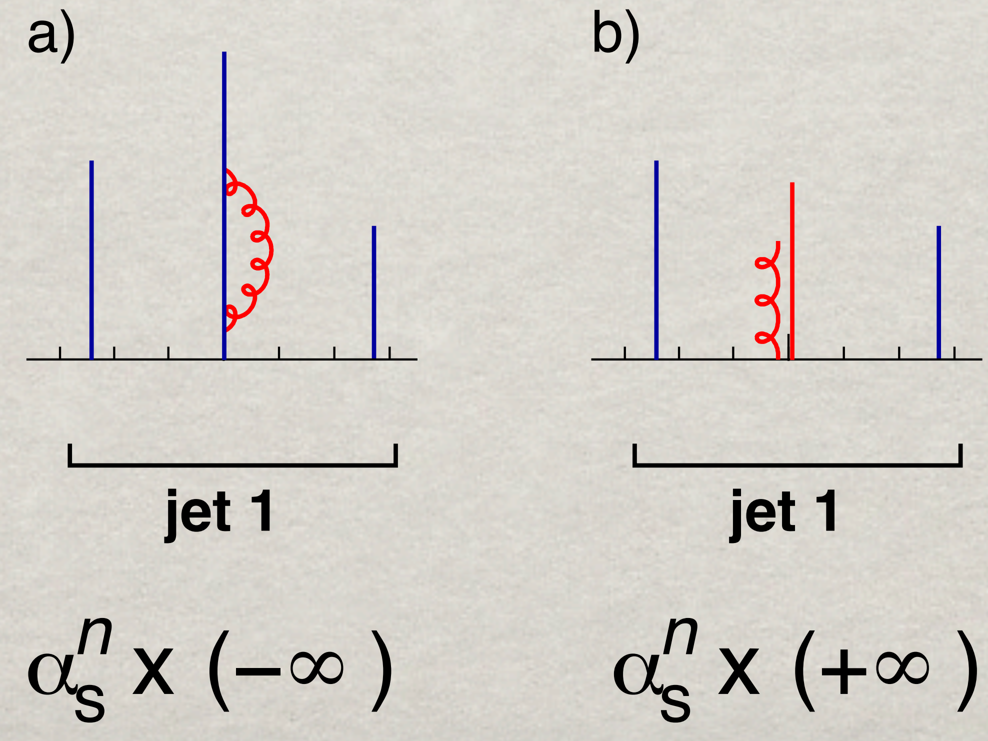


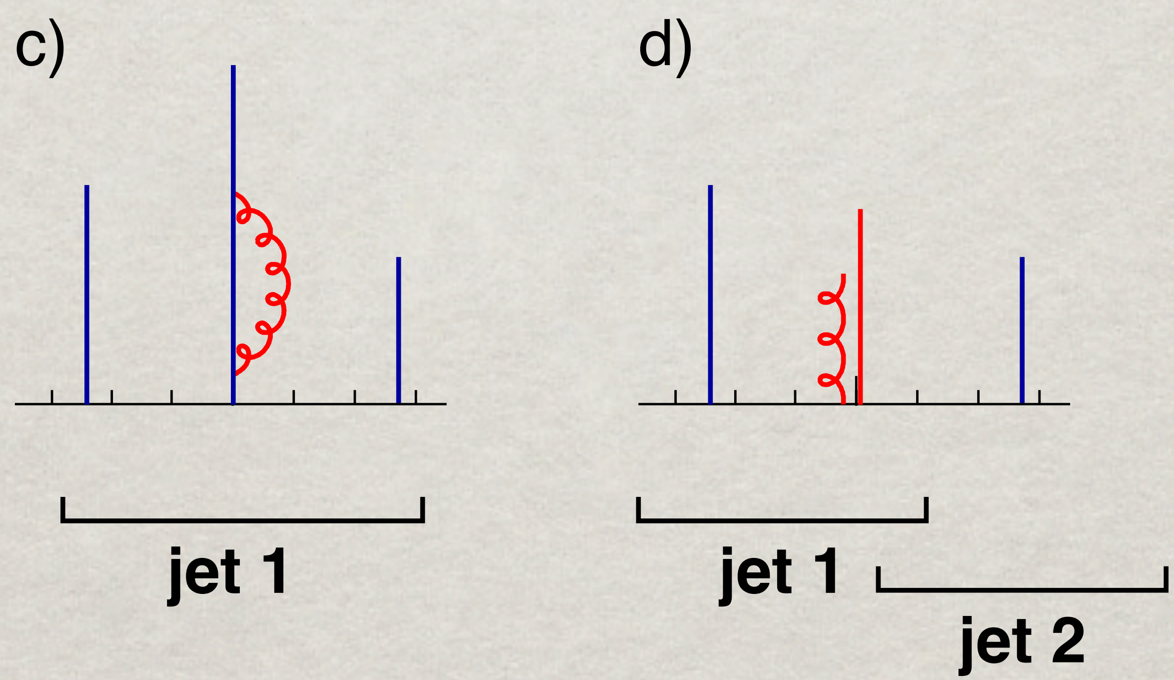
Issues with seeded cones

- Any IC procedure should by supplemented with the answers to the following questions
  - How does one choose the seeds?
  - How does one deal with "overlapping cones?"
- The stable cones found after IC procedure should not change after the addition of an extra-soft particle or a collinear splitting $\Rightarrow$ IRC safety



IRC unsafe cones (I)

- IC with progressive removal (IC-PR) uses the hardest particle as a trial seed, finds a stable cone, and then removes all the particles inside the found jet
- The "hardest particle" is not a collinear safe concept!

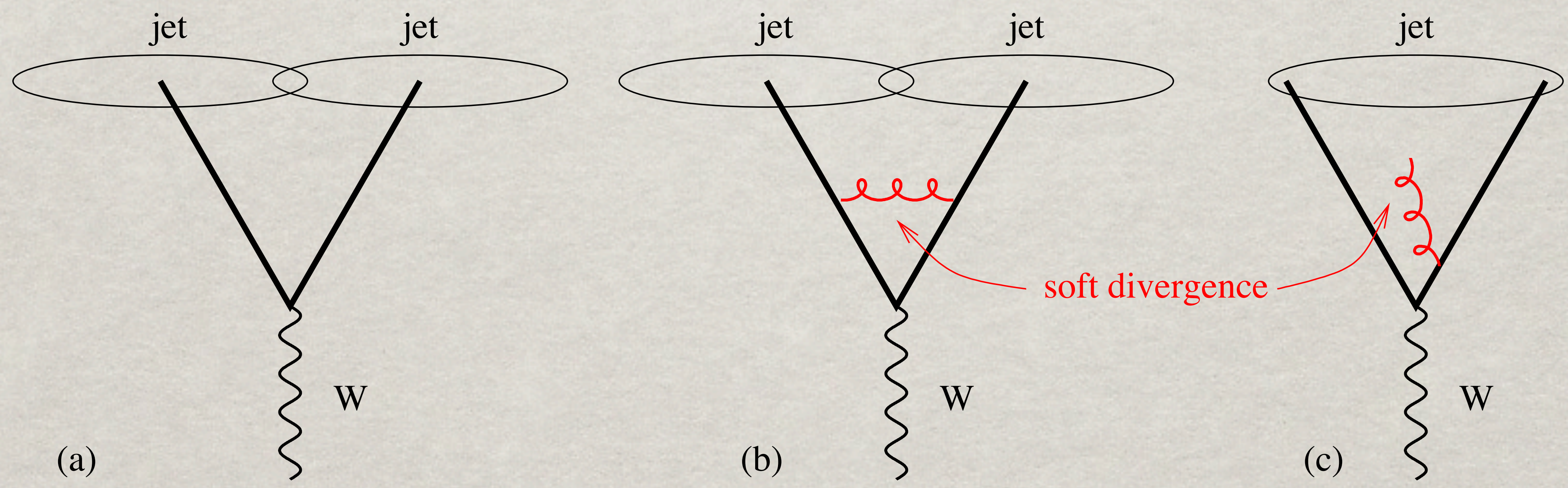


$\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$

- Real and virtual corrections give rise to different stable cones IC-PR jet cross sections diverge!

IRC unsafe cones (II)

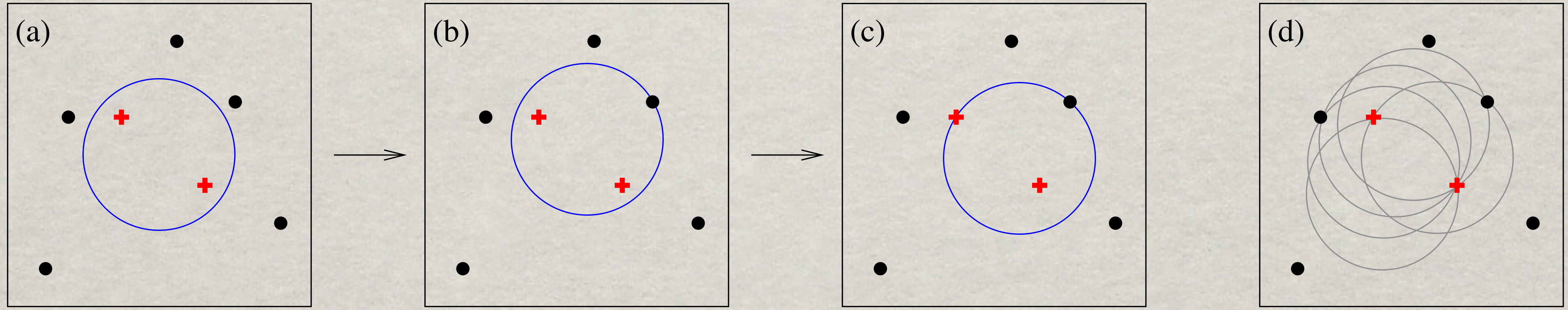
- IC-SM algorithm tries to find all stable cones using all particles as seeds
- Many cones have overlapping regions: split-merge (SM) procedure to assign particles to a single jet
- New stable cones are found after the addition of a number of extra-soft particles $\Rightarrow$IR unsafety



- At the moment there is no IRC safe seeded cone algorithm

Seedless cones

- The final solution to IRC safety is to devise a procedure that finds all stable cones
- Natural solution: consider all subset of particles and check if each corresponds to a stable cone
- Problem: for N particles computing time grows as $N2^N$, i.e. clustering 100 particles would take 10^{17} years!
- New solution: use computational geometry to find all stable cones $\Rightarrow$ computing time grows as $N^2 \ln N$!

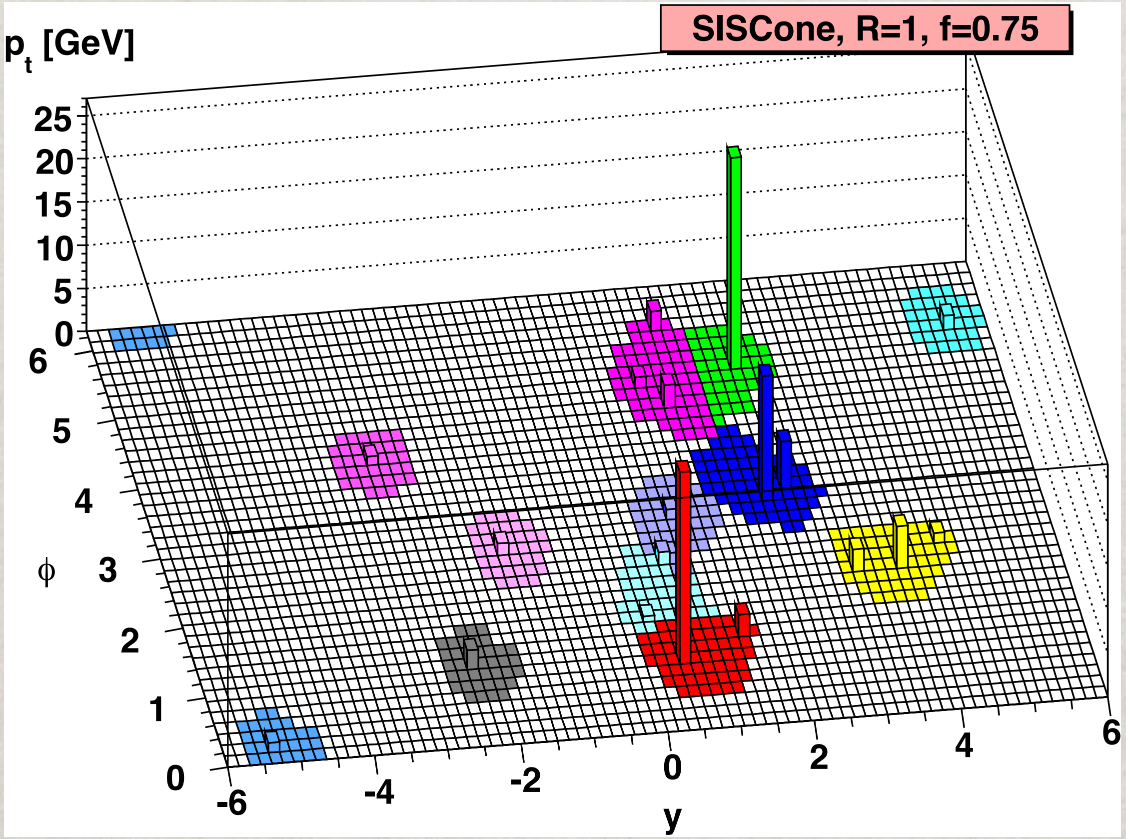


- This is the Seedless Infrared Safe Cone algorithm (SISCone)

[Salam Soyez JHEP 05 (2007) 086]

Cone highlights

- Cone algorithms are tied to the idea that in hard QCD events the energy is distributed around definite directions
- Hope is that particles clustered within the same jet form a circle of radius R in the y-ϕ plane $\Rightarrow$ easy subtraction of a uniform background



- However, in real life, due to the split-merge procedure, jets from cone algorithms do not usually have circular area

Sequential algorithms

- Sequential algorithms are iterative procedures that try to reverse the pattern of QCD multi-gluon emissions
- Example: Durham algorithm in e^+e^- annihilation
  [Catani Dokshitzer Olsson Turnock Webber PLB 269 (1991) 432]
  - For each pair of particles p_i, p_j compute the distance

$$\boxed{y_{ij} = 2\frac{\min\{E_i^2, E_j^2\}}{Q^2}(1-\cos\theta_{ij}) \simeq k_{t,ij}^2}$$

  - If $y_{ij} < y_{\rm cut}$ cluster the two particles, for instance via the replacement $p_i \to p_i + p_j$ (E scheme)
  - Repeat the procedure until all $y_{ij} > y_{\rm cut}$
- The Durham distance measure y_{ij} is tailored to the QCD soft and collinear emission probability ⇒ IRC safe by construction

$$dP_{q\to q_i\,g_j} \sim \frac{dE_j}{E_j}\frac{d\cos\theta_{ij}}{1-\cos\theta_{ij}} \sim \frac{dz}{1-z}\frac{d\theta_{ij}^2}{\theta_{ij}^2}$$

The kt algorithm

- The generalisation of the Durham algorithm to hadron collisions is the k_t-algorithm

[Ellis Soper PRD 48 (1993) 3160]

- Inclusive version of the k_t-algorithm
  - For any pair of particles p_i, p_j find the minimum of

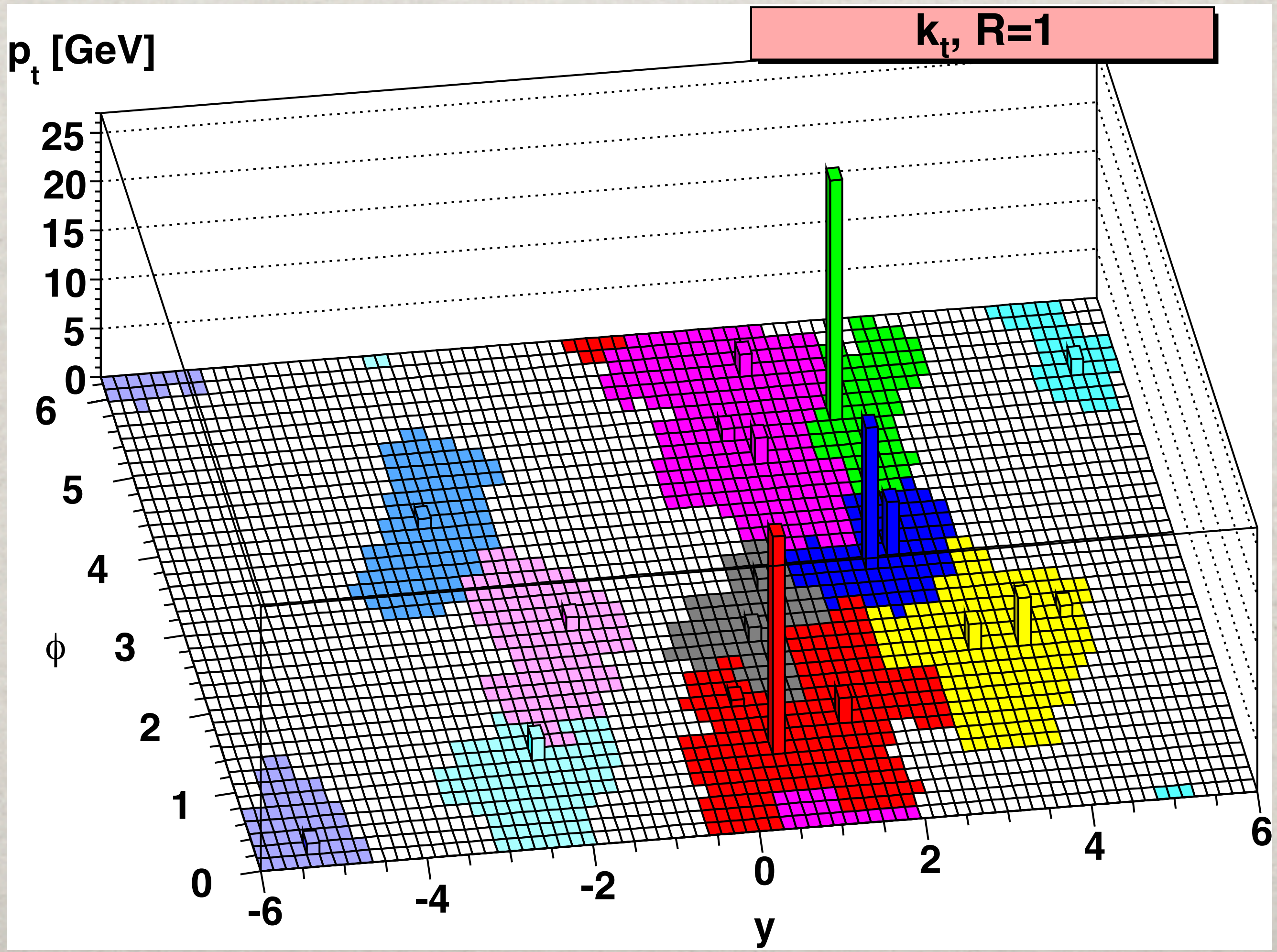
$$\boxed{d_{ij} = \frac{\min\{k_{ti}^2, k_{tj}^2\}}{R^2}\Delta R_{ij}^2 \simeq k_{t,ij}^2, \quad d_{iB} = k_{ti}^2, \quad d_{jB} = k_{tj}^2}$$

  - If the minimum distance is d_{iB} or d_{jB}, particle p_i or p_j is a jet and is removed from the list of particles, otherwise p_i and p_j are merged
  - Repeat the procedure until no particles are left

- Note: as for cones the IRC safe quantity is the number of jets with transverse momentum above $p_{t,\min}$

Issues with kt algorithm

- Drawbacks of k_t -algorithm with respect to cones
  - Speed: naive implementation of sequential algorithms has execution time that scales as N^3 $\Rightarrow$ use of computational geometry reduces computing time to $N \ln N$
  - Jet boundary has irregular shape $\Rightarrow$ subtracting a uniform soft-particle background becomes problematic

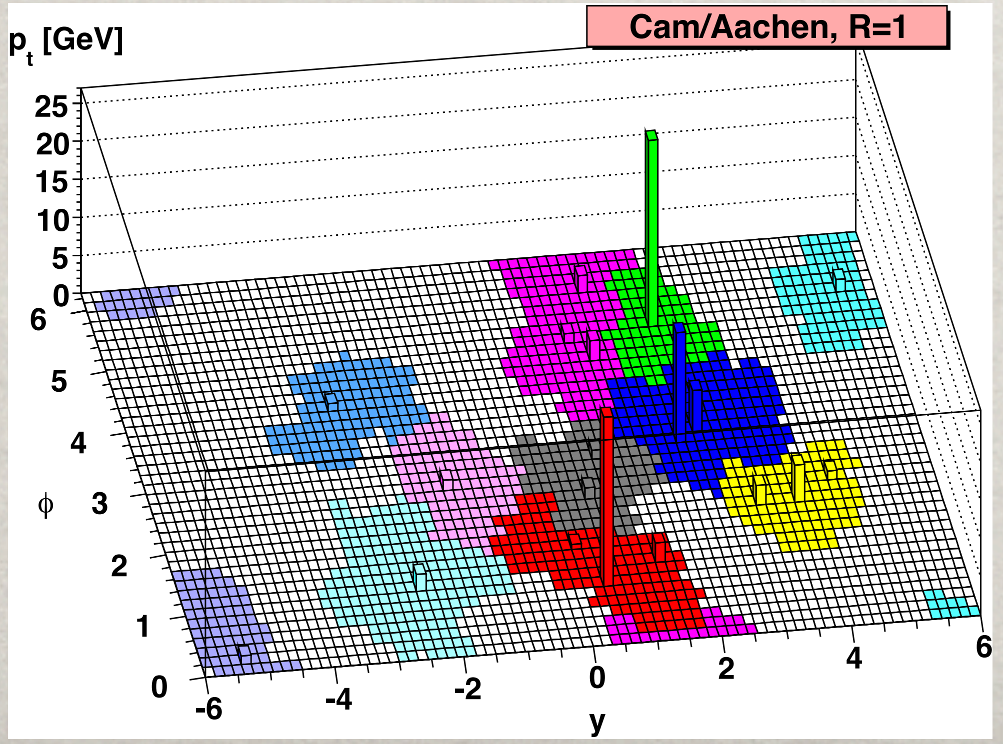


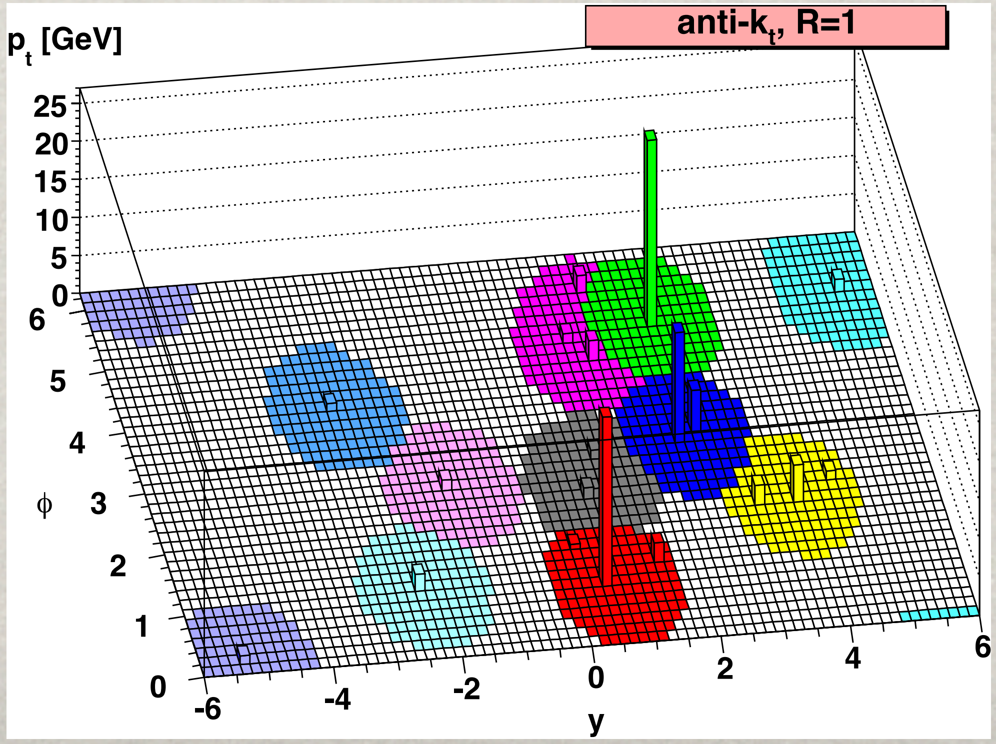
Generalised kt measures

- The k_t algorithm can be generalised by modifying the distance measures [Cacciari Salam Soyez JHEP 04 (2008) 063]

$$d_{ij} = \frac{\min\{k_{ti}^{2p}, k_{tj}^{2p}\}}{R^2} \Delta R_{ij}^2, \quad d_{iB} = k_{ti}^{2p}, \quad d_{jB} = k_{tj}^{2p}$$

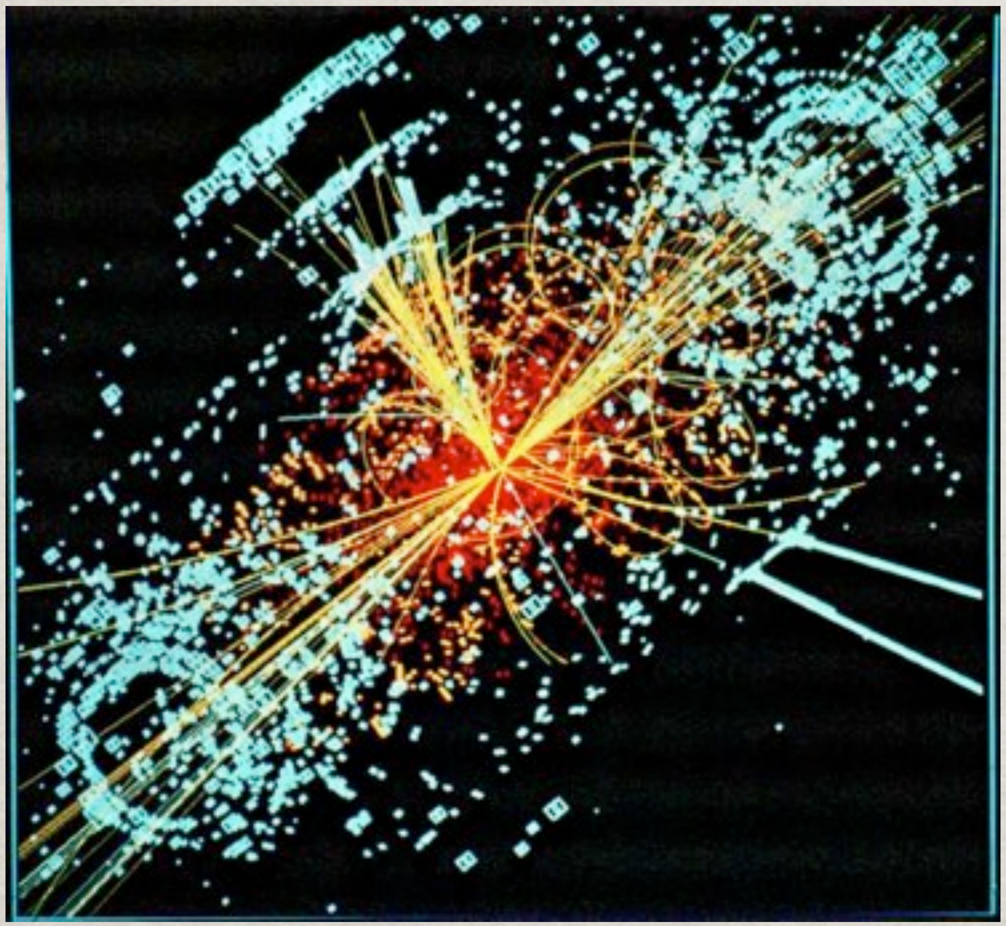
- $p = 0$: Cambridge-Aachen, distance variable is angle, reverts angular-ordered branching of QCD radiation
- $p = -1$: anti-k_t algorithm, particles are clustered around the hardest ones $\Rightarrow$ perfectly circular cones!

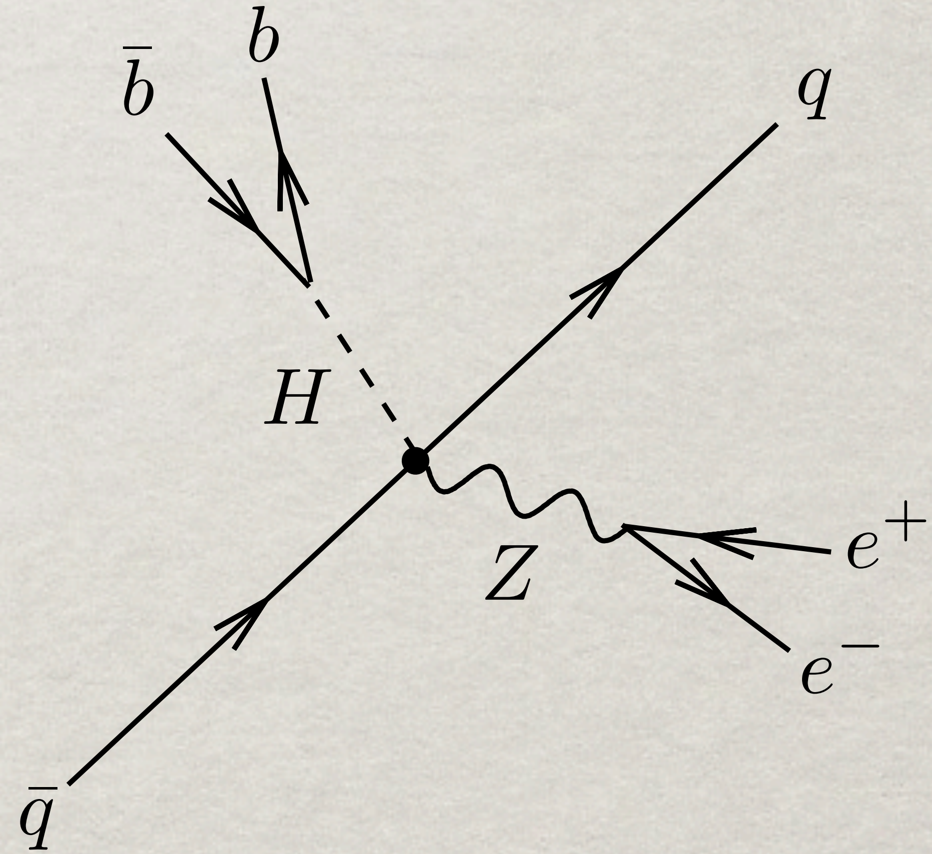




Jets at hadron colliders

- At the LHC, many procedures for finding New Physics exploit properties of jets

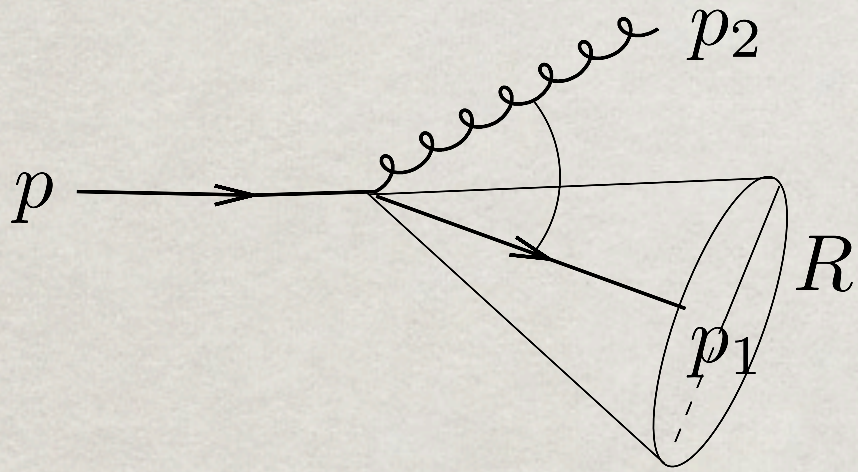




- One needs to understand how the characteristics of a jet, namely its transverse momentum and mass, are affected by both perturbative (QCD radiation) and non-perturbative effects (hadronisation, UE)

Pt of a jet in QCD

- In the small-R limit it is possible to relate the transverse momentum of the highest-p_t jet to that of its parent parton



$$p_{t1} = zp_t$$

$$p_{t2} = (1-z)p_t$$

- In any sequential algorithm, two particles p_1 and p_2 are not clustered together if and only if $\Delta R_{12} > R$
- After a collinear splitting, if recombination does not occur, the transverse momentum of the leading jet will be given by

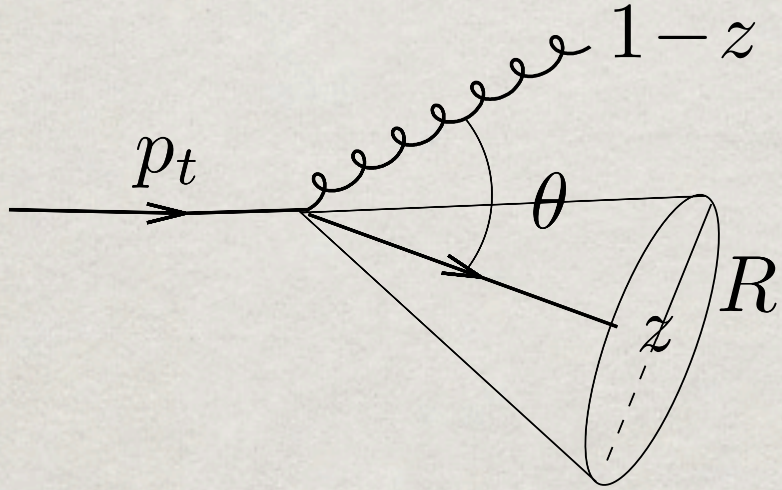
$$\boxed{p_{t,\text{jet}} = \max\{z, (1-z)\}p_t}$$

- Naive expectation: the smaller the radius, the larger the p_t loss due to gluon radiation

Pt loss of a quark jet

- Consider a jet initiated by a quark

[Dasgupta Magnea Salam JHEP 02 (2008) 055]



- We can then compute the average p_t loss of a quark jet in the small-R limit as (setting $\theta \equiv \Delta R_{12}$)

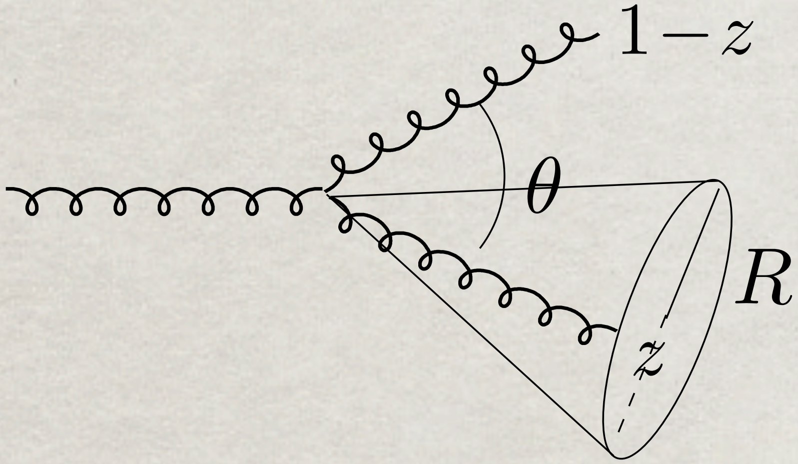
$$\langle \delta p_t \rangle_q = C_F \int \frac{d\theta^2}{\theta^2} dz\, \underbrace{p_t \min[z,(1-z)]}_{\delta p_t} \frac{\alpha_s[(1-z)\theta p_t]}{2\pi} \frac{1+z^2}{1-z} \Theta(\theta - R)$$

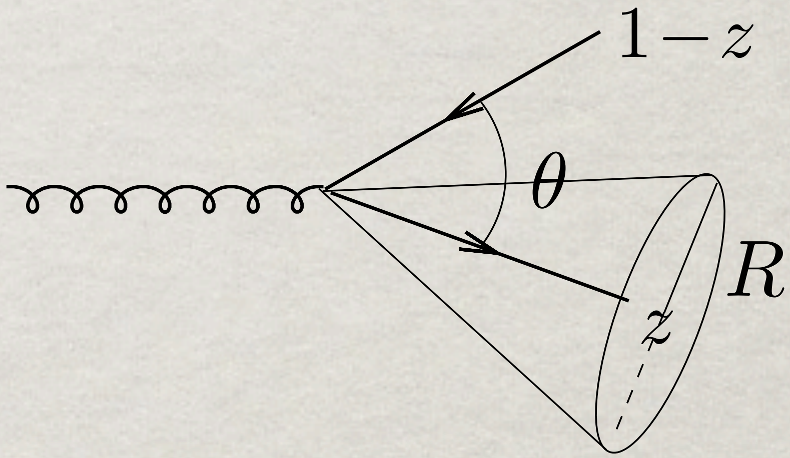
- This gives a relative transverse momentum loss

$$\boxed{\frac{\langle \delta p_t \rangle_q}{p_t} = -\frac{\alpha_s}{\pi} C_F \left(2\ln 2 - \frac{3}{8}\right) \ln\frac{1}{R} \simeq -0.43\,\alpha_s \ln\frac{1}{R}}$$

Pt loss of a gluon jet

- If a jet is initiated by a gluon one has to consider that a gluon can decay in gluons and quark-antiquark





$$dP_{g\to gg} \sim C_A\, dz \left[\frac{z}{1-z} + \frac{1-z}{z} + z\,(1-z)\right] \qquad dP_{g\to q\bar{q}} \sim T_R\, dz\, [z^2 + (1-z)^2]$$

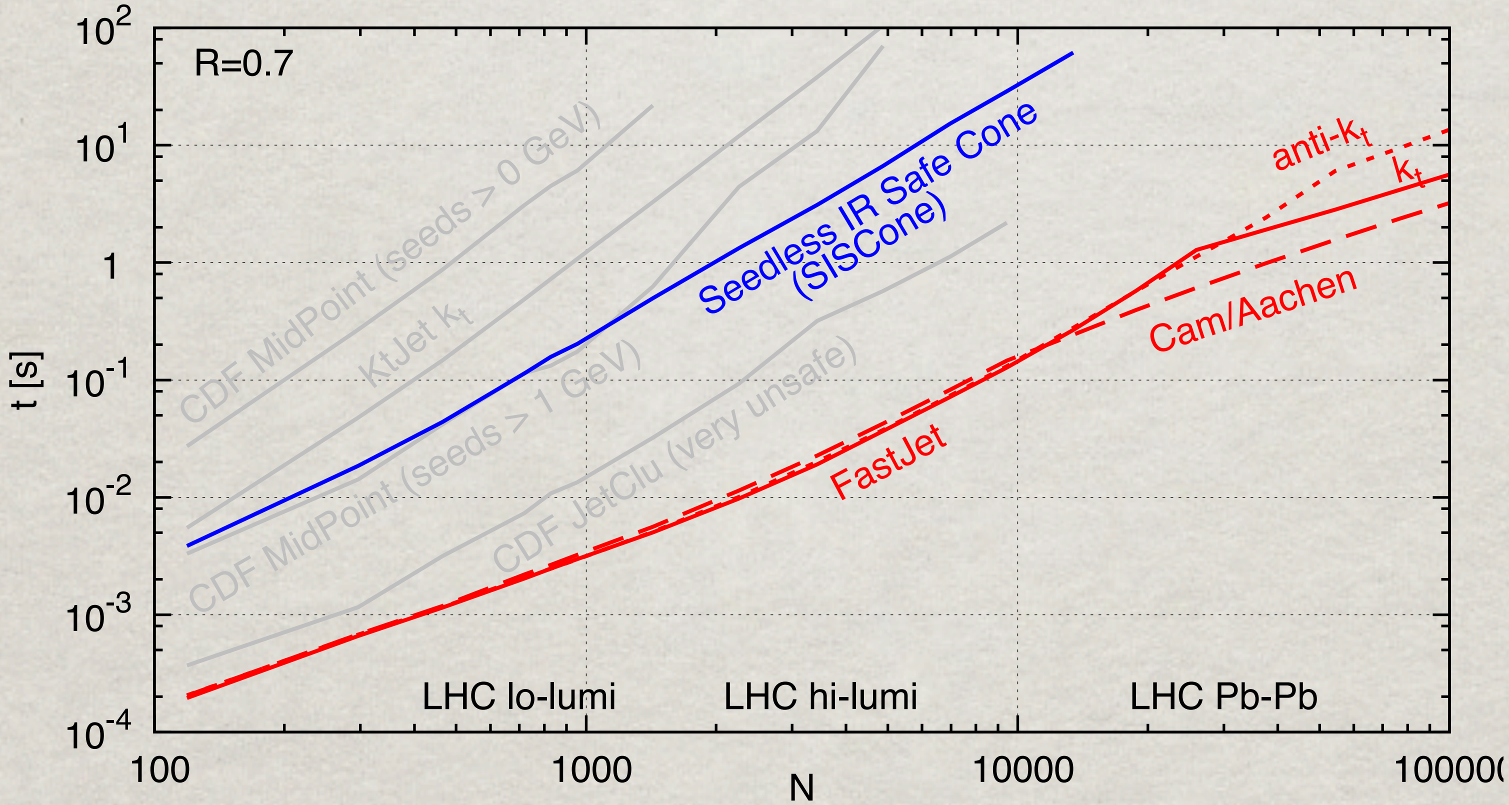
- Performing the same calculation as for quark jets

$$\boxed{\frac{\langle \delta p_t \rangle_g}{p_t} \simeq -(0.94 C_A + 0.15 T_R n_f)\frac{\alpha_s}{\pi} \ln\frac{1}{R} \simeq -1.02\,\alpha_s \ln\frac{1}{R}}$$

- For instance, for $R = 0.4$, a quark-jet has a p_t that is 4-5% smaller than that of the parent parton, while the corresponding loss for gluon jets is 8-10%

Summary

- We have a variety of efficient IRC safe jet algorithms for hadron collisions [Cacciari Salam Soyez http://fastjet.fr]



- Cone algorithms (SISCone, anti-k_t) and clustering algorithms (k_t, C/A) have complementary sequences of recombination (top-down vs bottom-up)
- Relevant properties of IRC safe jets (transverse momentum loss, mass) can be estimated using perturbative QCD