Topology in chaotic scattering

In chaotic scattering, an initially freely moving orbit (such as that of an atom or a star) enters a scattering region and evolves chaotically for a period of time before it escapes and returns to free motion[1]. We have looked at cases in which escape can occur in three or more distinct ways. Using a laboratory model, we demonstrate experimentally that the regions of state space (called basins) corresponding to different ways of escaping can have an interesting topological property that we call the Wada property, by which we mean that these regions of state space might be so convoluted that every point on the boundary of a basin is on the boundary of all basins[2,3].

Chaotic scattering enables us to model complex phenomena in several physical systems, including chemical reactions[4], celestial mechanics[5], fluid dynamics[6] and electron scattering in semiconductor microstructures[7]. In our laboratory model, we consider the path of a light ray as it reflects from a scatterer consisting of four mirrored balls[8] arranged as in Fig. 1a. A light ray entering the ‘inner chamber’ between the four balls can leave through one of four openings (Fig. 1a, b). Thus we can identify four basins, each consisting of the collection of light rays that leave through one of the four openings. The boundary separating these basins has the Wada property, which is important in chaotic scattering situations because it impedes one’s ability to predict the escape mode (for our model, the opening through which a light ray will leave the scatterer) from an initial condition.

It has previously been shown (reviewed in refs 3 and 9) that a division of space into three or more regions can have this strange topology. A theorem has been proved (reviewed in ref. 10) that implies that the boundary separating the basins (in the complex plane) of the four roots, found by using Newton’s method for a fourth-degree polynomial, has the topology of the basin boundary that we examine here.

The photograph in Fig. 2 shows a view looking in through one of the openings of the scatterer. Two of the colours, red and blue, are the reflections of poster boards placed outside two of the openings. Light reflects from a poster board; then, having taken the board’s colour, it reflects off the spheres, perhaps many times, before entering the camera lens. (Similar effects create the white patches: we passed light through a white diffusive filter before allowing it to pass into the scatterer by the third opening.) The black areas result from our having left the fourth (viewing) opening uncovered and the room dark. Had light been shone on the camera and photographer, their reflections would have been visible in the black areas.

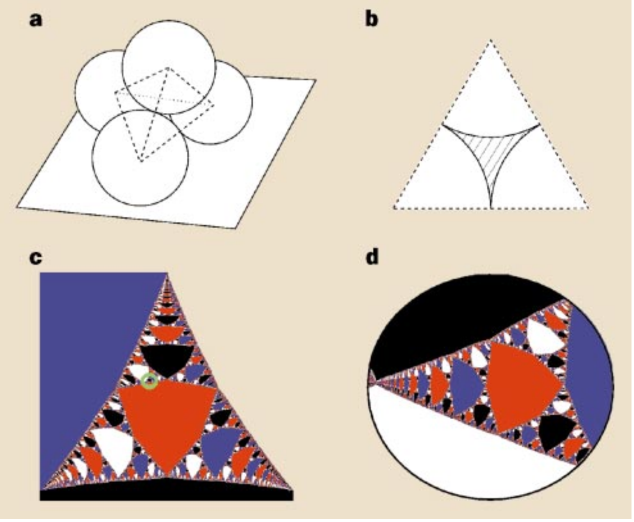

Figure 1 The light-scattering system used in our laboratory model. Four mirrored spheres (approximately 15 cm in radius) are stacked like cannonballs, three resting on the table and the fourth placed on top of the three so that they touch each other. **a,** A diagram of the scatterer. The dashed and dotted lines show how the sphere centres are located at the vertices of a tetrahedron. A light ray can enter or leave the inner chamber of the scatterer through one of four openings lying in the four faces of the tetrahedron. Once a light ray leaves, it cannot return to the inner chamber. **b,** Diagram of the shape (shaded) of one of the openings on a tetrahedron face. **c,** Plot of basins created by using numerical simulation. Light rays are initiated on a portion of the surface of one of the spheres that faces into the scatterer. Each light ray is aimed away from the viewer. **d,** If a small-diameter laser beam were shone on the circled region in **c**, parts of the beam would land in each of the four basins, as indicated by the presence of all four colours in this enlargement of the circled region. Enlargements of successively smaller sections of the boundary always contain all four colours. This suggests that the basins have the Wada property, where every point on the boundary of a basin is on the boundary of all basins.

In Fig. 2, the four colours create a map of the four basins. If an observer were to aim a narrow beam of light from his or her eye to a blue patch, for instance, that beam would hit the blue poster board after some reflections. It would leave the scatterer through the ‘blue’ opening and be said to have started in the blue basin. The boundary between the four colours is a fractal, and its fractal dimension, determined from numerical simulation, is approximately $D = 1.6$. Thus, our work provides a simple laboratory demonstration of an interesting type of fractal geometry.

To study this system numerically, we used the geometrical optics approximation (in other words, diffraction is neglected) in which the trajectory of a light ray reflecting from the spheres corresponds to that of a point particle bouncing off the spheres. We plotted a picture similar to that in Fig. 2 (see Fig. 1c) and magnified regions containing points in the basin boundary (for example, Fig. 1d). We find that no matter where, or by how much, we magnify around a boundary point, we always see all four basin colours. Thus, arbitrarily close to any boundary point we find points in all four basins, so every boundary point is on the boundary of all four basins.

Imagine performing an experiment on a scattering system with the Wada property. We would measure the initial conditions within some error bounds and record the outcome of the experiment. If the initial condition were sufficiently near the basin boundary then the small random error in the initial condition could push the experiment

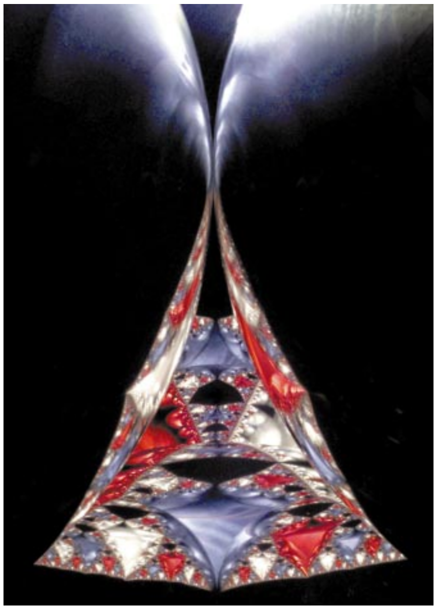

Figure 2 Photograph of the laboratory model of the scatterer described in Fig. 1. Red and blue poster boards were placed outside two of the upper openings and a white cloth was placed below. Their reflections created the coloured pattern seen through the fourth opening. The boundary separating the colours has a fractal dimension of ~1.6.

into any of the possible outcomes because of the Wada property.

We have verified the Wada property experimentally for our laboratory model by using a beam of 0.48 mm diameter from a 0.95 mW helium–neon laser. If the beam is shone on a boundary point, we find that the laser light can be seen through each of the openings. However, if the beam is aimed at the interior of one of the coloured regions, the light is seen through only one opening, casting a bright spot on the corresponding poster board.

We believe that Wada boundaries should be a typical feature in chaotic scattering systems that have more than two exit modes. We have verified the existence of Wada boundaries in a situation involving chemical reactions[4] with two and three degrees of freedom.

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Rainfall characteristics of hurricane Mitch

The hurricane or tropical storm known as Mitch struck Central America towards the end of October 1998. The subsequent flooding and landslides claimed approximately 11,000 lives. It was the most deadly hurricane to strike the Western Hemisphere in two centuries. We have measured rainfall totals during Mitch (from 1 to 48 hours) and find that they were not exceptional for hurricanes and tropical storms in the Atlantic basin. Rainfall totals and intensities measured over intervals of 1, 2, 5, 10, 30 and 60 minutes were less than values from the updated maximum potential rainfall curve[1,2]. The data suggest that extraneous factors, such as already saturated soils and denuded hillsides, were largely responsible for the damage caused.

Detailed rainfall data from hurricanes are often lacking because rainfall-recording instruments are washed away. The emphasis has been on recording wind strengths, which can underestimate the damage potential.

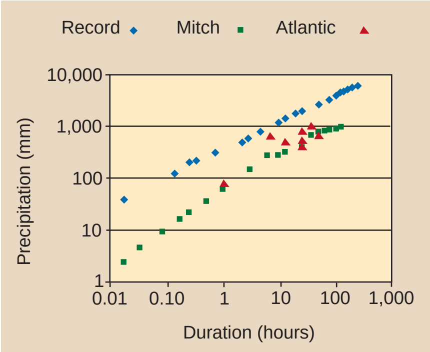

Figure 1 Plot of maximum rainfall amounts (squares) against duration from a rain gauge in southern Honduras during hurricane/tropical storm Mitch ($y=51.64x^{0.674}$, $R^2=0.99$). Also plotted are updated record rainfall events (diamonds) for different durations[12] that define the curve of maximum potential rainfall ($y=353.07x^{0.519}$, $R^2=0.99$) and data (triangles) from recent major Atlantic hurricanes and tropical storms[3–5].

Rainfall distribution is often estimated from radar and satellite data with little ground-truth support. Data from an autographic rain gauge in southern Honduras, where flooding and landslides were extensive, provide a unique insight into the rainfall distribution during Mitch. The tipping-bucket rain gauge (ELE DRG-52) was located at 87° 04′ W and 13° 17′ N on a hill crest (100 m above sea level) in the foothills of Cerro Guanacaure (1,007 m above sea level).

Mitch formed on 21 October 1998 in the southwest Caribbean and reached category 5 hurricane status on the Saffir/Simpson scale as it moved towards northern Honduras. On 29 October, Mitch was downgraded to a tropical storm and moved southwards and inland. The storm was declared to be over on 1 November.

Mitch produced torrential rains despite being downgraded from a hurricane. Between 18:00 (Honduran time, which is 6 hours before GMT) on 27 October and 21:00 on 31 October, there was 896 mm of rainfall. The most intense and prolonged rainfall was between 15:00 on 29 October and 07:00 on 31 October (a total of 41 hours). During this period, 698 mm of rain fell. There were two distinct periods of extreme rainfall intensities: 186 and 245 mm of rain fell during six-hour periods from 16:00 to 22:00 on 29 and 30 October, respectively. Maximum intensities ranged from 138 mm h^{-1} (2-minute period) to 58.4 mm h^{-1} (60-minute period). Landslides affected approximately 20% of surrounding hillsides and typically occurred during the two periods of most intense rainfall. Simultaneously, the Choluteca river flooded adjacent settlements and parts of the city of Choluteca. The river also altered its course.

Although Mitch caused extensive flooding and loss of life, Fig. 1 shows that rainfall in southern Honduras was comparable to the severest hurricanes and tropical storms in the Atlantic basin[3–5]. Rainfall was also substantially less than the updated maximum potential rainfall curve, which is defined largely by rainfall events from La Réunion, Indian Ocean[1,2], where there might be greater topographic forcing. The data suggest that extensive damage in Honduras and Nicaragua was accentuated by several factors: the storm struck at the end of the rainy season when the soil was saturated, resulting in catastrophic flooding and landslides; agricultural extension caused by land pressures had left many hillsides denuded; and the population was ill prepared because, before making landfall, it had been predicted that Mitch would move northwards.

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Analysis of telomere lengths in cloned sheep

The development of nuclear-transfer techniques using cultured somatic cells[1–4] allows animals to be produced without involving germline cells. This enables us to examine the importance of the repair of chromosome ends (telomeres) in the germ line and to test the telomere hypothesis of ageing[5–8].

To investigate whether telomere erosion is repaired after nuclear transfer, the telomere lengths of animals 6LL3 (Dolly)[1], 6LL6 and 6LL7 were compared with age-matched control sheep, donor mammary gland tissue and donor cells before and after culturing (Fig. 1a). Animal 6LL3 was derived from the transfer of a nucleus from sheep (ovine) mammary epithelial (OME) cells from a six-year-old Finn Dorset sheep; 6LL6 was derived from sheep embryonic cells (SEC 1) obtained from day 9 Poll Dorset embryos; and 6LL7 was derived from fibroblasts[1] from a day 25 Black Welsh fetus.