Problem Set 3

This problem set is due **at 11:59pm** on **Thursday, February 26, 2015.**

Please turn in each problem solution separately. Each submitted solution should start with your name, the course number, the problem number, your recitation section, the date, and the names of any students with whom you collaborated.

Exercise 3-1. Read CLRS, Section 20.3.

Exercise 3-2. Exercise 20-3.1.

Exercise 3-3. Exercise 20-3.2.

Problem 3-1. Variants on van Emde Boas [25 points]

For each of the following variants on the van Emde Boas data structures (presented in Lecture 4 and CLRS, Section 20.3), carefully describe what changes are needed to the pseudocode (from either lecture or the textbook), and analyze the costs of the vEB operations INSERT, DELETE, and SUCCESSOR, comparing them with the costs of the same operations for the original vEB structure.

(a) [7 points] Instead of dividing the structure into $u^{1/2}$ groups of $u^{1/2}$ numbers each, use $u^{1/3}$ groups of $u^{2/3}$ numbers each.

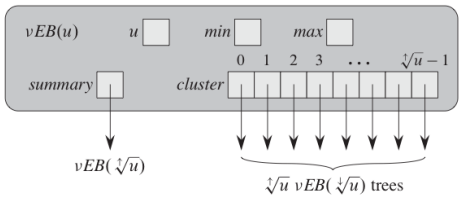
(b) [18 points] In addition to excluding the *minimum* element from lower-level vEB structures, also exclude the *maximum* element from lower-level vEB structures (and store it in the already existing max attribute). (Use the original division into $u^{1/2}$ groups of $u^{1/2}$ numbers here.)

20.3. van Emde Boas trees.

$T(u) \le T(\sqrt{u}) + O(1)$

$\Rightarrow T(u) = O(\lg \lg u)$

space: $O(u)$




```
VEB-TREE-SUCCESSOR(V, x)
 1  if V.u == 2
 2      if x == 0 and V.max == 1
 3          return 1
 4      else return NIL
 5  elseif V.min ≠ NIL and x < V.min
 6      return V.min
 7  else max-low = VEB-TREE-MAXIMUM(V.cluster[high(x)])
 8      if max-low ≠ NIL and low(x) < max-low
 9          offset = VEB-TREE-SUCCESSOR(V.cluster[high(x)], low(x))
10          return index(high(x), offset)
11      else succ-cluster = VEB-TREE-SUCCESSOR(V.summary, high(x))
12          if succ-cluster == NIL
13              return NIL
14          else offset = VEB-TREE-MINIMUM(V.cluster[succ-cluster])
15              return index(succ-cluster, offset)
```

look at current cluster

look at summary

P3.1.

a). $T(u) = T(u^{2/3}) + c.$

$T(u) = O(\lg \lg u)$

b). too !!! hard.

6.046J / 18.410J Design and Analysis of Algorithms
Spring 2015