

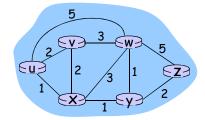
Graph abstraction

- Graph: G = (N,E)
- N = set of routers = { u, v, w, x, y, z }
- E = set of links ={ (u,v), (u,x), (v,x), (v,w), (x,w), (x,y), (w,y), (w,z), (y,z) }
- Remark: Graph abstraction is useful in other network contexts
- Example: P2P, where N is set of peers and E is set of TCP connections

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Graph abstraction: costs

- c(x,x') = cost of link (x,x')
 e.g., c(w,z) = 5
- cost could always be 1, or inversely related to bandwidth, or inversely related to congestion



- Cost of path $(x_1, x_2, x_3, ..., x_p) = c(x_1, x_2) + c(x_2, x_3) + ... + c(x_{p-1}, x_p)$
- Question: What's the least-cost path between u and z?
- Routing algorithm: algorithm that finds least-cost path

Routing Algorithm classification

Global or decentralized information?

Global:

- all routers have complete topology, link cost info
- "link state" algorithms

Decentralized:

- router knows physicallyconnected neighbors, link costs to neighbors
- iterative process of computation, exchange of info with neighbors
- "distance vector" algorithms

Static or dynamic?

Static:

routes change slowly over time

Dynamic:

- routes change more quickly
 - · periodic update
 - in response to link cost changes

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A Link-State Routing Algorithm

Dijkstra's algorithm

- net topology, link costs known to all nodes
 - accomplished via "link state broadcast"
 - all nodes have same info
- computes least cost paths from one node ('source") to all other nodes
 - gives forwarding table for that node
- iterative: after k iterations, know least cost path to k destinations

Notation:

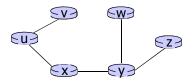
- C(x,y): link cost from node x to y; = ∞ if not direct neighbors
- D(v): current value of cost of path from source to dest. v
- p(v): predecessor node along path from source to v
- N': set of nodes whose least cost path definitively known

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Dijsktra's Algorithm
      1 Initialization:
     2 N' = \{u\}
     3 for all nodes v
     4
          if v adjacent to u
     5
             then D(v) = c(u,v)
     6
           else D(v) = \infty
     *8 Loop
          find w not in N' such that D(w) is a minimum
     10 add w to N'
     11 update D(v) for all v adjacent to w and not in N':
      12
           D(v) = \min(D(v), D(w) + c(w,v))
     13 /* new cost to v is either old cost to v or known
           shortest path cost to w plus cost from w to v */
      15 until all nodes in N'
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```

Dijkstra	a's algoritl	ım: exam	ple					
Step	N'	D(v),p(v)	D(w),p(w)	D(x),p(x)	D(y),p(y)	D(z),p(z)		
0	u	2,u	5,ú	1,ú	∞	∞		
1	ux ←	2,u	4,x		2,x	∞		
2	uxy←	2,u	3,y			4,y		
3	uxyv 🕶		3,y			4,y		
4	uxyvw 🗸					4,y		
5	uxyvwz 🗲							
	5 u 2 3 1 2 3 1 2							
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Dijkstra's algorithm: example (2)

Resulting shortest-path tree from u:



Resulting forwarding table in u:

destination	link		
V	(u,v)		
Χ	(u,x)		
У	(u,x)		
W	(u,x)		
Z	(u,x)		

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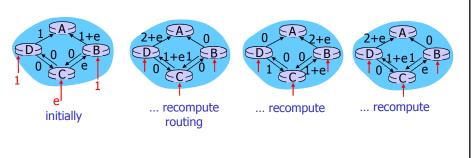
Dijkstra's algorithm, discussion

Algorithm complexity: n nodes

- each iteration: need to check all nodes, w, not in N
- n(n+1)/2 comparisons: O(n²)
- more efficient implementations possible: O(nlogn)

Oscillations possible:

• e.g., link cost = amount of carried traffic



Distance Vector Algorithm

Bellman-Ford Equation (dynamic programming)

• Define

 $d_x(y) := cost of least-cost path from x to y$

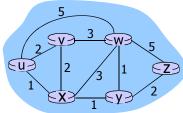
• Then

$$d_x(y) = \min \{c(x,v) + d_v(y)\}$$

where min is taken over all neighbors v of x

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Bellman-Ford example



Clearly,
$$d_v(z) = 5$$
, $d_x(z) = 3$, $d_w(z) = 3$

B-F equation says:

$$\begin{aligned} d_{u}(z) &= \min \big\{ \; c(u,v) + d_{v}(z), \\ &c(u,x) + d_{x}(z), \\ &c(u,w) + d_{w}(z) \; \big\} \\ &= \min \big\{ 2 + 5, \\ &1 + 3, \\ &5 + 3 \big\} \; = 4 \end{aligned}$$

Node that achieves minimum is next hop in shortest path → forwarding table

Distance Vector Algorithm

- $D_x(y)$ = estimate of least cost from x to y
- Distance vector: D_x = [D_x(y): y ∈ N]
- Node x knows cost to each neighbor v: c(x,v)
- Node x maintains D_x = [D_x(y): y ∈ N]
- Node x also maintains its neighbors' distance vectors
 - For each neighbor v, x maintains
 D_v = [D_v(y): y ∈ N]

Basic idea:

- Each node periodically sends its own distance vector estimate to neighbors
- When a node x receives new DV estimate from neighbor, it updates its own DV using B-F equation:

```
Dx(y) \leftarrow minv\{c(x,v) + Dv(y)\} for each node y \in N
```

 Under minor, natural conditions, the estimate Dx(y) converge to the actual least cost dx(y)

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Distance Vector Algorithm

Iterative, asynchronous: each

local iteration caused by:

- local link cost change
- DV update message from neighbor

Distributed:

- each node notifies neighbors only when its DV changes
 - neighbors then notify their neighbors if necessary

Each node:

wait for (change in local link cost of msg from neighbor)

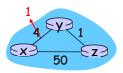
recompute estimates

if DV to any dest has changed, *notify* neighbors

Distance Vector: link cost changes

Link cost changes:

- node detects local link cost change
- updates routing info, recalculates distance vector
- if DV changes, notify neighbors



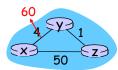
- "Good news travels fast"
 - At time t_0 , y detects the link-cost change, updates its DV, and informs its neighbors.
 - At time t_{J} , z receives the update from y and updates its table. It computes a new least cost to x and sends its neighbors its DV.
 - At time t₂, y receives z's update and updates its distance table. y's least costs
 do not change and hence y does not send any message to z.

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Distance Vector: link cost changes

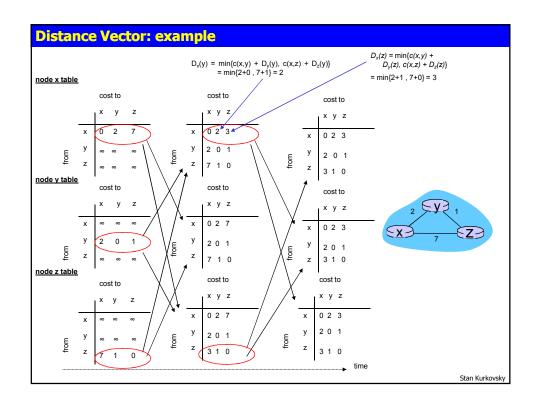
Link cost changes:

- good news travels fast
- bad news travels slow "count to infinity" problem!
- 44 iterations before algorithm stabilizes



Poissoned reverse:

- If Z routes through Y to get to X:
 - Z tells Y its (Z's) distance to X is infinite (so Y won't route to X via Z)
- will this completely solve count to infinity problem?



Comparison of LS and DV algorithms

Message complexity

- <u>LS:</u> with n nodes, E links, O(nE) msgs sent
- <u>DV:</u> exchange between neighbors only
 - · convergence time varies

Speed of Convergence

- LS: O(n²) algorithm requires O(nE) msgs
 - · may have oscillations
- **DV**: convergence time varies
 - · may be routing loops
 - count-to-infinity problem

Robustness: what happens if router malfunctions?

LS:

- node can advertise incorrect link cost
- each node computes only its own table

DV:

- DV node can advertise incorrect path cost
- each node's table used by others
 - error propagate thru network

Hierarchical Routing

Our routing study thus far - idealization

• all routers identical; network "flat" → ... not true in practice

scale: with 200 million destinations:

- can't store all dest's in routing tables!
- routing table exchange would swamp links!

administrative autonomy

- internet = network of networks
- each network admin may want to control routing in its own network
- aggregate routers into regions, "autonomous systems" (AS)
- routers in same AS run same routing protocol
 - "intra-AS" routing protocol
 - routers in different AS can run different intra-AS routing protocol

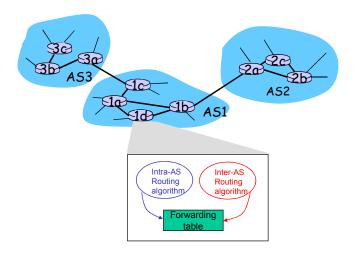
Gateway router

Direct link to router in another AS

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Interconnected ASes

- Forwarding table is configured by both intra- and inter-AS routing algorithm
 - Intra-AS sets entries for internal dests
 - Inter-AS & Intra-As sets entries for external dests



Inter-AS tasks

- Suppose router in AS1 receives datagram for which dest is outside of AS1
 - Router should forward packet towards one of the gateway routers, but which one?

AS1 needs:

- 1. to learn which dests are reachable through AS2 and which through AS3
- 2. to propagate this reachability info to all routers in $\ensuremath{\mathsf{AS1}}$

Job of inter-AS routing!

