

VINOBA INSTITUTE OF MATHEMATICS

REAL ANALYSIS

UNIT - I

ASSIGNMENT - II (Practice Sheet)

- 1) The function $f : \mathbb{R} \rightarrow [-1/2, 1/2]$ defined as $f(x) = \frac{x}{1+x^2}$, is
 - a. Surjective but not Injective
 - b. Neither injective nor surjective
 - c. Invertible
 - d. Injective but not surjective
- 2) For a real no. x let $[x]$ denote the greatest integer less than or equal to x .
Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 2x + [x] + \sin x \cos x$. Then f is
 - a. One-one but not onto
 - b. Onto but not one-one
 - c. Both one-one and onto
 - d. Neither one-one nor onto
- 3) The function $f : [0, 3] \rightarrow [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is
 - a. One-one and onto
 - b. Onto but not one-one
 - c. One-one but not onto
 - d. Neither one-one nor onto
- 4) For real x , let $f(x) = x^3 + 5x + 1$, then
 - a. f is one-one but not onto on \mathbb{R}
 - b. f is onto on \mathbb{R} but not one-one
 - c. f is one-one and onto on \mathbb{R}
 - d. f is neither one-one nor onto on \mathbb{R}
- 5) Let X and Y be two non-empty sets and $f : X \rightarrow Y$ be a function such that $f(C) = \{f(x) : x \in C\}$ for $C \subseteq X$ and $f^{-1}(D) = \{x : f(x) \in D\}$ for $D \subseteq Y$. If $A \subseteq X$ and $B \subseteq Y$, Then
 - a. $f^{-1}(f(A)) = A$
 - b. $f^{-1}(f(A)) = A$ only if $f(X) = Y$
 - c. $f(f^{-1}(B)) = B$ only if $B \subseteq f(X)$
 - d. $f(f^{-1}(B)) = B$
- 6) If $f(x) = \begin{cases} x, & x \in \mathbb{Q} \\ 0, & x \in \mathbb{Q}^c \end{cases}$ Then, $f - g$ is
 - a. One-one and into
 - b. Neither one-one nor onto
 - c. Many one and onto
 - d. One-one and onto
- 7) Let $f : \mathbb{N} \rightarrow Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in \mathbb{N} : y = 4x + 3 \text{ for some } x \in \mathbb{N}\}$. Then inverse of f is
 - a. $g(y) = \frac{y+3}{4}$
 - b. $g(y) = \frac{y-3}{4}$
 - c. $g(y) = \frac{3y+4}{3}$
 - d. $g(y) = 4 + \frac{y+3}{4}$
- 8) $f : \mathbb{N} \rightarrow \mathbb{Z}$

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$$
 is
 - a. Neither one-one nor onto
 - b. One-one but not onto
 - c. Onto but not one-one
 - d. One-one and onto both
- 9) The function $f : [0, \infty) \rightarrow \mathbb{R}$ given by $f(x) = \frac{x}{x+1}$, is
 - a. One-one and onto
 - b. One-one but not onto
 - c. Onto but not one-one
 - d. Neither one-one nor onto
- 10) If $f : [1, \infty) \rightarrow [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals
 - a. $\frac{x + \sqrt{x^2 - 4}}{2}$
 - b. $\frac{x}{1 + x^2}$
 - c. $\frac{x - \sqrt{x^2 - 4}}{2}$
 - d. $1 + \sqrt{x^2 - 4}$

