Numbers

Natural $\mathbb{N}$

Natural numbers are the counting numbers {1, 2, 3, ...} (positive integers) or the whole numbers {0, 1, 2, 3, ...} (non-negative integers). Mathematicians use the term "natural" in both cases.

Integer $\mathbb{Z}$

Integers are the natural numbers and their negatives {... −3, −2, −1, 0, 1, 2, 3 ...}. (Z is from German *Zahl*, "number".)

Rational $\mathbb{Q}$

Rational numbers are the ratios of integers, also called fractions, such as $\frac{1}{2}$ = 0.5 or $\frac{1}{3}$ = 0.3333... Rational decimal expansions end or repeat. (Q is from quotient.)

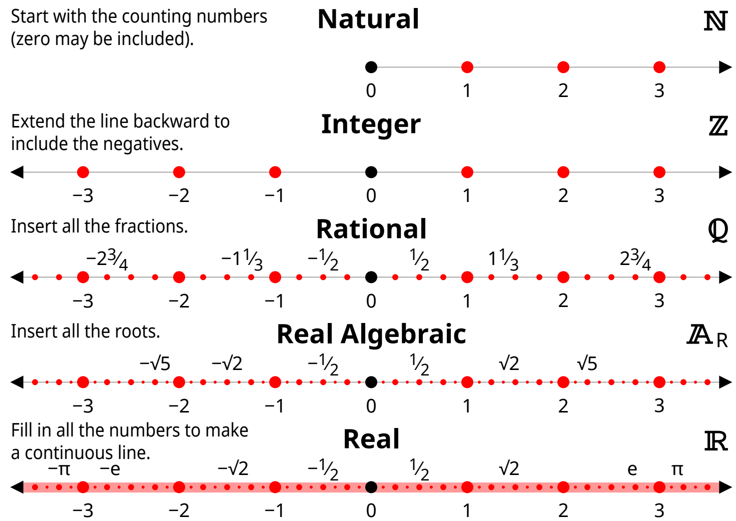
Real Algebraic $\mathbb{A}_R$

The real subset of the algebraic numbers: the real roots of polynomials. Real algebraic numbers may be rational or irrational. √2 = 1.41421... is irrational. Irrational decimal expansions neither end nor repeat.

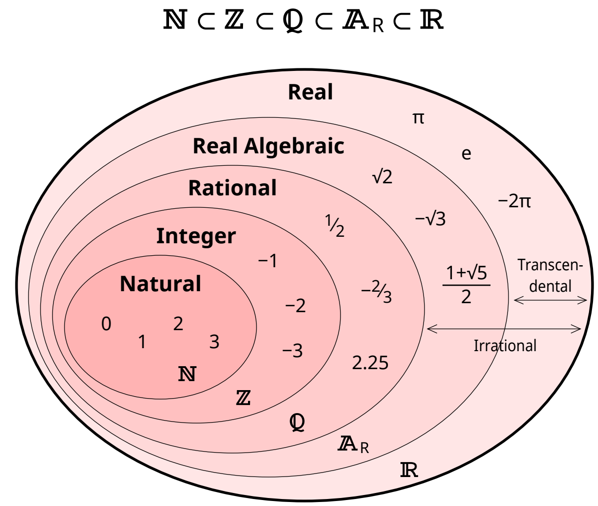
Real $\mathbb{R}$

Real numbers are all the numbers on the continuous number line with no gaps. Every decimal expansion is a real number. Real numbers may be rational or irrational, and algebraic or non-algebraic (transcendental). π = 3.14159... and e = 2.71828... are transcendental. A transcendental number can be defined by an infinite series.

Real Number Sets



Real Number Line



Real Number Set Diagram

Complex Number Sets

Imaginary

Imaginary numbers are numbers whose squares are negative. They are the square root of minus one, i = √−1, and all real number multiples of i such as 2i and i√2.

Algebraic $\mathbb{A}$

The roots of polynomials, such as $ax^3 + bx^2 + cx + d = 0$, with integer (or rational) coefficients. Algebraic numbers may be real, imaginary, or complex. For example, the roots of $x^2 - 2 = 0$ are ±√2, the roots of $x^2 + 4 = 0$ are ±2i, and the roots of $x^2 - 4x + 7 = 0$ are 2 ± i√3.

Complex $\mathbb{C}$

Complex numbers, such as 2 + 3i, have the form $z = x + iy$, where x and y are real numbers. x is called the real part and y the imaginary part. The set of complex numbers includes all the other sets of numbers. The real numbers are complex numbers with an imaginary part of zero.

Properties of the Number Sets

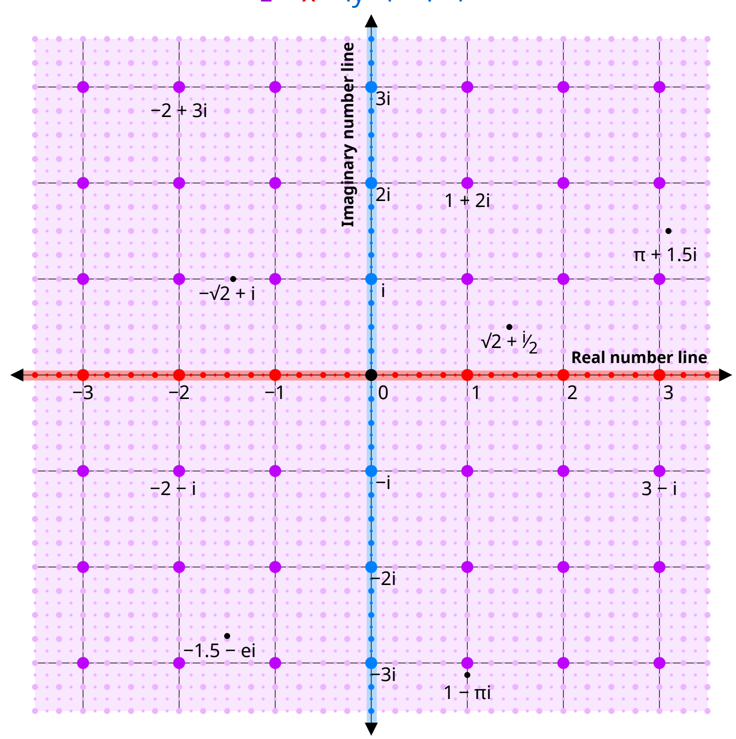
	ℕ	ℤ	ℚ	ℝ	𝔸	ℂ
Closed under Addition[1]	●	●	●	●	●	●
Closed under Multiplication[1]	●	●	●	●	●	●
Closed under Subtraction[1]		●	●	●	●	●
Closed under Division[1]			●	●	●	●
Dense[2]			●	●	●	●
Complete (Continuous)[3]				●		●
Algebraically Closed[4]					●	●

The complex numbers are the algebraic completion of the real numbers. This may explain why they appear so often in the laws of nature.

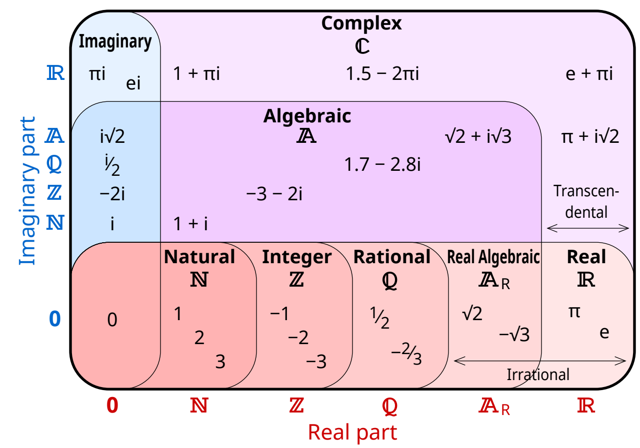
1. Closed under addition (multiplication, subtraction, division) means the sum (product, difference, quotient) of any two numbers in the set is also in the set.
2. Dense: Between any two numbers there is another number in the set.
3. Continuous with no gaps. Every sequence of numbers that keeps getting closer together (Cauchy sequence) will converge to a limit in the set.
4. Every polynomial with coefficients in the set has a root in the set.

Complex Number Plane

$z = x + iy \quad i = \sqrt{-1}$



Complex Number Set Diagram



$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{A}_R \subset \mathbb{R} \subset \mathbb{C}$

Note: This is an Euler diagram.

Infinity ∞

The integers, rational numbers, and algebraic numbers are countably infinite, meaning there is a one-to-one correspondence with the counting numbers. The real numbers and complex numbers are uncountably infinite, as Cantor proved.