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Implementation Issues Surrounding the New IAU Reference Systems for Astrodynamics

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Implementation Issues Surrounding the New IAU Reference Systems for Astrodynamics

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The realizations of celestial and terrestrial reference systems have grown increasingly accurate in recent years, due to advancements in observational technology. Because of its extremely high precision, implementation of the latest IAU-2000 reference systems may pose numerous challenges for users accustomed to the former systems. This paper reviews the relevant IAU resolutions, presents summary equations, discusses accuracy and timing, examines nomenclature necessary to discuss the new reference systems, and offers recommended practices for implementation. Examples are provided to demonstrate the numerical test results and the precision differences between various implementations.

INTRODUCTION

The application of astrodynamical principles requires both *celestial* and *terrestrial reference systems*. Differential equations of satellite motion are valid relative to the celestial system and its corresponding barycentric or geocentric reference frame. The terrestrial system defines a rotating, Earth-based reference frame where satellite observations may originate. For near-Earth orbit determination and other astrodynamical applications, it is convenient to adopt the geocentric celestial reference systems conventionally defined by astronomers. However, the improved accuracy of astronomical theory and observations is now requiring changes to the conventional transformation methods.

The *International Astronomical Union* (IAU) 1976 reference system and the *Fifth Fundamental Catalog* (FK5) were the bases of the IAU celestial reference system before 1998 (Kaplan 1981). The IAU 1976 Reference System is defined, in part, by the IAU 1976 Precession model, the IAU 1980 Theory of Nutation, the IAU 1982 Definition of Sidereal Time, the *Jet Propulsion Laboratory* (JPL) *Development Ephemeris* 200 (DE200), and the FK5 catalog, which was realized from observations of nearby stars observed at optical wavelengths.

Starting in January 1, 1998, the IAU formally adopted the *International Celestial Reference System* (ICRS) to replace the IAU 1976 FK5 reference system (Arias *et al*., 1995). The ICRS is an ideally fixed, epoch-less, barycentric reference system defined by extragalactic radio sources so it does not rotate with time. The reference system includes a new precession-nutation model and uses the latest values of astronomical constants, the Barycentric and Geocentric Coordinate Times (TCB and TCG), and the latest solar-system ephemerides. The fiducial directions of the ICRS were selected to align with the dynamical equinox and mean celestial pole at J2000.0 of the former IAU-76/FK5 system, to within the formal uncertainty of that system. The practical realization of the ICRS coordinate system is established through observations of extragalactic radio sources from *Very Long Baseline Interferometry* (VLBI) networks. This observed realization is technically known as the *International Celestial Reference Frame* (ICRF), and the alignment of radio and optical wavelengths is based on the adopted (optical) right ascension for radio source 3C 273 at epoch J2000.0 (Ma, 1998, Folkner *et al*., 1994). At optical wavelengths, the HIPPARCOS star catalog has been oriented on the ICRF and maintains continuity with the FK5 system

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(Kovalevsky *et al*., 1997). Other star catalogs, such as UCAC and USNO B1.0, provide densifications of the optical reference frame on the HIPPARCOS star catalog. The differences between the ICRF and the former reference system are about 20 milliarcseconds (mas), or about 10 cm at 1000 km.[*] As the estimated relative positions of the defining radio sources are improved, and as more defining sources are added, the ICRF will be maintained such that there is no net rotation introduced with respect to the previous realizations. The Earth-centered counterpart to the barycentric ICRF is known as the *Geocentric Celestial Reference Frame* (GCRF).

In 1991, the *International Union of Geodesy and Geophysics* (IUGG) adopted a specification for the *International Terrestrial Reference System* (ITRS) having its origin at the center of mass of the Earth. The physical location and axial orientation of the ITRS is practically realized by the adopted coordinates of defining fiducial stations on the surface of the Earth, and the realization is known as the *International Terrestrial Reference Frame* (ITRF). Since the relative station coordinates are affected by lateral tectonic motion on the order of centimeters per year, the ITRF is regularly re-estimated as a weighted, global combination of several analysis-center solutions. Sometimes, the ITRF solutions are identified by the calendar year of their publication, however, these solutions are constrained so there is no net rotation or frame shift with respect to previous realizations of the ITRF. For this reason, distinguishing between specific ITRF solutions is not usually required to describe locations in ITRF coordinates and changes in versions of the ITRF should be smaller than the observational errors for the purposes of orbit determination.[†]

The introduction of new systems and practices requires some new nomenclature, and unfortunately, minor differences in terminology and mathematical prescriptions now exist. While the IAU Working Group on *Nomenclature for Fundamental Astronomy* (NFA), the *International Earth Rotation and Reference System Service* (IERS), the Astronomical Almanac (2006), and the *US Naval Observatory* (USNO) Circular 179 (Kaplan, 2005) have introduced updated descriptors and definitions and further attempted to clarify many subtle technical issues, some legacy descriptors cannot precisely convey what is meant given the new reference systems. For example, reference to a "J2000 reference system" is ambiguous now because the 2000 IAU resolutions introduced an entirely new reference system relative to that epoch. For this reason, a select glossary of terms and acronyms has been appended to this paper.

THE IAU 2000 RESOLUTIONS

In the past, the basic concepts of Earth equator and ecliptic, temporal vernal equinox, precession and nutation, sidereal time, and polar motion have been used to define and realize astronomical and geodetic reference systems. A more relativistically consistent approach to such basic concepts as coordinate systems, time scales, and units of measure is now needed to satisfy the unprecedented requirements of ultra-high-precision instrumentation and theory. A series of IAU Resolutions in 1997 and 2000 were adopted to address these emerging requirements.

IAU Resolution B1.8 (2000) introduced a fundamentally new system based on Earth kinematics only, rather than solar-system dynamics. The new intermediate system would have a moving reference origin that would replace the equinox of date. By definition, the intermediate non-rotating origin resides on the instantaneous celestial equator and is constrained to only move perpendicularly to—but not along—the celestial equator. Unlike the equinox, such an origin is not defined geometrically, but is maintained by the accumulated motion of the celestial equator and the conventionally adopted initial point of departure (*i.e*., the dynamical equinox of J2000). Two such intermediate origins exist under such a system: a *Celestial Intermediate Origin* (CIO), and a *Terrestrial Intermediate Origin* (TIO). The TIO is the modern equivalent of the Greenwich meridian and is about 100m offset from the historical location (Kaplan 2005:54). The Earth-rotation angle between these two origins, and measured about the so-called *Celestial Intermediate*

[*] See **http://hpiers.obspm.fr/icrs-pc/icrs/icrs_fk5.gif**.

[†] The WGS-84 terrestrial frame is primarily used by the US Department of Defense and in GPS applications. The fundamental WGS-84 stations are actually constrained by their adopted ITRF coordinates during solution; thus, the axes of the modern WGS-84 and ITRF terrestrial frames spatially agree at the mas level.

Pole (CIP), supplants sidereal time under the new system. However, Resolution B1.8 also resolved that the IERS continue to provide users with data and algorithms for the conventional transformations, thus advocating two parallel methods to achieve (very nearly) the same outcome.

IAU Resolution B1.6 (2000) called for a new precession-nutation model, starting January 1, 2003, and designated the descriptor "IAU-2000A" to identify the so-called MHB-2000 nutation algorithm without a *Free-Core Nutation* (FCN, Matthews, 2002) model. There is not an adequate theory for FCN. FCN is observable, but not predictable, so this must be added as a small correction from the IERS. Thus, there is a difference between the potential accuracy for post-processing and that of pre-processing. The FCN effects are implemented by applying the dX/dY corrections to the CIO- and Equinox-based transformations. Note that the dX/dY must first be converted to an equivalent ($\delta\Delta\psi_{\mathrm{FCN}}$ and $\delta\Delta\varepsilon_{\mathrm{FCN}}$) for use with the equinox based theory (discussed later). There are probably few astrodynamical applications, except for satellite laser ranging and GPS, that qualify for this level of accuracy, as the corrections are generally less than 1 mas.

At the time of adoption, the nutation algorithm relied upon a corrected Lieske precession—the IAU-76 precession model (Lieske *et al*., 1977) with small corrections applied to the ecliptic obliquity and the precession rate. A deficiency of about 3 mas/yr in the IAU-adopted precession rate has been observed for some time, but there is no present agreement on a conventional value for obliquity (each set of solar-system ephemerides determines an obliquity value). Consequently, Resolution B1.6 encouraged the development of new conventional expressions for precession consistent with the IAU-2000A model. The new IAU precession model is anticipated to be the P03 model (Captaine et al. 2003) that will permit the continued use of an equinox-based system about as accurate as the non-rotating-origin formulation. Note that there is an apparent relativistic effect called geodesic precession and nutation which occurs due to the non-linear translation of the geocentric reference frame around the barycentric reference frame. Since the GCRF has been conventionally defined to be "kinematically non-rotating" with respect the BCRF, the geodesic precession and nutation must be included in the IAU-2000A precession-nutation model and the rotational transformation between the ITRF and the GCRF.

CELESTIAL AND TERRESTRIAL FRAME TRANSFORMATIONS

The celestial frame is related to a time-dependent terrestrial frame through an Earth orientation model, nowadays accomplished by the standard matrix-multiplication sequence of transformational rotations:

$$\mathbf{r}_{\mathrm{GCRF}} = [\mathbf{BPN}(t)][\mathbf{R}(t)][\mathbf{W}(t)]\mathbf{r}_{\mathrm{ITRF}} \qquad (1)$$

where $\mathbf{r}_{\mathrm{GCRF}}$ is location with respect to the GCRF, **BPN** is the bias-precession-nutation matrix of date t, **R** is the sidereal-rotation matrix of date t, **W** is the polar-motion matrix of date t, and $\mathbf{r}_{\mathrm{ITRF}}$ is location with respect to the ITRF. The conventional Earth orientation models may be sub-divided into partial rotational sequences, where intermediate frames are sometimes defined between these partial sequences. The BPN matrix may be sub-divided into separate matrices or maintained as a single matrix operator, depending on the theory adopted.

Observational data may need to be further reduced from the barycentric ICRS to the *Geocentric Reference System* (GCRS). Figure 1 illustrates some of the possible operations required to transform between an "observed place" and the ICRS. ICRS to GCRS transformations are used for the reduction of stellar observations or interplanetary spacecraft measurements.

ITRS to GCRS Transformation options

There are at least three primary and two secondary competing methods currently available to perform the transformations: the newer CIO-based approach, an equivalent equinox-based approach compatible with IAU-2000A (with one variation called IAU-2000B), and an equinox-based approach using the former IAU-76/FK5 based technique (also with one variation). These are shown in Figure 2. The challenge is to transform between these coordinate frames in a consistent and accurate manner.

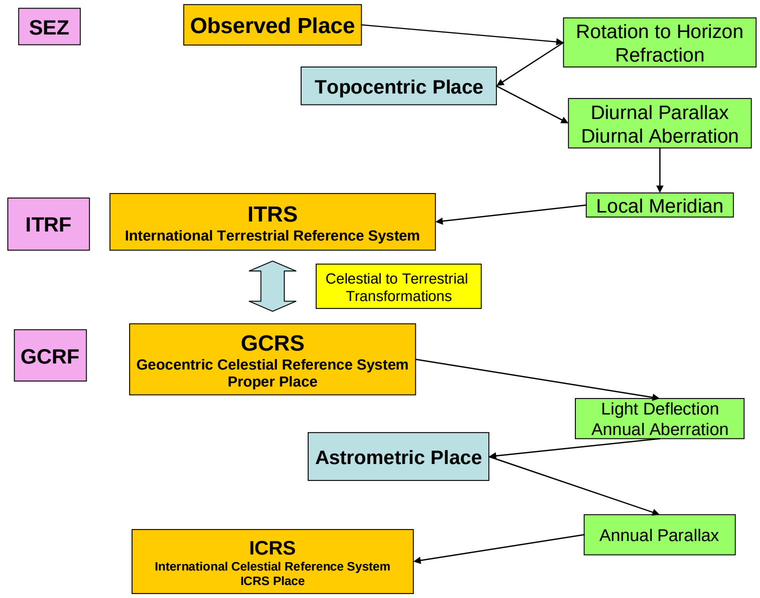

Figure 1. Observed Place to ITRS to GCRS to ICRS Transformations. Operations for transforming vectors between the observed place and ICRS reference systems, most important for stellar applications. Additional processing for observations (observed place to ITRS) is also shown for orbit determination and propagation operations. Adapted from http://danof.obspm.fr/iauWGnfa/NFA5_B1.pdf.

In this paper, several issues related to these developments are highlighted: the technical definitions, test cases, accuracy, and implementation recommendations. The nomenclature, equations, implementations, advantages, and disadvantages are discussed with each technique. Note that TT and Julian centuries of TT (T_{TT}) are used here in place of many previous relations, which showed the use of TDB. According to Kaplan (2005:43), these two quantities are sufficiently close so as not to introduce any noticeable error in the solutions.

Corrected IAU-76 FK5 Reference System based-Transformations Using IAU-76 Precession, IAU-80 Nutation, and IAU-82 Sidereal Time (Method 1)

The transformation that uses the IAU-76 Precession, the IAU-80 Theory of Nutation, IAU-82 sidereal time, TDT (TT) and TDB time scales, equinox correction, polar motion, and the FK5 star catalog* is referenced here . Because of ambiguity with similarly named systems, we refer to this as the IAU-76 FK5 Reference System, sometimes adding the qualifier at J2000.0. Since the adoption of the ICRS by the IAU in 1998, tabular corrections have existed for the precession constant, nutation parameters, and sidereal time equation, that permit continued use of the IAU-76 FK5 transformation. These values are typically very small (10-70 mas). Figure 2 shows the transformations for various operations. For astrodynamics, we are particularly interested in the ITRF to GCRF transformations. Notice that use of the IAU-76 FK5 Reference System without the corrections is discouraged, because the frame bias varies with time, and it is becoming increasingly inaccurate.

The FK5 Star Catalog technically defines an origin of right ascension for the specific catalog defining that system. The Dynamical Mean Equator and Equinox of J2000.0 refers to a fiducial direction implied by the DE200 ephemeris (or, another standard ephemeris) – the equinox is determined from the

* The HIPPARCOS star catalog is now recommended in place of the less accurate and smaller FK5 star catalog.

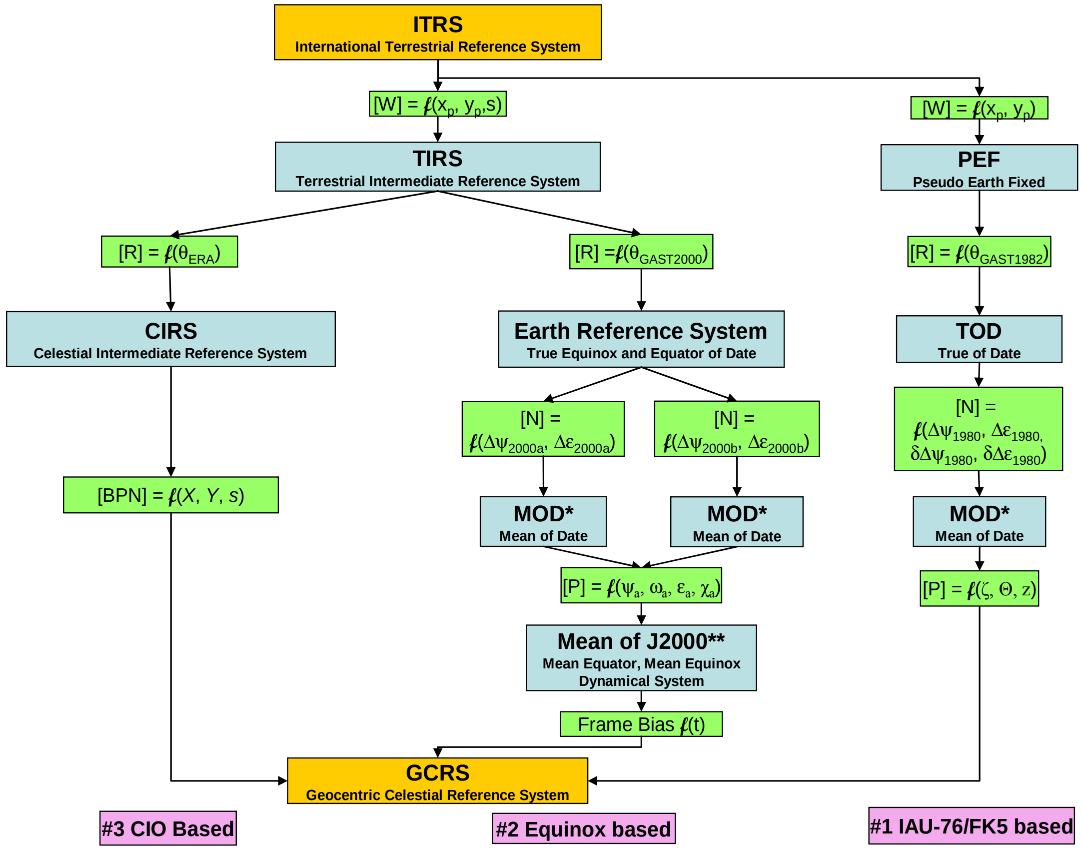

Figure 2. Celestial to Terrestrial Coordinate Transformations. Three general approaches (and two variations) transform vectors between terrestrial (ITRS) and celestial coordinate systems (GCRS). Although names (*e.g.* polar motion, mean-of-date (*), etc.) are the same, the formulae are different. IAU-76/FK5 precession/nutation corrections are found in the EOP data. There are also two approaches for the equinox-based precession/nutation models, IAU-2000A and IAU-2000B.

ephemerides as the location of the intersection of the ecliptic and the mean equator of the epoch. It is now necessary to make a distinction between the FK5 Star Catalog, the Dynamical Mean Equator and Equinox of J2000.0, and the ICRS (GCRF/ICRF) (Astronomical Almanac 2006:B27, and Kaplan, 2005:28). The origin (*i.e.*, 0,0) of coordinates defined by the extragalactic radio catalog adopted before 2000 did not align with the origin of the dynamical reference frame of J2000.0, so there is a bias between observations and convention. However, it is the conventional directions of the radio catalog that define the ICRF. Such a bias is similar to the difference between the conventional 90 degrees latitude point on Earth versus the actual observed rotational pole (the difference being mostly polar motion). The difference between the former system at J2000 and the adopted ICRF is simply that - a difference.

There are also two possible dynamical-equinox definitions.. The uniformly rotating system was the historical basis for the Newcomb equinox used until about 1984, and the DE200 equinox used until 2000 (Standish, 1981). The IAU resolutions of 1992 specify its use via footnotes. LeVerrier introduced the inertial equinox in the mid-nineteenth century, which was not widely used otherwise. The IERS now uses the inertial equinox in the precession-nutation model. The difference between these two equinoxes is about 0.1". Hence, one must be careful about which convention is being used. The conversion values from the ICRF to the dynamical equinox are given for each case by different sources.

A frame bias matrix (**B**) enables conversion from one conventional frame to another using the offsets from the ICRS pole (ξ_o and η_o) and the shift in the origin ($\delta\alpha_O$) (Ast Alm 2006:B27, Kaplan, 2005:27). The frame bias matrix (**B** in Fig. 2) that permits conversion from the J2000.0 dynamical equinox to the GCRF is:

$$\delta\alpha_O = -0.0146''$$
$$\xi_o = \delta\psi_B \sin(\varepsilon_O) = -0.041775'' \sin(84381.448) = -0.016617''$$
$$\eta_o = -0.0068192''$$

$$\begin{aligned}[\mathbf{B}]_{IAU76/FK5-GCRF} &= ROT3(-\delta\alpha_o)ROT2(-\delta\psi_B SIN(\varepsilon_o))ROT1(\delta\varepsilon_B) \\ &= \begin{bmatrix} 0.99999999999999 & 0.00000007078280 & -0.00000008056149 \\ -0.00000007078280 & 1.000000000000000 & -0.00000003306041 \\ 0.00000008056149 & 0.00000003306041 & 1.000000000000000 \end{bmatrix}\end{aligned} \tag{2}$$

The IAU-76 precession model includes precession angles (ζ, Θ, z) as a function of Julian centuries of Terrestrial time (T_{TT} = (TT – 2451545.5)/36525.0)* (McCarthy, 1992:29)

$$\begin{aligned} \zeta &= 2306.2181'' T_{TT} + 0.30188 T_{TT}^2 + 0.017998 T_{TT}^3 \\ \Theta &= 2004.3109'' T_{TT} - 0.42665 T_{TT}^2 - 0.041833 T_{TT}^3 \\ z &= 2306.2181'' T_{TT} + 1.09468 T_{TT}^2 + 0.018203 T_{TT}^3 \\ [\mathbf{P}]_{MOD-J2000} &= ROT3(\zeta)ROT2(-\Theta)ROT3(z) \end{aligned} \tag{3}$$

The form involving alternative precession angles (ψ_a, ω_a, ε_a, χ_a) is more commonly seen, including the obliquity of the ecliptic, ε_a = 84381.448″. (McCarthy and Petit, 2003:45)

$$\begin{aligned} \psi_a &= 5038.7784'' T_{TT} - 1.07259 T_{TT}^2 - 0.001147 T_{TT}^3 \\ \omega_a &= \varepsilon_o + 0.05127'' T_{TT}^2 - 0.007726 T_{TT}^3 \\ \varepsilon_a &= \varepsilon_o - 46.8150'' T_{TT} - 0.00059 T_{TT}^2 + 0.001813 T_{TT}^3 \\ \chi_a &= 10.5526'' T_{TT} - 2.38064 T_{TT}^2 - 0.001125 T_{TT}^3 \\ [\mathbf{P}]_{MOD-J2000} &= ROT1(-\varepsilon_o)ROT3(\psi_A)ROT1(\omega_A)ROT3(-\chi_A) \end{aligned} \tag{4}$$

The IAU-80 nutation uses so-called Delaunay variables and coefficients to calculate nutation in longitude ($\Delta\psi_{1980}$) and nutation in the obliquity of the ecliptic ($\Delta\varepsilon_{1980}$). (McCarthy, 1992:32)

$$\begin{aligned} M_(&= 134.96298139° + 1717915922.6330'' T_{TT} + 31.31 T_{TT}^2 + 0.064 T_{TT}^3 \\ M_O &= 357.52772333° + 129596581.2240'' T_{TT} - 0.577 T_{TT}^2 + 0.012 T_{TT}^3 \\ \mu_(&= 93.27191028° + 1739527263.1370'' T_{TT} - 13.257 T_{TT}^2 - 0.011 T_{TT}^3 \\ D_O &= 297.85036306° + 1602961601.3280'' T_{TT} - 6.891 T_{TT}^2 + 0.019 T_{TT}^3 \\ \Omega_(&= 125.04452222° - 6962890.5390'' T_{TT} + 7.455 T_{TT}^2 + 0.008 T_{TT}^3 \end{aligned} \tag{5}$$

The nutation parameters are then found using (McCarthy, 1992:33)

$$\begin{aligned} &a_{p_i} = a_{n1_i} M_(+ a_{n2_i} M_O + a_{n3_i} \mu_(+ a_{n4_i} D_O + a_{n5_i} \Omega_(\\ &\Delta\psi = \sum_{i=1}^{106} (A_{p_i} + A_{pl_i} T_{TDB}) \sin(a_{p_i}) \quad \Delta\varepsilon = \sum_{i=1}^{106} (A_{e_i} + A_{el_i} T_{TDB}) \cos(a_{p_i}) \end{aligned} \tag{6}$$

* McCarthy and Petit (2003:45) note that TDB and TT are nearly equivalent for most applications.

Corrections to the nutation parameters ($\delta\Delta\psi_{1980}$ and $\delta\Delta\varepsilon_{1980}$) supplied as Earth orientation parameters (EOP) from the IERS are simply added to the resulting values in Eq. 7 to provide a transformation to the GCRF from the mean equator and equinox of J2000.0 without use of the frame bias transformation. These corrections also include FCN and correct the errors in the IAU-76 precession and IAU-80 nutation, and use the mean obliquity of the ecliptic ($\bar{\varepsilon}$). The parameters let one find the true obliquity of the ecliptic, ε. (McCarthy, 1992:29-31)

$$
\begin{aligned}
&\Delta\psi_{1980} = \Delta\psi + \delta\Delta\psi_{1980} \qquad \Delta\varepsilon_{1980} = \Delta\varepsilon + \delta\Delta\varepsilon_{1980} \\
&\bar{\varepsilon} = 84381.448'' - 46.8150 T_{TT} - 0.00059 T_{TT}^2 + 0.001813 T_{TT}^3 \\
&\varepsilon = \bar{\varepsilon} + \Delta\varepsilon_{1980} \\
&[\mathbf{N}]_{TOD-MOD} = ROT1(-\bar{\varepsilon}) ROT3(\Delta\Psi) ROT1(\varepsilon)
\end{aligned}
\tag{7}
$$

For sidereal time, Greenwich mean (GMST) and apparent sidereal (GAST) times are needed, along with the equation of the equinoxes (*Eqe*). Note that "PEF" implies the *pseudo-Earth-fixed* frame, where polar motion has not yet been applied (Vallado, 2004: 217). From McCarthy (1992:30)

$$
\begin{aligned}
&Eqe = \Delta\psi_{1980}\cos(\bar{\varepsilon}) + 0.00264''\sin(\Omega_{(}) + 0.000063\sin(2\Omega_{(}) \\
&\theta_{GMST1982} = 67310.54841^{s} + (876600^{h} + 8640184.812866^{s}) T_{UT1} + 0.093104 T_{UT1}^2 - 6.2x10^{-6} T_{UT1}^3 \\
&\theta_{GAST1982} = \theta_{GMST1982} + Eqe \\
&[\mathbf{R}]_{PEF-TOD} = ROT3(-\theta_{GAST1982})
\end{aligned}
\tag{8}
$$

Polar motion is then applied as follows: (McCarthy, 1992:29)

$$
[\mathbf{W}]_{ITRF-PEF} = ROT1(y_p) ROT2(x_p) \tag{9}
$$

Operations with satellite orbits require a precise method to transform velocity vectors in addition to position vectors. A simple representation is as follows (two equivalent approaches are shown).

$$
\begin{aligned}
\vec{v}_{ITRF} &= [\mathbf{W}]^T \left\{ [\mathbf{R}]^T \, [\mathbf{BPN}]^T \, \vec{v}_{GCRF} - \vec{\omega}_{\oplus} \times \vec{r}_{PEF} \right\} \\
&= [\mathbf{W}]^T [\mathbf{R}]^T [\mathbf{BPN}]^T \vec{v}_{GCRF} + [\mathbf{W}]^T [\dot{\mathbf{R}}]^T [\mathbf{BPN}]^T \vec{r}_{GCRF} \\
\vec{v}_{GRCF} &= [\mathbf{BPN}] \, [\mathbf{R}] \, \left\{ [\mathbf{W}] \, \vec{v}_{ITRF} + \vec{\omega}_{\oplus} \times \vec{r}_{PEF} \right\} \\
&= [\mathbf{BPN}] \, [\mathbf{R}] \, [\mathbf{W}] \, \vec{v}_{ITRF} + [\mathbf{BPN}] \, [\dot{\mathbf{R}}] \, [\mathbf{W}] \, \vec{r}_{GCRF}
\end{aligned}
\tag{10}
$$

The true angular rotation of the Earth varies, and different values can cause differences in resulting propagation runs. The so-called excess length of day (*xlod*) accounts for this in the matrix derivative (rad/s). Note that each transformation approach should ideally use the appropriate angular derivative ($\theta_{GAST1982}$, $\theta_{GAST2000}$, and θ_{ERA}), respectively for the three methods discussed in this paper .

$$
\begin{aligned}
&\omega_{\oplus} = 7.292115146706979x10^{-5}\left(1 - \frac{xlod}{86400}\right) \\
&[\dot{\mathbf{R}}]^T = \begin{bmatrix} -\omega_{\oplus}\sin(\theta_{GAST1982}) & \omega_{\oplus}\cos(\theta_{GAST1982}) & 0 \\ -\omega_{\oplus}\cos(\theta_{GAST1982}) & -\omega_{\oplus}\sin(\theta_{GAST1982}) & 0 \\ 0 & 0 & 0 \end{bmatrix}
\end{aligned}
\tag{11}
$$

Advantages of Method 1

1. It is easiest to implement in systems currently relying on the former reference system.
2. Published EOP-like corrections are expected to continue for a few more years.

Disadvantages of Method 1

1. The IERS will eventually discontinue this service. Although the ($\delta\Delta\psi_{1980}$ and $\delta\Delta\varepsilon_{1980}$) corrections permit a temporary bridge from the former reference systems to the current, it's unknown at this time, how long these parameters will be maintained.

Equinox-Based Transformation Using IAU-2000A and Realized Using Classical Variables (Method 2)

Currently, the ICRS equinox-based transformation uses the ICRF, a corrected IAU-76 precession theory, the new nutation model based on IAU-2000A without the precession theory that is in it, and a new sidereal time expression.

The equations for the transformations in Fig. 2 begin with the frame bias, using Eq (2). Precession terms have additional terms over those in McCarthy and Petit (2003:43), and they also assume a different obliquity of the ecliptic constant, ε_o = 84381.406. We anticipate these expressions will be formally named the P03 precession theory, and it will be associated with the Equinox and CIO-based transformations. The precession angles are (Kaplan, 2005:43)

$$\begin{aligned}
\psi_a &= 5038.481507'' T_{TT} - 1.0790069 T_{TT}^2 - 0.00114045 T_{TT}^3 + 0.000132851 T_{TT}^4 - 0.0000000951 T_{TT}^5 \\
\omega_a &= \varepsilon_o - 0.025754'' T_{TT} + 0.0512623 T_{TT}^2 - 0.00772503 T_{TT}^3 - 0.000000467 T_{TT}^4 + 0.0000003337 T_{TT}^5 \\
\chi_a &= 10.556403'' T_{TT} - 2.3814292 T_{TT}^2 - 0.00121197 T_{TT}^3 + 0.000170663 T_{TT}^4 - 0.0000000560 T_{TT}^5 \\
[\mathbf{P}]_{MOD-J2000} &= ROT1(-\varepsilon_o) ROT3(\delta\psi_A) ROT1(\omega_A) ROT3(-\chi_A)
\end{aligned} \tag{12}$$

The equinox-based method has two approaches to find the nutation (IAU-2000A is shown here). The IAU-2000A uses 678 nutation and 687 planetary terms, while the less-accurate IAU-2000B uses only 77 nutation terms. The IAU-2000A fits the observations to an accuracy of 0.01 mas and the IAU-2000B fits IAU-2000A to an accuracy of 1 mas. There are no corrections to the nutation parameters ($\delta\Delta\psi_{\text{FCN}}$ and $\delta\Delta\varepsilon_{\text{FCN}}$). The consolidated EOP files (Vallado and Kelso, 2005) consolidate FCN corrections to the CIO based transformations (ΔX and ΔY), and corrections to the older IAU-76/FK5 Reference System ($\delta\Delta\psi_{1980}$ and $\delta\Delta\varepsilon_{1980}$), but not FCN corrections for the equinox-based approach.

The IERS Conventions (McCarthy and Petit, 2003:41) list equations to convert the corrections (EOP data) of the CIO coordinates (dX and dY) to equivalent equinox-based approach corrections.* The primary effect included in the (dX, dY) corrections is from FCN. The conversion to equivalent ($\delta\Delta\psi_{\text{FCN}}$ and $\delta\Delta\varepsilon_{\text{FCN}}$) corrections should not be confused with the published EOP data ($\delta\Delta\psi_{1980}$ and $\delta\Delta\varepsilon_{1980}$) that enables transformation to the GCRF from the older IAU-76/FK5 Reference System (hence the difference in nomenclature).

The corrections are related to the nutation parameters through the following equation. Solving the relation to permit transformation in both directions,

$$\begin{aligned}
dX &= \delta\Delta\psi_{\text{FCN}} \sin(\varepsilon_A) + (\psi_A \cos(\varepsilon_o) - \chi_A)\delta\varepsilon \\
dY &= \delta\Delta\varepsilon_{\text{FCN}} - (\psi_A \cos(\varepsilon_o) - \chi_A)\delta\psi \sin(\varepsilon_A) \\
\delta\Delta\psi_{FCN} &= \frac{dX}{\sin(\varepsilon_A)} - \frac{dY(\psi_A \cos(\varepsilon_A) - \chi_A)}{\sin(\varepsilon_A)} \\
\delta\Delta\varepsilon_{\text{FCN}} &= dY + dX * (\psi_A * \cos(\varepsilon_A) - \chi_A)
\end{aligned} \tag{13}$$

---

* See http://aa.usno.navy.mil/kaplan/dXdY_to_dpsideps.pdf

Under the new conventions, the Delaunay variables have additional terms from those originally published with the IAU-76/FK5 Reference System (Eq 6). (Kaplan, 2005:46)

$$
\begin{aligned}
M_{(} &= 485868.249036'' + 1717915923.2178 T_{TT} + 31.8792 T_{TT}^2 + 0.051635 T_{TT}^3 - 0.00024470 T_{TT}^4 \\
M_{O} &= 1287104.79305'' + 129596581.0481 T_{TT} - 0.5532 T_{TT}^2 + 0.000136 T_{TT}^3 - 0.00001149 T_{TT}^4 \\
\mu_{(} &= 335779.526232'' + 1739527262.8478 T_{TT} - 12.7512 T_{TT}^2 - 0.001037 T_{TT}^3 + 0.00000417 T_{TT}^4 \\
D_{O} &= 1072260.70369'' + 1602961601.2090'' T_{TT} - 6.3706 T_{TT}^2 + 0.006593 T_{TT}^3 - 0.00003169 T_{TT}^4 \\
\Omega_{(} &= 450160.398036'' - 6962890.5431 T_{TT} + 7.4722 T_{TT}^2 + 0.007702 T_{TT}^3 - 0.00005939 T_{TT}^4
\end{aligned} \tag{14}
$$

The nutation parameters are (Kaplan, 2005:45-46)

$$
\begin{aligned}
& a_{p_i} = a_{n1_i} M_{(} + a_{n2_i} M_{O} + a_{n3_i} \mu_{(} + a_{n4_i} D_{O} + a_{n5_i} \Omega_{(} + \\
& a_{pl6_i} \lambda_{Mx1} + a_{pl7_i} \lambda_{Mx2} + a_{pl8_i} \lambda_{Mx3} + a_{pl9_i} \lambda_{Mx4} + a_{pl10_i} \lambda_{Mx5} + a_{pl11_i} \lambda_{Mx6} + \\
& a_{pl12_i} \lambda_{Mx7} + a_{pl13_i} \lambda_{Mx8} + a_{pl14_i} \lambda_{Mx9} \\
& \Delta\psi = \sum_{i=1}^{678} \{(A_{ps_i} + A_{pls_i} T_{TT}) \sin(a_{p_i}) + (A_{ps_i} + A_{pls_i} T_{TT}) \cos(a_{p_i})\} \\
& \Delta\varepsilon = \sum_{i=1}^{678} \{(A_{ec_i} + A_{elc_i} T_{TT}) \cos(a_{p_i}) + (A_{ec_i} + A_{elc_i} T_{TT}) \sin(a_{p_i})\}
\end{aligned} \tag{15}
$$

and the planetary terms are (Kaplan, 2005:45-46)

$$
\begin{aligned}
\Delta\psi_{2000\,pl} &= \sum_{i=1}^{687} \{A_{ps_i} \sin(a_{p_i}) + A_{ps_i} \cos(a_{p_i})\} \\
\Delta\varepsilon_{2000\,pl} &= \sum_{i=1}^{687} \{A_{ec_i} \cos(a_{p_i}) + A_{ec_i} \sin(a_{p_i})\}
\end{aligned} \tag{16}
$$

Small corrections, primarily due to FCN, may eventually be applied to the theoretical transformation, although, these parameters ($\delta\Delta\psi_{\mathrm{FCN}}$, $\delta\Delta\varepsilon_{\mathrm{FCN}}$) are not available directly from online EOP data files. The mean obliquity of the ecliptic ($\bar{\varepsilon}$) has additional terms, and the obliquity of the ecliptic, ε_o, is now 84381.406″. From McCarthy and Petit (2003:43), this may be confusing, as they acknowledge the former value 84381.448″ on p. 45 for obliquity calculations, and otherwise cites 84381.4059″ on p. 12. (Kaplan, 2005:44-46)

$$
\begin{aligned}
& \Delta\psi_{2000} = \Delta\psi + \Delta\psi_{2000\,pl} + \delta\Delta\psi_{FCN} \qquad \Delta\varepsilon_{2000} = \Delta\varepsilon + \Delta\varepsilon_{2000\,pl} + \delta\Delta\varepsilon_{FCN} \\
& \bar{\varepsilon} = 84381.406'' - 46.836769 T_{TT} - 0.0001831 T_{TT}^2 + 0.00200340 T_{TT}^3 \\
& \quad - 0.000000576 T_{TT}^4 - 0.0000000434 T_{TT}^5 \\
& \varepsilon = \bar{\varepsilon} + \Delta\varepsilon_{2000} \\
& [\mathbf{N}]_{ERS-MOD} = ROT1(-\varepsilon_A) ROT3(-\Delta\Psi_{2000}) ROT1(\bar{\varepsilon})
\end{aligned} \tag{17}
$$

The GAST formulation is different from Eq (9). The GMST needs to be compatible with the precession theory, such that the GMST coefficients are slightly updated from McCarthy and Petit (2003:38 and 47), and the mean obliquity of the ecliptic in Eq (13) is used in place of ε_a. (Kaplan, 2005:16-17). The ten (10) terms for the equation of the equinoxes (Eqe) shown in the circular should be adequate for astrodynamical applications (only 4 are shown here).

$$
\theta_{ERA} = 2\pi(0.779057273264 + 1.00273781191135448(JD_{UT1} - 2451545.0) \quad rad
$$

$$
\begin{aligned}
&Eqe = \Delta\psi_{2000}\cos(\bar{\varepsilon}) + 0.00264096''\sin(\Omega_{(}) + 0.00006352\sin(2\Omega_{(}) + \ldots - 0.00000087T_{TT}\sin(\Omega_{(}) + \ldots \\
&\theta_{GMST2000} = \theta_{ERA} + 0.014506'' + 4612.156534T_{TT} + 1.3915817T_{TT}^{2} - 0.00000044T_{TT}^{3} \\
&\qquad\qquad - 0.000029956T_{TT}^{4} - 0.0000000368T_{TT}^{5} \\
&\theta_{GAST2000} = \theta_{GMST2000} + Eqe \\
&[\mathbf{R}]_{TIRS-ERS} = ROT3(-\theta_{GAST2000})
\end{aligned}
\tag{18}
$$

The polar motion rotation is from McCarthy and Petit (2003:35), and the Astronomical Almanac (2006:B76). The equations are (McCarthy and Petit, 2003:38 and Kaplan, 2005:64)

$$
\begin{aligned}
&s' = -0.000047''\,T_{TT} \\
&[\mathbf{W}]_{ITRF-TIRS} = ROT3(-s')ROT2(x_p)ROT1(y_p)
\end{aligned}
\tag{19}
$$

Advantages of Method 2

1. This is potentially more readily implemented in existing software supporting/expecting the classical equinox and/or classical intermediate frames.
2. The mean of date positions are retained.
3. Some software already exists, including a simplified IAU-2000B model.
4. The current IAU-2000A standard is MHB-2000.
5. FCN corrections are tiny, and are probably not required.

Disadvantages of Method 2

1. The classical equinox based approach is becoming less favored within the precision Earth-orientation community and FCN is not directly supported in this form. It requires a preprocessor to convert IERS-supplied *dX*, *dY* to $\delta\Delta\psi_{1980}$ and $\delta\Delta\varepsilon_{1980}$, compatible with MHB-2000 (not difficult, just inconvenient).
2. Corrected Lieske precession in MHB-2000 will be officially replaced, probably with P03 precession after 2006, thus creating another classical-variable form.
3. It introduces significant additional complexity by creating another class of intermediate frames.

ICRS CIO-Based Transformations Using IAU-2000A Realized Using Direction-Cosine Variables (X,Y)

This transformation marks the newest technique, but one that is also the most different from the previous descriptions. The reason lies in the use of the CIO, which effectively eliminates the distinctions of the ‘equinox’ and ‘mean position of date’. The process rests with determining the coordinates of the CIO; *X*, *Y*, and *s*. These are the CIP coordinates in the GCRS. Sidereal time is replaced by *Earth Rotation Angle* (ERA). The Greenwich meridian is replaced by an International Terrestrial Reference Frame (ITRF), and a Terrestrial Intermediate Origin (TIO) is used for the rotation frame. These, in turn, form a transformation matrix that is used to convert vectors between systems. There are no “mean” positions - only the ICRF and true positions - in this system.

The CIO transformation is shown graphically in Fig. 2. Polar motion (Eq 20) and sidereal time are found as in the ICRS Equinox based method. Notice that the ERA [Eq (19)] is used alone, instead of determining the GAST as in Eq (19). The transformation rests on the coordinates of the CIO - *X*, *Y*, and *s*. There are considerable calculations involved with these parameters, but they are reasonably efficient relative to the MHB2000 nutation model. Note that the frame bias terms are included in the first term of the *X* and *Y* expressions. From Kaplan, 2005:48 and McCarthy and Petit, 2003:36

$$
a = \frac{1}{1+\sqrt{1-X^2-Y^2}} \cong \frac{1}{2} + \frac{1}{8}(X^2+Y^2)
\tag{20}
$$

The CIO geometry is shown in Fig. 3.

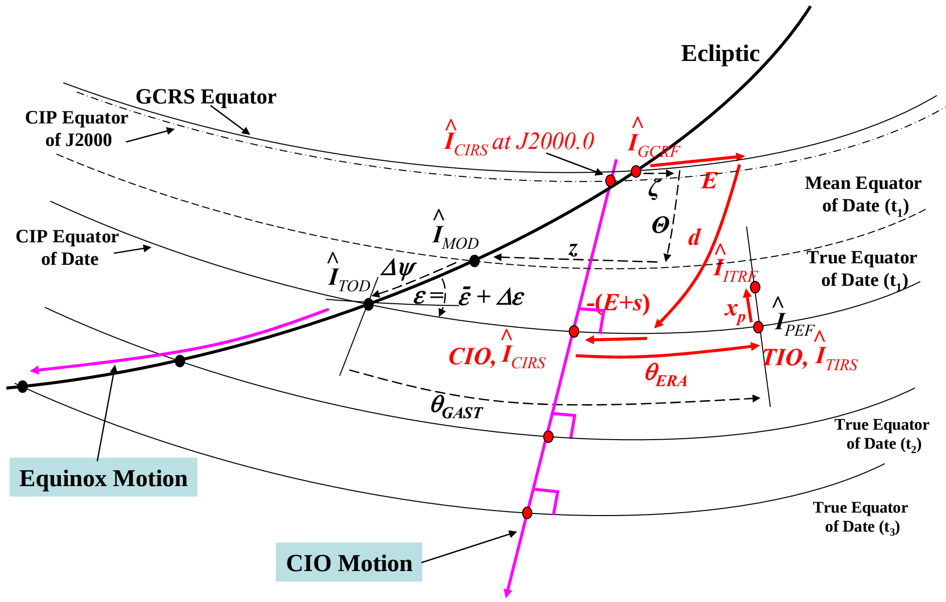

Figure 3. CIO and Equinox Geometry. This exaggerated figure shows a comparison of the equinox and CIO based approaches to transform between celestial (GCRF) and terrestrial coordinate systems (ICRF). Note that the CIO based method (red) has fewer intermediate points. The intermediate vectors are numerically close, but not identical. The motion of the CIO (CIRS) is perpendicular to the CIP equator at each time, but not coincident with the equinox (TOD). The rotation for the y_p component of polar motion is not shown. The contribution of s' to polar motion is small and is therefore not shown.

From McCarthy and Petit (2003:39), the coordinates of the CIO are

$$
\begin{aligned}
X = & -0.01661699'' + 2004.19174288 T_{TT} - 0.42721905 T_{TT}^2 - 0.19862054 T_{TT}^3 \\
& - 0.00004605 T_{TT}^4 + 0.00000598 T_{TT}^5 + \sum_{i=1}^{1306} A_{xso_i} \sin(a_{p_i}) + A_{xco_i} \cos(a_{p_i}) + \\
& \sum_{i=1}^{253} A_{xs1_i} T_{TT} \sin(a_{p_i}) + A_{xc1_i} T_{TT} \cos(a_{p_i}) + \sum_{i=1}^{36} A_{xs2_i} T_{TT}^2 \sin(a_{p_i}) + A_{xc2_i} T_{TT}^2 \cos(a_{p_i}) + \\
& \sum_{i=1}^{4} A_{xs3_i} T_{TT}^3 \sin(a_{p_i}) + A_{xc3_i} T_{TT}^3 \cos(a_{p_i}) + \sum_{i=1}^{1} A_{xs4_i} T_{TT}^4 \sin(a_{p_i}) + A_{xc4_i} T_{TT}^4 \cos(a_{p_i})
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
Y = & -0.00695078'' - 0.02538199 T_{TT} - 22.40725099 T_{TT}^2 + 0.00184228 T_{TT}^3 \\
& + 0.00111306 T_{TT}^4 + 0.00000099 T_{TT}^5 + \sum_{i=1}^{962} A_{yso_i} \sin(a_{p_i}) + A_{yco_i} \cos(a_{p_i}) + \\
& \sum_{i=1}^{277} A_{ys1_i} T_{TT} \sin(a_{p_i}) + A_{yc1_i} T_{TT} \cos(a_{p_i}) + \sum_{i=1}^{30} A_{ys2_i} T_{TT}^2 \sin(a_{p_i}) + A_{yc2_i} T_{TT}^2 \cos(a_{p_i}) + \\
& \sum_{i=1}^{5} A_{ys3_i} T_{TT}^3 \sin(a_{p_i}) + A_{yc3_i} T_{TT}^3 \cos(a_{p_i}) + \sum_{i=1}^{1} A_{ys4_i} T_{TT}^4 \sin(a_{p_i}) + A_{yc4_i} T_{TT}^4 \cos(a_{p_i})
\end{aligned} \tag{22}
$$

An expression for s including T^4 and T^5 terms is shown in the Astronomical Almanac (2006:B57). The final precession-nutation matrix is from Kaplan (2005:48).

$$
\begin{aligned}
s = & -\frac{XY}{2} + 0.000094'' + 0.00380835 T_{TT} - 0.00011994 T_{TT}^2 - 0.07257409 T_{TT}^3 \\
& + 0.000027707 T_{TT}^4 + 0.00001561 T_{TT}^5 + \sum_{i=1}^{33} A_{sso_i} \sin(a_{p_i}) + A_{sco_i} \cos(a_{p_i}) + \\
& \sum_{i=1}^{3} A_{ss1_i} T_{TT} \sin(a_{p_i}) + A_{sc1_i} T_{TT} \cos(a_{p_i}) + \sum_{i=1}^{25} A_{ss2_i} T_{TT}^2 \sin(a_{p_i}) + A_{sc2_i} T_{TT}^2 \cos(a_{p_i}) + \\
& \sum_{i=1}^{4} A_{ss3_i} T_{TT}^3 \sin(a_{p_i}) + A_{sc3_i} T_{TT}^3 \cos(a_{p_i}) + \sum_{i=1}^{1} A_{ss4_i} T_{TT}^4 \sin(a_{p_i}) + A_{sc4_i} T_{TT}^4 \cos(a_{p_i}) + \\
& 0.00000171 T_{TT} \sin(\Omega_{(}) + 0.00000357 T_{TT} \cos(2\Omega_{(}) + 0.00074353 T_{TT}^2 \sin(2\Omega_{(}) - \\
& 0.00000885 T_{TT}^2 \sin(2\Omega_{(}) + 0.00005691 T_{TT}^2 \sin(2(\mu_{M(} - D_O + \Omega_{(})) + \\
& 0.00000984 T_{TT}^2 \sin(2(\mu_{M(} + \Omega_{(}))
\end{aligned}
\tag{23}
$$

$$
[\mathbf{PN}]_{CIRS-GCRF} = \begin{bmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a(X^2 + Y^2) \end{bmatrix} ROT3[s] \tag{24}
$$

Small corrections are eventually applied to the theoretical transformation, primarily due to FCN. These EOP parameters, ΔX and ΔY, add to the calculated values of X and Y. They are typically very small (< 5 mas).

Advantages of Method 3

1. As this is the IERS-preferred method, adoption now may mean improved support for the future.
2. The method is operationally simpler for those not requiring the classical intermediate frames of date (*e.g.*, mean of date).
3. Some software already exists.

Disadvantages of Method 3

1. Potentially, this method could have the most effort to implement in pre-existing systems, depending on how the computer code is organized.
2. There is confusing terminology still developing, a limited bibliography, and it has all the uncertainties associated with a newly developed standard.
3. There is no simplified modeling option.

Time Scales

International Atomic Time (TAI) is a physical time scale at the surface of the Earth, based on the *SI* definition of the second, and affected by Earth's gravitational and rotational potential. It is maintained by the *Bureau International des Poids et Mesures* (BIPM) using a weighted average of various international frequency standards. The BIPM publishes small corrections *post facto* for individually contributing reference standards, such as the USNO master clocks, and these corrections must be added to the historical clock time to recover official TAI time. Consequently, TAI is not the most convenient time scale for civil timekeeping; instead, broadcast *Coordinated Universal Time* (UTC) has become the most widely recognized standard, as it provides both astronomical time of day and atomic-time interval. UTC is kept within ±0.9 s of *Universal Time* (UT1, the modern-day successor of Greenwich mean solar time) by the introduction of leap seconds. Therefore, UTC is always offset from TAI by an integer number of seconds, and is thus a carrier of precision frequency and time interval (Seago and Seidelmann, 2004). *Terrestrial Time* (TT) is a theoretically ideal time, practically realized as

$$\mathrm{TT} = \mathrm{TAI} + 32.184\mathrm{s} \tag{25}$$

To the accuracy of the UTC clock used, a user may infer TAI and TT for any UTC time tag after 1972-01-01 using a historical table of leap-second offsets.

Recently, Seidelmann and Seago (2005) discussed general time-scale usage in the context of relativistic transformations between the geocentric and barycentric coordinate systems, which can be important for some astrodynamical applications. *Geocentric Coordinate Time* (TCG) is a theoretical coordinate time, and *Barycentric Coordinate Time* (TCB) is the solar-system barycentric coordinate time, and TCB differs from TCG by both secular and periodic terms. The complicated relationships between TT, TCG, and TCB are based on relativistic Lorentz transformations, although approximate expressions for (TCB – TCG) exist (Fairhead and Bertagnon, 1990).

The TT, TCG, and TCB timescales were defined to be equivalent at the TAI epoch of t_0 = 1977-01-01T00:00:00 (JD_{TAI} = 2443144.5 precisely). TT, TCG, TCB at t_0 equal 1977-01-01T00:00:32.184 (*JD* = 2443144.5003725 precisely); that is to say, the epoch of these three (3) timescales read ahead of TAI by exactly 32.184 seconds at t_0 by convention. With an originating epoch prescribed,

$$\mathrm{TT} - \mathrm{TCG} = -L_G \times \left\{ \left(JD_{\mathrm{TCG}} - t_0 \right) \times 86400 \right\} \text{ in TT seconds} \tag{26}$$

with $L_G \equiv 6.969290134 \times 10^{-10}$.* The elapsed epoch difference in seconds is indicated by TT–TCG, while the Julian date epoch is indicated with JD_{TCG} (due to our use of Julian date, a scale factor of 86400 converts elapsed Julian-date interval from t_0 into units of second). This expression is consistent with Kaplan (2005:13), but the form may be inconvenient, since TCG is a coordinate time representing the independent argument of the equations of motion of bodies in its frame and will not be ordinarily kept by any physically real clock. It is more convenient to have a relationship in terms of TT, which is realized via TAI and UTC. It remains possible to first collect terms of TT, expand the denominator in a binomial series, and obtain an inverse expression:

$$\mathrm{TCG} - \mathrm{TT} = \left\{ \left(JD_{\mathrm{TT}} - t_0 \right) \times 86400 \right\} \times \left(L_G + L_G^2 + L_G^3 + \cdots \right) \text{ in TCG seconds} \tag{27}$$

Terms beyond L_G^2 are negligibly small while using double-precision (8-byte) words.

McCarthy and Petit (2003) provide a slightly different recommendation having TAI as the independent variable, which some may find convenient:

$$\begin{aligned} \mathrm{TCG} - \mathrm{TT} &= +L_G \times \left\{ \left(JD_{\mathrm{TAI}} - t_0 \right) \times 86400 \right\} \\ \mathrm{TT} - \mathrm{TCG} &= -L_G \times \left\{ \left(JD_{\mathrm{TAI}} - t_0 \right) \times 86400 \right\} \end{aligned} \tag{28}$$

The precision differences in using TAI versus TCG and TT as independent variables appears on the order of tens of nanoseconds. This difference is within the uncertainty of broadcast UTC itself, so TAI tends to be a sufficiently accurate proxy when TT will be realized from atomic-clock readings.

The Independent Argument of Ephemerides

In preparing ephemerides of the solar system, as is done by the JPL, the relativistic equations of motion for the numerical integrations must use an independent variable called *time*. Such must be a form of barycentric coordinate time, but it does not necessarily have a perfectly defined relation to an Earth-defined time. When an integrated ephemeris is fit to planetary observations, appropriate relativistic transformations

* L_G is a scale constant accounting for the Earth's gravitational and rotational potential affecting the rate of clocks according to the IAU-specified relativistic metric. IAU Resolution B1.9 (2000) recommends L_G as a defining constant, so the relationship cannot change.

are made to reconcile the different coordinate systems involved. After several iterations, the result is a barycentric ephemeris whose originating epoch and rate of barycentric coordinate time can differ from both TCB and TT. JPL adjusts the rate and epoch of this time scale to match TT over the span of the ephemeris, and calls it T_{eph}. T_{eph} thus becomes almost the same as TT over the time span of the ephemeris.

T_{eph} is functionally equivalent to *Barycentric Dynamical Time* (TDB). TDB was originally intended to be an independent argument of barycentric ephemerides and equations of motion, and was defined by an IAU 1976 resolution to differ from TT only by periodic terms

$$TDB \approx TT + 0.001658\text{s} \sin(M) + 0.000014 \sin(2M) \tag{29}$$

where the mean anomaly of the Earth is $M = 357.53° + 0.9856003(JD_{TT} - 2451545.0)$. Later it became clear that this condition cannot be rigorously satisfied, so IAU 1991 resolutions recommended TDB be defined as linearly related to TCB, but without fixing the exact rate ratio L_B.*

$$\mathrm{TCB} - \mathrm{TDB} = L_B \times \{(JD_{\mathrm{TAI}} - t_0) \times 86400\} \text{ in seconds} \tag{30}$$

Kaplan (2005:14) uses the Julian centuries of TT and shows

$$\begin{aligned} \mathrm{T}_{eph} &\approx \mathrm{TDB} \approx \mathrm{TT} + 0.001658\text{s} \sin(628.3076T_{TT} + 6.2401) \\ &+ 0.000022 \sin(575.3385T_{TT} + 4.2970) + 0.000014 \sin(1256.6152T_{TT} + 6.1969) \\ &+ 0.000005 \sin(606.9777T_{TT} + 4.0212) + 0.000005 \sin(52.9691T_{TT} + 0.4444) \\ &+ 0.000002 \sin(21.3299T_{TT} + 5.5431) + 0.000010T_{TT} \sin(628.3076T_{TT} + 4.2490) + \ldots \end{aligned} \tag{31}$$

The Astronomical Almanac (2006) shows the following (accurate to about 50 µs during 1980-2050)

$$\mathrm{TDB} = \mathrm{TT} + 0.001658\text{s} \sin(M) + 0.000021 \sin(\Delta M_\lambda) \tag{32}$$

where ΔM_λ is the difference in the mean longitudes of the Earth and Jupiter, $= 246.11° + 0.90255617(JD_{TT} - 2451545.0)$, and $M = 357.53° + 0.98560028(JD_{TT} - 2451545.0)$.

In practice, the time difference TT– T_{eph} never exceeds 1.7 ms. Thus, any error resulting from the use of TT instead of TDB or T_{eph} in the JPL planetary ephemerides is insignificant for almost all near-Earth orbit-determination applications. Since T_{eph} remains an awkwardly subscripted label, and since there is a need for a barycentric time scale for star catalogs, the label TDB is being revived for those purposes in the Astronomical Almanac, to be used as in the past. However, note that this paper uses TT and Julian centuries of TT (T_{TT}) in many relations where prior literature indicates TDB. According to Kaplan (2005, p. 43), TDB and TT are sufficiently close so as not to introduce any noticeable error in the solutions.†

The latest versions of the JPL ephemerides, DE-405 and DE-406, are referenced to, and thus compatible with, the ICRF. However, the luni-solar nutation tables supplied with the DE-405/406 are the former IAU 1980 nutation model and are *not* compatible with the IAU 2000 resolutions. Also, some constants adopted to develop the ephemerides may not be consistent with other sets of constants.

Software Implementations

There are three main sources of software that implement the IAU Resolutions. The USNO includes computer software with the IERS Conventions (2003).‡ This includes the MHB 2000 theory. It

* McCarthy and Petit (2003:12) provides an average value for $L_B = 1.55051976772 \times 10^{-8}$.

† It is correct to say that each ephemeris realizes its own version of TDB, but this level of distinction is inconsequential for practically every astrodynamical purpose.

‡ http://maia.usno.navy.mil/conv2000/chapter5/IAU2000A.f

includes FCN and is equivalent, less additional comments, to the original MHB code[*] at Thomas Herring's web page. The IERS Conventions (2003) also list a nutation-only portion, identified as IAU-2000A Nutation[†]. This does not include a FCN model. The BPN2000 routine uses the CIO coordinates (*XYs*) directly in determining the transformation matrix. This differs from the SOFA routines, discussed next.

The second source for software is the Standards of Fundamental Astronomy (SOFA) library, maintained by the IAU.[‡] These routines are highly modularized with many individual routines[§] (Astronomical Almanac, 2006:B57-B58).

$$E = ATAN2(Y, X)$$
$$d = ATAN\left(\sqrt{\frac{X^2+Y^2}{1-(X^2+Y^2)}}\right) \tag{33}$$
$$[\mathbf{PN}]_{CIRS-GCRF} = ROT3(-E)ROT2(-d)ROT3(E+s)$$

The third source of software is the NOVAS software from the USNO.[**] NOVAS is an integrated package of subroutines to calculate many common astrometric quantities and transformations. It provides the software for all three methods described above.

TEST CASES, TIMING, AND ACCURACY

A few test cases are provided to demonstrate the expected numerical differences with various methods of solution. To begin, we provide a short example of the different time scales that can be used for operations. For the given epoch we find the various times. Note the times are expressed in seconds from midnight at the beginning of the day of interest.

```
April 6, 2004, 7:51:28.386009 UTC
dut1 -0.439962 s dat  32 s

ut1 28287.9460470000 tut1   0.042623611411 jdut1 2453101.82740678310 hms   7  51 27.946047
utc 28288.3860090000                                                 hms   7  51 28.386009
tai 28320.3860090000                                                 hms   7  52  0.386009
tt  28352.5700090000 ttt    0.042623631889 jdtt  2453101.82815474550 hms   7  52 32.570009
tdb 28352.5716651154 ttdb   0.042623631890 jdtdb 2453101.82815476460
tcg 28353.1695861742                                                 hms   7  52 33.169586
tcb 28365.9109901113                                                 hms   7  52 45.910990
```

The state vectors are in km and km/s while the differences (position only) are in meters. As expected, differences from various approaches are more pronounced for higher altitude satellite orbits, but the angular separation remains about the same. The ITRF vector is used as the initial starting point for all tests. Although the EOP data are usually interpolated[††], these tests are intended to be static, and hence the single values.

LEO test case

Input:

```
April 6, 2004, 7:51:28.386009 UTC, dut1 -0.439962 s dat  32 s lod 0.001556 s
xp    -0.140682 yp     0.333309, ddpsi -0.052195 ddeps -0.003875, dx    -0.000199 dy    -0.000252
```

Output:

---

[*] http://bowie.mit.edu/~tah/mhb2000/MHB_2000.f

[†] http://maia.usno.navy.mil/conv2000/chapter5/NU2000A.f

[‡] http://www.iau-sofa.rl.ac.uk/

[§] In SOFA subroutine C2IXYS, the *XYs* parameters are input, but the trigonometric *E* and *d* parameters are first calculated, and then used to calculate the transformation matrix in Eq (24). This form appears to add computational time.

[**] http://aa.usno.navy.mil/software/novas/novas_info.html

[††] Interpolation assists recovery of the sub-diurnal values in UT and polar motion. These effects are removed to produce the EOP data and must be added back for operations.

	position vector components (km)			velocity vector components (km/s)			Position delta in m	Position arcsec
ITRF	-1033.4793830	7901.2952754	6380.3565958	-3.225636520	-2.872451450	5.531924446		
PEF iau76	-1033.4750313	7901.3055856	6380.3445328	-3.225632747	-2.872442511	5.531931288		
TIRS	-1033.4750312	7901.3055856	6380.3445328	-3.225632747	-2.872442511	5.531931288	0.0001	0.00000
TOD iau76	5094.5147804	6127.3664612	6380.3445328	-4.746088567	0.786077222	5.531931288		
ERS Eq	5094.5146278	6127.3665880	6380.3445328	-4.746088587	0.786077104	5.531931288	0.1984	0.00401
CIRS	5100.0184047	6122.7863648	6380.3445328	-4.745380330	0.790341453	5.531931288	7160.1092	144.67558
MOD iau76	5094.0290167	6127.8709363	6380.2478885	-4.746262495	0.786014149	5.531791025		
MOD iau76 w corr	5094.0283745	6127.8708164	6380.2485164	-4.746263052	0.786014045	5.531790562	0.9061	0.01831
J2000 iau76	5102.5096000	6123.0115200	6378.1363000	-4.743219600	0.790536600	5.533756190		
J2000 Eq a	5102.5090383	6123.0119758	6378.1363118	-4.743219766	0.790536344	5.533756084	0.7235	0.01462
J2000 Eq b	5102.5090383	6123.0119733	6378.1363142	-4.743219766	0.790536342	5.533756085	0.7219	0.01459
CIRS	5100.0184047	6122.7863648	6380.3445328	-4.745380330	0.790341453	5.531931288	3336.6212	67.41904
GCRF CIO based	5102.5089530	6123.0113955	6378.1369371	-4.743220161	0.790536492	5.533755724		
GCRF CIO dx,dy=0	5102.5089592	6123.0114033	6378.1369247	-4.743220156	0.790536498	5.533755728	0.0159	0.00032
GCRF iau 2000a	5102.5089579	6123.0114038	6378.1369252	-4.743220156	0.790536497	5.533755728	0.0153	0.00031
GCRF iau 2000b	5102.5089579	6123.0114012	6378.1369277	-4.743220156	0.790536495	5.533755729	0.0120	0.00024
GCRF iau76 w corr	5102.5089579	6123.0114007	6378.1369282	-4.743220157	0.790536497	5.533755727	0.0114	0.00023
GCRF iau76 wfb	5102.5095196	6123.0109480	6378.1369135	-4.743219990	0.790536753	5.533755834	0.7224	0.01460

GEO test case

Input:

```
June 1, 2004, 0:00:0.000000 UTC, dut1 -0.470905 s dat  32 s lod 0.000000 s
xp    -0.083853 yp     0.467217, ddpsi -0.053614 ddeps -0.004494, dx    -0.000199 dy    -0.000252
```

Output:

	position vector components (km)			velocity vector components (km/s)			Position delta in m	Position arcsec
ITRF	24796.9192915	-34115.8709234	10.2260621	-0.000979178	-0.001476538	-0.000928776		
PEF iau76	24796.9192956	-34115.8709001	10.2932583	-0.000979178	-0.001476540	-0.000928772		
TIRS	24796.9192953	-34115.8709004	10.2932583	-0.000979178	-0.001476540	-0.000928772	0.0004	0.00000
TOD iau76	-40577.4277501	-11500.0961306	10.2932583	0.837552338	-2.957524176	-0.000928772		
ERS Eq	-40577.4275475	-11500.0968457	10.2932583	0.837552390	-2.957524162	-0.000928772	0.7432	0.00363
CIRS	-40588.1581236	-11462.1670709	10.2932583	0.834787843	-2.958305669	-0.000928772	39417.6925	192.77694
MOD iau76	-40576.8226385	-11502.2315013	9.7382304	0.837708020	-2.957480118	-0.000814275		
MOD iau76 w corr	-40576.8226395	-11502.2315015	9.7337842	0.837708020	-2.957480117	-0.000814253	4.4462	0.02174
J2000 iau76	-40588.1503620	-11462.1670280	27.1476490	0.834787457	-2.958305691	-0.001173016		
J2000 Eq a	-40588.1495482	-11462.1699118	27.1468462	0.834787667	-2.958305632	-0.001172963	3.1021	0.01517
J2000 Eq b	-40588.1495481	-11462.1699118	27.1468613	0.834787667	-2.958305632	-0.001172968	3.0983	0.01515
CIRS	-40588.1581236	-11462.1670709	10.2932583	0.834787843	-2.958305669	-0.000928772	16854.3925	82.42844
GCRF CIO based	-40588.1503644	-11462.1670302	27.1431447	0.834787457	-2.958305691	-0.001172996		
GCRF CIO dx,dy=0	-40588.1503643	-11462.1670302	27.1431979	0.834787457	-2.958305691	-0.001172993	0.0532	0.00026
GCRF iau 2000a	-40588.1503617	-11462.1670397	27.1431974	0.834787458	-2.958305691	-0.001172993	0.0536	0.00026
GCRF iau 2000b	-40588.1503617	-11462.1670397	27.1432125	0.834787458	-2.958305691	-0.001172999	0.0685	0.00034
GCRF iau76 w corr	-40588.1503649	-11462.1670282	27.1432028	0.834787457	-2.958305691	-0.001172994	0.0581	0.00028
GCRF iau76 wfb	-40588.1511755	-11462.1641560	27.1440002	0.834787248	-2.958305750	-0.001173047	3.1066	0.01519

There are two sections in each test. First, results of the new reference systems are compared with the existing IAU-76/FK5 transformations because it is assumed most programs currently have some version of this transformation. The most noticeable difference is the intermediate CIRS frame which, as expected, is quite different from the TOD and MOD realizations. The second section compares the various approaches to realizing the GCRF. Here, each case is compared to the CIO based result. There is excellent similarity (< 1 mas) in the CIO-based method, IAU-2000A/B method, and corrected IAU-76/FK5 method.

We also examined the timing of the various approaches. The interpolated IAU-76/FK5 Reference System was used as a baseline for each variation. Over 12,000 different transformations were run with each method. The first series of tests implemented the series solutions directly. Another test series with a simple linear interpolation of tabulated parameters ($X/Y/s$ parameters, and $\Delta\psi_{2000}$ and $\Delta\varepsilon_{2000}$, etc) was conducted. The interpolation was a simple operation to minimize the number of series evaluations. The timing results should be viewed qualitatively in that run-times varied slightly. However, interpolation shows all the approaches are feasible for implementation. Notice that the IAU-2000A routine with full series was quite expensive computationally, but the IAU-2000B is much less intensive. The interpolated equinox-based approaches were slightly longer because the precession terms were still calculated. The CIO based approach combines precession and nutation, and is therefore very efficient. One could potentially interpolate the terms of the **PN** matrix, although this would add memory and complexity as the number of terms would expand by a factor of 3.

Technique	Direct Implementation	*Interpolated Approach*
IAU-76 Reference System	2.4	*1.0*
ICRS Equinox based 2000A	40.8	*1.8*
ICRS Equinox based 2000B	3.8	*1.9*
ICRS CIO based	49.9	*0.8*

Several conclusions can be drawn from these tests about the accuracy of the various approaches:

- The [uncorrected] IAU-76/FK5 Reference System yielded a result that is off by about 1 – 4 m (about 15 mas) compared to the ICRS options. This will grow with dates further from the epoch.
- The IAU-76/FK5 Reference System is about as accurate (<1 mas) as the IAU-2000A and IAU-2000B options when using the EOP nutation and precession corrections. This will grow in time.
- Including the dX /dY corrections is about the same as the difference between the ICRS CIO-based and equinox-based options.
- The ICRS equinox and CIO-based systems with full series have comparable precision (< 1 mas). These conclusions are roughly consistent with the stated accuracies of the new systems.
- The IAU-2000B precession-nutation model gives an accuracy of 1 mas from the IAU-2000A approach over a century centered on J2000.0.
- Failure to correct for FCN introduces an error of about 0.2 mas RMS relative to the observation residuals.

IMPLEMENTATION RECOMMENDATIONS

Considering the accuracy and timing implications mentioned above, the following recommendations are posed:

- Reference the GCRF and ITRF for the exchange of satellite locations and ephemerides.
- New software, and legacy software well positioned for changing coordinate-system methods, should make use of the CIO-based transformations. While the corrected equinox-based method can give equivalent accuracy and may be easier to implement today, it may not be operationally supported indefinitely.
- Existing programs that want to be minimally invasive can apply $\delta\Delta\psi_{1980}$ and $\delta\Delta\varepsilon_{1980}$ and precession corrections to the IAU-76/80/82 conventions to find the GCRF.
- Existing programs should anticipate the adoption of the P03 precession model around 2006 to replace the corrected Lieske precession now used with MHB2000, if requirements dictate the equinox forms.
- Computing platforms that are computationally limited should examine the interpolation of the $X/Y/s$ parameters, or $\Delta\psi_{2000}$ and $\Delta\varepsilon_{2000}$, for equinox-based approaches. Cubic splines or polynomial interpolation (5th order) are likely sufficient, but additional testing is required.

Ultimately, any decision on how to modernize will depend on accuracy, speed, and program requirements.

GLOSSARY

These definitions are extracted in part from the following sources:

http://www.iers.org/iers/earth/acronyms/acronyms.html and http://syrte.obspm.fr/iauWGnfa/

Astrometric place: Observed positions with respect to GCRF reference stars at some epoch. The positions have not been corrected for annual or planetary aberration, or light bending.

Barycentric Celestial Reference Frame: Equivalent to the ICRF.

Catalog place: A star catalog position.

Celestial Intermediate Origin (CIO): Origin for right ascension on the intermediate equator in the celestial intermediate reference system. It is the non-rotating origin in the GCRS that is recommended by the IAU 2000 Resolution B 1.8, where it was designated the **Celestial Ephemeris Origin**. (Note that CIO can also refer to the *Conventional International Origin*, the now-obsolete name for 90° terrestrial latitude, the reference point for polar motion).

Celestial Intermediate Pole: The pole established by the IAU-2000A precession–nutation model transforming between the ICRF and ITRF.

Celestial Intermediate Reference System: Reference system established by the CIO and CIP.

Dynamical Mean Equator and Equinox of J2000.0: Celestial reference frame defined by the mean equator and equinox of J2000.0 as determined from ephemerides.

Earth Rotation Angle (ERA): The angle measured along the intermediate equator of the Celestial Intermediate Pole (CIP) between the Terrestrial Intermediate Origin (TIO) and the Celestial Intermediate Origin (CIO), positively in the retrograde direction. It is related to UT1 by a conventionally adopted expression in which ERA is a linear function of UT1 (see IAU Resolution B1.8). Its time derivative is the Earth's angular velocity. Previously, it has been referred to as the *stellar angle*.

Free Core Nutation: Component of nutation due to the free wobble of the Earth's core for which there is no adequate predictive theory. It is not modeled in the IAU-2000A precession-nutation formula and is provided by IERS as a *post-facto* correction.

Geocentric Celestial Reference Frame (GCRF): The geocentric frame whose orientation is aligned with the Barycentric ICRF. Since the GCRF has been defined to be kinematically non-rotating with respect the BCRF, the geodesic precession and nutation is included in the IAU-2000A precession-nutation model and is included in the transformation between the ITRF and the GCRF, not between the GCRF and BCRF.

Geocentric Celestial Reference System (GCRS): A system of geocentric space-time coordinates within the framework of General Relativity with metric tensor specified by the IAU 2000 Resolution B1.3. The GCRS is defined such that the transformation between BCRS and GCRS spatial coordinates contains no rotational component, so that GCRS is kinematically non-rotating with respect to BCRS. The motion of, for example, an Earth satellite, with respect to the GCRS will appear to be affected by relativistic Coriolis forces that come mainly from geodesic precession. The spatial orientation of the GCRS is defined by that of the BCRS, that is unless otherwise stated, by the orientation of the ICRS.

Geodesic precession and nutation: A relativistic effect which occurs due to the motion of the geocentric reference frame around the barycentric reference frame.

IAU-2000A: An official name for the latest IAU precession-nutation theory relating the GCRF and ITRF. At the time of the resolution adoption, this title referred to the so-called MHB-2000 model. Since then, 'IAU-2000A' has also referred to a competing realization based on direction cosines, published in IERS TN32 and elsewhere. Some authors claim the direction-cosine form exclusively when using the official title "IAU 2000A", and relegate 'MHB-2000' to refer to the classical-variable form. However, note that the P03 precession model is expected to replace the corrected Lieske precession model employed by MHB-2000 around 2006, thus creating a slightly different classical-variable form of the IAU-2000A model.

IAU-2000B: An abridged version of MHB-2000 officially endorsed by the IAU, for those who require an orientation model at the 1 mas level.

International Celestial Reference Frame (ICRF): A set of extragalactic objects whose adopted positions and uncertainties realize the ICRS axes and their uncertainties. It is also the name of the radio catalog whose 212 defining sources is currently the most accurate realization of the ICRS. The orientation of the ICRF catalog was carried over from earlier IERS radio catalogs and was within the errors of the standard stellar and dynamic frames at the time of adoption. Successive revisions of the ICRF are intended to minimize rotation from its original orientation. There are also optical realizations of the ICRS based on the HIPPARCOS Catalog and other fainter optical catalogs for densification, such as UCAC and USNO B1.0.

International Celestial Reference System (ICRS): The ICRS is designed to be fixed in orientation regardless of epoch. It is kinematically non-rotating with respect to the ensemble of distant extragalactic objects. It has no intrinsic orientation, but was aligned close to the mean equator and dynamical equinox of J2000.0 for continuity with previous fundamental reference systems. Its orientation is independent of epoch, ecliptic, or equator, and is realized by a list of adopted coordinates of extragalactic sources. The system includes a generalized set of constants, ephemerides, precession-nutation model, Earth rotation, and polar motion. A Barycentric Celestial Reference System (BCRS) and a Geocentric Celestial Reference System (GCRS) are defined based on the appropriate potentials, along with Barycentric Coordinate Time (TCB) and Geocentric Coordinate Time (TCG), according to relativistic metrics. These reference systems and the transformations between them are consistent with the theory of relativity. The ICRF, determined from extragalactic radio sources, is the practical realization of the ICRS.

ICRS place: A direction in ICRS coordinates, (*e.g*. ICRS right ascension, the right ascension measured from the ICRS origin on the ICRS equator, and ICRS declination, the declination measured from the ICRS equator).

International Terrestrial Reference Frame (ITRF): A realization of ITRS by a set of instantaneous coordinates (and velocities) of reference points distributed on the topographic surface of the Earth (mainly space geodetic stations and related markers). Currently the ITRF provides a model for estimating, to high accuracy, the instantaneous positions of these points, which is the sum of conventional corrections provided by the IERS convention center (solid Earth tides, pole tides, ...) and of a regularized position. At present, the latter is modeled by a piecewise linear function, the linear part accounting for such effects as tectonic plate motion, post-glacial rebound, and the piecewise aspect representing discontinuities such as seismic displacements. The initial orientation of ITRF is that of the BIH Terrestrial System at epoch 1984.0.

International Terrestrial Reference System (ITRS): A specific GTRS for which the co-rotation condition is defined as no residual rotation with regard to the Earth's surface and the geocenter is understood as the center of mass of the whole Earth system, including oceans and atmosphere (IUGG Resolution 2, Vienna 1991). It was aligned close to the mean equator of 1900 and the Greenwich meridian, for continuity with previous terrestrial reference systems. The ITRS is the recommended system to express positions on the Earth.

J2000.0: An epoch defined in the framework of General Relativity by IAU Resolution C7 (1994) as being the event (epoch) at the geocenter and at the date 2000 January 1.5 TT = Julian Date 245 1545.0 TT. Note that this event has a different numerical representation within non-TT time scales.

MHB-2000: Matthews *et al*. (2003) introduced the MHB-2000 precession-nutation model that references IAU-76 precession theory and includes secular terms in the nutation. It is the basis of the official IAU-2000A model according to IAU 2000 Resolution B1.6. Some realizations of the MHB-2000 include a preliminary model for Free Core Nutation (FCN), but this component is not part of the official IAU-2000A model. Instead, the IERS provides observed FCN as a small, tabulated correction called *dX* and *dY*, not unlike other Earth orientation parameters such as UT1 and polar motion. MHB-2000 does not include geodesic precession.

Observed place: A position as observed without any corrections.

Pseudo Earth fixed (PEF): A term implying a basis close to the ITRF, except that polar motion has been removed, *i.e*., the difference between the ITRF and PEF is the application of polar motion.

Proper place: The direction of an object in the GCRS (e.g. right ascension and declination); geocentric place that is corrected for light-time, light deflection, annual parallax and annual aberration.

South-East-Zenith (SEZ): A right-handed coordinate basis useful for referencing topocentric orientation; a common alternative is East-North-Up.

Terrestrial Intermediate Reference System (TIRS): A terrestrial reference system with respect to the Celestial Intermediate pole. The ITRS has been corrected for polar motion from the TIRS.

Topocentric place: The observed position has been corrected for refraction.

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