CME 241: Assignment 10: Problem 1

Let us evaluate the conditional expectation $\beta = \mathbb{E}(-e^{-\gamma(W+IS_T)} \mid (t, S_t))$ with $S_T \sim N(S_t, \sigma^2(T-t))$

$\beta = -e^{-\gamma W}\, \mathbb{E}(e^{-\gamma I S_T} \mid (t, S_t))$

$\beta = -e^{-\gamma W}\, e^{-\gamma I S_t + \frac{\sigma^2(T-t) \times \gamma^2 I^2}{2}}$

In the end, this gives us:

$V(t, S_t, W, I) = e^{-\gamma\left(W + IS_t - \gamma \frac{I^2 \sigma^2 (T-t)}{2}\right)}$

By definition of the indifference Bid price:

$V(t, S_t, W - Q^{(b)}, I+1) = V(t, S_t, W, I)$

Hence: $e^{-\gamma\left(W - Q^{(b)} + (I+1)S_t - \gamma \frac{(I+1)^2 \sigma^2 (T-t)}{2}\right)} = e^{-\gamma\left(W + IS_t - \gamma I^2 \frac{\sigma^2 (T-t)}{2}\right)}$

This gives us: $-Q^{(b)} + (I+1)S_t - \gamma \frac{(I+1)^2 \sigma^2 (T-t)}{2} = IS_t - \gamma I^2 \frac{\sigma^2 (T-t)}{2}$

$Q^{(b)} = S_t - \gamma \sigma^2 \frac{(T-t)}{2} \left((I+1)^2 - I^2\right)$

$\underline{Q^{(b)} = S_t - \gamma \sigma^2 \frac{(T-t)}{2} (2I+1) = Q^{(b)}(t, S_t, I)}$

Similarly, using that $V(t, S_t, W + Q^{(a)}, I-1) = V(t, S_t, W, I)$, we get that:

$\underline{Q^{(a)}(t, S_t, I) = S_t - \frac{\gamma \sigma^2 (T-t)}{2} (2I-1)}$

My code for this question can be found in the `assignment10_code.py` file.

We printed the final reward, the final inventory as well as the final bid and ask prices for the Optimal and Naive policies across the different traces we generated.

Empirically, we did not manage to get an outstanding evidence that the Optimal Policy does indeed perform better than the naive policy. In particular, the final reward was not substantially higher on average.

The graph of the probability distribution of time steps to finish the game is can be found here: