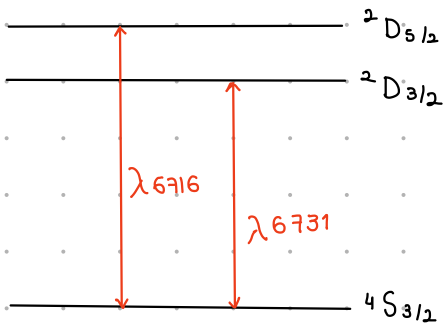
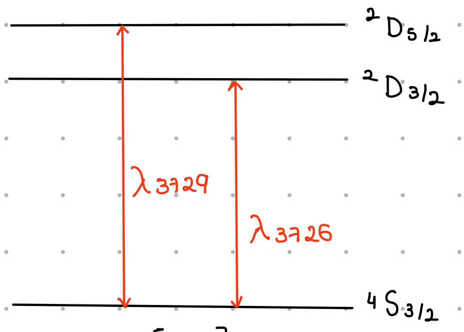
Ecuaciones de balanceo:



[SII]



[OII]

Para OII:

6716:

$$g(^4S_{3/2} \rightarrow {}^2D_{5/2}) + g(^2S_{3/2}, {}^2D_{5/2}) = g(^2D_{5/2} \rightarrow {}^4S_{3/2}) + g(^2D_{5/2} \rightarrow {}^2D_{3/2}) + g(^2D_{5/2} \rightarrow {}^4S_{3/2})$$

Con las definiciones de las tasas de excitación, desexcitación y emisión espontánea se tiene,

$$\frac{C N_e N_{^4S_{3/2}} \Omega(^4S_{3/2}, {}^2D_{5/2})\, e^{-E(^4S_{3/2}, {}^2D_{5/2})/kT}}{g(^4S_{3/2})} + \frac{C N_e N_{^2D_{3/2}} \Omega(^2D_{3/2}, {}^2D_{5/2})\, e^{-E(^2D_{3/2}, {}^2D_{5/2})/kT}}{g(^2D_{3/2})} =$$

$$\frac{C N_e N_{^2D_{5/2}} \Omega(^2D_{5/2}, {}^4S_{3/2})}{g(^2D_{5/2})} + \frac{C N_e N_{^2D_{5/2}} \Omega(^2D_{5/2}, {}^2D_{3/2})}{g(^2D_{5/2})} + N_{^2D_{5/2}} A_{6716}$$

Despejando los términos que contienen a $N_{^4S_{3/2}}$ se obtiene

$$\frac{C N_e N_{^4S_{3/2}} \Omega(^4S_{3/2}, {}^2D_{5/2})\, e^{-E(^4S_{3/2}, {}^2D_{5/2})/kT}}{g(^4S_{3/2})} = - \frac{C N_e N_{^2D_{3/2}} \Omega(^2D_{3/2}, {}^2D_{5/2})\, e^{-E(^2D_{3/2}, {}^2D_{5/2})/kT}}{g(^2D_{3/2})}$$

$$+ \frac{C N_e N_{^2D_{5/2}} \Omega(^2D_{5/2}, {}^4S_{3/2})}{g(^2D_{5/2})} + \frac{C N_e N_{^2D_{5/2}} \Omega(^2D_{5/2}, {}^2D_{3/2})}{g(^2D_{5/2})} + N_{^2D_{5/2}} A_{6716}$$

$$\frac{N_{^4S_{3/2}}\, e^{-E(^4S_{3/2}, {}^2D_{5/2})/kT}}{g(^4S_{3/2})} = - \frac{N_{^2D_{3/2}} \Omega(^2D_{3/2}, {}^2D_{5/2})\, e^{-E(^2D_{3/2}, {}^2D_{5/2})/kT}}{g(^2D_{3/2})\; \Omega(^4S_{3/2}, {}^2D_{5/2})}$$

$$+ \frac{N_{^2D_{5/2}} \Omega(^2D_{5/2}, {}^4S_{3/2})}{g(^2D_{5/2})\; \Omega(^4S_{3/2}, {}^2D_{5/2})} + \frac{N_{^2D_{5/2}} \Omega(^2D_{5/2}, {}^2D_{3/2})}{g(^2D_{5/2})\, \Omega(^4S_{3/2}, {}^2D_{5/2})} + \frac{N_{^2D_{5/2}} A_{6716}}{C N_e\; \Omega(^4S_{3/2}, {}^2D_{5/2})}$$

6731:

$$g\left({}^4S_{3/2} \rightarrow {}^2D_{3/2}\right) + g\left({}^2D_{5/2}, {}^2D_{3/2}\right) = g\left({}^2D_{3/2} \rightarrow {}^4S_{3/2}\right) + g\left({}^2D_{3/2} \rightarrow {}^2D_{5/2}\right) + g\left({}^2D_{3/2} \rightarrow {}^4S_{3/2}\right)$$

Con las definiciones de las tasas de excitación, desexcitación y emisión espontánea se tiene,

$$\frac{C N_e N_{^4S_{3/2}}\, \Omega\left({}^4S_{3/2}, {}^2D_{3/2}\right) e^{-E\left({}^4S_{3/2}, {}^2D_{3/2}\right)/kT}}{g\left({}^4S_{3/2}\right)} + \frac{C N_e N_{^2D_{5/2}}\, \Omega\left({}^2D_{5/2}, {}^2D_{3/2}\right)}{g\left({}^2D_{5/2}\right)} =$$

$$\frac{C N_e N_{^2D_{3/2}}\, \Omega\left({}^2D_{3/2}, {}^4S_{3/2}\right)}{g\left({}^2D_{3/2}\right)} + \frac{C N_e N_{^2D_{3/2}}\, \Omega\left({}^2D_{3/2}, {}^2D_{5/2}\right) e^{-E\left({}^2D_{3/2}, {}^2D_{5/2}\right)/kT}}{g\left({}^2D_{3/2}\right)} + N_{^2D_{3/2}} A_{6731}$$

Despejando los términos que contienen a $N_{^4S_{3/2}}$ se obtiene

$$\frac{C N_e N_{^4S_{3/2}}\, \Omega\left({}^4S_{3/2}, {}^2D_{3/2}\right) e^{-E\left({}^4S_{3/2}, {}^2D_{3/2}\right)/kT}}{g\left({}^4S_{3/2}\right)} = -\frac{C N_e N_{^2D_{5/2}}\, \Omega\left({}^2D_{5/2}, {}^2D_{3/2}\right)}{g\left({}^2D_{5/2}\right)}$$

$$+\frac{C N_e N_{^2D_{3/2}}\, \Omega\left({}^2D_{3/2}, {}^4S_{3/2}\right)}{g\left({}^2D_{3/2}\right)} + \frac{C N_e N_{^2D_{3/2}}\, \Omega\left({}^2D_{3/2}, {}^2D_{5/2}\right) e^{-E\left({}^2D_{3/2}, {}^2D_{5/2}\right)/kT}}{g\left({}^2D_{3/2}\right)} + N_{^2D_{3/2}} A_{6731}$$

$$\frac{N_{^4S_{3/2}}\, e^{-E\left({}^4S_{3/2}, {}^2D_{3/2}\right)/kT}}{g\left({}^4S_{3/2}\right)} = -\frac{N_{^2D_{5/2}}\, \Omega\left({}^2D_{5/2}, {}^2D_{3/2}\right)}{\Omega\left({}^4S_{3/2}, {}^2D_{3/2}\right) g\left({}^2D_{5/2}\right)}$$

$$+\frac{N_{^2D_{3/2}}\, \Omega\left({}^2D_{3/2}, {}^4S_{3/2}\right)}{g\left({}^2D_{3/2}\right) \Omega\left({}^4S_{3/2}, {}^2D_{3/2}\right)} + \frac{N_{^2D_{3/2}}\, \Omega\left({}^2D_{3/2}, {}^2D_{5/2}\right) e^{-E\left({}^2D_{3/2}, {}^2D_{5/2}\right)/kT}}{g\left({}^2D_{3/2}\right) \Omega\left({}^4S_{3/2}, {}^2D_{3/2}\right)} + \frac{N_{^2D_{3/2}} A_{6731}}{C N_e\, \Omega\left({}^4S_{3/2}, {}^2D_{3/2}\right)}$$

Encontrando la razón entre los dos términos se tiene:

$$\cancelto{1}{\dfrac{\dfrac{N_{^4S_{3/2}}\, e^{-E(^4S_{3/2},\,^2D_{5/2})/kT}}{g(^4S_{3/2})}}{\dfrac{N_{^4S_{3/2}}\, e^{-E(^4S_{3/2},\,^2D_{3/2})/kT}}{g(^4S_{3/2})}}} = \dfrac{-\dfrac{N_{^2D_{3/2}}\,\Omega(^2D_{3/2},\,^2D_{5/2})\, e^{-E(^2D_{3/2},\,^2D_{5/2})/kT}}{g(^2D_{3/2})\ \Omega(^4S_{3/2},\,^2D_{5/2})} + \dfrac{N_{^2D_{5/2}}\,\Omega(^2D_{5/2},\,^4S_{3/2})}{g(^2D_{5/2})\ \Omega(^4S_{3/2},\,^2D_{5/2})} + \dfrac{N_{^2D_{5/2}}\,\Omega(^2D_{5/2},\,^2D_{3/2})}{g(^2D_{5/2})\,\Omega(^4S_{3/2},\,^2D_{5/2})} + \dfrac{N_{^2D_{5/2}}\,A_{6716}}{CN_e\ \Omega(^4S_{3/2},\,^2D_{5/2})}}{-\dfrac{N_{^2D_{5/2}}\,\Omega(^2D_{5/2},\,^2D_{3/2})}{\Omega(^4S_{3/2},\,^2D_{3/2})\,g(^2D_{5/2})} + \dfrac{N_{^2D_{3/2}}\,\Omega(^2D_{3/2},\,^4S_{3/2})}{g(^2D_{3/2})\ \Omega(^4S_{3/2},\,^2D_{3/2})} + \dfrac{N_{^2D_{3/2}}\,\Omega(^2D_{3/2},\,^2D_{5/2})\ e^{-E(^2D_{3/2},\,^2D_{5/2})/kT}}{g(^2D_{3/2})\ \Omega(^4S_{3/2},\,^2D_{3/2})} + \dfrac{N_{^2D_{3/2}}\,A_{6731}}{CN_e\ \Omega(^4S_{3/2},\,^2D_{3/2})}}$$

Lo anterior asumiendo que $e^{-E(^4S_{3/2},\,^2D_{5/2})} \sim e^{-E(^4S_{3/2},\,^2D_{3/2})}$

$$-\dfrac{N_{^2D_{5/2}}\,\Omega(^2D_{5/2},\,^2D_{3/2})}{\Omega(^4S_{3/2},\,^2D_{3/2})\,g(^2D_{5/2})} + \dfrac{N_{^2D_{3/2}}\,\Omega(^2D_{3/2},\,^4S_{3/2})}{g(^2D_{3/2})\ \Omega(^4S_{3/2},\,^2D_{3/2})} + \dfrac{N_{^2D_{3/2}}\,\Omega(^2D_{3/2},\,^2D_{5/2})\ e^{-E\ ^2D_{3/2},\,^2D_{5/2}\ kT}}{g(^2D_{3/2})\ \Omega(^4S_{3/2},\,^2D_{3/2})}$$

$$+\dfrac{N_{^2D_{3/2}}\,A_{6731}}{CN_e\ \Omega(^4S_{3/2},\,^2D_{3/2})} = -\dfrac{N_{^2D_{3/2}}\,\Omega(^2D_{3/2},\,^2D_{5/2})\ e^{-E(^2D_{3/2},\,^2D_{5/2})/kT}}{g(^2D_{3/2})\ \Omega(^4S_{3/2},\,^2D_{5/2})} + \dfrac{N_{^2D_{5/2}}\,\Omega(^2D_{5/2},\,^4S_{3/2})}{g(^2D_{5/2})\ \Omega(^4S_{3/2},\,^2D_{5/2})}$$

$$+\dfrac{N_{^2D_{5/2}}\,\Omega(^2D_{5/2},\,^2D_{3/2})}{g(^2D_{5/2})\,\Omega(^4S_{3/2},\,^2D_{5/2})} + \dfrac{N_{^2D_{5/2}}\,A_{6716}}{CN_e\ \Omega(^4S_{3/2},\,^2D_{5/2})}$$

Ahora bien, para $h\nu \ll kT$ se tiene $e^{-E(^2D_{3/2},\,^2D_{5/2})/kT} \approx 1$, por lo cual:

$$-\dfrac{N_{^2D_{5/2}}\,\Omega(^2D_{5/2},\,^2D_{3/2})}{\Omega(^4S_{3/2},\,^2D_{3/2})\,g(^2D_{5/2})} + \dfrac{N_{^2D_{3/2}}\,\Omega(^2D_{3/2},\,^4S_{3/2})}{g(^2D_{3/2})\ \Omega(^4S_{3/2},\,^2D_{3/2})} + \dfrac{N_{^2D_{3/2}}\,\Omega(^2D_{3/2},\,^2D_{5/2})}{g(^2D_{3/2})\ \Omega(^4S_{3/2},\,^2D_{3/2})} + \dfrac{N_{^2D_{3/2}}\,A_{6731}}{CN_e\ \Omega(^4S_{3/2},\,^2D_{3/2})} =$$

$$-\dfrac{N_{^2D_{3/2}}\,\Omega(^2D_{3/2},\,^2D_{5/2})}{g(^2D_{3/2})\ \Omega(^4S_{3/2},\,^2D_{5/2})} + \dfrac{N_{^2D_{5/2}}\,\Omega(^2D_{5/2},\,^4S_{3/2})}{g(^2D_{5/2})\ \Omega(^4S_{3/2},\,^2D_{5/2})} + \dfrac{N_{^2D_{5/2}}\,\Omega(^2D_{5/2},\,^2D_{3/2})}{g(^2D_{5/2})\ (^4S_{3/2},\,^2D_{5/2})} + \dfrac{N_{^2D_{5/2}}\,A_{6716}}{CN_e\,\Omega(^4S_{3/2},\,^2D_{5/2})}$$

Organizando los términos:

$$\dfrac{N_{^2D_{5/2}}\,\Omega(^2D_{5/2},\,^2D_{3/2})}{\Omega(^4S_{3/2},\,^2D_{3/2})\,g(^2D_{5/2})} + \dfrac{N_{^2D_{5/2}}\,\Omega(^2D_{5/2},\,^4S_{3/2})}{g(^2D_{5/2})\ \Omega(^4S_{3/2},\,^2D_{5/2})} + \dfrac{N_{^2D_{5/2}}\,\Omega(^2D_{5/2},\,^2D_{3/2})}{g(^2D_{5/2})\ (^4S_{3/2},\,^2D_{5/2})} + \dfrac{N_{^2D_{5/2}}\,A_{6716}}{CN_e\,\Omega(^4S_{3/2},\,^2D_{5/2})} =$$

$$\dfrac{N_{^2D_{3/2}}\,\Omega(^2D_{3/2},\,^2D_{5/2})}{g(^2D_{3/2})\ \Omega(^4S_{3/2},\,^2D_{5/2})} + \dfrac{N_{^2D_{3/2}}\,\Omega(^2D_{3/2},\,^4S_{3/2})}{g(^2D_{3/2})\ \Omega(^4S_{3/2},\,^2D_{3/2})} + \dfrac{N_{^2D_{3/2}}\,\Omega(^2D_{3/2},\,^2D_{5/2})}{g(^2D_{3/2})\ \Omega(^4S_{3/2},\,^2D_{3/2})} + \dfrac{N_{^2D_{3/2}}\,A_{6731}}{CN_e\ \Omega(^4S_{3/2},\,^2D_{3/2})}$$

Encontrando la razón entre $N_{^2D_{5/2}}/N_{^2D_{3/2}}$

$$N^2_{5/2}\left(\frac{\Omega(^2D_{5/2},{}^2D_{3/2})}{\Omega(^4S_{3/2},{}^2D_{3/2})\,g(^2D_{5/2})}+\frac{\Omega(^2D_{5/2},{}^4S_{3/2})}{g(^2D_{5/2})\,\Omega(^4S_{3/2},{}^2D_{5/2})}+\frac{\Omega(^2D_{5/2},{}^2D_{3/2})}{g(^2D_{5/2})\,(^4S_{3/2},{}^2D_{5/2})}+\frac{A_{6716}}{CN_e\,\Omega(^4S_{3/2},{}^2D_{5/2})}\right)=$$

$$N^2_{3/2}\left(\frac{\Omega(^2D_{3/2},{}^2D_{5/2})}{g(^2D_{3/2})\,\Omega(^4S_{3/2},{}^2D_{5/2})}+\frac{\Omega(^2D_{3/2},{}^4S_{3/2})}{g(^2D_{3/2})\,\Omega(^4S_{3/2},{}^2D_{3/2})}+\frac{\Omega(^2D_{3/2},{}^2D_{5/2})}{g(^2D_{3/2})\,\Omega(^4S_{3/2},{}^2D_{3/2})}+\frac{A_{6731}}{CN_e\,\Omega(^4S_{3/2},{}^2D_{3/2})}\right)$$

$$\frac{N^2_{5/2}}{N^2_{3/2}}=\frac{\left(\frac{\Omega(^2D_{3/2},{}^2D_{5/2})}{g(^2D_{3/2})\,\Omega(^4S_{3/2},{}^2D_{5/2})}+\frac{\Omega(^2D_{3/2},{}^4S_{3/2})}{g(^2D_{3/2})\,\Omega(^4S_{3/2},{}^2D_{3/2})}+\frac{\Omega(^2D_{3/2},{}^2D_{5/2})}{g(^2D_{3/2})\,\Omega(^4S_{3/2},{}^2D_{3/2})}+\frac{A_{6731}}{CN_e\,\Omega(^4S_{3/2},{}^2D_{3/2})}\right)}{\left(\frac{\Omega(^2D_{5/2},{}^2D_{3/2})}{\Omega(^4S_{3/2},{}^2D_{3/2})\,g(^2D_{5/2})}+\frac{\Omega(^2D_{5/2},{}^4S_{3/2})}{g(^2D_{5/2})\,\Omega(^4S_{3/2},{}^2D_{5/2})}+\frac{\Omega(^2D_{5/2},{}^2D_{3/2})}{g(^2D_{5/2})\,(^4S_{3/2},{}^2D_{5/2})}+\frac{A_{6716}}{CN_e\,\Omega(^4S_{3/2},{}^2D_{5/2})}\right)}$$

$$\frac{N^2_{5/2}}{N^2_{3/2}}=\frac{\frac{1}{g(^2D_{3/2})}\left(\frac{\Omega(^2D_{3/2},{}^2D_{5/2})}{\Omega(^4S_{3/2},{}^2D_{5/2})}+\frac{\Omega(^2D_{3/2},{}^4S_{3/2})}{\Omega(^4S_{3/2},{}^2D_{3/2})}+\frac{\Omega(^2D_{3/2},{}^2D_{5/2})}{\Omega(^4S_{3/2},{}^2D_{3/2})}+\frac{A_{6731}\,g(^2D_{3/2})}{CN_e\,\Omega(^4S_{3/2},{}^2D_{3/2})}\right)}{\frac{1}{g(^2D_{5/2})}\left(\frac{\Omega(^2D_{5/2},{}^2D_{3/2})}{\Omega(^4S_{3/2},{}^2D_{3/2})}+\frac{\Omega(^2D_{5/2},{}^4S_{3/2})}{\Omega(^4S_{3/2},{}^2D_{5/2})}+\frac{\Omega(^2D_{5/2},{}^2D_{3/2})}{\Omega(^4S_{3/2},{}^2D_{5/2})}+\frac{A_{6716}\,g(^2D_{5/2})}{CN_e\,\Omega(^4S_{3/2},{}^2D_{5/2})}\right)}$$

Con $\Omega(j,k)=\Omega(k,j)$ se tiene:

$$\frac{N^2_{5/2}}{N^2_{3/2}}=\frac{g(^2D_{5/2})\left(1+\frac{\Omega(^2D_{3/2},{}^2D_{5/2})}{\Omega(^4S_{3/2},{}^2D_{5/2})}+\frac{\Omega(^2D_{3/2},{}^2D_{5/2})}{\Omega(^4S_{3/2},{}^2D_{3/2})}+\frac{A_{6731}\,g(^2D_{3/2})}{CN_e\,\Omega(^4S_{3/2},{}^2D_{3/2})}\right)}{g(^2D_{3/2})\left(1+\frac{\Omega(^2D_{5/2},{}^2D_{3/2})}{\Omega(^4S_{3/2},{}^2D_{3/2})}+\frac{\Omega(^2D_{5/2},{}^2D_{3/2})}{\Omega(^4S_{3/2},{}^2D_{5/2})}+\frac{A_{6716}\,g(^2D_{5/2})}{CN_e\,\Omega(^4S_{3/2},{}^2D_{5/2})}\right)}$$

Usando la ecuación $j_\nu = N_{k\downarrow}A_{kj}\frac{h\nu_{kj}}{4\pi}$ y sabiendo que $h\nu_{6716}\approx h\nu_{6731}$ para [SII]

$$\frac{j_{6716}}{j_{6731}}=\frac{A_{6716}\,g(^2D_{5/2})\left(1+\frac{\Omega(^2D_{3/2},{}^2D_{5/2})}{\Omega(^4S_{3/2},{}^2D_{5/2})}+\frac{\Omega(^2D_{3/2},{}^2D_{5/2})}{\Omega(^4S_{3/2},{}^2D_{3/2})}+\frac{A_{6731}\,g(^2D_{3/2})}{CN_e\,\Omega(^4S_{3/2},{}^2D_{3/2})}\right)}{A_{6731}\,g(^2D_{3/2})\left(1+\frac{\Omega(^2D_{5/2},{}^2D_{3/2})}{\Omega(^4S_{3/2},{}^2D_{3/2})}+\frac{\Omega(^2D_{5/2},{}^2D_{3/2})}{\Omega(^4S_{3/2},{}^2D_{5/2})}+\frac{A_{6716}\,g(^2D_{5/2})}{CN_e\,\Omega(^4S_{3/2},{}^2D_{5/2})}\right)}$$

Con $A_{6716}\approx 2.6\cdot 10^{-4}\ s^{-1}$ y $A_{6731}\approx 8.8\cdot 10^{-4}\ s^{-1}$

De forma similar se encuentra la ecuación para OII:

$$\frac{j_{3729}}{j_{3726}} = \frac{A_{3729}\, g(^2D_{5/2})\left(1 + \frac{\Omega(^2D_{3/2},\, ^2D_{5/2})}{\Omega(^4S_{3/2},\, ^2D_{5/2})} + \frac{\Omega(^2D_{3/2},\, ^2D_{5/2})}{\Omega(^4S_{3/2},\, ^2D_{3/2})} + \frac{A_{3726}\, g(^2D_{3/2})}{C N_e\, \Omega(^4S_{3/2},\, ^2D_{3/2})}\right)}{A_{3726}\, g(^2D_{3/2})\left(1 + \frac{\Omega(^2D_{5/2},\, ^2D_{3/2})}{\Omega(^4S_{3/2},\, ^2D_{3/2})} + \frac{\Omega(^2D_{5/2},\, ^2D_{3/2})}{\Omega(^4S_{3/2},\, ^2D_{5/2})} + \frac{A_{3729}\, g(^2D_{5/2})}{C N_e\, \Omega(^4S_{3/2},\, ^2D_{5/2})}\right)}$$

Para graficar en Python se toman los siguientes valores:

```
#SII
#Valores de A
a6716=2.6e-4
a6731=8.8e-4

#Omega
d32_d52=7.47
s32_d52=4.14
s32_d32=2.76
d52_d32=7.47

#Pesos
g52=6
g32=4

T=10000
C=8.62e-6/np.sqrt(T)
ne=np.linspace(1,100000,100000)
```

[S II]

```
#OII
#Valores de A
a3729=3.6e-5
a3726=1.6e-4

#Omegas
o_d32_d52=1.17
o_s32_d52=0.804
o_s32_d32=0.536
o_d52_d32=1.17
```

[O II]

Las ecuaciones en el código quedan de la siguiente forma:

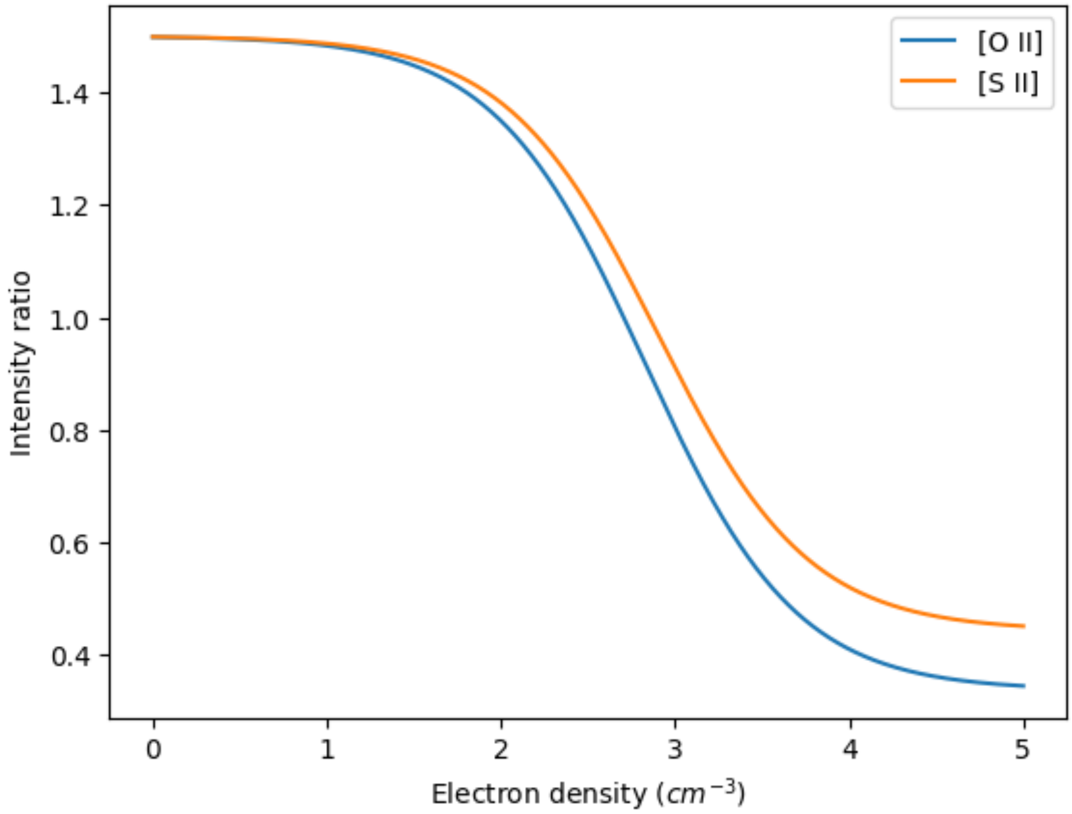
```
n=(a6716*g52)*(1+(d32_d52/s32_d52)+(d32_d52/s32_d32)+((a6731*g32)/(C*ne*s32_d32)))

d=(a6731*g32)*(1+(d52_d32/s32_d32)+(d52_d32/s32_d52)+((a6716*g52)/(C*ne*s32_d52)))

j=n/d
```

[S II]

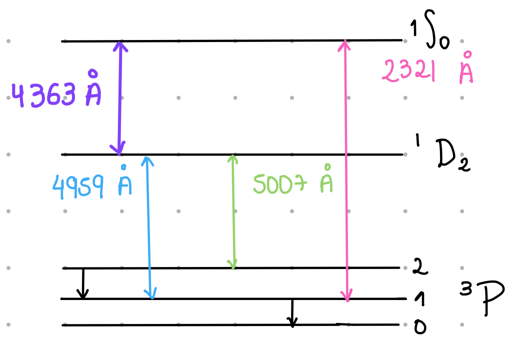
```
n_o=(a3729*g52)*(1+(o_d32_d52/o_s32_d52)+(o_d32_d52/o_s32_d32)+((a3726*g32)/(C*ne*o_s32_d32)))

d_o=(a3726*g32)*(1+(o_d52_d32/o_s32_d32)+(o_d52_d32/o_s32_d52)+((a3729*g52)/(C*ne*o_s32_d52)))

j_o=n_o/d_o
```

[O II]

Con lo anterior se obtiene



Para [O III]·



Sus ecuaciones de balanceo son:

Nivel S:

$$q_{P\to S} + q_{D\to S} = q^e_{S\to D} + q^e_{S\to P} + q_{S\to D} + q_{S\to P}$$

$$N_P\, C N_e \frac{\Omega_{PS}}{g_P} e^{-E_{PS}/kT} + N_D\, C N_e \frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT} = N_S A_{SD} + N_S A_{SP} + N_S\, C N_e \frac{\Omega_{SD}}{g_S} + N_S\, C N_e \frac{\Omega_{SP}}{g_S}$$

$$N_P\, C N_e \frac{\Omega_{PS}}{g_P} e^{-E_{PS}/kT} = -N_D\, C N_e \frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT} + N_S A_{SD} + N_S A_{SP} + N_S\, C N_e \frac{\Omega_{SD}}{g_S} + N_S\, C N_e \frac{\Omega_{SP}}{g_S}$$

Nivel D:

$$q_{P\to D} + q_{S\to D} + q^e_{S\to D} = q_{D\to S} + q_{D\to P} + q^e_{D\to P}$$

$$N_P\, C N_e \frac{\Omega_{PD}}{g_P} e^{-E_{PD}/kT} + N_S\, C N_e \frac{\Omega_{SD}}{g_S} + N_S A_{SD} = N_D\, C N_e \frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT} + N_D\, C N_e \frac{\Omega_{DP}}{g_D} + N_D A_{DP}$$

$$N_P\, C N_e \frac{\Omega_{PD}}{g_P} e^{-E_{PD}/kT} = -N_S\, C N_e \frac{\Omega_{SD}}{g_S} - N_S A_{SD} + N_D\, C N_e \frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT} + N_D\, C N_e \frac{\Omega_{DP}}{g_D} + N_D A_{DP}$$

Hay que hallar la razón N_D/N_S, por lo que hacemos la razón entre las últimas expresiones halladas para cada nivel:

$$\frac{\cancel{N_P C N_e} \frac{\Omega_{PD}}{\cancel{g_P}} e^{-E_{PD}/kT}}{\cancel{N_P C N_e} \frac{\Omega_{PS}}{\cancel{g_P}} e^{-E_{PS}/kT}} = \frac{-N_S\, C N_e \frac{\Omega_{SD}}{g_S} - N_S A_{SD} + N_D\, C N_e \frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT} + N_D\, C N_e \frac{\Omega_{DP}}{g_D} + N_D A_{DP}}{-N_D\, C N_e \frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT} + N_S A_{SD} + N_S A_{SP} + N_S\, C N_e \frac{\Omega_{SD}}{g_S} + N_S\, C N_e \frac{\Omega_{SP}}{g_S}}$$

$$\frac{\Omega_{PD}\, e^{-E_{PD}/kT}}{\Omega_{PS}\, e^{-E_{PS}/kT}} = \frac{-N_S\, CNe\frac{\Omega_{SD}}{g_S} - N_S A_{SD} + N_D\, CNe\frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT} + N_D\, CNe\frac{\Omega_{DP}}{g_D} + N_D A_{DP}}{-N_D\, CNe\frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT} + N_S A_{SD} + N_S A_{SP} + N_S\, CNe\frac{\Omega_{SD}}{g_S} + N_S\, CNe\frac{\Omega_{SP}}{g_S}}$$

$$\Omega_{PD}\, e^{-E_{PD}/kT}\left(-N_D\, CNe\frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT} + N_S A_{SD} + N_S A_{SP} + N_S\, CNe\frac{\Omega_{SD}}{g_S} + N_S\, CNe\frac{\Omega_{SP}}{g_S}\right) =$$

$$\Omega_{PS}\, e^{-E_{PS}/kT}\left(-N_S\, CNe\frac{\Omega_{SD}}{g_S} - N_S A_{SD} + N_D\, CNe\frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT} + N_D\, CNe\frac{\Omega_{DP}}{g_D} + N_D A_{DP}\right)$$

Sacando factor común de N_D y N_S

$$N_D\left(-\Omega_{PD}\, e^{-E_{PD}/kT}\, CNe\frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT}\right) + N_S\left(A_{SD}\,\Omega_{PD}\, e^{-E_{PD}/kT} + A_{SP}\,\Omega_{PD}\, e^{-E_{PD}/kT}\right.$$

$$\left. + CNe\frac{\Omega_{SD}}{g_S}\Omega_{PD}\, e^{-E_{PD}/kT} + CNe\frac{\Omega_{SP}}{g_S}\Omega_{PD}\, e^{-E_{PD}/kT}\right) = N_D\left(CNe\frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT}\,\Omega_{PS}\, e^{-E_{PS}/kT}\right.$$

$$\left. + CNe\frac{\Omega_{DP}}{g_D}\Omega_{PS}\, e^{-E_{PS}/kT} + A_{DP}\,\Omega_{PS}\, e^{-E_{PS}/kT}\right) + N_S\left(-CNe\frac{\Omega_{SD}}{g_S}\Omega_{PS}\, e^{-E_{PS}/kT} - A_{SD}\,\Omega_{PS}\, e^{-E_{PS}/kT}\right)$$

Organizando los términos de N_D y N_S

$$N_D\left(-\Omega_{PD}\, e^{-E_{PD}/kT}\, CNe\frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT} - CNe\frac{\Omega_{DS}}{g_D} e^{-E_{DS}/kT}\,\Omega_{PS}\, e^{-E_{PS}/kT} - CNe\frac{\Omega_{DP}}{g_D}\Omega_{PS}\, e^{-E_{PS}/kT}\right.$$

$$\left. - A_{DP}\,\Omega_{PS}\, e^{-E_{PS}/kT}\right) = N_S\left(-CNe\frac{\Omega_{SD}}{g_S}\Omega_{PS}\, e^{-E_{PS}/kT} - A_{SD}\,\Omega_{PS}\, e^{-E_{PS}/kT} - A_{SD}\,\Omega_{PD}\, e^{-E_{PD}/kT}\right.$$

$$\left. - A_{SP}\,\Omega_{PD}\, e^{-E_{PD}/kT} - CNe\frac{\Omega_{SD}}{g_S}\Omega_{PD}\, e^{-E_{PD}/kT} - CNe\frac{\Omega_{SP}}{g_S}\Omega_{PD}\, e^{-E_{PD}/kT}\right)$$

$$\frac{N_D}{N_S} = \frac{-CNe\frac{\Omega_{SD}}{g_S}\Omega_{PS}e^{-E_{PS}/kT} - A_{SD}\Omega_{PS}e^{-E_{PS}/kT} - A_{SD}\Omega_{PD}e^{-E_{PD}/kT} - A_{SP}\Omega_{PD}e^{-E_{PD}/kT} - CNe\frac{\Omega_{SD}}{g_S}\Omega_{PD}e^{-E_{PD}/kT} - CNe\frac{\Omega_{SP}}{g_S}\Omega_{PD}e^{-E_{PD}/kT}}{-\Omega_{PD}e^{-E_{PD}/kT}\,CNe\frac{\Omega_{DS}}{g_D}e^{-E_{DS}/kT} - CNe\frac{\Omega_{DS}}{g_D}e^{-E_{DS}/kT}\,\Omega_{PS}e^{-E_{PS}/kT} - CNe\frac{\Omega_{DP}}{g_D}\Omega_{PS}e^{-E_{PS}/kT} - A_{DP}\cdot\Omega_{PS}e^{-E_{PS}/kT}}$$

$$\frac{N_D}{N_S} = \frac{\Omega_{PS}\left(-CNe\frac{\Omega_{SD}}{g_S}e^{-E_{PS}/kT} - A_{SD}e^{-E_{PS}/kT} - A_{SD}\frac{\Omega_{PD}}{\Omega_{PS}}e^{-E_{PD}/k} - A_{SP}\frac{\Omega_{PD}}{\Omega_{PS}}e^{-E_{PD}/kT} - \frac{CNe}{g_S}\frac{\Omega_{SD}}{\Omega_{PS}}\Omega_{PD}e^{-E_{PD}/kT} - \frac{CNe}{g_S}\frac{\Omega_{SP}}{\Omega_{PS}}\Omega_{PD}e^{--PD\ kT}\right)}{\Omega_{PS}\left(-\frac{\Omega_{PD}}{\Omega_{PS}}e^{-E_{PD}/kT}\,CNe\frac{\Omega_{DS}}{g_D}e^{-E_{DS}/kT} - CNe\frac{\Omega_{DS}}{g_D}e^{-E_{DS}/kT}\,e^{-E_{PS}/kT} - CNe\frac{\Omega_{DP}}{g_D}e^{-E_{PS}/kT} - A_{DP}e^{-E_{PS}\ kT}\right)}$$

$$\frac{N_D}{N_S} = \frac{-CNe\frac{\Omega_{SD}}{g_S}e^{-E_{PS}/kT} - A_{SD}e^{-E_{PS}/kT} - A_{SD}\frac{\Omega_{PD}}{\Omega_{PS}}e^{-E_{PD}/kT} - A_{SP}\frac{\Omega_{PD}}{\Omega_{PS}}e^{-E_{PD}/kT} - \frac{CNe}{g_S}\frac{\Omega_{SD}}{\Omega_{PS}}\Omega_{PD}e^{-E_{PD}/kT} - \frac{CNe}{g_S}\frac{\Omega_{SP}}{\Omega_{PS}}\Omega_{PD}e^{-E_{PD}/kT}}{-\frac{\Omega_{PD}}{\Omega_{PS}}e^{-E_{PD}/kT}\,CNe\frac{\Omega_{DS}}{g_D}e^{-E_{DS}/kT} - CNe\frac{\Omega_{DS}}{g_D}e^{-E_{DS}/kT}\,e^{-E_{PS}/kT} - CNe\frac{\Omega_{DP}}{g_D}e^{-E_{PS}/kT} - A_{DP}e^{-E_{PS}/kT}}$$

Multiplicando por $1/CNe$

$$\frac{N_D}{N_S} = \frac{-\frac{\Omega_{SD}}{g_S}e^{-E_{PS}/kT} - \frac{A_{SD}}{CNe}e^{-E_{PS}/kT} - \frac{A_{SD}}{CNe}\frac{\Omega_{PD}}{\Omega_{PS}}e^{-E_{PD}/kT} - \frac{A_{SP}}{CNe}\frac{\Omega_{PD}}{\Omega_{PS}}e^{-E_{PD}/kT} - \frac{1}{g_S}\frac{\Omega_{SD}}{\Omega_{PS}}\Omega_{PD}e^{-E_{PD}/kT} - \frac{1}{g_S}\frac{\Omega_{SP}}{\Omega_{PS}}\Omega_{PD}e^{-E_{PD}/kT}}{-\frac{\Omega_{PD}}{\Omega_{PS}}e^{-E_{PD}/kT}\,\frac{\Omega_{DS}}{g_D}e^{-E_{DS}/kT} - \frac{\Omega_{DS}}{g_D}e^{-E_{DS}/kT}\,e^{-E_{PS}/kT} - \frac{\Omega_{DP}}{g_D}e^{-E_{PS}/kT} - \frac{A_{DP}}{CNe}e^{-E_{PS}/kT}}$$

$$\frac{N_D}{N_S}=\frac{e^{-E_{PD}/kT}\left(\frac{\Omega_{SD}}{g_S}e^{(-E_{PS}+E_{PD})/kT}-\frac{A_{SD}}{CNe}e^{(-E_{PS}+E_{PD})/kT}-\frac{A_{SD}}{CNe}\frac{\Omega_{PD}}{\Omega_{PS}}-\frac{A_{SP}}{CNe}\frac{\Omega_{PD}}{\Omega_{PS}}-\frac{1}{g_S}\frac{\Omega_{SD}}{\Omega_{PS}}\Omega_{PD}-\frac{1}{g_S}\frac{\Omega_{SP}}{\Omega_{PS}}\Omega_{PD}\right)}{e^{-E_{PD}/kT}\left(-\frac{\Omega_{PD}}{\Omega_{PS}}\frac{\Omega_{DS}}{g_D}e^{-E_{DS}/kT}-\frac{\Omega_{DS}}{g_D}e^{(-E_{DS}-E_{PS}+E_{PD})/kT}-\frac{\Omega_{DP}}{g_D}e^{(-E_{PS}+E_{PD})/kT}-\frac{A_{DP}}{CNe}e^{(-E_{PS}+E_{PD})/kT}\right)}$$

$$\frac{N_D}{N_S}=\frac{-\frac{\Omega_{SD}}{g_S}e^{(-E_{PS}+E_{PD})/kT}-\frac{A_{SD}}{CNe}e^{(-E_{PS}+E_{PD})/kT}-\frac{A_{SD}}{CNe}\frac{\Omega_{PD}}{\Omega_{PS}}-\frac{A_{SP}}{CNe}\frac{\Omega_{PD}}{\Omega_{PS}}-\frac{1}{g_S}\frac{\Omega_{SD}}{\Omega_{PS}}\Omega_{PD}-\frac{1}{g_S}\frac{\Omega_{SP}}{\Omega_{PS}}\Omega_{PD}}{-\frac{\Omega_{PD}}{\Omega_{PS}}\frac{\Omega_{DS}}{g_D}e^{-E_{DS}/kT}-\frac{\Omega_{DS}}{g_D}e^{(-E_{DS}-E_{PS}+E_{PD})/kT}-\frac{\Omega_{DP}}{g_D}e^{(-E_{PS}+E_{PD})/kT}-\frac{A_{DP}}{CNe}e^{(-E_{PS}+E_{PD})/kT}}$$

$$\frac{N_D}{N_S}=\frac{e^{(-E_{PS}+E_{PD})/kT}\left(-\frac{\Omega_{SD}}{g_S}-\frac{A_{SD}}{CNe}\right)-\frac{1}{CNe}\frac{\Omega_{PD}}{\Omega_{PS}}(A_{SD}+A_{SP})-\frac{1}{g_S}\frac{\Omega_{PD}}{\Omega_{PS}}(\Omega_{SD}+\Omega_{SP})}{-\frac{\Omega_{DS}}{g_D}e^{-E_{DS}/kT}\left(\frac{\Omega_{PD}}{\Omega_{PS}}+e^{(-E_{PS}+E_{PD})/kT}\right)-e^{(-E_{PS}+E_{PD})/kT}\left(\frac{\Omega_{DP}}{g_D}+\frac{A_{DP}}{CNe}\right)}$$

Factor de Ω_{PD}/g_S

$$\frac{N_D}{N_S}=\frac{\frac{\Omega_{PD}}{g_S}\left[\frac{e^{(-E_{PS}+E_{PD})/kT}g_S}{\Omega_{PD}}\left(-\frac{\Omega_{SD}}{g_S}-\frac{A_{SD}}{CNe}\right)-\frac{1}{CNe}\frac{g_S}{\Omega_{PS}}(A_{SD}+A_{SP})-\frac{1}{\Omega_{PS}}(\Omega_{SD}+\Omega_{SP})\right]}{\frac{\Omega_{PD}}{g_S}\left[-\frac{g_S}{\Omega_{PD}}\frac{\Omega_{DS}}{g_D}e^{-E_{DS}/kT}\left(\frac{\Omega_{PD}}{\Omega_{PS}}+e^{(-E_{PS}+E_{PD})/kT}\right)-e^{(-E_{PS}+E_{PD})/kT}\left(\frac{\Omega_{DP}}{g_D}+\frac{A_{DP}}{CNe}\right)\frac{g_S}{\Omega_{PD}}\right]}$$

$$\frac{N_D}{N_S}=\frac{\frac{e^{(-E_{PS}+E_{PD})/kT}g_S}{\Omega_{PD}}\left(-\frac{\Omega_{SD}}{g_S}-\frac{A_{SD}}{CNe}\right)-\frac{1}{CNe}\frac{g_S}{\Omega_{PS}}(A_{SD}+A_{SP})-\frac{1}{\Omega_{PS}}(\Omega_{SD}+\Omega_{SP})}{-\frac{g_S}{\Omega_{PD}}\frac{\Omega_{DS}}{g_D}e^{-E_{DS}/kT}\left(\frac{\Omega_{PD}}{\Omega_{PS}}+e^{(-E_{PS}+E_{PD})/kT}\right)-e^{(-E_{PS}+E_{PD})/kT}\left(\frac{\Omega_{DP}}{g_D}+\frac{A_{DP}}{CNe}\right)\frac{g_S}{\Omega_{PD}}}$$

$$\frac{N_D}{N_S}=\frac{\frac{e^{(-E_{PS}+E_{PD})/kT}g_S}{\Omega_{PD}}\left(-\frac{\Omega_{SD}}{g_S}-\frac{A_{SD}}{CNe}\right)-\frac{1}{CNe}\frac{g_S}{\Omega_{PS}}(A_{SD}+A_{SP})-\frac{1}{\Omega_{PS}}(\Omega_{SD}+\Omega_{SP})}{\frac{g_S}{g_D}e^{-E_{DS}/kT}\left(-\frac{\Omega_{DS}}{\Omega_{PS}}-\frac{\Omega_{DS}}{\Omega_{PD}}e^{(-E_{PS}+E_{PD})/kT}\right)-e^{(-E_{PS}+E_{PD})/kT}\left(\frac{\Omega_{DP}}{g_D}\frac{g_S}{\Omega_{PD}}+\frac{A_{DP}}{CNe}\frac{g_S}{\Omega_{PD}}\right)}$$

Con $e^{-E_{DS}/kT} = e^{(-E_{PS}+E_{PD})/kT}$

$$\frac{N_D}{N_S} = \frac{\dfrac{e^{-E_{DS}/kT}\, g_S}{\Omega_{PD}}\left(\dfrac{\Omega_{SD}}{g_S} - \dfrac{A_{SD}}{CN_e}\right) - \dfrac{1}{CN_e}\dfrac{g_S}{\Omega_{PS}}\left(A_{SD}+A_{SP}\right) - \dfrac{1}{\Omega_{PS}}\left(\Omega_{SD}+\Omega_{SP}\right)}{\dfrac{g_S}{g_D}\, e^{-E_{DS}/kT}\left(-\dfrac{\Omega_{DS}}{\Omega_{PS}} - \dfrac{\Omega_{DS}}{\Omega_{PD}}\, e^{-E_{DS}/kT} - \dfrac{\Omega_{DP}}{\Omega_{PD}} - \dfrac{A_{DP}\, g_D}{CN_e\,\Omega_{PD}}\right)}$$

Equivalente a:

$$\frac{N_D}{N_S} = \frac{\dfrac{(A_{SD}+A_{SP})}{N_e}\dfrac{g_S}{C\,\Omega_{PS}} + \dfrac{(\Omega_{SD}+\Omega_{SP})}{\Omega_{PS}} + \dfrac{\Omega_{SD}}{\Omega_{PD}}\, e^{-E_{SD}/kT} + \dfrac{A_{SD}}{N_e}\dfrac{e^{-E_{SD}/kT}\, g_S}{C\,\Omega_{PD}}}{\dfrac{\Omega_{DS}}{g_D}\, e^{-E_{DS}/kT}\dfrac{e^{-E_{SD}/kT}\, g_S}{\Omega_{PD}} + \dfrac{\Omega_{DP}}{g_D}\dfrac{e^{-E_{SD}/kT}\, g_S}{\Omega_{PD}} + \dfrac{A_{DP}}{N_e}\dfrac{e^{-E_{SD}/kT}\, g_S}{C\,\Omega_{PD}}}$$

$$\frac{1}{\dfrac{C\Omega_{PD}}{g_D}\dfrac{\Omega_{DS}}{\Omega_{PS}}\;\dfrac{e^{-E_{SD}/kT}\, g_S}{C\,\Omega_{PD}}}$$

Sacando factor común

$$\frac{N_D}{N_S} = \frac{\dfrac{(A_{SD}+A_{SP})}{N_e}\dfrac{g_S}{C\,\Omega_{PS}} + \dfrac{(\Omega_{SD}+\Omega_{SP})}{\Omega_{PS}} + \dfrac{\Omega_{SD}}{\Omega_{PD}}\, e^{-E_{SD}/kT} + \dfrac{A_{SD}}{N_e}\dfrac{e^{-E_{SD}/kT}\, g_S}{C\,\Omega_{PD}}}{\dfrac{g_S}{g_D}\, e^{-E_{SD}/kT}\left(\dfrac{\Omega_{DS}}{\Omega_{PD}}\, e^{-E_{DS}/kT} + 1 + \dfrac{g_D\, A_{DP}}{CN_e\,\Omega_{PD}}\right)}$$

Con $\gamma = \dfrac{\Omega_{DS}+\Omega_{PS}}{\Omega_{PS}}$.

$$\frac{N_D}{N_S} = \frac{\dfrac{(A_{SD}+A_{SP})}{N_e C}\dfrac{g_S}{\Omega_{PS}} + \gamma + \dfrac{\Omega_{SD}}{\Omega_{PD}}\, e^{-E_{SD}/kT} + \dfrac{A_{SD}}{N_e}\dfrac{e^{-E_{SD}/kT}\, g_S}{C\,\Omega_{PD}}}{\dfrac{g_S}{g_D}\, e^{-E_{SD}/kT}\left(\dfrac{\Omega_{DS}}{\Omega_{PD}}\, e^{-E_{DS}/kT} + \dfrac{A_{DP}\, g_D}{CN_e\,\Omega_{PD}} + \dfrac{\Omega_{DS}}{\Omega_{PS}} + 1\right)}$$

$$\frac{N_D}{N_S} = \frac{\left(\frac{(A_{SD} + A_{SP})}{\gamma N_e C} \frac{g_S}{\Omega_{PS}} + 1 + \frac{\Omega_{SD}}{\gamma \Omega_{PD}} e^{-E_{SD}/kT} + \frac{A_{SD}}{N_e} \frac{e^{-E_{SD}/kT} g_S}{\gamma C \Omega_{PD}} \right) \gamma}{\frac{g_S}{g_D} e^{-E_{SD}/kT} \left(\frac{\Omega_{DS}}{\Omega_{PD}} e^{-E_{DS}/kT} + \frac{A_{DP}\, g_D}{C N_e \Omega_{PD}} + \frac{\Omega_{DS}}{\Omega_{PS}} + 1 \right)}$$

$$\frac{N_D}{N_S} = \frac{\frac{(A_{SD} + A_{SP})}{\gamma N_e C} \frac{g_S}{\Omega_{PS}} + 1 + \frac{\Omega_{SD}}{\gamma \Omega_{PD}} e^{-E_{SD}/kT} + \frac{A_{SD}}{N_e} \frac{e^{-E_{SD}/kT} g_S}{\gamma C \Omega_{PD}}}{\frac{g_S}{g_D} e^{-E_{SD}/kT} \left(\frac{\Omega_{DS}}{\Omega_{PD} \gamma} e^{-E_{DS}/kT} + \frac{A_{DP}\, g_D}{C N_e \Omega_{PD} \gamma} + \frac{\Omega_{DS}}{\Omega_{PS} \gamma} + \frac{1}{\gamma} \right)}$$

$$\frac{N_D}{N_S} = \frac{\frac{(A_{SD} + A_{SP})}{\gamma N_e C} \frac{g_S}{\Omega_{PS}} + 1 + \frac{\Omega_{SD}}{\gamma \Omega_{PD}} e^{-E_{SD}/kT} + \frac{A_{SD}}{N_e} \frac{e^{-E_{SD}/kT} g_S}{\gamma C \Omega_{PD}}}{\frac{g_S}{g_D} e^{-E_{SD}/kT} \left(\frac{\Omega_{DS}}{\Omega_{PD} \gamma} e^{-E_{DS}/kT} + \frac{A_{DP}\, g_D}{C N_e \Omega_{PD} \gamma} + \frac{\Omega_{DS}}{\Omega_{PS} \gamma} + \frac{\Omega_{PS}}{\Omega_{PS} \gamma} \right)}$$

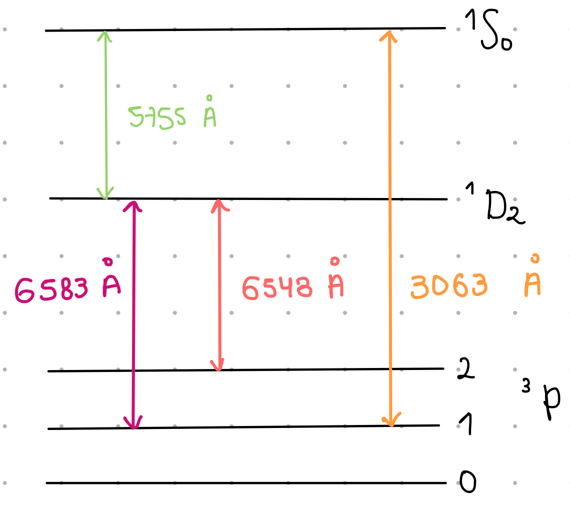
$$\frac{N_D}{N_S} = \frac{\frac{(A_{SD} + A_{SP})}{\gamma N_e C} \frac{g_S}{\Omega_{PS}} + 1 + \frac{\Omega_{SD}}{\gamma \Omega_{PD}} e^{-E_{SD}/kT} + \frac{A_{SD}}{N_e} \frac{e^{-E_{SD}/kT} g_S}{\gamma C \Omega_{PD}}}{\frac{g_S}{g_D} e^{-E_{SD}/kT} \left(\frac{\Omega_{DS}}{\Omega_{PD} \gamma} e^{-E_{DS}/kT} + \frac{A_{DP}\, g_D}{C N_e \Omega_{PD} \gamma} + 1 \right)}$$

Usando la ecuación $j_\nu = N_{k\downarrow} A_{kj} \frac{h \nu_{kj}}{4\pi}$ y sabiendo que $h_S \approx h_D$.

$$\frac{j_{DP}}{j_{DS}} = \frac{A_{DP}\, E_{DP}}{A_{DS}\, E_{DS}} \; \frac{\frac{(A_{SD} + A_{SP})}{\gamma N_e C} \frac{g_S}{\Omega_{PS}} + 1 + \frac{\Omega_{SD}}{\gamma \Omega_{PD}} e^{-E_{SD}/kT} + \frac{A_{SD}}{N_e} \frac{e^{-E_{SD}/kT} g_S}{\gamma C \Omega_{PD}}}{\frac{g_S}{g_D} e^{-E_{SD}/kT} \left(\frac{\Omega_{DS}}{\Omega_{PD} \gamma} e^{-E_{DS}/kT} + \frac{A_{DP}\, g_D}{C N_e \Omega_{PD} \gamma} + 1 \right)}$$

El valor de A varía de acuerdo a la transición.

De forma similar se halla para N II:



[N II]

Para O III se tendria el cociente de $\frac{j_{5006} + j_{4958}}{j_{4363}}$ y para N II

$$\frac{j_{6583} + j_{6548}}{j_{5755}}$$

Para graficar se toman en cuenta los siguientes valores:

```
c=299792458 #m/s
h=6.626070e-34 #J/S
k=1.380649e-23 #J/K
T_2=np.linspace(2000,20000,10000)
C_2=8.62e-6/np.sqrt(T_2)
```

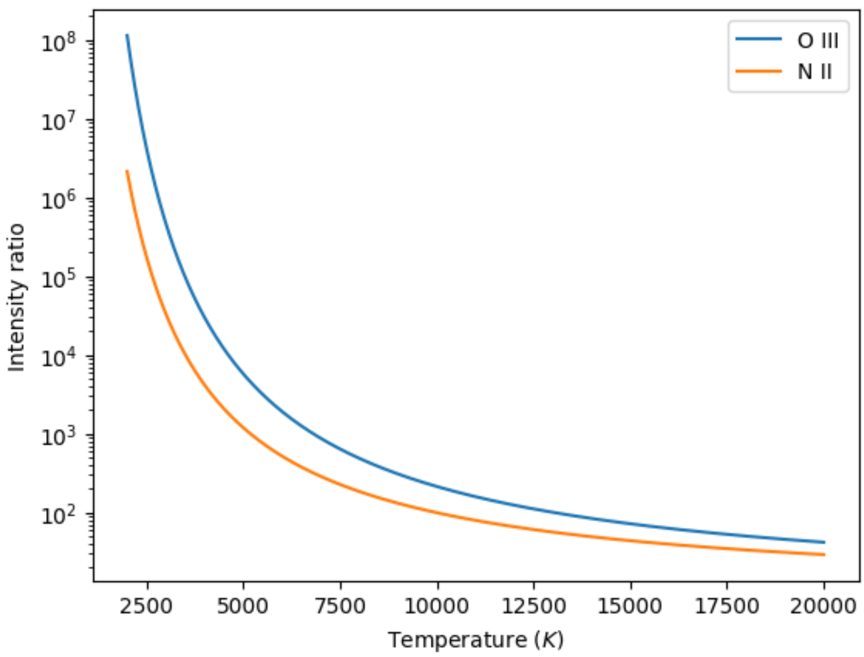
```
#Valores de A
adp1_o3=2e-2
adp2_o3=6.8e-3
asd_o3=1.6
asp_o3=2.3e-1

#Omegas
omg_ds_o3=0.58
omg_ps_o3=0.29
omg_pd_o3=2.29

#Gamma
gamma_o3=(omg_ds_o3+omg_ps_o3)/omg_ps_o3

#Pesos
gs=1
gd=5

#Exponencial y energias
lsd_4363=4363.2e-10
e_sd_o3=(h*c)/lsd_4363
exp_o3=np.exp(-(e_sd_o3/(k*T_2)))
nu_sd_o3=(c)/lsd_4363

ldp1_5007=5006.9e-10
e_dp1_o3=c/ldp1_5007

ldp2_4959=4958.9e-10
e_dp2_o3=c/ldp2_4959
```

[O III]

```
#Valores de A
adp1_n2=3e-3
adp2_n2=9.8e-4
asd_n2=1
asp_n2=3.3e-2

#Omegas
omg_ds_n2=0.27
omg_ps_n2=0.15
omg_pd_n2=1.36

#Gamma
gamma_n2=(omg_ds_n2+omg_ps_n2)/omg_ps_n2

#Pesos
gs=1
gd=5

#Exponencial y energias
lsd_5755=5754.6e-10
e_sd_n2=(h*c)/lsd_5755
exp_n2=np.exp(-(e_sd_n2/(k*T_2)))
nu_sd_n2=(c)/lsd_5755

ldp1_6583=6583.4e-10
e_dp1_n2=c/ldp1_6583

ldp2_6548=6548e-10
e_dp2_n2=c/ldp2_6548
```

[N II]

Con lo anterior la gráfica queda:



Valores para galaxias SDSS

Con el siguiente código se realiza la búsqueda en la base de datos del SDSS:

```
SELECT TOP 500
el.specObjID, s.class, s.z, s.zwarning,
Flux_OII_3726, Flux_OII_3726_Err,
Flux_OII_3728, Flux_OII_3728_Err,
Flux_SII_6716, Flux_SII_6716_Err,
Flux_SII_6730, Flux_SII_6730_Err,
Flux_OIII_4363, Flux_OIII_4363_Err,
Flux_OIII_4958, Flux_OIII_4958_Err,
Flux_OIII_5006, Flux_OIII_5006_Err,
Flux_NII_6547, Flux_NII_6547_Err,
Flux_NII_6583, Flux_NII_6583_Err
FROM emissionLinesPort as el
JOIN SpecObj as s
ON s.specObjID = el.specObjID
WHERE (s.class = 'QSO'
OR s.class = 'GALAXY')
AND s.zwarning = 0.0
AND Flux_OII_3726 > Flux_OII_3726_Err *5.
AND Flux_OII_3728 > Flux_OII_3728_Err *5.
AND Flux_SII_6716 > Flux_SII_6716_Err *5.
AND Flux_SII_6730 > Flux_SII_6730_Err *5.
AND Flux_OIII_4363 > Flux_OIII_4363_Err *5.
AND Flux_OIII_4958 > Flux_OIII_4958_Err *5.
AND Flux_OIII_5006 > Flux_OIII_5006_Err *5.
AND Flux_NII_6547 > Flux_NII_6547_Err *5.
AND Flux_NII_6583 > Flux_NII_6583_Err *5.
AND Flux_OII_3726_Err >0.
AND Flux_OII_3728_Err >0.
AND Flux_SII_6716_Err >0.
AND Flux_SII_6730_Err >0.
AND Flux_OIII_4363_Err >0.
AND Flux_OIII_4958_Err >0.
AND Flux_OIII_5006_Err >0.
AND Flux_NII_6547_Err >0.
AND Flux_NII_6583_Err >0.
```

En Python se importan, calculan y filtran los datos con el siguiente código:

```python
file='Skyserver_SQL11_9_2023 7_52_38 PM.csv'

cat = pd.read_csv(file,delimiter=',',header=1)

#Seleccion 5 objetos
row_ind = [337,50,326,218,321]
cat2 = cat.iloc[row_ind]

js2_t=cat2['Flux_SII_6716']/cat2['Flux_SII_6730'] #Flujo para SII
jo2_t=cat2['Flux_OII_3728']/cat2['Flux_OII_3726'] #Flujo para OII
jo3_t=(cat2['Flux_OIII_5006']+cat2['Flux_OIII_4958'])/cat2['Flux_OIII_4363'] #Flujo para OIII

#Se crea una nueva tabla con los valores de interés
data = {'js2':js2_t,'jo2':jo2_t,'jo3':jo3_t}
j = pd.DataFrame(data)

#Filtro para tener datos de acuerdo a los valores obtenidos en las gráficas
filtro = j[(j['js2'] >= 0.4) & (j['js2'] <= 1.5)]

#Convertir valores en arreglos
js2_arr = np.array(js2_t)
jo3_arr = np.array(jo3_t)
```

La selección de las 5 galaxias tiene los siguientes datos:

	ID	SII 6716	SII 6731	OIII 5007	OIII 4959	OIII 4636
337	961533403662936064	1127.05100	1100.14400	12231.750	4281.1110	198.11990
50	311537583365766536	93.12099	67.95963	2448.920	857.1221	37.93814
326	1068577733480572928	128.76980	88.59130	3378.360	1182.4260	40.20818
218	6077670745025736704	56.89594	38.66472	1576.311	551.7089	21.93887
321	2987116977425770496	217.09560	163.33410	4805.343	1681.8700	45.19572

Para finalmente obtener los cocientes:

	js2	jo2	jo3
337	1.024458	1.125399	83.347816
50	1.370240	3.285921	87.142967
326	1.453526	1.443714	113.429307
218	1.471521	1.064142	96.997699
321	1.329150	0.880946	143.536003

Con estos valores se corre el siguiente código para encontrar los valores de T y Ne:

```
T=10000 #Valor inicial de T

#Función que recibe los cocientes de las galaxias del SDSS de [S II] y [O III]
#y el valor de temperatura inicial.
def valores_iniciales(js2_g,jo3_g,T):
#Se itera 3 veces para que las soluciones se estabilicen.
  for i in range(3):
    #Función que halla el valor del cociente de [S II] con el T dado.
    js2 = SII(T)
    #Se encuentra la diferencia de los valores con el de SDSS.
    js2_diferencia = np.abs(js2_g-js2)
    #Se encuentra el menor valor, que corresponde al indice del Ne que estamos buscando.
    ind_elegido_ne = np.argmin(js2_diferencia)
    #Con el índice se encuentra el valor de Ne.
    ne_elegido = ne[ind_elegido_ne]
    #Imprimimos ese valor.
    print('Ne :',ne_elegido,'cm^{-3}')
    #Con el Ne hallado encontramos el cociente para [O III].
    jo3 = OIII(ne_elegido)
    #Hallamos la diferencia de los valores con respecto al del SDSS.
    jo3_diferencia = np.abs(jo3_g-jo3)
    #Encontramos el índice del T que soluciona la ecuación.
    ind_elegido_T = np.argmin(jo3_diferencia)
    #Hallamos el valor de T.
    T = T_2[ind_elegido_T]
    #Imprimimos.
    print('T  :', T , 'K')
```

Los valores tomados son los mismos que el código mostrado para encontrar las gráficas. De esta forma se encuentran los siguientes valores:

js2	jo2	jo3	T [k]	Ne [cm^{-3}]
1.024458	1.125399	83.347816	769.0	14032.403240
1.370240	3.285921	87.142967	130.0	13783.978398
1.453526	1.443714	113.429307	41.0	12405.040504
1.471521	1.064142	96.997699	25.0	13189.918992
1.329150	0.880946	143.536003	163.0	11387.938794