

Single-round Two-player Geodesic Voronoi Games

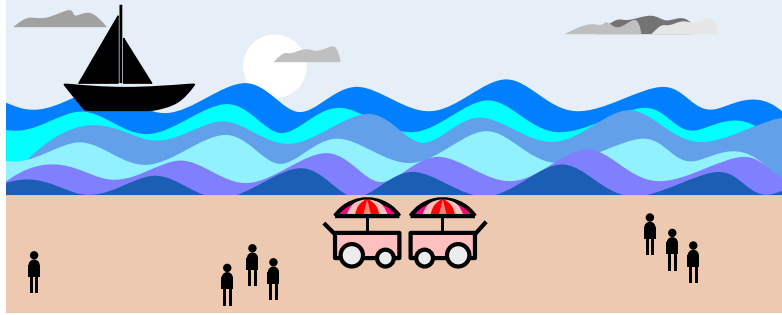
Sarvottamananda

January 19, 2026

Ajit Diwan Memorial Workshop

Ramakrishna Mission Vivekananda Educational and Research Institute

Abstract



In a Voronoi game, each player provides a common service or facility, i.e., there is no distinguishing feature to the service or facility they offer. This service or facility is provided from a location chosen individually by each player. Moreover, the premise is that the clients individually request the service or facility from their nearest service location. The clients' locations are predetermined in the game space, and the aim of each player is to maximize the clients that they serve by placing their facility at the best possible location.

Definition 1. Let Alice and Bob be two players. A two-player single-round Voronoi game \mathcal{G} is a 6-tuple $\langle \mathcal{M}, d_a, d_b, C, \mu_a, \mu_b \rangle$, where \mathcal{M} is a geometric space with metric distances, d_a and d_b , C is a set of clients, and μ_a and μ_b are the Voronoi payoff functions for Alice and Bob, respectively.

We explore geodesic Voronoi games played with respect to a polygon P using Euclidean or \mathbb{L}_1 metric by two players, Alice and Bob. Alice and Bob are individually restricted to the interior or exterior of the polygon, or may be unrestricted. For example, Alice may be unrestricted and Bob may be restricted externally.

We study two versions of this game, one is where the game is played on a simple polygon and the other is where the game is played on a convex polygon. The version of the game with a convex polygon has efficient solutions and bounds.

We study two problems related to this games, one is where we try to find the lower and upper bounds on the gains, and other is where we try to find the optimal placement of the players' optimal facility locations. The second problem is also called the designing of optimal strategies for players.

Generally, the bounds in case of simple polygons are not very interesting. We present the following results.

- There exist simple polygons such that a single interior player prohibits exterior players to gain more than one client each.
- There exist convex polygons such that a single interior player prohibits exterior players to gain more than one client each.
- There exist efficient strategies for players when we play the game on perimeter of convex polygons, Alice in the interior and Bob exterior.
- We can guarantee gains for all players if the clients are uniformly distributed on the perimeter of the convex polygon of the game in \mathbb{L}_1 .

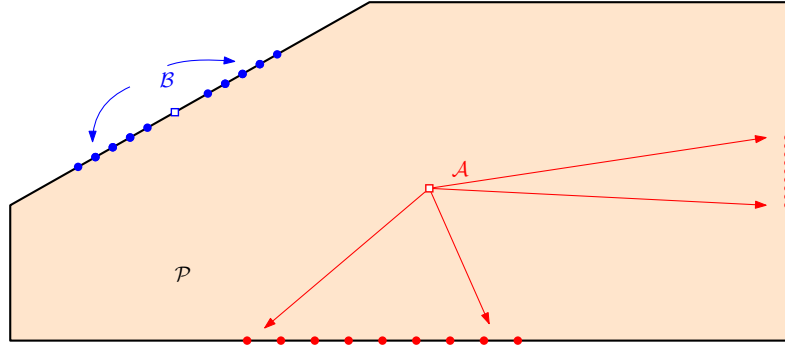


Figure 1: Example Voronoi game