Preliminaries and some computational aspects of discrete Morse theory

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Abstract

An *abstract simplicial complex* (ASC) is a (finite) family of finite sets (called *simplices*) with the downward-closedness property. These combinatorial objects occur naturally throughout discrete mathematics (e.g. matroids, clique or independence complexes of graphs, matching complexes of (hyper-)graphs, order complexes of posets, etc.) and in other areas of mathematics too. Often, the problem of interest lies in computing various important parameters (e.g. Euler characteristic, Betti numbers, homology groups, etc.) related to an ASC that characterize the shape and structure of the space associated with the ASC.

Discrete Morse theory (DMT) helps us capture the underlying structure of the space through an *efficient decomposition* (i.e. an "equivalent" space built with fewer building blocks compared to the number of simplices in the original ASC). The key concept of DMT is the notion of a discrete Morse function (DMF) defined on an ASC, as the structure of the space is determined by only the simplices that are "critical" with respect to the assigned DMF. However, the discrete optimization problem of finding a DMF on a given ASC that minimizes the number of critical simplices is NP-hard, in general. An *efficient* DMF also helps us compute the aforementioned parameters related to an ASC.

In this talk, we shall discuss some basics of DMT, including preliminary notions and results, a method of augmenting a DMF to a more efficient one, and some computations with examples. No specific prerequisites are needed.