Inductive graph invariants and its algorithmic applications

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Abstract

We introduce and study an inductively defined analogue $f_{\text{IND}}()$ of any increasing graph invariant $f()$. An invariant $f()$ is increasing if $f(H) \leq f(G)$ whenever H is an induced subgraph of G. This inductive analogue simultaneously generalizes and unifies known notions like degeneracy, inductive independence number, etc into a single generic notion.

We also explore the possibility of computing $f_{\text{IND}}()$ (and a corresponding optimal vertex ordering) and identify some pairs $(\mathcal{C}, f())$ for which $f_{\text{IND}}()$ can be computed efficiently for members of $\mathcal{C}$. In particular, it includes graphs of bounded $f_{\text{IND}}()$ values. Some specific examples have already been studied extensively.

We further extend this new notion by (i) allowing weighted graphs, (ii) allowing $f()$ take to take values from a totally ordered universe with a minimum and (iii) allowing the consideration of r-neighborhoods for arbitrary but fixed $r \geq 1$. Such a generalization is employed in designing efficient approximations of some graph optimization problems.

Precisely, we obtain efficient algorithms for approximating optimal weighted induced $\mathcal{P}$-subgraphs and optimal $\mathcal{P}$-colorings (for hereditary $\mathcal{P}$'s) within multiplicative factors of (essentially) k and $k/(m-1)$ respectively, where k denotes the inductive analogue (as defined in this work) of optimal size of an unweighted induced $\mathcal{P}$-subgraph of the input and m is the minimum size of a forbidden induced subgraph of $\mathcal{P}$. Our results generalize the previous result on efficiently approximating independent sets and minimum colorings on graphs of bounded inductive independence number, to arbitrary hereditary classes $\mathcal{P}$.

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