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Q1

(a) $u+v = (-1, 2) + (3, 4) = (2, 6)$

$ku = 3(-1, 2) = (0, 6)$

(b) u & v is in V

$u+v = (u_1+v_1, u_2+v_2)$ is in V

therefore V is closed under ~~scalar multiplication~~ addition.

$u = (u_1, u_2)$ is in V

$ku = (0, ku_2)$ is in V

therefore V is cloused under scalar multiplication.

(c) 1-5 : Axioms hold for V

because they are known to hold for R^2.

(d) Axiom 7 : $k((u_1, u_2) + (v_1, v_2)) = k(u_1, u_2) + k(v_1+v_2)$

$k(u+v) = ku + kv$

$= k(u_1+v_1, u_2+v_2)$

$= (0, k(u_2+v_2))$

$= (0, ku_2) + (0, kv_2)$

$= k(u_1, u_2) + k(v_1, v_2)$

Axiom 8 : $(k+m)u = ku + mu$

$= (k+m)(u_1, u_2)$

$= (0, (k+m)u_2)$

$= (0, ku_2 + mu_2)$

$= (0, ku_2) + (0, mu_2)$

$= k(u_1, u_2) + m(u_1, u_2)$

Axiom 9 : $k(mu) = m(ku)$

$= k(m(u_1, u_2))$

$= k(0, mu_2)$

$= (0, kmu_2)$

$= (km)(u_1, u_2)$

(e) $1(u) \neq u$

$1(u_1, u_2) = (0, u_2)$

$(0, u_2) \neq (u_1, u_2)$ therefore it doesnot hold.

& V is not a vector space.

Q2.

(a) $u+v = (2,2)$ $ku = (0,8)$

(b) $0 + u = u$ (4)

$(0,0) + (u_1, u_2) = (u_1+1, u_2+1)$

$(u_1, u_2) \neq (u_1+1, u_2+1)$

therefore $(0,0)$ is not vector 0 in axiom 4.

(c) $(u_1, u_2) + (-1, -1) = (u_1, u_2)$

$(-1, -1) + (u_1, u_2) = (u_1, u_2)$

(d) $u + (-u) = 0$ $u = (u_1, u_2)$

$-u = (-2-u_1, -2-u_2)$

$u + (-u) = (-1, -1) = 0$

$(-u) + (u) = 0$ holds.

(e) axiom 7 $k(u+v) \neq ku + kv$

axiom 8 $(k+m)u \neq ku + mu$

Q3. all axioms hold. - vector space.

Q4 "

Q5. Axiom 5 fails. : $(x, y) + (x', y') = (0,0)$

since $x' \geq 0$

Axiom 6 fails when $K < 0$ $KU = UK$.

Therefore not a vector space.

Q6 All axioms hold - vector space.

Q7. Axiom 8 fails.

$(K+m)^2 u \neq (k^2 + m^2) u$

Q8. Axiom 1 fails $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Axiom 6 fails - matrix mult.

$\therefore$ not a vector space.

Q9. All axioms hold - vector space

Q10. "

Q11. "

Q12. All axiom hold - vector space.

Ex 4.2

Q1)

(a) ~~a≠0~~ $(a, 0, 0)$

$u = (u_1, 0, 0)$ $v = (v_1, 0, 0)$

$u+v = (u_1+v_1, 0, 0)$

$ku = k(u_1, 0, 0) = (ku_1, 0, 0) \in W$

∴ subspace.

(b) $(a, 1, 1)$

$u = (u_1, 1, 1)$ $v = (v_1, 1, 1)$

$u+v = (u_1+v_1, 2, 2) \notin W$

∴ not a subspace

(c) (a, b, c) $b = a+c$

$u = (u_1, u_1+u_3, u_3)$

$v = (v_1, v_1+v_3, v_3)$

$u+v = (u_1+v_1, u_1+u_3+v_1+v_3, v_3) \in W$

$ku = ku_1, k(u_1+u_3), ku_3) \in W$

Q2)

(a) $u = (u_1, u_1+u_3+1, u_3)$ $v = (v_1, v_1+v_3+1, v_3)$

$u+v = (u_1+v_1, \underbrace{u_1+u_3+v_1+v_3+2}, u_3+v_3) \notin W$

doesnot = sum.

(b) $u+v = (u_1+v_1, u_2+v_2, 0) \in W$

holds ∴ since last component is 0.

$ku = (ku_1, ku_2, 0) \in W$

holds ∴ since last 0.

(c) $u+v = (u_1+v_1, u_2+v_2, u_3+v_3) \notin W$

$u_1+v_1+u_2+v_2 =$

$\underbrace{u_1+u_2}_{7} + \underbrace{v_1+v_2}_{7} = 14$

$7 \neq 14$ ∴ not ~~in~~ a subspace.

Q3-4

(a)

i) ① $\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}+b_{11} & 0 \\ 0 & a_{22}+b_{22} \end{bmatrix}$

② $K \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} = \begin{bmatrix} Ka_{11} & 0 \\ 0 & Ka_{22} \end{bmatrix}$

W is a subspace of M_{nn}.

(b)

$\underset{A}{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}} + \underset{B}{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\det \neq 0$

W is not a subspace of M_{nn}.

(c)

$tr(A) = a_{11} + a_{22} \dots a_{nn} = 0$

$tr(B) = b_{11} + b_{22} \dots b_{nn} = 0$

$tr(A+B) = (a_{11}+b_{11}) + (a_{22}+b_{22}) + \dots$

$= 0 + 0 \dots + 0 \Rightarrow 0$

$tr(KA) = ka_{11} + ka_{22} \dots + ka_{nn}$

$= k(a_{11} + a_{22} \dots + a_{nn})$

$= k(0) \Rightarrow 0$

Therefore W is a subspace of M_{nn}.

(d)

W is a set of all invertible matrix

A^{-1} exist

This set is not closed under scalar multiplication

when scalar $(K) = 0$

Therefore W is not a subspace of M_{nn}.

Q5)

(a) $a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$a_0 = 0$ $\qquad a = 0 + a_1 x + a_2 x^2 + a_3 x^3$

(1) $a+b = 0 + (a_1+b_1)x + (a_2+b_2)x^2 + (a_3+b_3)x^3$

(2) $k(0 + a_1 x + a_2 x^2 + a_3 x^3) = 0 + ka_1 x + ka_2 x^2 + ka_3 x^3$

therefore W is a subspace of P_2

(b)

$-a_1 - a_2 - a_3 + a_1 x + a_2 x^2 + a_3 x^3$

(1) $a+b = (-a_1 - a_2 - a_3 - b_1 - b_2 - b_3) + (a_1+b_1)x + (a_2+b_2)x^2 + (a_3+b_3)x^3$

$\Rightarrow 0$

(2) $k(-a_1 - a_2 - a_3 + a_1 x + a_2 x^2 + a_3 x^3) = (-ka_1 - ka_2 - ka_3) + (ka_1)x + (ka_2)x^2 + (ka_3)x^3$

W is a subspace.

$\Rightarrow 0$

Q12)

(a) $u+v$ test

$$A\begin{bmatrix}1\\-1\end{bmatrix} + B\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix} \Rightarrow \begin{bmatrix}0\\0\end{bmatrix}$$

closed under add.

ku test

$$k\left[A\begin{bmatrix}1\\-1\end{bmatrix}\right]$$

$$k\begin{bmatrix}0\\0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}0\\0\end{bmatrix}$$ closed under scalar mult.

// subspace proved

(b) $$A\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}A$$

$u+v$

$$A\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} + B\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}A + \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}B$$

closed under add.

ku

$$kA\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} = k\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}$$

closed under scalar mult.

// subspace proved.

Q13)

(a) $\underline{\underline{U+V}}$

$U = (a, a^2, a^3, a^4)$
$V = (b, b^2, b^3, b^4)$

$U+V = (a+b, a^2+b^2, a^3+b^3, a^4+b^4)$

det $(a, a^2, a^3, a^4) = (1, 1, 1, 1)$

det $(U+V) = (2, 2, 2, 2)$

// therefore not a subspace.

(b) $U = (a, 0, b, 0)$
$V = (a', 0, b', 0)$

(1) $U+V = (a+a', 0, b+b', 0)$

det $a = 1$, $b = 1$

$U+U = (2, 0, 2, 0)$

True.

(2) kU

$= (ka, 0, kb, 0)$

closed under scaler mult.

// subspace.

Q19.

(a) $A = \begin{bmatrix} -1 & 1 & 1 \\ 3 & -4 & 0 \\ 2 & -4 & 5 \end{bmatrix}$ REF $\rightarrow$ $\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 3/2 \\ 0 & 0 & 0 \end{bmatrix}$

$x_3 = t$

$x_2 = -3/2\, t$

$x_1 = -1/2\, t$

line passing through origin $\Rightarrow$

$(-1/2\, t, -3/2\, t, t)$

(b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ REF $\rightarrow$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$\Rightarrow (0, 0, 0)$

The solution space is the only solution & it is the origin.

(*) Ex 4.3 Q 3, 4, 9, 11

Q 3.

(a)
$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & 6 \\ 0 & -1 & 2 & -8 \\ -2 & 2 & 1 & -1 \\ -2 & 3 & 4 & -8 \end{array}\right]$$

REF ↓

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

consistent
∴ linear combination.

(b)
$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ -2 & 2 & 1 & 0 \\ -2 & 3 & 4 & 0 \end{array}\right]$$

REF ↓

$$\begin{bmatrix} -1 & 0 & -1/2 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & -11 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

consistent
∴ linearly combination.

(c)
$$\begin{bmatrix} 4 & 1 & 0 & -1 \\ 0 & -1 & 2 & 5 \\ -2 & 2 & 1 & 7 \\ -2 & 3 & 4 & 1 \end{bmatrix}$$

REF ↓

$$\begin{bmatrix} 1 & 0 & 0 & 1/18 \\ 0 & 1 & 0 & 4/3 \\ 0 & 0 & 1 & 19/6 \\ 0 & 0 & \boxed{0} & \boxed{-73} \end{bmatrix}$$

inconsistent

∴ therefore not a linear combination.

Q4.
$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

(a)
$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 \\ 1 & -1 & 0 & 0 \end{array}\right] \xrightarrow{REF} \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1/4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1/2 \\ 0 & 1 & 0 & -1/4 \\ 0 & 0 & 1 & 1/4 \end{bmatrix}$$ → linear combination.

(b)
$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 1 & -1 & 0 & 1 \end{array}\right] \cancel{\xrightarrow{RREF}}$$

Using determinant method.

$$2(2)-1(-2)+1(-1) = 5$$

$$5 \neq 0$$

Hence it is a linear combination

(c)
$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 \\ 1 & -1 & 0 & 1 \end{array}\right]$$

$$2(2)-1(-2)+1(-1)$$

$$= 5 \qquad 5 \neq 0$$

hence it is linear combination

(9).
$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ -1 & 1 & -1 & -2 \\ 2 & 0 & 4 & 2 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 5 & 2 & b_0 \\ -1 & 1 & -1 & -2 & b_1 \\ 2 & 0 & 4 & 2 & b_2 \end{array}\right]$$

$$\left[\begin{array}{ccccc} 1 & 3 & 5 & -2 & b_0 \\ 0 & 1 & 1 & 0 & \frac{b_0+b_1}{4} \\ 0 & 0 & 0 & 6 & 6\left(\frac{b_0+b_1}{4}\right)-2b_0-b_2 \end{array}\right]$$

$$\left[\begin{array}{ccccc} -1 & 0 & -2 & 2 & 3\left(\frac{b_0+b_1}{4}\right)-b_0 \\ 0 & 1 & 1 & 0 & \frac{b_0+b}{4} \\ 0 & 0 & 0 & 0 & 6\frac{(b_0+b)}{4}-2b_0-b_2 \end{array}\right]$$

$0 = n$

not consistent.

not linearly dependent.

does not span.

Q11)

(a) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

$\det = 1\{1(-1)\}$

$-1 - 1\{-1(1-0)\}$

$-1 + 1 \Rightarrow 0$

$\det \neq 0$

therefore doesnot span. ∴ not linearly dependent.

Basis $\begin{cases} \text{linearly indep.} & k_1 = 0 \ k_2 = 0 \ k_3 = 0 \\ \text{span} & (\det(a) \mathrel{!=} 0) \end{cases}$

Example 3

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{bmatrix}$$

$$1(0) - 2(8) + 1(6 - 27) \neq 0$$

~~#6~~ // shown $\quad \det(A) \neq 0$.

linearly independent

Ex 4.4

Q2. $\left[\begin{array}{ccc|c} 3 & 2 & 1 & 0 \\ 1 & 5 & 4 & 0 \\ -4 & 6 & 8 & 0 \end{array}\right] \xrightarrow{REF} \left[\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 0 \\ 0 & 1 & 11/13 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$(0,0,0)$ $\quad x_3 = 0$

// Hence, linearly independent $\quad x_2 = 0$

$x_1 = 0$

Spanning

$\text{Det} = 3(16) - 2(24) + 1(26)$

$\Rightarrow 26$ $\quad$ Since $\det \neq 0$ the vectors form a basis for R^3.

✓ spanning.

Q4. To show linear independence.

$$c_1(1+x) + c_2(1-x) + c_3(1-x^2) + c_4(1-x^3) = 0$$

For spanning

RHS: $\Rightarrow a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$(c_1 + c_2 + c_3 + c_4) + (c_1 x - c_2 x) + (-c_3)x^2 - c_4 x^3$.

$$\det(A) = 0 \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix} - 0 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix} + 0 \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} - 1 \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$= -3$

✓ span.

Since $\det \neq 0$ the polynomial has trivial solution for homogenous system

And non homogenous system is consistent. They form basis for R^3.

RREF $\rightarrow$ $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$.

$k_1 = 0 \quad k_2 = 0 \quad k_3 = 0 \quad k_4 = 0$

Basis formed !!!!

Q5. $\begin{bmatrix} 3 & 0 & 0 & 1 \\ 6 & -1 & -8 & 0 \\ 3 & -1 & -12 & -1 \\ -6 & 0 & -4 & 2 \end{bmatrix}$

$\det(A) = 3(28-16) - 1(-12) + 3(12) - 1(-12)$

$= 48$

$\neq 0$

✓ span.

RREF $\rightarrow$ $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$

$k_1 = 0 \quad k_2 = 0 \quad k_3 = 0 \quad k_4 = 0$

✓ linearly independent

Basis formed !!!

Ex 4.5

Q1) $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ -2 & -1 & 2 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right]$

RREF $\rightarrow$ $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

$x_1 - x_3 = 0$

$x_1 = x_3$

$x_2 = 0$

$x_3 = t \quad x_1 = t.$

$t(1, 0, 1)$

$\boxed{dim = 1}$

Q3) $\left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 0 & 5 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$x_1 = 0, \ x_2 = 0, \ x_3 = 0$

$\boxed{Dim = 0}$

Q4) $\begin{bmatrix} 1 & -4 & 3 & -1 \\ 2 & -8 & 6 & 2 \end{bmatrix}$.

$\begin{bmatrix} 1 & -4 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_2 = r \quad x_3 = s \quad x_4 = t$

$x = 4r - 3s + t$

$$\begin{cases} r = (4, 1, 0, 0) \\ s = (-3, 0, 1, 0) \\ t = (1, 0, 0, 1) \end{cases}$$

Q7) (a, b, c)

$(a, a+c, c)$

$a(1,1,0) + c(0,1,1)$

$S = \{(1,1,0), (0,1,1)\}$

linearly independent.

& span hence form R^2.

Ex 4.7

Q3.

(a) $\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 3 & 2 \end{array}\right]$

$\begin{bmatrix} -1 & 0 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$.

$0 = 3$ inconsistent.

$\therefore$ column space doesnt exist

Q4 (a)

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ -1 & 1 & -1 & 0 \\ -1 & -1 & -1 & 0 \end{array}\right]$.

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & -2 & 0 & 2 \end{array}\right]$.

$0 = 2$

inconsistent

$\therefore$ not a column space.

Q8. (a) $\left[\begin{array}{cccc|c} 1 & -2 & 1 & 2 & -1 \\ 2 & -4 & 2 & 4 & -2 \\ -1 & 2 & -1 & -2 & 1 \\ 3 & -6 & 3 & 6 & 3 \end{array}\right]$.

$$\begin{bmatrix} 1 & -2 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 2x_2 - x_3 - 2x_4 + x_5$

$x_2 = s$

$x_3 = v$

$x_4 = t$

$x_5 = u$

$x_1 = 2s - v - 2t + u$

$x_2 = s(2, 1, 0, 0)$

$x_3 = v(-1, 0, 1, 0)$

$x_4 = u(1, 0, 0, 1)$

Q11 (a) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

~~(1, 0, 2)~~

$\left.\begin{matrix} [1 \quad 0 \quad 2] \\ [0 \quad 0 \quad 1] \end{matrix}\right\}$ row space.

$\underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}}_{\text{colomn space}}$

(b) row space

$[1 \quad -3 \quad 0 \quad 0]$

$[0 \quad 1 \quad 0 \quad 0]$.

column space

$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

Q 12

(a) row space

$[1 \quad 2 \quad 4 \quad 5]$

$[0 \quad 1 \quad -3 \quad 0]$

$[0 \quad 0 \quad 1 \quad -3]$

$[0 \quad 0 \quad 0 \quad 1]$

column space

$$\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix} \begin{bmatrix}2\\1\\0\\0\\0\end{bmatrix} \begin{bmatrix}4\\-3\\1\\0\\0\end{bmatrix} \begin{bmatrix}5\\0\\-3\\1\\0\end{bmatrix}$$

(b) row space

$[1 \quad 2 \quad -1 \quad 5]$

$[0 \quad 1 \quad 4 \quad 3]$

$[0 \quad 0 \quad 1 \quad -7]$

$[0 \quad 0 \quad 0 \quad 1]$

column space

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix} \begin{bmatrix}2\\1\\0\\0\end{bmatrix} \begin{bmatrix}-1\\4\\1\\0\end{bmatrix} \begin{bmatrix}5\\3\\-7\\1\end{bmatrix}$$

Q14).

$$\begin{bmatrix}1 & 2 & 2\\1 & 0 & -1\\-4 & 2 & 3\\-3 & -2 & 2\end{bmatrix}.$$

RREF $\rightarrow$ $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix} \rightarrow$ invalid row

form the basis for column space

but not for row space.

Ex 4.8

Q 3

(a) rank = 3 Nullity = 0

(b) rank + nullity = row

0 + 3 = 3 proved!!

(c) leading 1s = 3

Parametere = 0.

Q 6.

(a) rank = 3 Nullity = 1

(b) rank + nullity = rows

3 + 1 = 4.

4 = 4 proved!!

(c) leading 1s = 3

parameter = 1

Q 8. rank = m when all rows have leading 1s.

smallest nullity = 0.

Q13.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Rank = 3

Nullity = 0.

Ex 5.1

Q3.

$$Ax = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 10 \\ 5 \end{bmatrix}$$

$5 \times x$.

Hence it is an eigen vector.
because it is a multiple.

Q7.

$$\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}.$$

$= \lambda I - A$

$$= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$A' = \begin{bmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{bmatrix}$$

$\det(A') = (\lambda-4)(\lambda-1)^2 + 2\lambda - 2$

$\det(A') = \cancel{-4+2\lambda-2}\ \lambda^2 - 4\lambda^2 - 2\lambda^2 + 8\lambda + \lambda - 4$

$\lambda = 1 \quad \lambda = 2 \quad \lambda = 3$

for $\lambda = 1$

$$\begin{bmatrix} -3 & 0 & -1 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1/3 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 0 & 2/3 \\ 0 & 0 & 2/3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 0$

$x_2 = t$

$x_3 = 0$

$\Rightarrow t = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

for $\lambda = 2$

$$\begin{bmatrix} -2 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

RREF $\rightarrow$ $$\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = \tfrac{1}{2} x_3$ $\qquad$ $x_1 = \tfrac{1}{2} t$

$x_2 = x_3$ $\qquad$ $x_2 = t$ $\qquad$ $\Rightarrow t \begin{bmatrix} 1/2 \\ 1 \\ 1 \end{bmatrix}$.

$x_3 = t$

for $\lambda = 3$

$$\begin{bmatrix} -1 & 0 & -1 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

REF $\rightarrow$ $$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = -x_3$ $\qquad$ $x_1 = -t$

$x_2 = x_3$ $\qquad$ $x_2 = t$ $\qquad$ $\Rightarrow t \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

$x_3 = t$

Q 8

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

$\det(\lambda I - A) = 0$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} \lambda-1 & 0 & 2 \\ 0 & \lambda & 0 \\ 2 & 0 & \lambda-4 \end{bmatrix}$$

$(\lambda-1)(\lambda^2-4\lambda) - 4\lambda = 0$

$\lambda^3 - 5\lambda^2 = 0$

$\underline{\lambda = 5}$ $\underline{\lambda = 0}$

(for $\lambda = 0$)

$$\begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 2x_3$ $x_1 = 2t$

$x_2 = 0$

$x_3 = t$ $\Rightarrow t \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

(for $\lambda = 5$)

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$x_2 = 0$

$x_1 + \frac{1}{2}x_3 = 0$

$$\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = -\frac{1}{2}x_3$

$x_1 = -\frac{1}{2}t$ $\Rightarrow t \begin{bmatrix} -1/2 \\ 0 \\ 1 \end{bmatrix}$

$x_3 = t$

Q13. $\begin{bmatrix} 3 & 0 & 0 \\ -2 & 7 & 0 \\ 4 & 8 & 1 \end{bmatrix}$ $\cancel{\lambda=2}$ $\cancel{\lambda=1}$

λ -values

3, 7 & 1

$(\lambda-3)(\lambda-7)(\lambda-1)=0$

Q14). λ values

9, -1, 3, 7

$(\lambda-9)(\lambda+1)(\lambda-3)(\lambda-7)=0$

Q16). $\begin{bmatrix} 2 & 1 & -1 \\ -1 & 0 & 1 \\ -1 & -1 & 2 \end{bmatrix}$

$\lambda I - A$

$\begin{bmatrix} \lambda-2 & 1 & -1 \\ -1 & \lambda & 1 \\ -1 & -1 & \lambda-2 \end{bmatrix}$

$(\lambda-2)\{\lambda^2-2\lambda+1\}-1\{\lambda-2+1\}-1\{1+\lambda\}$

$\lambda^3-4\lambda^2+5\lambda=0$

$\lambda=2$ $\lambda=1$

for $\lambda=2$

$\begin{bmatrix} 0 & 1 & -1 \\ -1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$

REF $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1+x_3=0$

$x_2=-x_3$

$\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

for $\lambda=1$

$\begin{bmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $s\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$x_1=x_2+x_3$ $t\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$x_2=s$

$x_3=t$

(b) Nullity = 2

(Ans)