Q1

a) $0^i 1^j 2^k \mid i, j, k >= 0$ and either $i=j$ or $i=k$

Taking string 00112222 ; $i=j$

case 1:

$x = \epsilon$
$y = 00$
$z = 11222$.

$x y^i z$

$i=0$.

$11222 \notin L$

$\therefore$ contradiction.

case 2:

$x = 00$
$y = 11$
$z = 222$.

$x y^i z$.

$i=0$

$00222 \notin L$

$\therefore$ contradiction.

case 3:

$x = 0011$
$y = 222$
$z = \lambda$

$i=0$.

$0011222 \in L$
since $i \neq j$
$i \neq k$.
$\therefore$ contradiction

case 4:

$x = 0$
$y = 01$.
$z = 1222$

$i = 2$.

$0\,0101\,1222 \notin L$

$\therefore$ contradiction

Thus by pumping lemma, we conclude L_1 is not regular language.

b) $w \in (0,1,2)^*$ | number of 0's = number of 2's.

Taking string : 00122.

c1 $x = \lambda$ $y = 001$ $z = 22$ $\xrightarrow{y=0}$ $22 \notin L$

c2 $x = 001$ $y = 22$ $z = \lambda$ $\xrightarrow{y=2}$ $00122222 \notin L$

c3 $x = 0$ $y = 01$ $z = 22$ $\xrightarrow{y=2}$ $0010122 \notin L$

} contradiction.

Thus by pumping lemma we conclude L_2 is not regular language.

c) $L_3 = \{ 0^n 1^m \mid n \leq m \}$

Taking 00111 $\leftarrow$ $0^{p+1} 1^p$

c1 $x = \lambda$ $y = 00$ $z = 111$ $\longrightarrow$ cannot pump : length of 0 cannot be > 1

c2 $x = 0$ $y = 01$ $z = 11$ $\xrightarrow{i=2}$ $00101\,11 \notin L$

c3 $x = 0$ $y = 0111$ $z = \lambda$ $\xrightarrow{i=2}$ $0\,01110111 \in L$

} contradiction.

Thus L_3 not Regular Language.

(d) $L_4 = \{ w \in \{0,1\}^+ \mid w \text{ has more 1s than 0.} \}$

Assume $0^{P-5}\,1^{P+1}$ $|s| > P$.

xy^iz

c1 if $x=\lambda$ $y=0^{P-5}$ $z=1^{P+1}$ $\xrightarrow{i=2}$ $0^{P^2-2P}\,1^{P+1} \notin L$.

c2 if $x=\lambda$ $y=0$ $z=11$ $\xrightarrow{i=2}$ $0011 \notin L$.

c3 if $x=\lambda$ $y=0^{P-5}$ $z=(0111)^P$ $\xrightarrow{i=2}$ $000111 \in L$

} contradiction

L_4 is not regular : contradiction.

(e) $L_5 = \{0^{n^2} \mid n \geq 0\}$

$n=P$ $\quad 0^{P^2} \rightarrow 0^P 0^P$

c1 $xy^0z = \longrightarrow 0^P \notin L$.
$x=\lambda$ $y=0^P$ $z=0^P$

c2 $xy^2z \longrightarrow 0^{P^3} \notin L$.
$x=\lambda$ $y=0^P$ $z=0^P$

c3 $x=0^P$ $y=0^P$ $z=\lambda \longrightarrow 0^P \notin L$
xy^0z

} contradiction.

L_5 is not regular.