21K-3278 Sohaib Shamsi

Q1 i) a) $M \hookrightarrow N$

b) $N \rightarrow K$

c) $\neg K \vee I$

d) $\neg M \rightarrow I$

ii) $((p \vee q) \wedge (p \rightarrow r)) \rightarrow (q \vee r)$

$((p \vee q) \wedge (p \rightarrow r)) \rightarrow (q \vee r)$

$\neg((p \vee q) \wedge (\neg p \vee r)) \vee (q \vee r)$

$\neg(p \vee q) \vee \neg(\neg p \vee r) \vee (q \vee r)$

$(\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (q \vee r)$

$(\neg p \vee q) \wedge (\neg q \vee q) \vee (p \vee r) \wedge (r \vee r)$

$((\neg p \vee q \wedge T) \vee (a \vee \neg)) \wedge T$

$(\neg p \vee q) \vee (p \vee r)$

$T \vee b \vee c$

T

iv) a) Some students in your class understand all examples in lecture notes.

b) All set of examples are understood by some students in your class.

v) a) $\forall p \exists p F(p,q)$

b) $\exists q \forall p F(p,q)$

vi) a) True

b) False

Q2) i) $X-(X \cap Y) = (X-Y)$

$X \cap (\bar{X} \cup \bar{Y}) = (X-Y)$

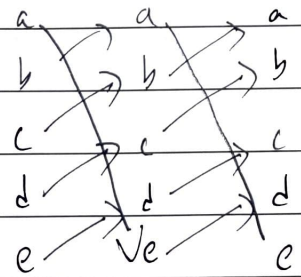
$(X \cap \bar{X}) \cup (X \cap \bar{Y}) = (X-Y)$

$X \cap \bar{X} = \phi$

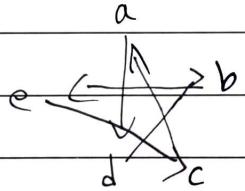
$\phi \cup X \cap \bar{Y} = X-Y$

$X \cap \bar{Y} = X-Y$

ii)



$\{(a,d), (b,e), (c,a), (d,b), (e,c)\}$



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iii) Equivalence Relation:-

Reflexive: a player cannot play himself
: So not reflexive.

Symmetric:- u beats v, so v cannot beat u
: So not symmetric

Transitive: u beats v, v beats w, u beats w
: not Transitive.

Antisymmetric, u beats v but v does not beat u
: So it is antisymmetric.

iv) $g(n) = n^2 + 1$ $\quad g(1) = 2$

$= 1^2 + 1 = 2$

not equal as $g(1)$ is an element and $g(\{1\})$ is a set

v) $f(n+1) = f(n) + 2n + 1$

~~$f(6) = f(5) + 2(5) + 1$~~ $\quad$ 36

$f(6) = f(5+1) = 25 + 11 =$

$f(1) = f(0+1) = 0+1 = 1$

$f(2) = f(1+1) = 1+3 = 4$

$f(3) = f(2+1) = 4 + 4 + 1 = 9$

$f(4) = f(3+1) = 9 + 6 + 1 = 16$

$f(5) = f(4+1) = 16 + 8 + 1 = 25$

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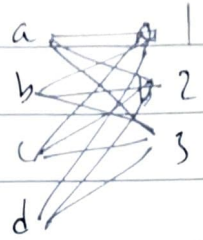
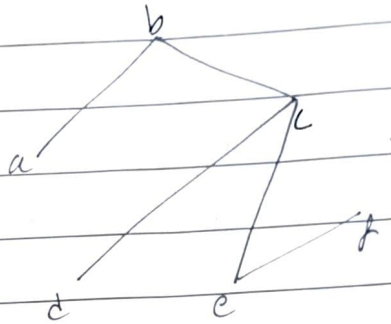
vi)



$3 \times 3 \times 3 \times 3 = 81$

Q3) c) $\{a,b\} = 3$
$\{b,c\} = 2$
$\{c,d\} = 1$
$\{c,e\} = 4$
$\{e,f\} = 3$

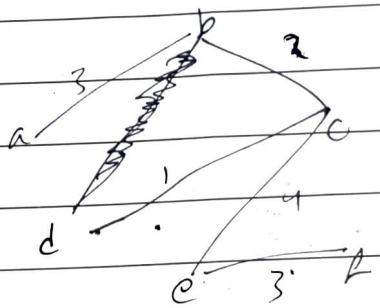
prims:



cost = 13

Kruskal

b) $1 = \{c,d\}$
$2 = \{b,c\}$
$3 = \{a,b\}$
$3 = \{e,f\}$
$4 = \{c,e\}$
$10 = \{a,f\}$



c, d = 1
b, c = 2
a, b = 3
e, f = 3
c, e = 4

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ii)

u	A	B	D	E
E	(1, C)	7, C	2, C	∞
CA		4, A	(2, C)	∞
CAD		(4, A)		9, D
CADB				(5, B)
CADBE				

iii) hamiltan cycle

iv) ~~h(1)=s, h(2)≠t, h(3)=u, h(4)≠v, h(5)=w, h(6)=x, h(7)=y~~
h(8)=z

v) Pre-order

60, ~~53~~, 95

~~33~~ ~~57~~ ~~78~~

25 42 71

60, 53, 33, 57, ~~95~~, 78

25 42 71

60, 53, 33, 25, 42, 57, ~~95~~, 78, 71

Post-order:

25, 42, 33, 57, 53, 71, 78, 95, 60

In-order:

25, 33, 42, 53, 57, 60, 71, 78, 95

vi) Pre-fix: - * + ABC * - DE + FG

Post-fix: AB + C * DE - FG + * -

Q4) i. $\frac{6!}{6^5}$

ii. $\frac{5!}{6^4}$

iii) ${}^5C_5 \times {}^6C_4$

iv) $C(n,k) - C(n-2, k-2)$

v) ${}^4C_2 = 6$

vi) a) $1 \times 26 \times 26 \times 10 \times 10 \times 1 = 67600$

b) $26 \times 25 \times 24 \times 10 \times 9 \times 8 = 11,232,000$

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25) i) a, b, c, d

$2 \pmod 3$
$4 \pmod 5$
$5 \pmod 7$
$1 \pmod{11}$

$a_1 = 2, a_2 = 4, a_3 = 5, a_4 = 1$

$m_1 = 3, m_2 = 5, m_3 = 7, m_4 = 11 = 1155$

$M_1 = \frac{m}{m_1} = \frac{1155}{3} = 385$

$M_2 = \frac{m}{m_2} = \frac{1155}{5} = 231$

$M_3 = \frac{m}{m_3} = \frac{1155}{7} = 165$

$M_4 = \frac{m}{m_4} = \frac{1155}{11} = 105$

$y_1 = M_1^{-1} \bmod m_1$

$= 385^{1} \bmod 3$

$385 = (128)(3) + (1)$

$1 = 1 \cdot 385 - 128 \cdot 3$

$1 = (1)(385) + (-128)(3)$

$\boxed{y_1 = 1}$

$y_2 = M_2^{-1} \bmod m_2$

$= 231^{-1} \bmod 5$

$231 = (46)(5) + (1)$

$1 = 1 \cdot 231 + (-46)(5)$

$\boxed{y_2 = 1}$

$y_3 = M_3^{-1} \bmod m_3$

$165^{-1} \bmod 7$

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$y_3 = 2$

$y_4 = 2$

231.

$((2 * 385 * 1) + (4 \times 231 \times 1) + (5 \times 165 \times 2) + (1 \times 6 \times 2) \bmod$

$= 3554 \pmod{1155} = 89.$

ii) $a^{2574} \bmod 79$

$a^{p-1} \equiv 1 \bmod p$

$a^{78} \equiv 1 \bmod 79$

$a = qd + r$

$2574 = (33)(78) + (5)$

$a^{33 \times 78 + 5} \bmod 79$

$59049 \bmod 79$

$= 36$

iii) $(x+3y)^9$

$9C_2 \times 1^7 \cdot x^7 \times 3^2 \cdot y^2$

$9C_2 \times 1^7 \times 3^2$

$= 324 x^7 y^2$

iv) Supposition - $\leq$ 8 freshman,

≤ 8 junior, ≤ 8 Sophomore

All together nor more than 24

assumption disapproved.

v) Supposition : ≤ 2 F, ≤ 18, Soplone, ≤ 4 junior

All together no more than 24

assumption disapproved.

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vi) $3\times6+9+3\times2+7+3\times7+1+3\times4+8+3\times1+1+3\times6+x_{12} \equiv 0 \pmod{10}$.

$119+x_{12} \equiv 0 \pmod{10}$

check digit is 1.

Q6) i) $a-2+3 = 3k$

$a+1 = 3(k+1)$

$a^2-1 = 3m$

$m = 3(k+1)(a-1)$

ii) $x^2 = 4y+2$

$x = 2k+1$

$2k^2 = (2y+1)$

Supposition is wrong

iii) $a = 2k \quad , \quad b = 2k$

$a+b = 2k+2k$

$= 4k$

Supposition is wrong

iv) $1^3+2^3+3^3+\ldots n^3 \left(\frac{n(n+1)}{2}\right)^2$

$n=1 : \left(\frac{1(1+1)}{2}\right)^2 = 1$ $\qquad n=k : \left(\frac{k(k+1)}{2}\right)^2$

$\left(\frac{k^2+k}{2^2}\right)^2 = \frac{(k^2+k)^2}{4}$

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$n = k+1$: $\left(\frac{(k+1)(k+1)+1}{2}\right)^2$

$\left(\frac{((k+1)(k+2)}{2}\right)^2 = \left(\frac{(k+1)(k+2)}{2}\right)$

proved.

v) $1^2+2^2 = 1+4 = 5$ = True

$3^2+4^2 = 9+16 = 25$ = True

$1^2+3^2 = 1+9 = 10$ = False

iv) $N = 55$ $\quad 1 = 40 - 3(13)$

$p = 5$ $\quad 1 = 40(1) + (3)(-13)$

$q = 11$ $\quad d = -13 \bmod 40$

$= 27$