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Assignment 2 (4 weightage)
Deadline: 5/october/2021
Time: 11.59 pm
Course: Calculus and analytical geometry
Course code: MT-1003
Book: Calculas by Howard anton
Instructions:
Kindly upload file using cam scanner by converting in one pdf file.

Rules and techniques of differentiation.	**2.3** (65,66,67)
Product and quotient rule. Derivative of trigonometric and logarithm function, Chain rule	**2.4** (21 to 24) **2.5** (15 to 18,21,22,23) **2.6** (31 to 36)
Implicit differentiation.	**3.1** (25-28)

65. Show that
$$f(x) = \begin{cases} x^2 + x + 1, & x \le 1 \\ 3x, & x > 1 \end{cases}$$
is continuous at $x = 1$. Determine whether f is differentiable at $x = 1$. If so, find the value of the derivative there. Sketch the graph of f.

f is continous at 1 because

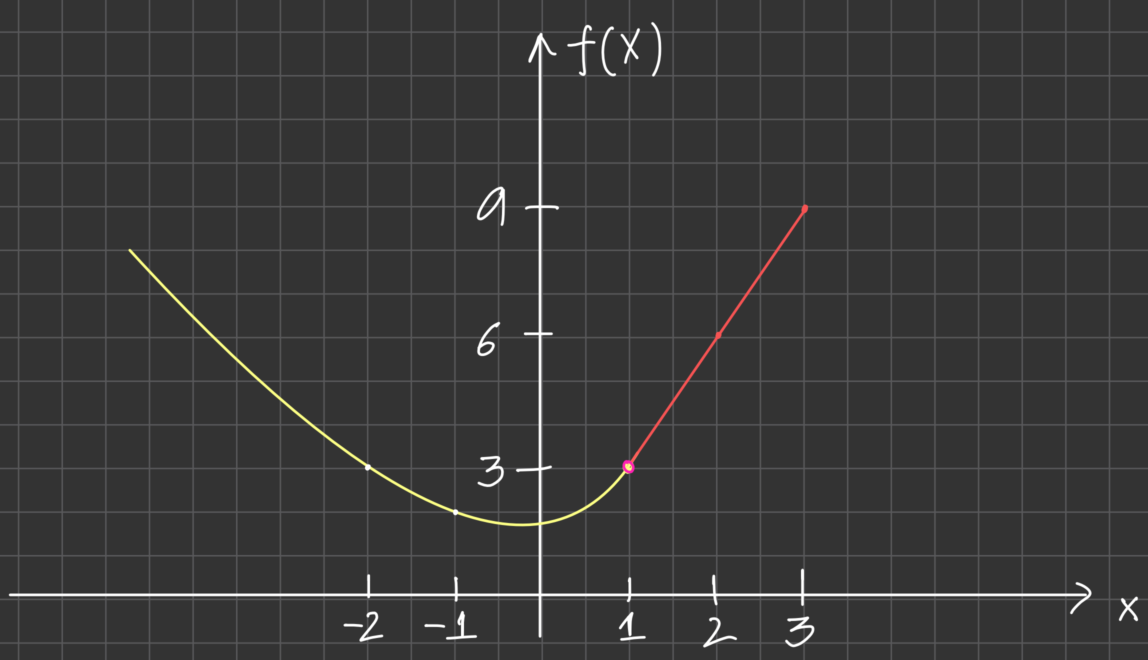
$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1) = 3$$

$$f'(x) = \begin{cases} 2x + 1 \\ 3 \end{cases}$$

$$\lim_{x \to 1^-} (2x + 1) = 3$$

$$\lim_{x \to 1^-} (3) = 3$$

f is differentiable at 1 and the derivative is 3.



66. Let

$$f(x) = \begin{cases} x^2 - 16x, & x < 9 \\ \sqrt{x}, & x \geq 9 \end{cases}$$

Is f continuous at $x = 9$? Determine whether f is differentiable at $x = 9$. If so, find the value of the derivative there.

$$\lim_{x \to 9^-} f(x) = -63$$

f is not continous at $x = 9$

$$\lim_{x \to 9^+} f(x) = 3$$

Since $f(x)$ is not continous ∴ it cannot be differentiable at $x = 9$.

67. Let

$$f(x) = \begin{cases} x^2, & x \leq 1 \\ \sqrt{x}, & x > 1 \end{cases}$$

Determine whether f is differentiable at $x = 1$. If so, find the value of the derivative there.

$$\lim_{x \to 1^-} f(x) = 1 \qquad \lim_{x \to 1^+} f(x) = 1$$

$f(x)$ is continous.

$$\lim_{x \to 1} 2x = 2$$

$$f'(x) = \begin{cases} 2x \\ \dfrac{1}{2\sqrt{x}} \end{cases}$$

$$\lim_{x \to 1} \frac{1}{2\sqrt{x}} = \frac{1}{2}$$

Since the derivative is not equal ∴ f is not differentiable at $x = 1$.

21. $y = \dfrac{2x - 1}{x + 3}$

$$\frac{dy}{dx} = \frac{(x+3)\cdot 2 - (2x-1)\cdot 1}{(x+3)^2} = \frac{7}{(x+3)^2}$$

$x = 1$: $\dfrac{7}{16}$ (ANS).

22. $y = \dfrac{4x + 1}{x^2 - 5}$

$$\frac{dy}{dx} = \frac{(x^2-5)\cdot 4 - (4x+1)\cdot 2x}{(x^2+5)^2}$$

$$\frac{dy}{dx} = \frac{-4x^2 - 2x - 20}{(x^2+5)^2}$$

$x = 1$: $-\dfrac{13}{8}$ (Ans)

23. $y = \left(\dfrac{3x+2}{x}\right)(x^{-5}+1)$

$$\frac{dy}{dx} = \left(\frac{3x+2}{x}\right)\cdot -5x^{-6} + (x^{-5}+1)\,\frac{d}{dx}\left(\frac{3x+2}{x}\right)$$

A

$$\frac{dy}{dx} = \frac{x.3-(3x+2).1}{x^2}$$

$$\frac{dy}{dx} = \frac{-2}{x^2}$$

$$\frac{dy}{dx} = -5x^{-6} + (x^{-5}+1)\left(\frac{-2}{x^2}\right)$$

$x=1$: $-5+(2)(-2) = -9$ (Ans)

24. $y = (2x^7 - x^2)\left(\dfrac{x-1}{x+1}\right)$

$$\frac{dy}{dx} = (2x^7-x^2)\cdot\frac{d}{dx}\left(\frac{x-1}{x+1}\right) + \left(\frac{x-1}{x+1}\right)\cdot(14x^6-2x)$$

A

$$\frac{d}{dx}\left(\frac{x-1}{x+1}\right) = \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$

$$\frac{dy}{dx} = (2x^7-x^2)\cdot\left(\frac{2}{(x+1)^2}\right) + \left(\frac{x-1}{x+1}\right)\cdot(14x^6-2x)$$

$x=1$: $(2-1)\cdot(2/4) + 0 = \dfrac{1}{2}$

15. $f(x) = \sin^2 x + \cos^2 x$

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{d}{dx}(1) = 0$$

16. $f(x) = \sec^2 x - \tan^2 x$

$$\frac{dy}{dx} = 2\sec x \tan x \sec x - 2\tan x \sec^2 x$$

$$\frac{dy}{dx} = \frac{2\sin \cancel{x}}{\cancel{\cos}^3 x} - \frac{2\sin \cancel{x}}{\cancel{\cos}^3 x}$$

$$\frac{dy}{dx} = 0$$

17. $f(x) = \dfrac{\sin x \sec x}{1 + x \tan x}$

$$\frac{(1+x\tan x).\sec^2 x - \tan x[x(\sec^2 x) + (\tan x)(1)]}{(1+x\tan x)^2}$$

$$\frac{\underbrace{\sec^2 x - \tan^2 x}_{=1}}{(1+x\tan x)^2} = \frac{1}{(1+x\tan x)^2}$$

18. $f(x) = \dfrac{(x^2+1)\cot x}{3 - \cos x \csc x}$ $= \dfrac{(x^2+1)\cot x}{3 - \cot x}$

$$f'(x) = \frac{(3-\cot x)\left[2x\cot x - (x^2+1)\csc^2 x\right] - (x^2+1)\cot x \csc^2 x}{(3-\cot x)^2}$$

$$f'(x) = \frac{6x\cot x - 2x\cot^2 x - 3(x^2+1)\csc^2 x}{(3-\cot x)^2}$$

21. $y = x \sin x - 3\cos x$

$$\frac{dy}{dx} = (x\cos x + \sin x) + 3\sin x$$

$$\frac{dy}{dx} = x\cos x + 4\sin x$$

$$\frac{d^2y}{dx^2} = x \cdot -\sin x + \cos x + 4\cos x$$

$$\frac{d^2y}{dx^2} = -x\sin x + 5\cos x$$

22. $y = x^2 \cos x + 4 \sin x$

$$\frac{dy}{dx} = x^2 . -\sin x + 2x\cos x + 4\cos x$$

$$\frac{d^2y}{dx^2} = x^2 . -\cos x + (-\sin \cancel{x . 2x})$$

$$+ 2x \cancel{. -\sin x} + \cos x . 2 \quad -4\sin x$$

$$\frac{d^2y}{dx^2} = \cos x (2 - x^2) - 4\sin x$$

23. $y = \sin x \cos x$

$$\frac{dy}{dx} = \sin x . -\sin x + \cos x . \cos x$$

$$\frac{dy}{dx} = -\sin^2 x + \cos^2 x$$

$$\frac{d^2y}{dx^2} = -2\sin x . \cos x \quad + 2\cos x . -\sin x$$

$$\frac{d^2y}{dx^2} = -4\sin x \cos x$$

31. $y = \cos(\cos x)$

$$\frac{dy}{dx} = -\sin(\cos x) \cdot -\sin x$$

$$\frac{dy}{dx} = \sin x(\cos x)\ \sin x$$

32. $y = \sin(\tan 3x)$

$$\frac{dy}{dx} = \cos(\tan 3x) \cdot \sec^2 3x \cdot 3$$

$$\frac{dy}{dx} = 3\cos(\tan 3x) \cdot \sec^2 3x$$

33. $y = \cos^3(\sin 2x)$

$$\frac{dy}{dx} = 3\cos^2(\sin 2x) \cdot -\sin(\sin 2x) \cdot 2\cos 2x$$

$$\frac{dy}{dx} = -6\cos^2(\sin 2x) \cdot \sin(\sin 2x) \cdot \cos 2x$$

34. $y = \dfrac{1 + \csc(x^2)}{1 - \cot(x^2)}$

$$\frac{dy}{dx} = \frac{\left(\cancel{1} - \cot(x^2)\right)\left[-2x\csc(x^2)\cot(x^2)\right] - \left(\cancel{1} + \csc x^2\right)\left(2x\csc^2(x^2)\right)}{\left[1 - \cot(x^2)\right]^2}$$

$$= -2x\csc^2(x^2)\left[\cot^2(x^2) + \csc(x^2)\right] \quad \text{(Answer)}$$

35. $y = (5x + 8)^7 \left(1 - \sqrt{x}\right)^6$

$$\frac{dy}{dx} = (5x+8)^7 \cdot 6(1-\sqrt{x})^5\left(-\frac{1}{2\sqrt{x}}\right) + 7(5x+8)^6(5).(1-\sqrt{x})^6$$

$$\frac{dy}{dx} = \frac{-3}{2\sqrt{x}}(5x+8)^7(1-\sqrt{x})^5 + 35(5x+8)^6(1-\sqrt{x})^6$$

36. $y = (x^2 + x)^5 \sin^8 x$

$$\frac{dy}{dx} = (x^2+x)^5 \cdot 8\sin^7 x \cdot \cos x + \sin^8 x \cdot 5(x^2+x)^4(2x+1)$$

$$\frac{dy}{dx} = 8(x^2+x)^5 \cdot \sin^7 x \cdot \cos x + 5(x^2+x)^4(2x+1)\sin^8 x$$

25. $x^4 + y^4 = 16$; $(1, \sqrt[4]{15})$ [***Lamé's special quartic***]

$$4x^3 + 4y^3 \cdot \frac{dy}{dx} = 0 \quad ; \quad \frac{dy}{dx} = \frac{-x^3}{y^3}$$

apply values. : $\frac{-1}{15^{3/4}} = 0.131$

26. $y^3 + yx^2 + x^2 - 3y^2 = 0$; $(0, 3)$ [***trisectrix***]

$$3y^2 \cdot \frac{dy}{dx} + x^2 \cdot \frac{dy}{dx} + 2xy + 2x - 6y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x - 2xy}{3y^2 + x^2 - 6y} \quad (0, 3)$$

$$\frac{dy}{dx} = 0$$

27. $2(x^2 + y^2)^2 = 25(x^2 - y^2)$; $(3, 1)$ [***lemniscate***]

$$4(x^2+y^2)\left[2x+2y\cdot\frac{dy}{dx}\right] = 25\left(2x-2y\cdot\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{x\left[25-4(x^2+y^2)\right]}{y\left[25+4(x^2+y^2)\right]} \leftarrow \text{at } (3,1)$$

$$\frac{dy}{dx} = \frac{-9}{13}$$

28. $x^{2/3} + y^{2/3} = 4$; $(-1, 3\sqrt{3})$ [***four-cusped hypocycloid***]

$$\frac{2}{3}\left(x^{-1/3} + y^{-1/3}\cdot\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = \frac{-x^{-1/3}}{y^{-1/3}}$$

$$\frac{dy}{dx} = \frac{y^{1/3}}{x^{1/3}} \longleftarrow \text{at } (-1, 3\sqrt{3})$$

$$\frac{dy}{dx} = \sqrt{3}$$