Q1 $f(x) = \sqrt{x} - \cos x$

$f(0) = -1$
$f(1) = 0.45969$

n	a	b	$c = \frac{(b+a)}{2}$	abs error $= |C_{n+1} - C_n|$
01	0	1	0.5	
02	0.5	1	0.75	0.25
03	0.5	0.75	0.625	0.125
04	0.625	0.75	0.6875	0.0625
05	0.625	0.6875	0.65625	0.03125
06	0.625	0.65625	0.640625	0.015625
07	0.640625	0.65625	0.6484375	7.8125×10^{-3}

$x = 0.6484375 \approx 0.64844$ (Ans)

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Q_2.

$f(0) = 0.6435$

$f(1) = 0.45$

n	a	b	$c = a - \dfrac{f(a)(b-a)}{f(b)-f(a)}$	abs error. $2\lvert c_{n+1} - c_n \rvert$
01	0	1	0.6850733573	
02	0	0.6850733573	0.6503949801	0.0346783772
03	0	0.6503949801	$\boxed{0.6435566552}$	6.8384281×10^{-3}
04	0	0.6435566552	0.6421098576	$1.446694413 \times 10^{-3}$

$$x = 0.643556552 \approx 0.64356 \quad \text{(Ans)}$$

bisection :—

Q3. $f(x) = 2e^{-x}$ $x = 2e^{-x}$

$0 = 2e^{-x} - x$

$f(1) = 0.7357588823$ (−0.2642411177)

$f(0) = 2$

% relative approx error.

$$= \frac{|C_{n+1} - C_n|}{C_{n+1}}$$

n	a	b	c	
01	0	1	0.5	
02	0	0.7130613194	0.3565306597	
03	0	0.3565306597	0.1782653299 0.5218356764	
04	0	0.1782653299	0.08913266495	
05	0	0.08913266495	0.04456633248	

n	a	b	c	% relative approx error
01	0	1	0.5	
02	0.5	1	0.75	0.3333
03	0.75	1	0.875	0.0625
04	0.75	0.875	0.8125	0.07692307692
05	0.8125	0.875	0.84375	0.03703703704
06	0.84375 0.8125	0.875	0.859375	0.01818181818
07	0.84375	0.859375	0.8515625	$9.174311927 \times 10^{-3} \to 0.91743\%$

← bisection $x \approx 0.85156$

regular falsi :—

n	a	b	$c = a - \frac{f(a)(b-a)}{f(b)-f(a)}$	
01	0	1	0.8832981542	
02	0	0.8832981542	0.8590456379	0.02823192998 (2.82319%)
03	0	0.8590456379	0.8539588662	5.0867717×10^{-3} (0.50867717%)
04	0.8539588662	0.8590456379	0.852603498	— regular falsi

$x \approx 0.85396$

Q4. $N(t) = t^3 - 7t^2 + 14t + 10$

$f(0) = -6$
$f(1) = 2$

$16 = t^3 - 7t^2 + 14t + 10 \implies t^3 - 7t^2 + 14t - 6 = 0$

n	a	b	$c = a - \dfrac{f(a)(b-a)}{f(b)-f(a)}$	abs error
01	0	1	0.75	
02	0	0.75	0.644295302	0.105704698
03	0	0.644295302	0.605751653	0.038543649
04	0	0.605751653	0.5924946631	0.0132569899
05	0	0.5924946631	0.5880284877	4.4661754×10^{-3}
06	0	0.5880284877	0.5865344541	1.4940336×10^{-3}
07	0	0.5865344541	0.5860358506	$6.913095998 \times 10^{-4}$

$x = 0.5865358506 \approx 0.58653$ (Ans)

Q5. $f(x) = x^4 - x - 10$ $x_0 = 4$

$g(x)$ $\longrightarrow$ $x^4 - 10 = x$ $\bigcirc$

$g(x) = x$

$$x^4 = x + 10$$
$$x = (x+10)^{1/4}$$

$g(x) = (x+10)^{1/4}$ $4x^3$ 4

$\bigcirc$

n	x_i	$g(x_i)$	abs error.
01	4	1.93433642	2.06566358
02	1.93433642	1.858658358	0.075678062
03	1.858658358	1.855704793	2.953565×10^{-3}
04	1.855704793	1.855589234	1.15559×10^{-4}
05	1.855589234	1.855584713	4.5212271×10^{-6}

$x = 1.855589234 \approx 1.85558$ (Ans)

Q6 $x = \cos t - 3t + 3$ $x_0 = 0$

$2 = \cos t - 3t + 3$

$0 = \cos t - 3t + 1$

$3t = \cos t + 1$

$t = \dfrac{\cos t + 1}{3}$

x_0	$g(x_0)$	
0	$0.\dot{6}6\dot{6}$	
$0.\dot{6}66$	0.5952957536	0.0713709124
0.5952957536	0.6093275634	0.0140318098
0.6093275634	0.6066776832	2.6498802×10^{-3}
0.6066776832	0.607182246	$5.045627971 \times 10^{-4}$
0.607182246	0.6070863205	$9.592553545 \times 10^{-5}$

Q7.

$$15 = 75 e^{-1.5t} + 20 e^{-0.075t} \qquad t = 6$$

$$f(t) = 75 e^{-1.5t} + 20 e^{-0.075t} - 15$$

$$f'(t) = -112.5 e^{-1.5t} - 1.5 e^{-0.075t}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$x_{i+1} = t - \frac{75 e^{-1.5t} + 20 e^{-0.075t} - 15}{-112.5 e^{-1.5t} - 1.5 e^{-0.075t}}$$

x_i	abs error
6	
3.693371482	2.306628518
3.981896208	0.288524726
4.001563209	0.019667001
4.001634067	0.0000708

$$x = 4.001634067 \qquad (Ans)$$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

Q8. $v(t) = t\cos(t) + \sin(t)$

$$c = b + \frac{f(b)(a-b)}{f(a) - f(b)} \left\{ t_{i+1} = t_i - \frac{f(t_i)(t_{i-1} - t_i)}{f(t_{i-1}) - f(t_i)} \right.$$

i	t_{i-1} a	t_i b	t_{i+1}	$c = b - \frac{f(b)(b-a)}{f(b) - f(a)}$
1	2	3 .	2.02649946	
	3	2.02649946	2.028594812	
	2.02649946	2.028594812	2.028757961	
	2.028594818	2.028757961	2.028757838	

error $= 1.22 \times 10^{-7}$

Q9. $f(x) = x^{1/2} + x^{-1/2} - 3$

$$f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$$

$$\cancel{c = b - \frac{f(b)(a-b)}{f(a) - f(b)}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

i	x_i
01	2.
02	6.9705627
03	6.85377713
04	6.85 4101964
05	6.854101966

$$\boxed{x = 6.85410}$$

041630   Q10.

$$S = a + kf + \sin t + t - 3.5.$$

$$\sin t + t = 1.5$$

$$t = 1.5 - \sin t \qquad\qquad t' = -\cos t$$

$$\boxed{t_{i+1} = 1.5 - \sin t_i} \qquad\qquad < 1$$

i	t_i	
01	2.	
02	0.5907025732	
	~~0.9430513178~~	
03	~~0.5907025732~~	
04	0.6906486818	
05	0.8629665127	
06	0.7402253105	
07	0.8255457208	
08	0.7650820245	
09	0.8074038327	
10	0.7775053178	
11	0.7984962707	0.014803
12	0.783692378	0.0148062707
		0.01041
13	0.7941004194	$7.3 \times 10^{-3} \to 0.0073$
14	0.7867666464	(stop).
15	0.7919262176	$\boxed{x = 0.786767}$

Q11

$$P = c(h - P_0)^2 + \ln(h) - 2$$

↑
1.5

$c = 1$

$P = 2$

$$2 = (h - 1.5)^2 + \ln(h) - 2$$

$$4 = (h - 1.5)^2 + \ln(h)$$

$$h = e^{(h - 1.5)^2 - 4}$$

$$\ln(h) = h^2 - 3h - \frac{7}{4}$$

$$\frac{1}{h} = 2h - 3$$

$$f(x) \rightarrow h^2 + \ln(h) - 4$$

$$f(x) = (h - \circ)^2 + \ln(h) - 4 = 0$$

$$X_0 = 1.5$$

#i	w_i
0	1.5
1	1.866691334
2	1.841228281
3	1.841097062
4	1.841097058
5	1.841097058