24. Since g' is continuous at p and $|g'(p)| > 1$, by letting $\epsilon = |g'(p)| - 1$ there exists a number $\delta > 0$ such that $|g'(x) - g'(p)| < |g'(p)| - 1$ whenever $0 < |x - p| < \delta$. Hence, for any x satisfying $0 < |x - p| < \delta$, we have

$$|g'(x)| \geq |g'(p)| - |g'(x) - g'(p)| > |g'(p)| - (|g'(p)| - 1) = 1.$$

If p_0 is chosen so that $0 < |p - p_0| < \delta$, we have by the Mean Value Theorem that

$$|p_1 - p| = |g(p_0) - g(p)| = |g'(\xi)||p_0 - p|,$$

for some ξ between p_0 and p. Thus, $0 < |p - \xi| < \delta$ so $|p_1 - p| = |g'(\xi)||p_0 - p| > |p_0 - p|$.

Exercise Set 2.3, page 71

1. $p_2 = 2.60714$
2. $p_2 = -0.865684$; If $p_0 = 0$, $f'(p_0) = 0$ and p_1 cannot be computed.
3. (a) 2.45454 (b) 2.44444 (c) Part (a) is better.
4. (a) -1.25208 (b) -0.841355
5. (a) For $p_0 = 2$, we have $p_5 = 2.69065$.
   (b) For $p_0 = -3$, we have $p_3 = -2.87939$.
   (c) For $p_0 = 0$, we have $p_4 = 0.73909$.
   (d) For $p_0 = 0$, we have $p_3 = 0.96434$.
6. (a) For $p_0 = 1$, we have $p_8 = 1.829384$.
   (b) For $p_0 = 1.5$, we have $p_4 = 1.397748$.
   (c) For $p_0 = 2$, we have $p_4 = 2.370687$; and for $p_0 = 4$, we have $p_4 = 3.722113$.
   (d) For $p_0 = 1$, we have $p_4 = 1.412391$; and for $p_0 = 4$, we have $p_5 = 3.057104$.
   (e) For $p_0 = 1$, we have $p_4 = 0.910008$; and for $p_0 = 3$, we have $p_9 = 3.733079$.
   (f) For $p_0 = 0$, we have $p_4 = 0.588533$; for $p_0 = 3$, we have $p_3 = 3.096364$; and for $p_0 = 6$, we have $p_3 = 6.285049$.
7. Using the endpoints of the intervals as p_0 and p_1, we have:

   (a) $p_{11} = 2.69065$ (b) $p_7 = -2.87939$ (c) $p_6 = 0.73909$ (d) $p_5 = 0.96433$
8. Using the endpoints of the intervals as p_0 and p_1, we have:

   (a) $p_7 = 1.829384$ (b) $p_9 = 1.397749$

(c) $p_6 = 2.370687; p_7 = 3.722113$ (d) $p_8 = 1.412391; p_7 = 3.057104$
(e) $p_6 = 0.910008; p_{10} = 3.733079$
(f) $p_6 = 0.588533; p_5 = 3.096364; p_5 = 6.285049$

9. Using the endpoints of the intervals as p_0 and p_1, we have:

(a) $p_{16} = 2.69060$ (b) $p_6 = -2.87938$ (c) $p_7 = 0.73908$ (d) $p_6 = 0.96433$

10. Using the endpoints of the intervals as p_0 and p_1, we have:

(a) $p_8 = 1.829383$ (b) $p_9 = 1.397749$
(c) $p_6 = 2.370687; p_8 = 3.722112$ (d) $p_{10} = 1.412392; p_{12} = 3.057099$
(e) $p_7 = 0.910008; p_{29} = 3.733065$
(f) $p_9 = 0.588533; p_5 = 3.096364; p_5 = 6.285049$

11. (a) Newton's method with $p_0 = 1.5$ gives $p_3 = 1.51213455$.
The Secant method with $p_0 = 1$ and $p_1 = 2$ gives $p_{10} = 1.51213455$.
The Method of False Position with $p_0 = 1$ and $p_1 = 2$ gives $p_{17} = 1.51212954$.

(b) Newton's method with $p_0 = 0.5$ gives $p_5 = 0.976773017$.
The Secant method with $p_0 = 0$ and $p_1 = 1$ gives $p_5 = 10.976773017$.
The Method of False Position with $p_0 = 0$ and $p_1 = 1$ gives $p_5 = 0.976772976$.

12. (a)

	Initial Approximation	Result	Initial Approximation	Result
Newton's	$p_0 = 1.5$	$p_4 = 1.41239117$	$p_0 = 3.0$	$p_4 = 3.05710355$
Secant	$p_0 = 1, p_1 = 2$	$p_8 = 1.41239117$	$p_0 = 2, p_1 = 4$	$p_{10} = 3.05710355$
False Position	$p_0 = 1, p_1 = 2$	$p_{13} = 1.41239119$	$p_0 = 2, p_1 = 4$	$p_{19} = 3.05710353$

(b)

	Initial Approximation	Result	Initial Approximation	Result
Newton's	$p_0 = 0.25$	$p_4 = 0.206035120$	$p_0 = 0.75$	$p_4 = 0.681974809$
Secant	$p_0 = 0, p_1 = 0.5$	$p_9 = 0.206035120$	$p_0 = 0.5, p_1 = 1$	$p_8 = 0.681974809$
False Position	$p_0 = 0, p_1 = 0.5$	$p_{12} = 0.206035125$	$p_0 = 0.5, p_1 = 1$	$p_{15} = 0.681974791$

13. For $p_0 = 1$, we have $p_5 = 0.589755$. The point has the coordinates $(0.589755, 0.347811)$.

14. For $p_0 = 2$, we have $p_2 = 1.866760$. The point is $(1.866760, 0.535687)$.

15. The equation of the tangent line is

$$y - f(p_{n-1}) = f'(p_{n-1})(x - p_{n-1}).$$

To complete this problem, set $y = 0$ and solve for $x = p_n$.

16. Newton's method gives $p_{15} = 1.895488$, for $p_0 = \frac{\pi}{2}$; and $p_{19} = 1.895489$, for $p_0 = 5\pi$. The sequence does not converge in 200 iterations for $p_0 = 10\pi$. The results do not indicate the fast convergence usually associated with Newton's method.