21k-3278
Sohaib Shamsi
LA A10

Q1

$$x^2 + y^2 + z^2 = 6$$
$$x^2 - y^2 + 2z^2 = 2$$
$$2x^2 + y^2 - z^2 = 3$$

$$X + Y + Z = 6$$
$$X - Y + 2Z = 2$$
$$2X + Y - Z = 3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 2 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$R_2 - R_1 \to R_2$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -4 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$R_3 - 2R_1 \to R_3$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 6 \\ 0 & -2 & 1 & -4 \\ 0 & -1 & -3 & -9 \end{array}\right]$$

$R_2 \times -\frac{1}{2} \to R_2$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 6 \\ 0 & 1 & -1/2 & 2 \\ 0 & -1 & -3 & -9 \end{array}\right] = \left[\begin{array}{cccc} 1 & 1 & 1 & 6 \\ 0 & 1 & -1/2 & 2 \\ 0 & 0 & -7/2 & -7 \end{array}\right]$$

$R_3 + R_2 \to R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -1/2 & 2 \\ 0 & 0 & -7/2 & -7 \end{bmatrix}$$

$$R_3 \times -\frac{2}{7} \rightarrow R_3$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -1/2 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix} \text{---} 3$$

$(Z = 2)$

$Y - \frac{1}{2}Z = 2$

$Y - 1 = 2$

$(Y = 3)$

$X + 3 + 2 = 6$

$(X = 1)$

$\sqrt{x^2} = \sqrt{1}$

$\boxed{x = \pm 1}$

$z^2 = 2$

$\boxed{z = \pm\sqrt{2}}$

$y^2 = 3$

$\boxed{y = \pm\sqrt{3}}$

(a)

Q2) $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 3 & 4 & 5 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$\begin{bmatrix} 3 & 4 & 5 \\ 0 & -2 & -29 \\ 2 & 1 & 3 \end{bmatrix}$

$R_1 - R_3 \rightarrow R_1$

$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -2 & -29 \\ 2 & 1 & 3 \end{bmatrix}$

$R_3 - 2R_1 \rightarrow R_3$

$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -2 & -29 \\ 0 & -5 & -1 \end{bmatrix}$

$\boxed{R_2 \times -2 \rightarrow R_2}$

$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & -58 \\ 0 & -5 & -1 \end{bmatrix}$

$R_2 + R_3 \rightarrow R_2$

$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & -59 \\ 0 & -5 & -1 \end{bmatrix}$

$R_2 \times -1 \rightarrow R_2$

$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 59 \\ 0 & -5 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 59 \\ 0 & -5 & -1 \end{bmatrix}$$

$R_3 + 5R_2 \rightarrow R_3$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 59 \\ 0 & 0 & 294 \end{bmatrix}$$

$R_3 \times \frac{1}{294} \rightarrow R_3$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 59 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 - 59R_3 \rightarrow R_2$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 - 3R_2 \rightarrow R_1$

$R_1 - 2R_3 \rightarrow R_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q3. (a)

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix}$$

$R_2 + 2R_3$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 5 & -10 & 0 & 0 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix}$$

$R_3 + R_1 \rightarrow R_3$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 5 & -10 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix}$$

$R_4 - 3R_1$ $\qquad$ $R_2 \times \frac{1}{5} \rightarrow R_2$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1\,\cancel{5} & \cancel{-10}\,-2 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix}$$

$R_4 - 3R_2$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 - R_2 \rightarrow R_1$

(Ans)

Q3

(b) $\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 1 & 4 & 2 & 0 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 2 & -4 & 1 & 1 & 0 \\ 1 & -2 & -1 & 1 & 0 \end{bmatrix}$

$R_2 - R_1 \to R_2$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 2 & -4 & 1 & 1 & 0 \\ 1 & -2 & -1 & 1 & 0 \end{bmatrix}$$

$R_4 - 2R_5 \to R_4$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 1 & -2 & -1 & 1 & 0 \end{bmatrix}$$

$R_5 - R_1 \to R_5$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 0 & -5 & -1 & 0 & 0 \end{bmatrix}$$

$R_5 + 5R_2 \to R_5$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 0 & 0 & 9 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 2 & -3 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 0 & 0 & 9 & -5 & 0 \end{bmatrix}$$

$R_3 + 2R_2 \to R_3$ $\qquad$ $R_3 \times \frac{1}{2} \to R_3$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 0 & 0 & 9 & -5 & 0 \end{bmatrix}$$

$R_5 - 3R_4 \to R_5$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & -2 & 0 \end{bmatrix}$$

$R_4 - 3R_3$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 7/2 & 0 \\ 0 & 0 & 0 & -2 & 0 \end{bmatrix}$$

$R_4 \times 2/7 \to R_4$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 \end{bmatrix}$$

$R_5 + 2R_4 \to R_5$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 + \frac{3}{2}R_4 \to R_3$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$(R_4 + R_2 \to R_2)$ & $(R_2 - 2R_3 \to R_2)$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 - R_4 \to R_1$

$R_1 - 3R_2 \to R_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(Ans).

Q4.

(a) $\begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}$ $\quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$5R_1 + R_2 \rightarrow R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ E_1

$R_2 \times \frac{1}{2} \rightarrow R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}$ E_2

$A^{-1} = E_2 . E_1$

$A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 5/2 & 1/2 \end{bmatrix}$ (Ans)

(b) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2 \times \frac{1}{4} \rightarrow R_2$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3/4 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ E_1

$2R_3 + R_1 \rightarrow R_1$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3/4 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ E_2

$R_2 - 3/4 R_3 \rightarrow R_2$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3/4 \\ 0 & 0 & 1 \end{bmatrix}$ E_3

$A^{-1} = E_3 E_2 E_1$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3/4 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(Ans)

$A^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1/4 & -3/4 \\ 0 & 0 & 1 \end{bmatrix}$

Q5 (a)

$$A = \left[\begin{array}{cccc|c} -1 & -4 & 2 & 1 & -32 \\ 2 & -1 & 7 & 9 & 14 \\ -1 & 1 & 3 & 1 & 11 \\ 1 & -2 & 1 & -4 & -4 \end{array}\right] \qquad B = \begin{bmatrix} -32 \\ 14 \\ 11 \\ -4 \end{bmatrix}$$

$$|A| = -1\begin{vmatrix} -1 & 7 & 9 \\ 1 & 3 & 1 \\ -2 & 1 & -4 \end{vmatrix} + 4\begin{vmatrix} 2 & 7 & 9 \\ -1 & 3 & 1 \\ 1 & 1 & -4 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 & 9 \\ -1 & 1 & 1 \\ 1 & -2 & -4 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 & 7 \\ -1 & 1 & 3 \\ 1 & -2 & 1 \end{vmatrix}$$

$$\begin{aligned} |A| = & -1\{-1(-12-1)-7(-4+2)+9(1+6)\} \\ & +4\{2(-12-1)-7(4-1)+9(-1-3)\} \\ & +2\{2(-4+2)+1(4-1)+9(2-1)\} \\ & -1\{2(1+6)+1(-1-3)+7(2-1)\} \end{aligned}$$

$$|A| = -1[90]+4[-83]+2[28]-1[17] \Rightarrow -423$$

$$A_1 = \begin{vmatrix} -32 & -4 & 2 & 1 \\ 14 & -1 & 7 & 9 \\ 11 & 1 & 3 & 1 \\ -4 & -2 & 1 & -4 \end{vmatrix} = -2115 \qquad x_1 = \frac{|A_1|}{|A|} = \frac{-2115}{-423} = 5$$

$$x_2 = \frac{|A_2|}{|A|} = \frac{-3384}{-423} = 8$$

$$A_2 = \begin{vmatrix} -1 & -32 & 2 & 1 \\ 2 & 14 & 7 & 9 \\ -1 & 11 & 3 & 1 \\ 1 & -4 & 1 & -4 \end{vmatrix} = -3384$$

$$x_3 = \frac{|A_3|}{|A|} = \frac{-1269}{-423} = 3$$

$$A_3 = \begin{vmatrix} -1 & -4 & -32 & 1 \\ 2 & -1 & 14 & 9 \\ 1 & 1 & 11 & 1 \\ -1 & -2 & -4 & -4 \end{vmatrix} = -1269$$

$$x_4 = \frac{423}{-423} = -1$$

$$A_4 = \begin{vmatrix} -1 & -4 & 2 & -32 \\ 2 & -1 & 7 & 14 \\ 1 & 1 & 3 & 11 \\ -1 & -2 & 1 & -4 \end{vmatrix} = 423$$

$(5, 8, 3, -1)$ or $\begin{bmatrix} 5 \\ 8 \\ 3 \\ -1 \end{bmatrix}$ (Ans)

Q5 (b)

(part a)
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2-14 & a+2 \end{bmatrix}$$

$R_2 - 3R_1 \to R_2$

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 4 & 1 & a^2-14 & a+2 \end{bmatrix}$$

$R_3 - 4R_1$

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & 10/7 \\ 0 & -7 & a^2-2 & a-14 \end{bmatrix}$$

$7R_2 + R_3 \to R_3$

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & 10/7 \\ 0 & 0 & a^2-16 & a-4 \end{bmatrix}$$

$a^2 - 16 = 0$

$a = \pm 4$

no solution for $a = 4, -4$

For all other values, infinitely many solutions.

$a^2 - 16 = a - 4$

$a^2 - a - 12 = 0$

$a = 1 \quad b = -1 \quad c = -12$

$b^2 - 4ac$

$(-1)^2 - 4(1)(-12)$

$1 + 48 \Rightarrow 49$

$a = 4$ gives $0 = 0$

$a = -3$ gives $0 = 0$

$b^2 - 4ac > 0$ infinite soln

$b^2 - 4ac = 0$ 1 soln

$b^2 - 4ac < 0$ no soln

At $a = 4$ & $a = -3$ there are infinitely many solutions

(part b)

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & -(a^2-3) & a \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 2 \\ 0 & -6 & 1 & -3 \\ 1 & 2 & 3-a^2 & a \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 2 \\ 0 & -6 & 1 & -3 \\ 0 & 2 & 3-a^2 & a \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 2 \\ 0 & 1 & -1/6 & 3/2 \\ 0 & 0 & 2-a^2 & -2a+a \end{array}\right]$$

$+2-a^2=0$

$a^2=2$

$a=\pm\sqrt{2}$

if $a=\sqrt{2}$ & $a=-\sqrt{2}$

no solution

for all other values

it has infinte many solutions

Q6

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\left[\begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{array}\right] \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$

$R_3 - R_1 \rightarrow R_?$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right]$

$R_3 - R_2$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & -1 & 1 \end{array}\right]$

$R_3 \times -\frac{1}{2}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{array}\right]$

$R_3 - R_2$

$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{array}\right]$ (Ans)

Q6 (b) $\begin{bmatrix} \sqrt{2} & 3\sqrt{2} & 0 \\ -4\sqrt{2} & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} \sqrt{2} & 3\sqrt{2} & 0 & 1 & 0 & 0 \\ -4\sqrt{2} & \sqrt{2} & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \times \frac{1}{\sqrt{2}}$ & $4\sqrt{2}R_1 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1/\sqrt{2} & 0 & 0 \\ 0 & 13\sqrt{2} & 0 & 4 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\frac{13}{\sqrt{2}} \times R_2$ & $3R_2 - R_1$ & $(R_1 \times -1)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \sqrt{2}/26 & -3\sqrt{2}/26 & 0 \\ 0 & 1 & 0 & 4/13\sqrt{2} & 1/13\sqrt{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

∴ Inverse exist .

Q6 (c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{bmatrix}$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 3 & 5 & 0 & 0 & 0 & 1 & 0 \\ 1 & 3 & 5 & 7 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1 - R_2$, $R_1 - R_3$, $R_1 - R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & +1/3 & 0 & 0 \\ 0 & -3 & -5 & 0 & 1 & 0 & -1 & 0 \\ 0 & -3 & -5 & -7 & 1 & 0 & 0 & -1 \end{array}\right]$$

$R_2 \times -\frac{1}{3}$

$3R_2 + R_3$ $\quad 3R_2 + R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/5 & +1/5 & 0 \\ 0 & 0 & -5 & -7 & 0 & 1 & 0 & -1 \end{array}\right]$$

$R_3 \times -\frac{1}{5}$

$5R_3 + R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/5 & 1/5 & 0 \\ 0 & 0 & 0 & -7 & 0 & 0 & 1 & -1 \end{array}\right]$$

$R_4 \times -\frac{1}{7}$

$$\Rightarrow \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ -1/3 & 1/3 & 0 & 0 \\ 0 & -1/5 & 1/5 & 0 \\ 0 & 0 & -1/7 & 1/7 \end{array}\right]$$ (Ans).

Q7)

(part a)

$$|A| = \underset{(1+1)}{-1}\begin{vmatrix} 3 & 2 & 1 \\ 2 & 3 & 4 \\ -3 & -2 & 1 \end{vmatrix} \underset{(1+2)}{+2}\begin{vmatrix} 4 & 2 & 1 \\ 1 & 3 & 4 \\ -4 & -2 & 1 \end{vmatrix} \underset{(1+3)}{-3}\begin{vmatrix} 4 & 3 & 1 \\ 1 & 2 & 4 \\ -4 & -3 & -1 \end{vmatrix} \underset{(1+4)}{+4}\begin{vmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ -4 & -3 & -2 \end{vmatrix}$$

$$|A| = -1(0) + 2(0) - 3(0) + 4(0) \Rightarrow 0.$$

$C_{11} = 0$

$C_{12} = 0$

$C_{13} = 0$

$C_{14} = 0$

$$C_{21} = 4(-1)^{2+1}\begin{vmatrix} -2 & -3 & -4 \\ 2 & 3 & 4 \\ -3 & -2 & -1 \end{vmatrix} = 0$$

$$C_{22} = 3(-1)^{2+2}\begin{vmatrix} -1 & -3 & -4 \\ 1 & 3 & 4 \\ -4 & -2 & -1 \end{vmatrix} = 0$$

$$C_{23} = 2(-1)^{2+3}\begin{vmatrix} -1 & -2 & -4 \\ 1 & 2 & 4 \\ -4 & -3 & -1 \end{vmatrix} = 0$$

$$C_{24} = 2(-1)^{2+4}\begin{vmatrix} -1 & -2 & -3 \\ 1 & 2 & 3 \\ -4 & -3 & -2 \end{vmatrix} = 0$$

$C_{31} = 0$

$C_{32} = 0$

$C_{33} = 0$

$C_{34} = 0$

$C_{41} = 0$

$C_{42} = 0$

$C_{43} = 0$

$C_{44} = 0$

Q8).

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

3×3 $\quad$ 3×1

Standard matrix $= \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}$

$T(-1, 2, 4)$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} -3 + 10 - 4 \\ -4 - 2 + 4 \\ -3 + 4 - 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix}$$

$$\boxed{\begin{aligned} w_1 &= 3 \\ w_2 &= -2 \\ w_3 &= -3 \end{aligned}}$$