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Class = BCS - 1E

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Assignment 1 (3 weightage)

Deadline: 21/sep/2021

Time: 4 pm

Course: Calculus and analytical geometry

Course code: MT-1003

Instructions:

Kindly upload file using cam scanner by converting in one pdf file.

Exercise: 1.5

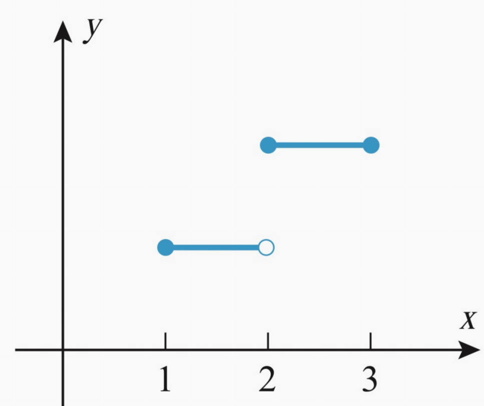
Q# 1 to 6, 11 to 22, 29,30,35,36

1–4 Let f be the function whose graph is shown. On which of the following intervals, if any, is f continuous?

(a) $[1, 3]$ (b) $(1, 3)$ (c) $[1, 2]$
(d) $(1, 2)$ (e) $[2, 3]$ (f) $(2, 3)$

For each interval on which f is not continuous, indicate which conditions for the continuity of f do not hold.

1.



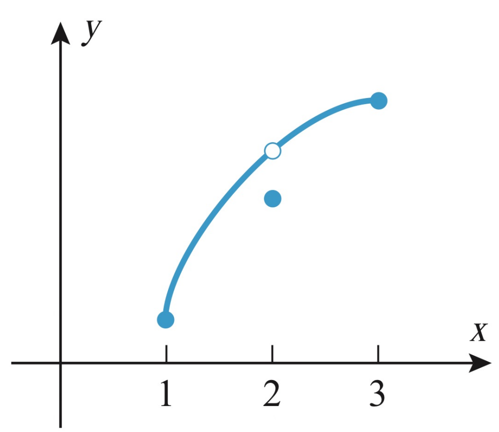
a. NO, $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

b. NO, $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

c. NO, $\lim_{x\to 2^-} f(x) \neq f(2)$

d. Yes e. Yes f. Yes

2.



a. NO, $\lim_{x\to 2} f(x) \neq f(2)$

b. NO, $\lim_{x\to 2} f(x) \neq f(2)$

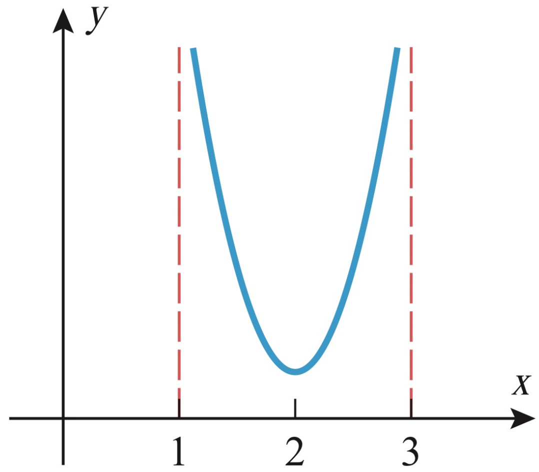
c. NO, $\lim_{x\to 2^-} f(x) \neq f(2)$

d. Yes

e. NO, $\lim_{x\to 2^+} f(x) \neq f(2)$

f. Yes

3.



a. NO, $f(1)$ and $f(3)$ are undefined.

b. Yes

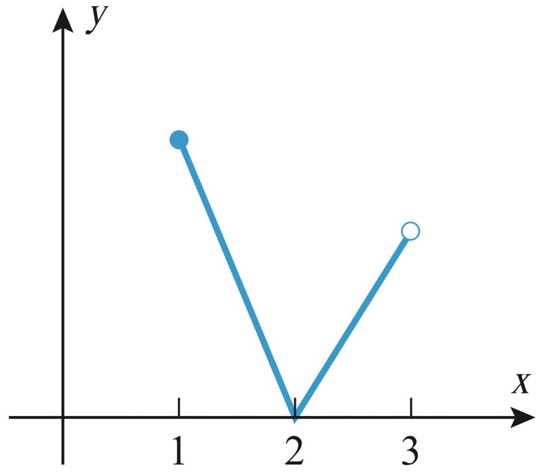
c. NO, $f(1)$ is undefined.

d. Yes

e. NO, $f(3)$ is undefined.

f. Yes

4.



a. No. $f(3)$ is undefined.

b. Yes

c. Yes

d. Yes

e. NO, $f(3)$ is undefined

f. Yes

5. Consider the functions

$$f(x) = \begin{cases} 1, & x \neq 4 \\ -1, & x = 4 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 4x - 10, & x \neq 4 \\ -6, & x = 4 \end{cases}$$

In each part, is the given function continuous at $x = 4$?

(a) $f(x)$ (b) $g(x)$ (c) $-g(x)$ (d) $|f(x)|$
(e) $f(x)g(x)$ (f) $g(f(x))$ (g) $g(x) - 6f(x)$

a. NO b. NO c. NO d. Yes

e. Yes f. NO g. Yes

$1 \times 6 = 6$

$-1 \times -6 = 6$

$-g(x) = \begin{cases} -4x+10 \\ 6 \end{cases}$

$4(1) - 10 = -6$

6. Consider the functions

$$f(x) = \begin{cases} 1, & 0 \leq x \\ 0, & x < 0 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 0, & 0 \leq x \\ 1, & x < 0 \end{cases}$$

In each part, is the given function continuous at $x = 0$?

(a) $f(x)$ (b) $g(x)$ (c) $f(-x)$ (d) $|g(x)|$
(e) $f(x)g(x)$ (f) $g(f(x))$ (g) $f(x) + g(x)$

a. NO b. NO c. NO d. NO

e. Yes f. Yes g. Yes

11–22 Find values of x, if any, at which f is not continuous.

11. $f(x) = 5x^4 - 3x + 7$

None ∴ continous $x \in R$

12. $f(x) = \sqrt[3]{x - 8}$

None ∴ continous $x \in R$

13. $f(x) = \dfrac{x + 2}{x^2 + 4}$

None ∴ continous $x \in R$

14. $f(x) = \dfrac{x + 2}{x^2 - 4}$

$$\frac{(x+2)}{(x-2)(x+2)}$$

not continous at $x = 2, -2$

$x = (-\infty, -2) \cup (-2, 2) \cup (2, +\infty)$

15. $f(x) = \dfrac{x}{2x^2 + x}$

$$\frac{x}{x(2x+1)}$$

not continous at $x = -\frac{1}{2}, 0$

$x = (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 0) \cup (0, +\infty)$

16. $f(x) = \dfrac{2x+1}{4x^2+4x+5}$

None ∴ continous $x \in \mathbb{R}$

17. $f(x) = \dfrac{3}{x} + \dfrac{x-1}{x^2-1}$

$(x+1)(x-1)$

not continous at $x = 1, -1, 0$

$x = (-\infty,-1) \cup (-1,0) \cup (0,1) \cup (1,+\infty)$

18. $f(x) = \dfrac{5}{x} + \dfrac{2x}{x+4}$

not continous at $x = 0, -4$

$x = (-\infty,-4) \cup (-4,0) \cup (0,+\infty)$

19. $f(x) = \dfrac{x^2+6x+9}{|x|+3}$

None ∴ continous $x \in \mathbb{R}$

20. $f(x) = \left|4 - \dfrac{8}{x^4+x}\right|$

Not continous at $x = 0, -1$

$x = (-\infty,-1) \cup (-1,0) \cup (0,+\infty)$

21. $f(x) = \begin{cases} 2x+3, & x \leq 4 \\ 7 + \dfrac{16}{x}, & x > 4 \end{cases}$

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$$

$$2(4)+3 = 7 + \frac{16}{4}$$

$$11 = 11$$

None, continous $\therefore x \in \mathbb{R}$

22. $f(x) = \begin{cases} \dfrac{3}{x-1}, & x \neq 1 \\ 3, & x = 1 \end{cases}$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$\frac{3}{0} \neq 3$$

Not continous at $x = 1$

$x = (-\infty, 1) \cup (1, +\infty)$

29–30 Find a value of the constant k, if possible, that will make the function continuous everywhere. ■

29. (a) $f(x) = \begin{cases} 7x - 2, & x \leq 1 \\ kx^2, & x > 1 \end{cases}$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$7(1) - 2 = K(1)^2$$

$K = 5$ ↳ $f(x)$ is continous for all values.

(b) $f(x) = \begin{cases} kx^2, & x \leq 2 \\ 2x + k, & x > 2 \end{cases}$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$K(2)^2 = 2(2) + K$$

$$3K = 4$$

$K = 4/3$ ↳ $f(x)$ is continous for all values

30. (a) $f(x) = \begin{cases} 9 - x^2, & x \geq -3 \\ k/x^2, & x < -3 \end{cases}$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$$

$$9 - 9 = k/9$$

$k = 0$ → $f(x)$ is continous for all values.

(b) $f(x) = \begin{cases} 9 - x^2, & x \geq 0 \\ k/x^2, & x < 0 \end{cases}$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$9 - 0^2 = k/0^2 \quad \infty$$

No such value of k that makes function continous everywhere

35–36 Find the values of x (if any) at which f is not continuous, and determine whether each such value is a removable discontinuity.

35. (a) $f(x) = \dfrac{|x|}{x}$: $x \neq 0$

$\dfrac{+x}{x} = 1$ $\quad \dfrac{-x}{x} = -1$

$\lim_{x \to 0^-} = 1 \neq -1 = \lim_{x \to 0^+}$

∴ Discontinous ↳ non-removable.

(b) $f(x) = \dfrac{x^2 + 3x}{x + 3}$ → $x \neq -3$

$\dfrac{x\cancel{(x+3)}}{\cancel{(x+3)}} = x$

$f(-3) = -3$ ∴ Discontinous ↳ is removable.

(c) $f(x) = \dfrac{x-2}{|x|-2}$

$x \neq -2, 2.$

$\dfrac{x-2}{x-2} = 1$

$\lim\limits_{x \to -2} \dfrac{x-2}{-x-2} = \dfrac{-4}{0} = -\infty$

$f(-2)$ doesnot exist

$\therefore$ Discontinous by non-removable.

36. (a) $f(x) = \dfrac{x^2-4}{x^3-8}$

$x \neq 2$

$$\frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(x^2+2x+4)}$$

$$\lim_{x \to 2} \frac{x+2}{x^2+2x+4} = \frac{4}{12} = \frac{1}{3}$$

$f(x)$ gets defined for $x = 2$.

$\therefore$ Discontinous by is removable.

(b) $f(x) = \begin{cases} 2x - 3, & x \leq 2 \\ x^2, & x > 2 \end{cases}$

$$\lim_{x \to 2^-} f(x) = 1$$

$1 \neq 4$

$\therefore$ Discontinous at $x = 2$

and non removable.

$$\lim_{x \to 2^+} f(x) = 4$$

(c) $f(x) = \begin{cases} 3x^2 + 5, & x \neq 1 \\ 6, & x = 1 \end{cases}$

$$\lim_{x \to 1} f(x) = 3(1)^2 + 5 \Rightarrow 8$$

$8 \neq 6$

$$\lim_{x \to 1} f(x) = 6$$

$\therefore$ Discontinous at $x = 1$

and non removable.