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Error And Analysis :-

True Value = approximation + Error,

Error = True value − approximation

Methods of Measuring Error.-

absolute error :- $|P - P^*|$

P = true value

P^* = approximate value.

$P = 2.5$ $P^* = 2.6$ $E_a = 0.1$ Ex

$P = 0.0005$ $P^* = 0.00049$ $E_a = 0.000001$

$P = 50,000$ $P^* = 50,001$ $E_a = 1$

relative error :-

$$\frac{|P - P^*|}{|P|}$$

Ex

$E_r = 0.1$

$E_r = 0.02$

$E_r = 2 \times 10^{-4}$

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Roundoff and chopping :-

2.635 | 9

Round off = 2.636

Chopping = 2.635

$\pi = \frac{22}{7}$

$\pi = 0.314159265... \times 10^1$

Accuray and Precision :-

Accuracy :- - refers to how closely a computed or measured value agrees with the true value.

~~Prece~~

Precision :- refers to how closely indivisual computed or measured value(s) agree with each other.

$$f(x_0) + \frac{f'(x_0)(x-x_0)^1}{1!} + \frac{f''(x_0)(x-x_0)^2}{2!} + \frac{f^{(k)}(x_0)(x-x_0)^k}{k!}$$

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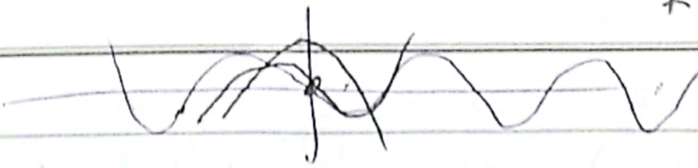

Truncation Error :-

(Remainder)

$$f(x) = P_n(x) + \boxed{R_n(x)} \leftarrow$$ Refer notes

where;

$$P_n(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

$$= \sum_{k=0}^{n} \frac{f^{(k)}(x_0)(x-x_0)^k}{k!}$$

$R_n(x)$

$f(x) = e^x$ $\quad f'(x) = e^x \quad f''(x) = e^x$

$f(x_0 = 0) = e^0 = 1$ $\quad x_0 = 0 \leftarrow$ Macclorence Series

$f'(0) = 1$

$$e^x = 1 + 1(x-0) + \frac{1(x-0)^2}{2!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \dots$$

$$\frac{f'''(x_0)}{3!}(x-x_0)^3$$

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$f(x) = \sin x$ $\quad f'(x) = \cos x$ $\quad f''(x) = -\sin x$

$f(0) = 0$ $\quad f'(x) = 1$ $\quad f''(x) = 0$

$f'''(x) = -\cos x$

$f'''(0) = -1$

$$\sin x = 0 + 1(x-0) + \frac{0}{2!}(x-0)^2 + \frac{-1}{3!}(x-0)^3 + \text{———}$$

$$\sin x = x - \frac{(x)^3}{3!} + \text{———}$$

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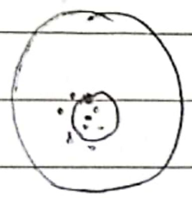
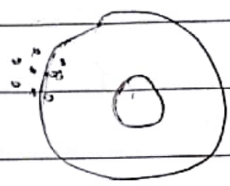
Accuracy

$x = 2$

Precission





Eg :-

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\boxed{e^{0.5} = 1.648721271}$$

$\rightarrow$ 1 term $= e^x = 1 \Rightarrow \boxed{e^{0.5} = 1}$

$\rightarrow$ 2 term $= e^x = 1 + x \Rightarrow e^{0.5} = 1 + 0.5 = 1.5$

$\rightarrow$ 3 term $= e^x = 1 + x + \frac{x^2}{2!}$ $\quad e^{0.5} = 1 + \frac{(0.5)^2}{2!} = 1.625$

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Terms	Result (Approximation)	absolute error $\|P-P^*\|$ P = true value P^* = Approx val	relative error $\frac{\|P-P^*\|}{\|P\|}$
1	1	$\|1.6487-1\| = 0.6487$	$\frac{\|1.6487-1\|}{\|1.6487\|} = 0.3934$
2	1.5	$\|1.6487-1.5\| = 0.1487$	$\frac{\|1.6487-1.5\|}{\|1.6487\|} = 0.0901$
3	1.625	$\|1.6487-1.625\| = 0.0237$	$\frac{\|1.6487-1.625\|}{\|1.6487\|} = 0.0144$

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Significant figures :-

1) 324.65 ← 5

2) 3002 ← 4
300.002 ← 6.

3) 0.54 ← 2
0.0032 ← 2

4) 92.00 ← 4

5) 540. ← 3

6) 540 ← 2

$f(x_0)$

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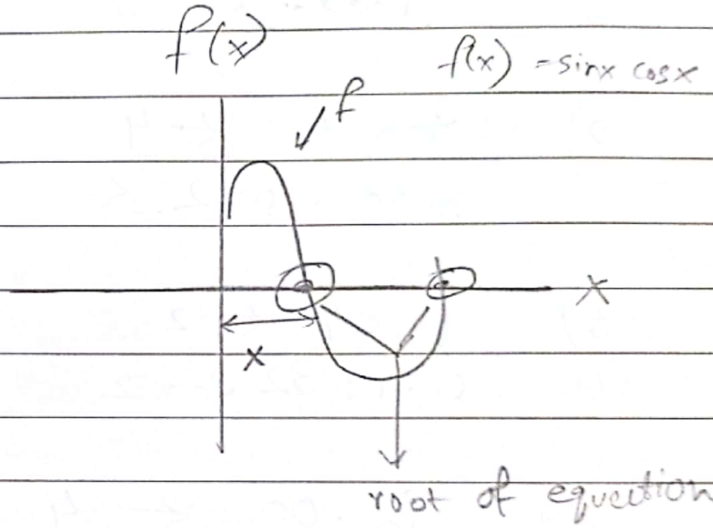
$f(x) = e^x$ $x_0 = 0$

Root Finding methods of Single variable Equation



1) Bracketing method:-

a) Bisection method
b) False position method

2) Open Methods:-

a) Newton Raphson

b) fixed Point Iteration
c) Secant method.

Bisection Method:-

Intermediate Value theorem:

If $f \in C[a,b]$ and K is any number b/w $f(a)$ and $f(b)$ then there exist a number c in (a,b) for which $f(c) = K$.

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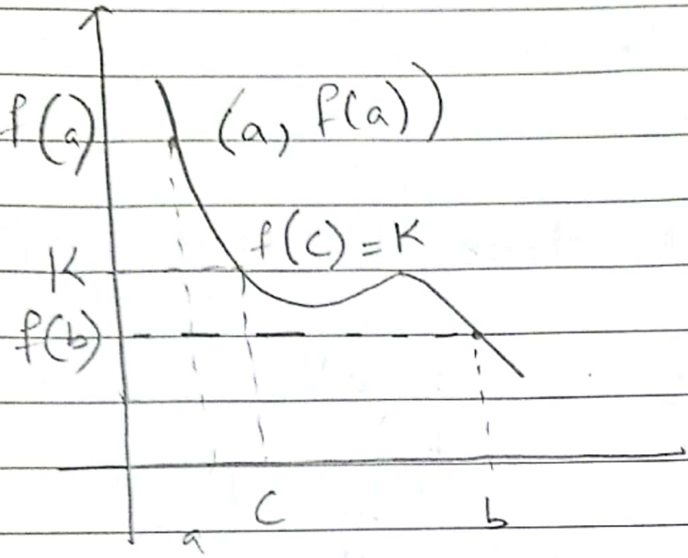
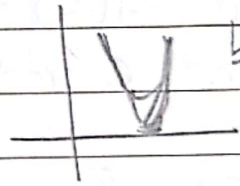


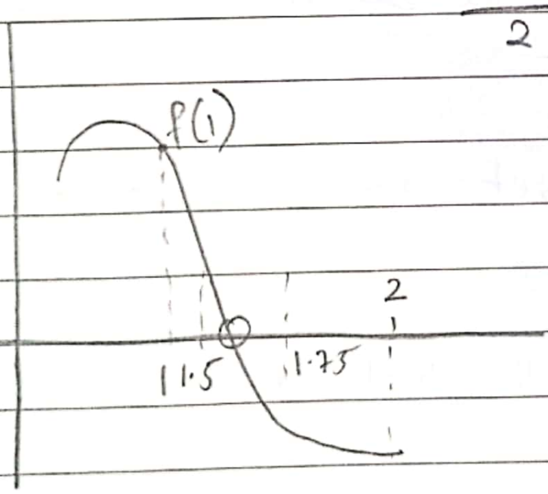
Bisection method:-



bisection method will not be applied

$$c = \frac{a+b}{2}$$

(1,2)
f(1) > 0
f(2) < 0
(1.5, 2)
(1.5, 1.75)



$$c = \frac{1+2}{2} = 1.5$$

$$c = \frac{1.5+2}{2} = 1.75$$

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Algorithm for bisection method:-

(1) Set the interval [a,b] and choose tol. level (tolerence level)

(2) Compute mid pt $c = \frac{a+b}{2}$

(3) Evaluate $f(c)$

(4) if $f(c) = 0$ OR reached tolerence level then c = root

(5) If $f(a) * f(c) < 0$ set $b = c$ and go to Step 2.

(6) if $f(b) * f(c) < 0$ set $a = c$ and go to Step 2

(7) Repeat the steps until the sol is in dessired tol.

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Stopping Criteria :-

① $|P_N - P_{N-1}| < \varepsilon$

② $\dfrac{|P_N - P_{N-1}|}{|P_N|} < \varepsilon$

③ $|f(P_N)| < \varepsilon$

Formula for obtaining no. of it iteration.

$$\frac{|b-a|}{2^n} = \text{absolute error}$$

$$n = \log_2 \left(\frac{|b-a|}{\text{absolute error}} \right)$$

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False Position Method :-

$$c = a - \frac{f(a)(b-a)}{f(b)-f(a)}$$

whole process is of bisection method will be followed except calculation of c by the given formula.

Derivation:-

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$(x_1, y_1) = (a, f(a))$

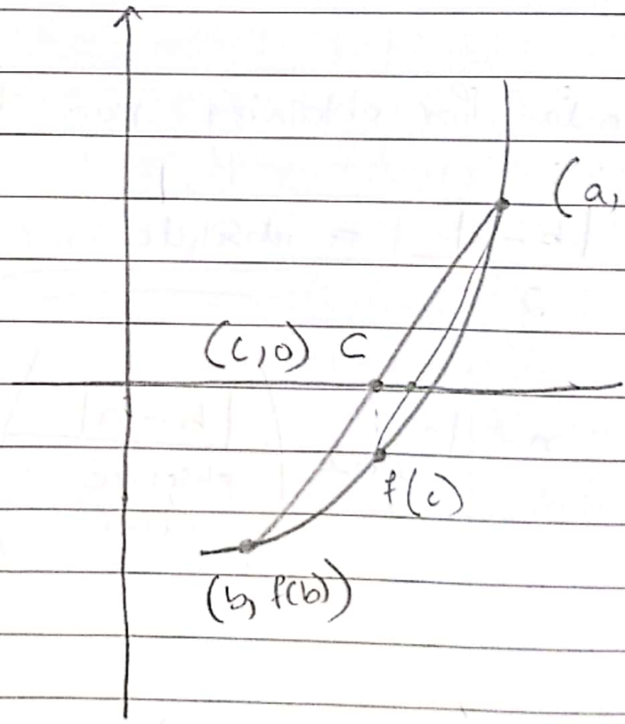
$(x_2, y_2) = (b, f(a))$

$$\frac{y-f(a)}{f(b)-f(a)} = \frac{x-a}{b-a}$$

$(x, y) = (c, 0)$

$$\frac{0-f(a)}{f(b)-f(a)} = \frac{c-a}{b-a}$$

$$\boxed{c = a - \frac{f(a)(b-a)}{f(b)-f(a)}}$$



for book reference

$x_u = a$

$x_L = b$

$x_r = c$

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Find root of $f(x) = x^3 + 4x^2 - 10 = 0$ b/w [1,2] by bisection and false position method upto the accuracy of 10^{-4} digits.

$x_t = 1.365234375$

n	Bisection				$x_t - c$	False Proposition			
	a	b	$c = \frac{a+b}{2}$	absolute error	absolute error	a	b	$c = a - \frac{f(a)(b-a)}{f(b)-f(a)}$	absolute error
	1	2	1.5						

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Iteration no 1

Bisection	False position
$a=1$	$a=1$
$b=2$	$b=2$
$f(1)=-5$	$f(1)=-5$
$f(2)=14$	$f(2)=14$
$c=\frac{1+2}{2}=1.5$	$c=1-\frac{f(1)(2-1)}{f(2)-f(1)}$
	$=1.263157895$
$f(1.5)=2.375$	$f(1.263157895)$
	$=-1.60227$

Iteration no.2

Bisection	False position
$a=1$	$a=1.263157895$
$b=1.5$	$b=2$
$f(a)=f(1)=-5$	$f(1.263157895)=-1.60227$
$f(1.5)=2.375$	$f(2)=14$
$c=\frac{1+1.5}{2}=1.25$	$c=1.26315-\frac{f(1.26315)(2-1.26315)}{f(2)-f(1.26315)}$
$f(c)=-1.795$	$c=1.338827653$