SIEMENS

Problem 1

(a) $\frac{\delta u^3}{\delta x^3} + \frac{\delta^2 u}{\delta y^2} + \left(\frac{\delta u}{\delta z}\right)^2 + ux^3 + uy^2 + uz = 0$

Partial differential equation ; non linear ∴ $\left(\frac{\delta u}{\delta z}\right)^2$

order = 3 ; Degree = 1

(b) $\left(\frac{dy}{dx}\right)^2 = \left(\frac{d^2y}{dx^2} + y\right)^{3/2}$; $(y')^4 = (y' + y)^3$

Ordinary differential equation ; non linear ∴ more than 1 power.

order = 2 ; Degree = 3

Problem 2.

(a) $y'' - y' - 12y = 0$; $y(0) = -2$, $y'(0) = 6$

$y = c_1 e^{4x} + c_2 e^{-3x}$ $\qquad -2 = c_1 + c_2$ — (1)

$y' = 4c_1 e^{4x} - 3c_2 e^{-3x}$ $\qquad 6 = 4c_1 - 3c_2$ — (2)

$y'' = 16c_1 e^{4x} + 9c_2 e^{-3x}$

$16e^{4x}c_1 + 9c_2e^{-3x} - 4c_1e^{4x} + 3c_2e^{-3x} - 12c_1e^{4x} - 12c_2e^{-3x} = 0$

$12e^{4x}c_1 + 12\cancel{c_2e^{-3x}} - 12e^{4x}c_1 - 12\cancel{c_2e^{-3x}} = 0$

$0 = 0$ ||proved||

Simultaneously solving eq (1) and (2).

$6 = 4(-2 - c_2) - 3c_2$ $\qquad c_1 = -2 - c_2 = -2 + 2 \Rightarrow 0$

$6 = -8 - 4c_2 - 3c_2$

$14 = -7c_2$ $\qquad c_2 = -2$ & $c_1 = 0$

$\Rightarrow y = -2e^{-3x}$ (Ans).

$y(2) = 2$
$y'(2) = 2$
$y''(2) = 6$

(b) $x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0$

$y = c_1 x + c_2 x^2 + c_3 x^3$
$y' = c_1 + 2c_2 x + 3c_3 x^2$
$y'' = 2c_2 + 6c_3 x$
$y''' = 6c_3$

$2 = 2c_1 + 4c_2 + 8c_3$ — (1)
$2 = c_1 + 4c_2 + 12c_3$ — (2)
$6 = 2c_2 + 12c_3$; $3 = c_2 + 6c_3$ — (3)
$c_2 = 3 - 6c_3$ — (4)

$6c_3x^3 - 3x^2(2c_2 + 6c_3x) + 6x(c_1 + 2c_2x + 3c_3x^2) - 6(c_1x + c_2x^2 + c_3x^3) = 0$
$\cancel{6c_3x^3} - 6c_2x^2 - \cancel{18c_3x^3} + 6c_1x + 12c_2x^2 + \cancel{18c_3x^3} - 6c_1x - 6c_2x^2 - \cancel{6c_3x^3} = 0$
$-12c_2x^2 + 12c_2x^2 + 6c_1x - 6c_1x = 0$
$0 = 0$ //proved//

Solving simultaneously

$2 = c_1 + 12 - 24c_3 + 12c_3$
$-10 = c_1 - 12c_3$
$c_1 = -10 + 12c_3$

$2 = 2(-10 + 12c_3) + 4(3 - 6c_3) + 8c_3$
$2 = -20 + \cancel{24c_3} + 12 - \cancel{24c_3} + 8c_3$
$10 = 8c_3$
$c_3 = 10/8$
$c_3 = 5/4$

$c_1 = -10 + 12(5/4)$
$c_1 = -10 + 15$
$c_1 = 5$

$c_2 = 3 - 6c_3$
$c_2 = 3 - 6(5/4)$
$c_2 = 3 - 15/2$
$c_2 = \frac{6 - 15}{2}$
$c_2 = -\frac{9}{2}$

$\Rightarrow y = 5x - \frac{9}{2}x^2 + \frac{5}{4}x^3$ (Ans).

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(c) $y'' + y = 0$ $\qquad y(0) = 1 \qquad y'(\pi/2) = -1$

$y = c_1 \sin x + c_2 \cos x$ $\qquad 1 = c_2 \qquad c_2 = 1$

$y' = c_1 \cos x - c_2 \sin x$ $\qquad -1 = c_1 \cos(\pi/2) - c_2 \sin(\pi/2)$

$y'' = -c_1 \sin x - c_2 \cos x$ $\qquad c_2 = 1$

$-c_1 \sin x - c_2 \cos x + c_1 \sin x + c_2 \cos x = 0$

$0 = 0$ //proved//

$y = \cos x$

Q3.

(a) $x^3 + y^3 = 3cxy$ – (1) $\qquad$ 1 arbitary constant.

$3x^2 + 3y^2 \frac{dy}{dx} = 3c\left[y + x \frac{dy}{dx} \right]$ – (2)

$\frac{3x^2 + 3y^2 y'}{y + xy'} = 3c$

putting in eq (1)

$\Rightarrow x^3 + y^3 = \frac{y + xy'}{3x + 3yy'}$ (Ans).

$x^3 + y^3 = \frac{3x^2 + 3y^2 y'}{y + xy'} xy$

$x^3 y + x^4 y' + y^4 + y^3 x y' = 3x^3 y + 3xy^3 y'$

$x^4 y' - 3xy^3 y' = 3x^3 y - y^4 - x^3 y$

$y' = \frac{2x^3 y - y^4}{x^4 - 3xy^3}$

(b) $3y = \frac{4x^3}{x^2+1} + \frac{3c}{x^2+1}$ – (1) $\qquad$ 1 arbitary constant

$3y(x^2+1) = 4x^3 + 3c$

$\frac{d}{dx}(3yx^2 + 3y) = \frac{d}{dx}(4x^3 + 3c)$

$6xy + 3x^2 y' + 3y' = 12x^2$

$2xy + x^2 y' + y' = 4x^2$

$y'(x^2+1) = 4x^2 - 2xy$

$\Rightarrow y' = \frac{4x^2 - 2xy}{x^2 + 1}$ (Ans).

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Q4.

(a) $(xy + 2x + y + 2)\,dx + (x^2 + 2x)\,dy = 0$

$$(x(y+2) + 1(y+2))\,dx = -(x^2+2x)\,dy$$

$$(y+2)(x+1)\,dx = -(x^2+2x)\,dy$$

$$\int \frac{x+1}{x^2+2x}\,dx = -\int \frac{1}{y+2}\,dy$$

$$\frac{x+1}{x^2+2x} = \frac{A}{x} + \frac{B}{x+2} \qquad A = 1/2 \qquad B = 1/2$$

$$\int \frac{1/2}{x}\,dx + \int \frac{1/2}{x+2}\,dx = -\int \frac{1}{y+2}\cdot dy$$

$$\frac{1}{2}\ln x + \frac{1}{2}\ln|x+2| + \ln c = -\ln|y+2|$$

$$\ln|y+2| = -\frac{1}{2}\ln x - \frac{1}{2}\ln|x+2| - \ln c$$

$$\ln|y+2| = \ln(x^{-1/2}\cdot(x+2)^{-1/2}) + \ln c$$

$$\Rightarrow y = (x(x+2))^{-1/2} + c - 2 \qquad \text{(Ans)}.$$

(b) $\dfrac{dy}{dx} + \dfrac{y}{x\ln x} = \dfrac{3x^2}{\ln x}$ Linear.

$$P = \frac{1}{x\ln x} \qquad Q = \frac{3x^2}{\ln x} \qquad IF = e^{\int + \frac{1}{x\ln x}} = e^{+\ln(\ln x)}$$

$$= \ln x$$

$$y \times \ln x = \int \frac{3x^2}{\ln x} \times \ln x + c$$

$$\Rightarrow y = \frac{x^3}{\ln x} + \frac{c}{\ln x} \qquad \text{(Ans)}.$$

(c) $e^x(y - 3(e^x+1)^2)\,dx + (e^x + x)\,dy = 0 \qquad y(0) = 4$

$$M_y = N_x$$

$$e^x + 0 = e^x \quad \therefore \text{ exact}.$$

$$c = \int N\,dy + h(x) \qquad h'(x) = M - \frac{\delta}{\delta x}\int N\,.\,dy$$

$$c = \int(e^x+1)\,dy + h(x) \qquad = e^x(y - 3(e^x+1)^2) - ye^x$$

$$c = e^x y + y + h(x) \qquad = e^x(-3(e^x+1)^2)$$

$$= -3e^x(e^x+1)^2$$

$$h(x) = \int h'(x)\,dx$$

$$= \int -(3e^{x})(e^{x}+1)^2\,dx$$

$$= -3\int e^{x}(e^{2x} + 2e^{x} + 1)$$

$$= -3 \int e^{3x} + 2e^{2x} + e^{x}$$

$$= -3\left(\frac{e^{3x}}{3} + e^{2x} + e^{x}\right)$$

$$= -e^{3x} - 3e^{2x} - 3e^{x}$$

$$h(x) = -(e^{x}+1)^3$$

$c = e^{x}y + y - (e^{x}+1)^3$ applying IVP $y(0) = 4$

$$c = e^{0}(4) + 4 - (e^{0}+1)^3$$

$$c = 4 + 4 - 8$$

$$c = 0$$

$$0 = e^{x}y + y - (e^{x}+1)^3$$

$$y\cancel{(e^{x}+1)} = (e^{x}+1)^{\cancel{3}\,2}$$

$\Rightarrow$ $y = (e^{x}+1)^2$ (Ans).