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Class : BCS 1-E

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1. A displacement vector in the *xy* plane is 7.3 m long and directed at angle of 30° in Fig.1. Determine (a) the *x* component and (b) the *y* component of the vector.

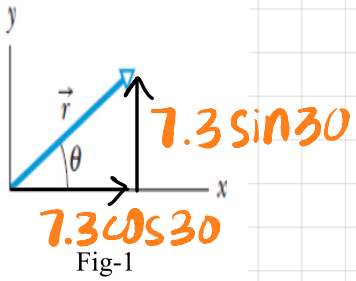


Fig-1

a. $R_x = 7.3 \cos 30$

$R_x = 6.32m$

b. $R_y = 7.3 \sin 30$

$R_y = 3.65m$

2. The two vectors ***a*** and ***b*** in Fig-2 have equal magnitudes of 10m and the angles are θ1= 30 and θ1= 105Find the (a) x and y components of their vector sum ***r*** (b) the magnitude of ***r*** and (c) the angle ***r*** makes with the positive direction of the x axis

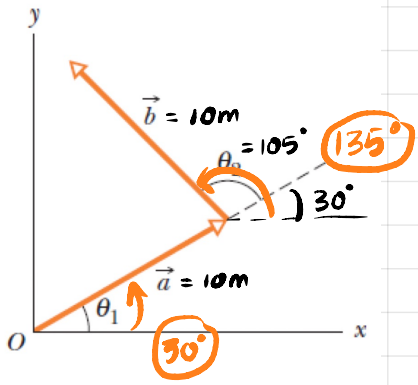


Fig-2

a. $a_x = 10 \cos 30$ $\quad a_y = 10 \sin 30$

$b_x = 10 \cos 135$ $\quad b_y = 10 \sin 135$

$r_x = 10\cos 30 - 10\cos 135 \Rightarrow 1.59\,m$

$r_y = 10\cos 135 + 10\sin 135 \Rightarrow 12.07m$

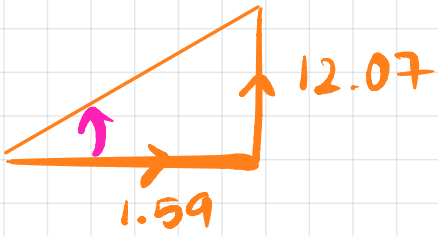
$\vec{r} = 1.59i + 12.07j$

b. $|\vec{r}| = \sqrt{(r_x)^2 + (r_y)^2}$

$|\vec{r}| = \sqrt{(1.59)^2 + (12.07)^2}$

$|\vec{r}| = 12.17m$

c.



$$\tan\theta = \frac{r_y}{r_x}$$

$$\theta = \tan^{-1}\left(\frac{12.07}{1.59}\right)$$

$$\theta = 82.5^\circ$$

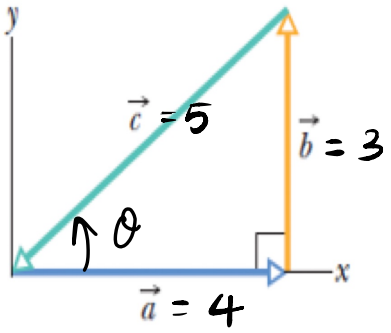
3. For the vectors in Fig. 3, with a = 4, b = 3, and c = 5, what are (a) the magnitude and the direction of ***a*** x ***b*** , (b) the magnitude and) the direction of ***a*** x ***c*** , and (c) the magnitude and the direction of ***b*** x ***c*** ?

a $a \times b = ab\sin\theta$

$= 4 \times 3 \times \sin 90$

$= 12$

magnitude = 12



b. $a \times c = ac\sin\theta$

$= 4 \times 5 \times \sin 36.9$

magnitude = 12

$A^2 = B^2 + C^2 - 2BC\cos\theta$

$3^2 = 5^2 + 4^2 - 2(5)(4)\cos\theta$

$$\theta = \cos^{-1}\left(\frac{5^2 + 4^2 - 3^2}{2(5)(4)}\right)$$

$\theta = 36.9^\circ$

The direction is out of the page by the use of right hand rule ∴ direction is positive z axis.

$\theta = \cos^{-1}(3/5)$

$\theta = 53.1^\circ$

c. $b \times c = bc\sin\theta$

$= 5 \times 3 \times \sin 53.1$

$= 12$

Direction is out of the page ∴ positive z axis.

4. By Considering the above problem -2 find the (a) $a \cdot b$ (b) $a \times b$ (c) angle between a and b

$a = 8.66i + 5j \qquad b = -7.07i + 7.07j$

a. $a \cdot b = ab\cos\theta$

$= (8.66)(-7.07) + (5)(7.07)$

$= -61.22 + 35.35$

$= -25.9$

b. $a \times b = ab\sin\theta$

$$\begin{vmatrix} i & j & K \\ 8.66 & 5 & 0 \\ -7.07 & 7.07 & 0 \end{vmatrix}$$

$$i\begin{vmatrix} 5 & 0 \\ 7.07 & 0 \end{vmatrix} - j\begin{vmatrix} 8.66 & 0 \\ -7.07 & 0 \end{vmatrix} + K\begin{vmatrix} 8.66 & 5 \\ -7.07 & 7.07 \end{vmatrix}$$

$i(0) - j(0) + K(96.6)$

$= 96.6\,K$

c. $a \cdot b = ab\cos\theta$

$|a| = \sqrt{8.66^2 + 5^2} = 10$

$|b| = \sqrt{-7.07^2 + 7.07^2} = 10$

$\theta = \cos^{-1}\left(\frac{a \cdot b}{ab}\right)$

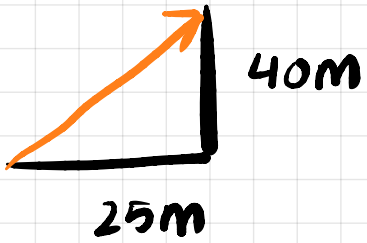
$\theta = \cos^{-1}\left(\frac{-25.9}{100}\right)$

$\theta = 105^\circ$

5. The x component of vector A is 25.0 m and the y component is 40.0 m. (a) What is the magnitude of A (b) What is the angle between the direction of and the positive direction of x?

a. $|A| = \sqrt{25^2 + 40^2}$

$= 47.2\text{m}$



b. $\theta = \tan^{-1}(A_y/A_x)$

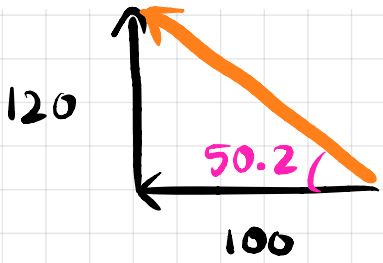
$= \tan^{-1}(40/25)$

$= 50^\circ$

6. A ship sets out to sail to a point 120 km due north. An unexpected storm blows the ship to a point 100 km due east of its starting point. (a) How far and (b) in what direction must it now sail to reach its original destination?

a. $\sqrt{H^2} = \sqrt{100^2 + 120^2}$

$H = 156.2\text{ km}$

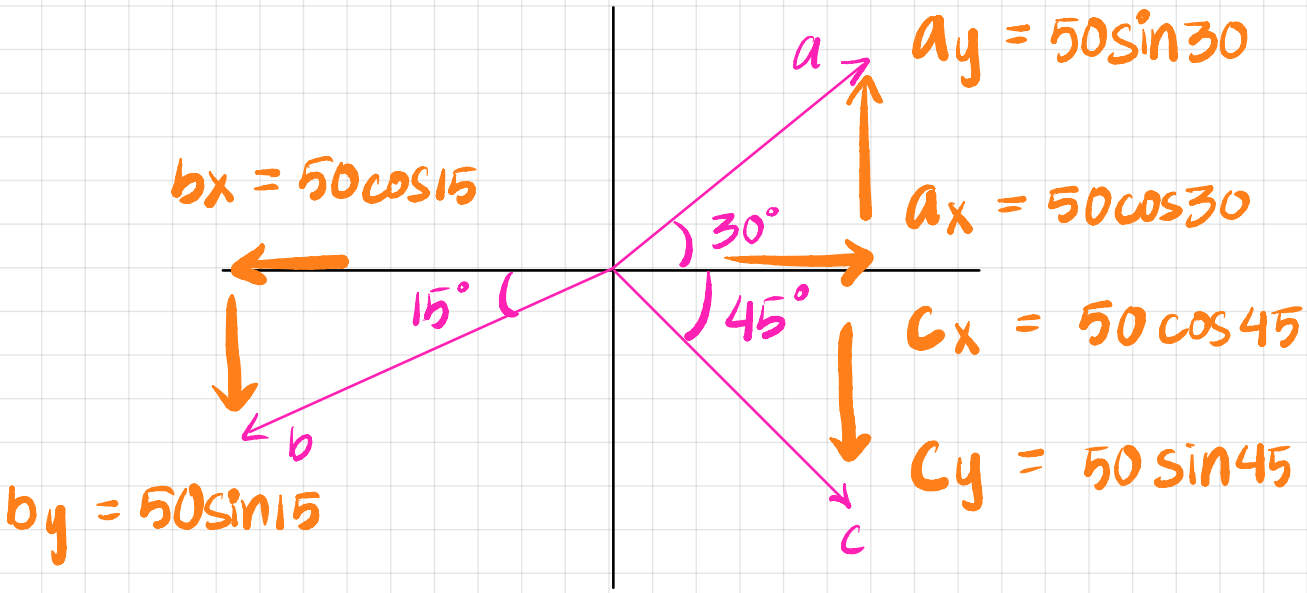


b. $\theta = \tan^{-1}(120/100)$

$\theta = 50.2$

$90 - 50.2 = 39.8$ west of due north.

7. Three vectors ***a*** , ***b*** and ***c*** each have a magnitude of 50 m and lie in an xy plane. Their directions relative to the positive direction of the x axis are 30°, 195°, and 315°, respectively. What are (i) the magnitude and the angle of the vector ***a+b+c*** , and (ii) the magnitude and the angle of ***a-b+c***? What are the (iii) magnitude and angle of a fourth vector ***d*** such that ***(a+b) – (c+d) = 0*** ?



a. $(a+b+c)_x = 50\cos 30 + 50\cos 45 - 50\cos 15$

$= 30.36\text{m}$ (towards right)

$(a+b+c)_y = 50\sin 15 + 50\sin 45 - 50\sin 30$

$= 23.3\text{ m}$ (towards bottom)

$= 30.36\,i - 23.3\,j$

$\sqrt{(30.36)^2 + (23.3)^2}$

$= 38.3\text{m}$

$\theta = \tan^{-1}\left(\frac{23.30}{30.36}\right) = -37.5$

b. $\vec{a} - \vec{b} + \vec{c}$

X: $50\cos 30 + 50\cos 45 + 50\cos 15 = 127$ (right)

y: $50\sin 30 + 50\sin 15 - 50\sin 45 = 2.59$ (top)

$= 1.27i + 2.59j$

$\sqrt{127^2 + 2.59^2} = 127.03\,m$

$\tan^{-1}\left(\frac{2.59}{127}\right) = 1.17°$

c. $a + b$ X: $50\cos 30 - 50\cos 15 = 4.99$ (Left)

y: $50\sin 30 - 50\sin 15 = 12.06$ (Top)

$-4.99i + 12.06j = (50\cos 45 + dx)i - (50\sin 45 + dy)j$

$dx = -40.35$ $dy = 47.4$

$|d| = \sqrt{(40.35)^2 + (47.4)^2} = 62.2\,m$

$\theta = \tan^{-1}\left(\frac{47.40}{40.35}\right) = 49.6$

Direction is 130° with positive axis.

$180 - 49.6 \simeq 130$

8. Find the angle betwenn the vector A= 2i -3j+5k and the x, y,and z axes, respectively.

With x axis :

$$\cos\theta = \frac{A \cdot i}{|A|}$$

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{30}}\right)$$

$$\theta = 71.07^\circ$$

With y axis

$$\cos\theta = \frac{A \cdot j}{|A|}$$

$$\theta = \cos^{-1}\left(\frac{-3}{\sqrt{30}}\right)$$

$$\theta = 119.2^\circ$$

With z axis :

$$\cos\theta = \frac{A \cdot z}{|A|}$$

$$\theta = \cos^{-1}\left(\frac{5}{\sqrt{30}}\right)$$

$$\theta = 35.8$$

9. Calculate the angle between "r" and the positive z-axis. (c) Find the angle between "a" and "b". where a = 5i +4j -6k , b= -2i +2j+3k and c = 4i+3j+2k , r = a+b+c.

$$\vec{r} = (5-2+4)i + (4+2+3)j + (-6+3+2)k$$

$$\vec{r} = 7i + 9j - k$$

$$|r| = \sqrt{7^2+9^2+1^2} = \sqrt{131}$$

$$\cos\theta = \frac{r \cdot z}{|r|}$$

$$\theta = \cos^{-1}\left(\frac{-1}{\sqrt{131}}\right)$$

$$\theta = 95.01$$

c.

$$|a| = \sqrt{5^2+4^2+6^2} = \sqrt{77} \qquad |b| = \sqrt{2^2+2^2+3^2} = \sqrt{17}$$

$$a \cdot b = \begin{bmatrix} 5 \\ 4 \\ -6 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix} = -10 + 8 - 18 = -20$$

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$\theta = \cos^{-1}\left(-20/\sqrt{17}\sqrt{17}\right)$$

$$\theta = 123.6$$

10. Vector A has a magnitude of 6 units, vector B has a magnitude of 7 units, and A.B has a value of 14. What is the angle between the direction of A and B?

$|A| = 6 \qquad |B| = 7$

$A.B = 14$

$A.B = AB\cos\theta$

$14 = 6 \times 7 \times \cos\theta$

$\theta = \cos^{-1}(1/3)$

$\theta = 70.53$