21k-3278

Q1. $E(x) = \sum x\, f(x)$

$E(x) = -5000(0.2) + 10000(0.5) + 30000(0.3)$

$\boxed{E(X) = 13000}$ (Ans)

Q2. **Mean (μ)**

$E(X) = \sum_{i=1}^{4} x\, f(x)$

$= 1\left(\frac{5-1}{10}\right) + 2\left(\frac{5-2}{10}\right) + 3\left(\frac{5-3}{10}\right) + 4\left(\frac{5-4}{10}\right)$

$\Rightarrow \boxed{2}$ (Ans)

Variance (σ^2)

$\sigma^2 = E(x^2) - \mu^2$

$= \sum_{i=1}^{4} x^2 f(x) - (2)^2$

$\Rightarrow 5 - 4 \Rightarrow \boxed{1}$ (Ans)

Q3.

Mean (μ)

$E(X) = \sum x\, f(x)$

$E(X) = 20(0.2) + 21(0.4) + 22(0.2) + 23(0.1) + 24(0.1)$

$E(X) = \boxed{21.5}$ $\rightarrow \boxed{21.5}$ (Ans)

Standard deviation (σ)

$\sigma = \sqrt{E(x^2) - \mu^2}$

$E(x^2) = \sum x^2 f(x)$

$= 20^2(0.2) + (21)^2(0.4) + 22^2(0.2) + 23^2(0.1) + 24^2(0.1)$

$= 463.7$

$\sigma = \sqrt{463.7 - (21.5)^2}$

$\sigma = \boxed{\cancel{\sqrt{2.396}}}$ (Ans)

$\boxed{1.204}$

Q4. 21k-3298

(i) $E(X) = \sum x\, f(x)$

$E(X) = (0\times0.5) + (10\times0.25) + (20\times0.15) + (30\times0.10)$

$E(X) = \boxed{8.5}$ (Ans)

(ii) → More data lies on left side of median (right skewed).

(iii) → $\sigma^2 = E(X^2) - \mu^2$

$E(x^2) = \sum x^2 f(x)$

$= (0^2\times0.5) + (10^2\times0.25) + (20^2\times0.15) + (30^2\times0.10)$

$= 175$

$\sigma = \sqrt{E(X^2) - \mu^2}$

$\sigma = \sqrt{175 - (8.5)^2}$

$\sigma = \boxed{10.137}$ (Ans)

(iv) $P(X \geq 20)$

$= f(20) + f(30)$

$= 0.15 + 0.10$

$\Rightarrow \boxed{0.25}$ (Ans).

Q5.

Marginal prob of X

(i) $g(x) = \int_0^\infty x e^{-x(1+y)}\, dy$

$x e^{-x} \int_0^\infty e^{[redacted]-xy}\, dy$

$x e^{-x} \left[\frac{e^{-xy}}{-x} \right]_0^\infty$

$+ e^{-x}(e^0)$

Note: we dont put infinity when trying limits (only real values).

$\boxed{g(x) = e^{-x} \quad x \geq 0}$

Marginal prob of Y

(ii) $h(y) = \int_0^\infty x e^{-x(1+y)}\, dx$ ~~$x e^{-x} \int_0^\infty e^{-xy}$~~ ~~$u = x(1+y)$~~

$\left[\frac{x e^{-x(1+y)}}{-(1+y)} - \frac{e^{-x(1+y)}}{(1+y)^2} \right]_0^\infty$

$+\ x$	$e^{-x(1+y)}$
$-\ 1$	$\frac{e^{-x(1+y)}}{-(1+y)}$
0	$\frac{e^{-x(1+y)}}{(1+y)(1+y)}$

$-\left(0 - \frac{1}{(1+y)^2}\right) \Rightarrow \frac{1}{(1+y)^2} \quad Y \geq 0.$

$P(X \leq 1$ and $Y \leq 1)$

$\boxed{h(y) = 1/(1+y)^2 \quad Y \geq 0}$

(iii) $\int_0^1 \int_0^1 x e^{-x(1+y)}\, dy\, dx = -\int_0^1 (e^{-2x} - e^{-x})\, dx = 0.1997 \leftarrow P(X \leq 1 \cap Y \leq 1)$

$P(X > 1 \cup Y > 1) = 1 - P(X \leq 1 \cap Y \leq 1).$

$= 1 - 0.1997 \Rightarrow$ ~~0.8662XXXXX~~,

$\boxed{0.8003}$

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Q6)

(i) $P(Y>X)$

$P(0,1) + P(0,2) + P(1,2)$

$0.05 + 0 + 0.05$

$= \boxed{0.1}$ (Ans)

(ii) $\text{cov}(X, Y)$

$E(XY) - E(X)E(Y)$

$\frac{\sum XY}{n} - \frac{\sum x}{n} \frac{\sum y}{n}$

$E(XY) = (1)(1)(0.15) + (1)(2)(0.05) + (2)(1)(0.05) + (2)(2)(\cancel{0.05}\ 0.1) + (3)(1)(0.05) + (3)(2)(0.05)$

$E(XY) = 1.2$

$E(XY) - E(X)E(Y)$

$1.2 - (1.2)(0.7)$

$\Rightarrow \boxed{0.36}$ (Ans)

$E(X) = \sum x\, g(x)$

$E(Y) = \sum y\, h(y)$

$E(XY) = \sum_{x=0}^{3} \sum_{y=0}^{2} xy\, f(x,y)$

$E(X) = 0 \times 0.2 + 1 \times 0.5 + 2 \times 0.2 + 3 \times 0.1$

$E(X) = 1.2$

$E(Y) = 0 \times 0.5 + 1 \times 0.3 + 2 \times 0.2 = 0.7$.

$E(Y) = 0.7$.

21K-3278

Q7. $n=5 \quad p=0.7 \quad q=0.3$

$x=0$: ${}^5C_0 (0.7)^0 (0.3)^5 = 2.43 \times 10^{-3}$

$x=1$: ${}^5C_1 (0.7)^1 (0.3)^4 = 0.02835$

$x=2$: ${}^5C_2 (0.7)^2 (0.3)^3 = 0.1323$

$x=3$: ${}^5C_3 (0.7)^3 (0.3)^2 = 0.3087$

Q8.

$\mu = 2$ $\mu = np$

$\sigma^2 = 1$ $\sigma^2 = npq$

$\sigma^2 = \mu q$

$\mu = np$ $1 = 2(q)$

$2 = n(0.5)$ $\underline{q = 0.5}$

$n = 4$ $p = 1-q$

$p = 1-0.5$

$P(X=2)$ $\underline{p = 0.5}$

$= {}^4C_2 (0.5)^2 (0.5)^2$

$= \boxed{0.375}$ (Ans)

Q9.

(i) ${}^7C_4 (0.2)^4 (0.8)^3 \Rightarrow \boxed{0.028672}$ (Ans)

(ii) $\mu = np = 7 \times 0.2 = 1.4$

$P(X < 1.4) = P(X=0) + P(X=1)$

$= {}^7C_0 (0.2)^0 (0.8)^7 + {}^7C_1 (0.2)^1 (0.8)^6$

$\Rightarrow \boxed{0.5767168}$ (Ans)

(iii) $P(X \geq 5) = P(X=5) + P(X=6) + P(X=7)$

$= {}^7C_5 (0.2)^5 (0.8)^2 + {}^7C_6 (0.2)^6 (0.8)^1 + {}^7C_7 (0.2)^7 (0.8)^0$

$= \boxed{4.672 \times 10^{-3}}$ (Ans)

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Q11.

$\cancel{n \to 7}$.

$\cancel{p \to 0.95}$.

$$\cancel{P(x/t) = \frac{e^{-\nu t} \cdot (\nu t)^x}{x!}}$$

$$\cancel{= e^{-(0.95 \times 2)} \cdot (0.95 \times 2)^7}$$

Trials are independent.

(i) $P(x=2) = {}^7C_2 (0.95)^2 (0.05)^5$

$\Rightarrow \boxed{0.00000592}$ (Ans)

(ii) $P(x \geq 5) = P(X=5) + P(X=6) + P(X=7)$

$= {}^7C_5 (0.95)^5 (0.05)^2 + {}^7C_6 (0.95)^6 (0.05)^1 + {}^7C_7 (0.95)^7 (0.05)^0$

$= \boxed{0.9962}$ (Ans)

(iii) $P(X=5) = {}^7C_5 (0.95)^5 (0.05)^2$

5 survived
2 died.

$\Rightarrow \boxed{0.0406}$ (Ans)

Q12. 21k-3278

(i) $P(X=3)$

$^{10}C_3\,(0.08)^3\,(0.92)^7$

$\Rightarrow \boxed{0.03427}$ (Ans)

(ii) $E(X) = np$

mean

$np = (10)(0.08)$

$= \boxed{0.8}$ (Ans)

(iii) $V(X) = npq$.

$= 10(0.08)(0.92)$

$= \boxed{0.736}$ (Ans)

Q13. $P(A) = 0.72$ $P(B) = 0.28$

$P(A_D) = 0.18$ $P(B_D) = 0.24$.

(i)

$P(A) \times P(A_D)$

0.72×0.18

$\Rightarrow \boxed{0.1296}$ (Ans)

(ii) $\{P(A) \times P(A_D)\} + \{P(B) \times P(B_D)\}$

$(0.72 \times 0.18) + (0.28 \times 0.24)$

$\Rightarrow \boxed{0.1968}$ (Ans)

(iii) $P(A \mid D)$

$$= \frac{P(A \cap D)}{P(D)}$$

$$= \frac{0.72 \times 0.18}{0.1968}.$$

$\Rightarrow \boxed{0.6585}$ (Ans)