HW3

Sohaib Syed

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Problem 3 Code

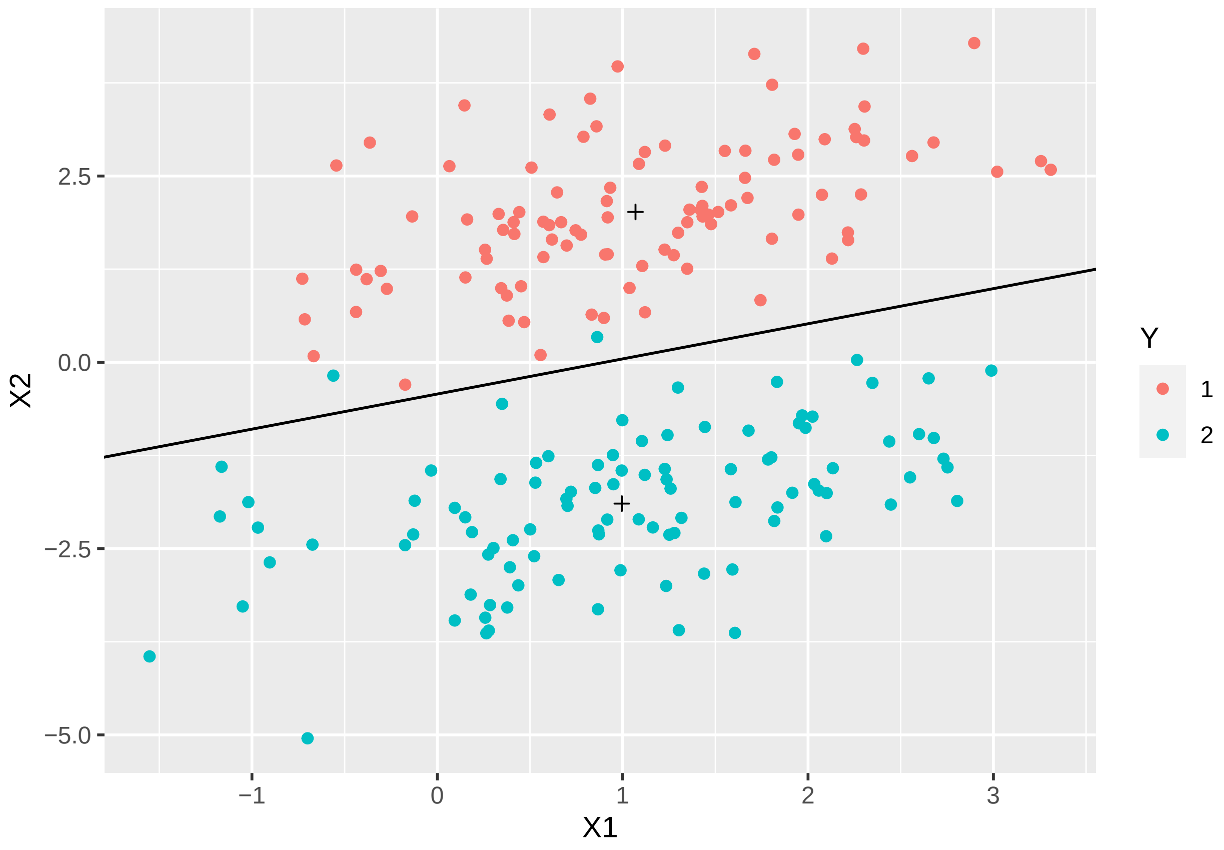
generating points and graph of points

```r
set.seed(1)
library(ggplot2)
library(MASS)
library(mvtnorm)


class1_samples=100
class2_samples=100
total_samples=class2_samples+class1_samples

var_covar = matrix(data = c(1, 0.5, 0.5, 1), nrow = 2)

# Random bivariate Gaussian samples for class +1
C1 <- rmvnorm(class1_samples, mean = c(1, 2), sigma = var_covar)

# Random bivariate Gaussian samples for class -1
C2 <- rmvnorm(class2_samples, mean = c(1, -2), sigma = var_covar)

# Samples for the dependent variable
Y_samples <- c(rep(1, class1_samples), rep(2, class2_samples))

# Combining the independent and dependent variables into a dataframe
dataset <- as.data.frame(cbind(rbind(C1, C2), Y_samples))
colnames(dataset) <- c("X1", "X2", "Y")
dataset$Y <- as.factor(dataset$Y)

# Plot the above samples and color by class labels
centroids <- aggregate(cbind(X1,X2)~Y,dataset,mean)
```

LDA computations Part 1

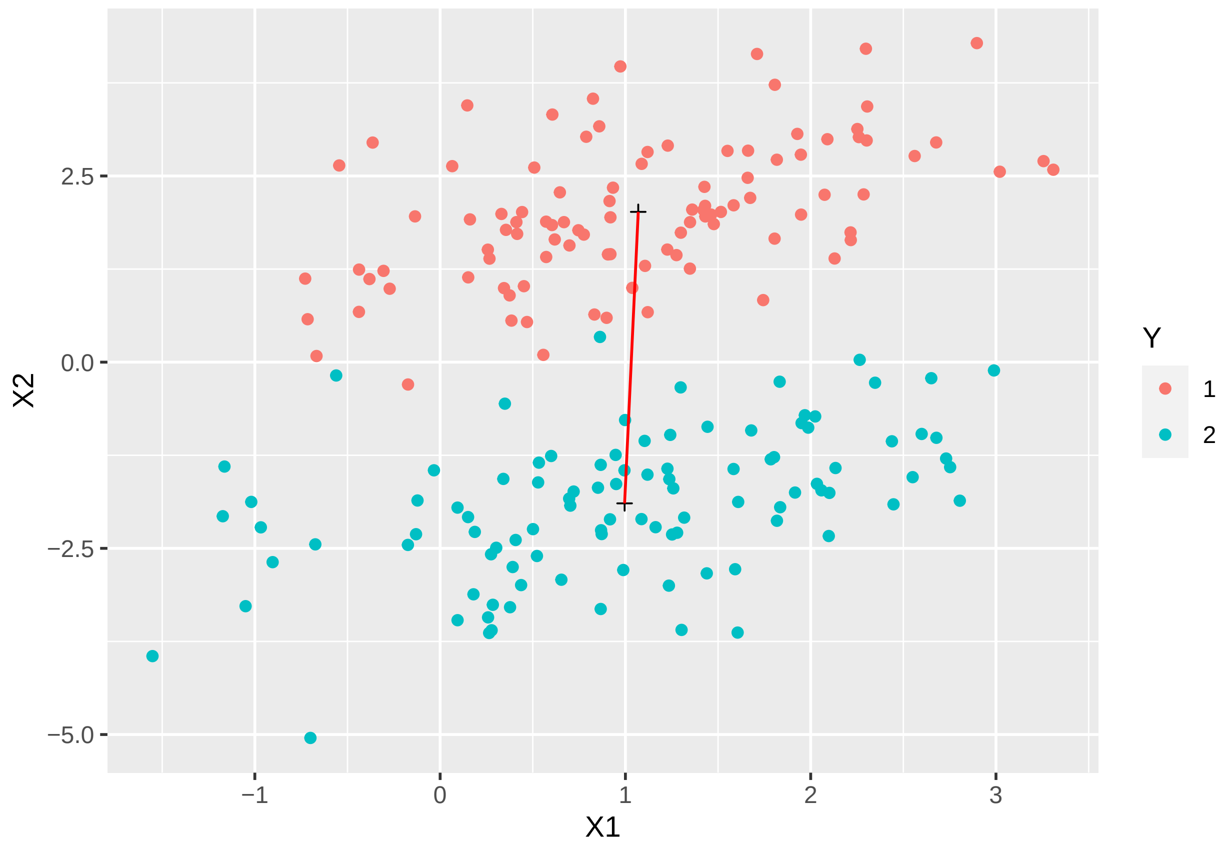
credit to: https://freakonometrics.hypotheses.org/53021

```
mu1<-rbind(sum(C1[,1])/class1_samples,sum(C1[,2])/class1_samples)
mu2<-rbind(sum(C2[,1])/class2_samples,sum(C2[,2])/class2_samples)
common_cov<-matrix(c(0,0,0,0),nrow=2,ncol=2)

x1_range <- seq(-2, 4, by = 0.05)
x2_range <- seq(-5.5, 5.5, by = 0.05)
combined_range <- expand.grid(X1 = x1_range, X2 = x2_range)

for (row in 1:nrow(C1) ){
  common_cov<-common_cov+((C1[row,]-mu1) %*% t(C1[row,]-mu1) / (total_samples-2))
};

for (row in 1:nrow(C2) ){
  common_cov<-common_cov+((C2[row,]-mu2) %*% t(C2[row,]-mu2) / (total_samples-2))
};

omega = solve(common_cov)%*%(mu2-mu1)

b = (t(mu2)%*%solve(common_cov)%*%mu2-t(mu1)%*%solve(common_cov)%*%mu1)/2

ggplot() + geom_point(data=dataset,aes(X1, X2, color = Y)) +
  geom_point(data=centroids,aes(X1,X2), shape =3,alpha=1) +geom_abline(slope=-omega[1]/omega[2],interce
```



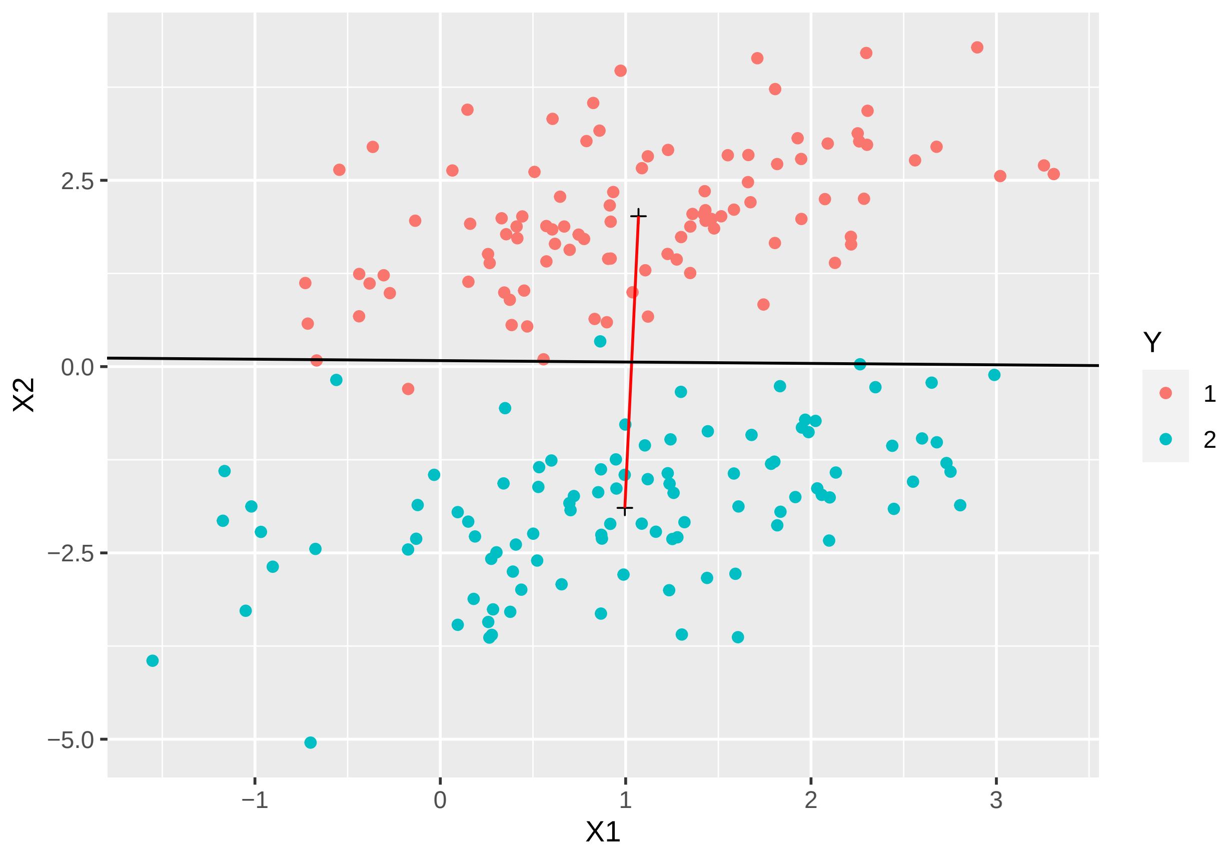
Part 2

```
ggplot() + geom_point(data=dataset,aes(X1, X2, color = Y)) +
  geom_point(data=centroids,aes(X1,X2), shape =3,alpha=1)+
  geom_line(data=centroids,aes(X1,X2),group=1,color='red')
```



# we see the line between the centroids, to get a decision boundary we will first compute the mid point of that red line, then calculate the formula of that red line, then find the line that is perpindicular to that line at the midpoint to get the decision boudary.

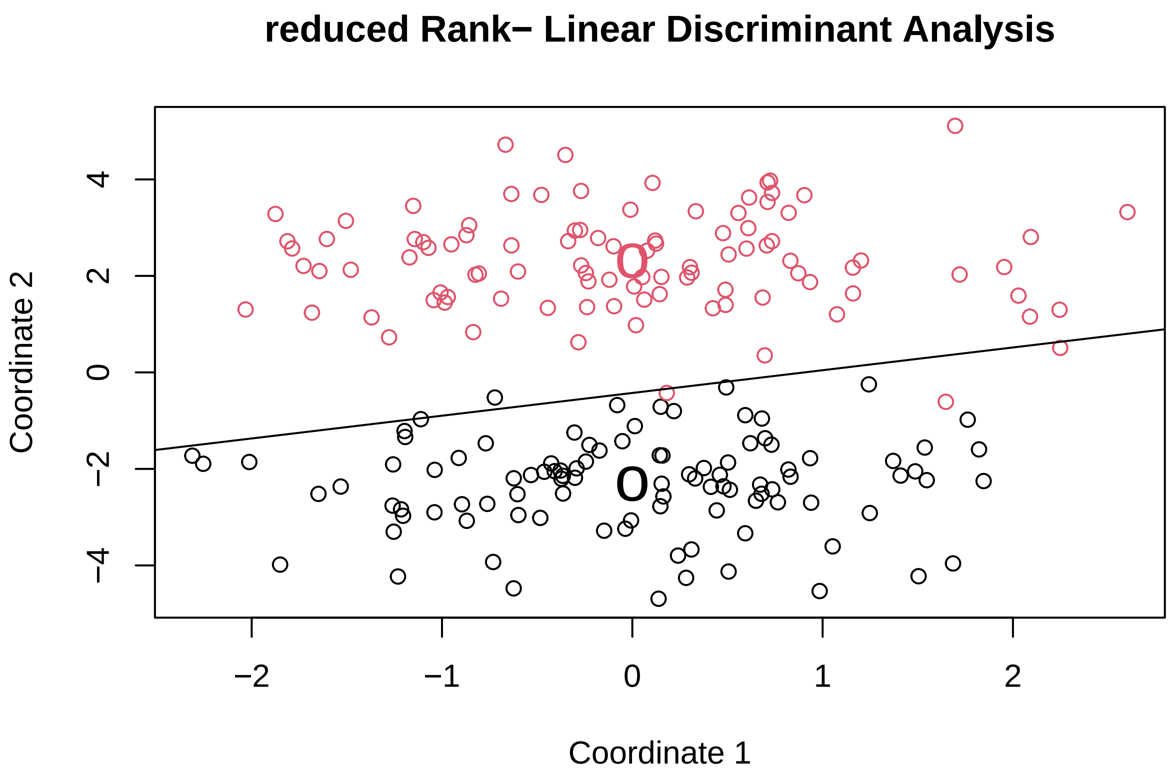
```
midpoint<-c(mean(centroids[-1]$X1),mean(centroids[-1]$X2))
slope_of_centroidline<-(centroids$X2[1]-centroids$X2[2])/(centroids$X1[1]-
  centroids$X1[2])
# using point slope formula y=mx+b, b= y-mx
intercept<- centroids$X2[1]-slope_of_centroidline*(centroids$X1[1])

new_perp_slope<- -1/slope_of_centroidline

#y=mx+b
new_inter<-midpoint[2]-new_perp_slope*midpoint[1]
ggplot() + geom_point(data=dataset,aes(X1, X2, color = Y)) +
  geom_point(data=centroids,aes(X1,X2), shape =3,alpha=1)+
  geom_line(data=centroids,aes(X1,X2),group=1,color='red')+ geom_abline(slope=new_perp_slope,intercept
```



Part 3

```
y<-as.numeric(dataset$Y)
x<-as.matrix(dataset[,-3])


K <- 2
p <- 2
n <- dim(dataset)[1]

M <- matrix(0, K, p)
for(i in 1:K) M[i,] <- apply(x[y==i,], 2, mean);

# calculate within-class covariance
W <- t(as.matrix(x) - M[y, ])%*%(as.matrix(x) - M[y, ])/(n-2)

utilda1<-(sqrt(eigen(W)$values)**-1)%*%t(eigen(W)$vectors)%*%mu1
utilda2<-(sqrt(eigen(W)$values)**-1)%*%t(eigen(W)$vectors)%*%mu2


# calculate Mstar = M W^{-1/2}
temp <- svd(W)       # singular value decomposition
Wn0p5 <- temp$u %*% diag(1/sqrt(temp$d)) %*% t(temp$v) # W^{-1/2}
Mstar <- M %*% Wn0p5
```

```r
temp <- Mstar
for(i in 1:2) temp[,i] <- temp[,i]-mean(temp[,i]);
Bstar <- t(temp)%*%temp/10

# eigen-decomposition of Bstar
temp <- eigen(Bstar)

Vstar <- temp$vectors   # columns are v^*_l
V <- Wn0p5 %*% Vstar    # columns are v_l

# discriminant variables
Z <- as.matrix(x) %*% V
for(i in 1:2) Z[,i] <- Z[,i] - mean(Z[,i]);
Z <- -Z
Z[,2] <- -Z[,2]
Mnew <- matrix(0, K, p)
for(i in 1:K) Mnew[i,] <- apply(Z[y==i,], 2, mean);


par(mfrow=c(1,1))
i1 <- 1
i2 <- 2
plot(Z[,i2],Z[,i1] , xlab="Coordinate 1",
     ylab="Coordinate 2",
     main="reduced Rank- Linear Discriminant Analysis", type="n")
for(i in 1:2) {
  points(Mnew[i,i2],Mnew[i,i1], col=i, type="p", pch="o", cex=2);
  points(Z[y==i,i2],Z[y==i,i1],  col=i, type="p", pch=21, cex=1);
}
abline(b=-omega[1]/omega[2],a=b/omega[2])
```

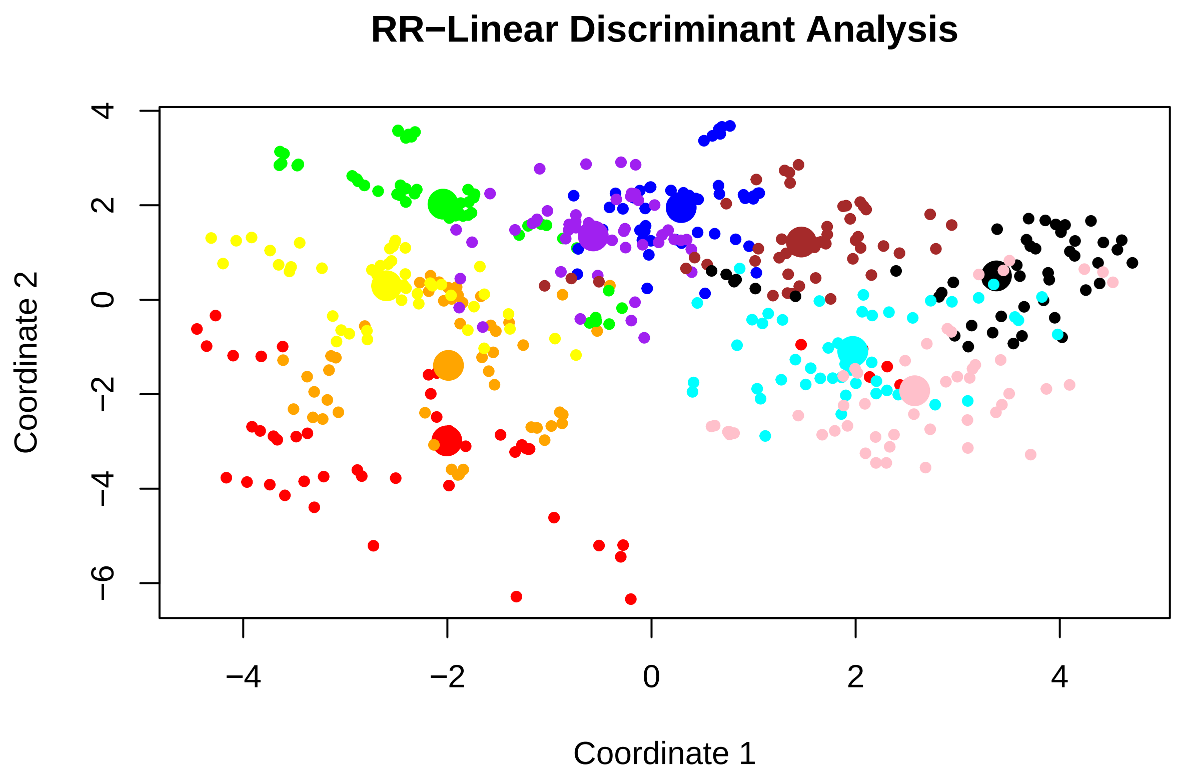


There is some missclassification but now the centroids are as far apart as is expected when plotting the first two discriminants

Problem 4.1

```
df<-read.csv("./vowel.train.csv")[-1]
Kvow <- 11               # 11 vowels or 11 classes
pvow <- 10               # dimension of input space
nvow <- dim(df)[1]                # number of observations
yvow <- df$y    # output or response, integer values 1, 2, ..., 11
Xvow <- df[,2:(pvow+1)]

# 11 class centroids in R^10
Mvow <- matrix(0, Kvow, pvow)
for(i in 1:Kvow) Mvow[i,] <- apply(Xvow[yvow==i,], 2, mean);

Wvow <- t(as.matrix(Xvow) - Mvow[yvow, ])%*%(as.matrix(Xvow) - Mvow[yvow, ])/
  (nvow-Kvow)

temp <- svd(Wvow)
W_half <- temp$u %*% diag(1/sqrt(temp$d)) %*% t(temp$v)
Mstarvow <- Mvow %*% W_half

temp <- Mstarvow
```

```r
for(i in 1:10) temp[,i] <- temp[,i]-mean(temp[,i]);
Bstarvow <- t(temp)%*%temp/10


temp <- eigen(Bstarvow)
Vstarvow <- temp$vectors
Vvow <- W_half %*% Vstarvow


Zvow <- as.matrix(Xvow) %*% Vvow
for(i in 1:10) Zvow[,i] <- Zvow[,i] - mean(Zvow[,i]);
Zvow <- -Zvow
Zvow[,2] <- -Zvow[,2]
Zvow[,10] <- -Zvow[,10]
Mnewvow <- matrix(0, Kvow, pvow)
for(i in 1:Kvow) Mnewvow[i,] <- apply(Zvow[yvow==i,], 2, mean);

par(mfrow=c(1,1))
i1vow <- 1
i2vow <- 2
colors=c("red",'orange','yellow','green','blue','purple','brown','black','cyan','pink')
plot(Zvow[,i1vow], Zvow[,i2vow], xlab="Coordinate 1", ylab="Coordinate 2",
     main="RR-Linear Discriminant Analysis", type="n")
for(i in 1:11) {
  points(Zvow[yvow==i,i1vow], Zvow[yvow==i,i2vow], col=colors[i], type="p", pch=20, cex=1);
   points(Mnewvow[i,i1vow], Mnewvow[i,i2vow], col=colors[i], type="p", pch=19, cex=2);
}
```



The procedure for page 114 is showed throughout the code above

Problem 4.2

help from https://notebook.community/MariaRigaki/STK4030/4.1_solution

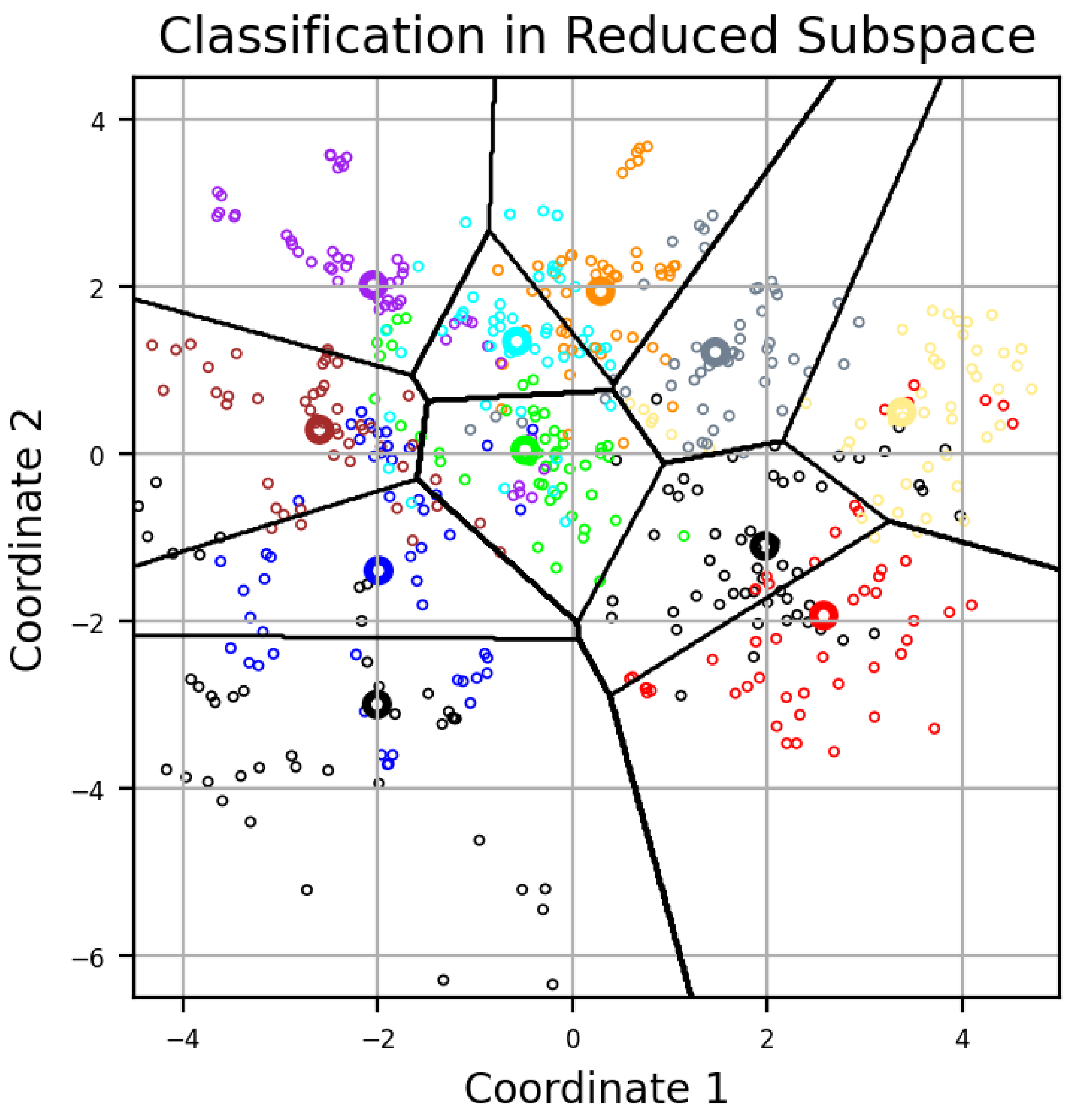
In [ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import operator
import matplotlib.pylab as plt
from sklearn import preprocessing
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis as LDA
from sklearn import linear_model
```

In [ ]:
```
vowel_train=pd.read_csv('./vowel.train.csv')
vowel_test = pd.read_csv('./vowel.test.csv')

y_train = vowel_train['y']
X_train = vowel_train.drop(['row.names', 'y'], axis=1)

y_test = vowel_test['y']
X_test = vowel_test.drop(['row.names', 'y'], axis=1)

scale_transformer = preprocessing.StandardScaler().fit(X_train)
X_trainscale = scale_transformer.transform(X_train)
X_testscale = scale_transformer.transform(X_test)
```

plotting lines help from: https://github.com/empathy87/The-Elements-of-Statistical-Learning-Python-Notebooks/blob/master/examples/Vowel.ipynb

In [ ]:
```
ldafit=LDA(n_components=2).fit(X_trainscale,y_train)

reducedX=ldafit.transform(X_trainscale)
Rlda = LDA().fit(reducedX[:, :2], y_train)
M=ldafit.transform(ldafit.means_)
grid_size = 500

X = np.transpose([np.tile(np.linspace(-4.5, 5, grid_size), grid_size),
                  np.repeat(np.linspace(-6.5, 4.5, grid_size), grid_size)])
y = Rlda.predict(X)

X0 = X[:, 0].reshape(grid_size, grid_size)
X1 = X[:, 1].reshape(grid_size, grid_size)
Y = y.reshape(grid_size, grid_size)


colors = np.array([
    '#000000', '#0000FF', '#A52A2A', '#A020F0', '#FF8C00', '#00FFFF',
    '#708090', '#FFEC8B', '#000000', '#FF0000', '#00FF00'])
fig, ax = plt.subplots(figsize=(4, 4), dpi=200)
ax.scatter(reducedX[:, 0], reducedX[:, 1], facecolors='none',
           edgecolors=colors[y_train-1], s=5, linewidth=0.6)
```

```
ax.scatter(M[:, 0], M[:, 1], facecolors='none', edgecolors=colors,
           s=5, linewidth=5)
for i in ax.get_yticklabels() + ax.get_xticklabels():
    i.set_fontsize(6)
plt.title('Classification in Reduced Subspace')
plt.xlabel('Coordinate 1')
plt.ylabel('Coordinate 2')
_ = ax.contour(X0, X1, Y, np.linspace(0, 9, 10)+0.5, linewidths=1,colors='black')
plt.grid()
plt.show()
```



In [ ]:
```
y_pred=ldafit.predict(X_trainscale)
ldamiss = np.sum(y_pred !=y_train )
print("Classification Accuracy is:", 1- (ldamiss/ y_train.shape[0]))
print("Mislassification is:", ldamiss/y_train.shape[0])
print("Mislassified:", ldamiss, "out of 528 points")
```

```
Classification Accuracy is: 0.6837121212121212
Mislassification is: 0.3162878787878788
Mislassified: 167 out of 528 points
```

Problem 5

help from https://notebook.community/MariaRigaki/STK4030/4.1_solution

Linear

In [ ]:
```
vowel_train=pd.read_csv('./vowel.train.csv')
vowel_test = pd.read_csv('./vowel.test.csv')

y_train = vowel_train['y']
X_train = vowel_train.drop(['row.names', 'y'], axis=1)

y_test = vowel_test['y']
X_test = vowel_test.drop(['row.names', 'y'], axis=1)


# Indicator matrix
lb = preprocessing.LabelBinarizer()
lb.fit(y_train)
y_bin = lb.transform(y_train)
ytest_bin = lb.transform(y_test)

# Add the column of ones in the training and test data
X_train_one = np.hstack((np.ones((len(X_train), 1)), X_train))
X_test_one = np.hstack((np.ones((len(X_test), 1)), X_test))


beta_hat = np.dot(np.linalg.inv(np.dot(X_train_one.T, X_train_one)), np.dot(X_train
f_x = np.dot(X_train_one, beta_hat)
f_x_test = np.dot(X_test_one, beta_hat)

# Find the index with the max value and assign it to the corresponding class
g_hat = []
for row in f_x:
        index, value = max(enumerate(row), key=operator.itemgetter(1))
        g_hat.append(index + 1) # Align the class because index starts from 0

g_hat_test = []
for row in f_x_test:
        index, value = max(enumerate(row), key=operator.itemgetter(1))
        g_hat_test.append(index + 1) # Align the class because index starts from 0
```

In [ ]:
```
print('Classification Accuracy training',sum((g_hat ==y_train)/ len(y_train)))
print('Classification Accuracy test', sum((g_hat_test== y_test)) / len(y_test))
```

```
Classification Accuracy training 0.5227272727272729
Classification Accuracy test 0.3333333333333333
 0.5227272727272729
Classification Accuracy test 0.3333333333333333
```

Logistic

In [ ]:

```
regr = linear_model.LogisticRegression(C=1.0)
y_pred = regr.fit(X_trainscale, y_train).predict(X_trainscale)
y_pred_test = regr.predict(X_testscale)
print('Classification Accuracy training',sum(y_pred==y_train)/len(y_train))
print('Classification Accuracy test',sum(y_pred_test==y_test)/len(y_test))
```

```
Classification Accuracy training 0.7272727272727273
Classification Accuracy test 0.45021645021645024
```