Question 2

1. **Proving FairSplit-D ∈ NPC using reduction from Subset Sum:**

First, we need to show that FairSplit-D is in NP. Given a certificate (a partition of S into A and B), we can easily verify in polynomial time whether the sum of the elements in A equals the sum of the elements in B.

Next, we need to show that FairSplit-D is NP-hard. We can do this by reducing the Subset Sum problem to FairSplit-D.

The Subset Sum problem is defined as follows:

Given a set of integers, is there a non-empty subset whose sum is zero? We can transform an instance of Subset Sum to an instance of FairSplit-D as follows:

- If the sum of all elements in the set is not even, then there is no solution to FairSplit-D because an odd number cannot be divided evenly.
- If the sum of all elements in the set is even, then we can divide the set into two subsets A and B such that the sum of the elements in A equals the sum of the elements in B. This is equivalent to finding a subset with sum equal to half of the total sum, which is the Subset Sum problem.

Therefore, if we can solve FairSplit-D, we can solve Subset Sum, which means FairSplit-D is NP-hard. Since FairSplit-D is both in NP and NP-hard, it is NP-complete.

2. **Proving FairSplit1000 ∈ NPC using reduction from FairSplit-D:**

First, we need to show that FairSplit1000 is in NP. Given a certificate (a partition of S into A and B), we can easily verify in polynomial time whether the absolute difference between the sum of the elements in A and the sum of the elements in B is less than 1000.

Next, we need to show that FairSplit1000 is NP-hard. We can do this by reducing FairSplit-D to FairSplit1000.

We can transform an instance of FairSplit-D to an instance of FairSplit1000 as follows:

- Add 1000 to each element in the set. This ensures that the absolute difference between the sum of the elements in any two partitions of the set is at least 1000, unless the two partitions are equal (which is the FairSplit-D problem).

Therefore, if we can solve FairSplit1000, we can solve FairSplit-D, which means FairSplit1000 is NP-hard. Since FairSplit1000 is both in NP and NP-hard, it is NP-complete.