التاريخ : / /   الموضوع :

Probability Distribution

Distribution : the possible values a variable can take and how frequently they occur.

- probabilities measure the likehood of an outcom, depending on how often it is featured in the sample space.

Two characteristics : → mean (average) μ → variance (spead out) σ^2

data kinds → population data "all the data" → Sample data "part of it"

- sample mean = $\overline{X}$
- sample variance = S^2
- sample STD = S
- population STD = σ

القيمه المتوقعه

- $\sigma^2 = E((Y-\mu)^2) \xrightarrow{e} = E(Y^2) - \mu^2$

Types of probability Distribution

1. Discrete distribution
   - finite number of outcomes (Die , picking a card)

2. Continuous distribution
   - infinitely many outcomes (time , Distance)

variable

$X \sim N(\mu, \sigma^2)$ → characteristics (May vary)

Telda   Type

Discrete distributions

- uniform Distribution
  all outcomes are equally likely
  → Equiprobable.
  (picking a card, Coin)
- Bernoulli Distribution
  Events with only two possible outcomes → true / False
- Binomial Distribution
  * Two outcomes per iteration
  * Many iterations.
  (Flipping a coin 3 times)
- poisson Distribution
  Test out how unusual an event frequency is for a given interval

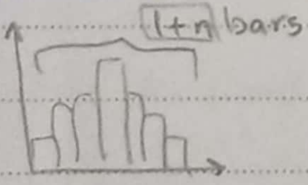
Continuous distributions

- Normal distribution
  often observed in nature
- Chi-Squared
  - Asymmetric.
  - only consists of non-negative value., alway starts from zero on the left
  - Doesn't often mirror real life events
  - The curve is skewed to left
  - used in Hypothesis Testing to help to determine How goodness of fit
- Exponential distribution
  - Events are rapidly changing early on.
- logistic distribution
  - useful in Forecast analysis
  - useful for determining a cut-off point for a successful outcome.

* Uniform distribution $U(\ )$
  - Each outcome is equally likely.
  - Expected value Provides us no relevant information because all outcomes have the same probability
  - Both the mean and the variance are uninterpretable.
  - No predictive power
* Binomial distribution $B(\ )$
  - It is a sequence of identical Bernoulli events

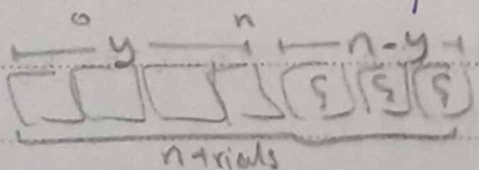
$B(n\ ,\ p)$ / , $Bern(p) = B(1,p)$

عدد المحاولات ← n ; p → احتمال نجاح كل مرة



$p(\text{desired outcome}) = p$

$p(\text{alternative outcome}) = 1-p$



- The number of ways in which 4 out of the 6 trials can be successful. $= C^6_4$

$$p(y) = \binom{n}{y} \cdot p^y \cdot (1-p)^{n-y}$$

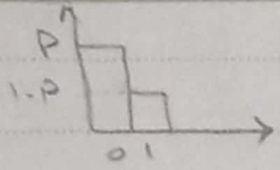
Expected value → $E(x) = X_0 \cdot p(X_0) + X_1 \cdot p(X_1) + \cdots X_n \cdot p(X_n)$

$$\sigma^2 = E(y^2) - E(y)^2 = n \cdot p \cdot (1-p)$$

* Bernoulli distribution $Bern(p)$

events with : → 1 Trial
→ 2 possible outcomes

p is known OR Past data indicating some experimental probability



* $1 \rightarrow p$
* $0 \rightarrow 1-p$

* $E(x) \begin{cases} p \\ 1-p \end{cases}$

* $\sigma^2 = p(1-p)$

* Poisson distribution $Po(\lambda)$

Deals with the frequency with which an event occurs

p^x $\lambda^r \longrightarrow$ عدد مرات حدوث الشيء خلال فترة زمنية أو مسافة محددة

$$P(Y) = \frac{\lambda^y e^{-\lambda}}{y!} \quad , \quad E(y) = y_0 \cdot p(y_0) + y_1 \cdot p(y_1) + ----$$

$$\mu = \sigma^2 = \lambda$$

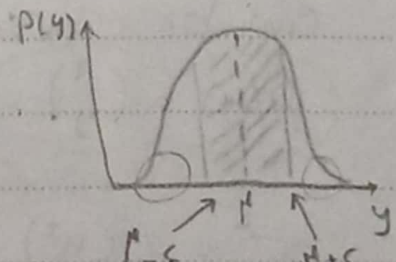
* Normal distribution $N(\mu, \sigma^2)$

Appearing in nature.

ex; The size of a full grown male lion.   150 : 250 kg

Distinct characteristics:

The most of the data is centered around the mean

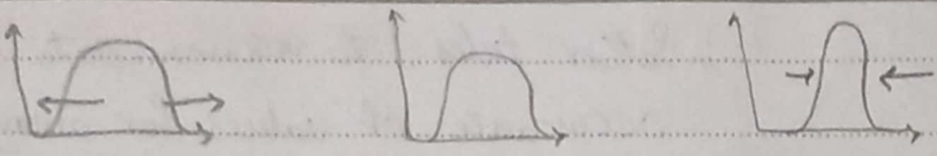


$E(x) = \mu$

$Var(x) = \sigma^2 \longleftarrow$ usually given

$Var(x) = E(x^2) - E(x)^2$

transformation: A way in which we can alter every element of a distribution to get a new distribution with similar charachtaristics.



$y = F\left(\frac{x}{c}\right)$ $y = f(x)$ $y = f(x.c)$

Standardizing: A special kind of transformation in which $\boxed{E(X) = 0}$, $\boxed{Var(X) = 1}$ we get Standard Normal distribution

Z-Score table: is statistical measure that describes a value's relationship to the mean of a group of valu

How to standardize?

$$y = F(x) \xrightarrow{\mu = 0} y = F(x - \mu) \xrightarrow{\sigma = 1} y = f\left(\frac{x - \mu}{\sigma}\right)$$

$$\left.\begin{aligned} Z &\sim N(0,1) \\ y &\sim N(\mu, \sigma^2) \end{aligned}\right\} \Rightarrow Z = \frac{y - \mu}{\sigma}$$ ← Transformation we use in standardizing

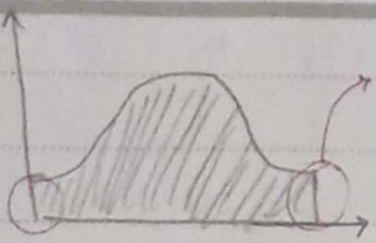
$$y = \mu + 2.3\sigma \Rightarrow Z = 2.3$$

Student's T distribution $t(k)$ → degrees of freedom

- small sample size approximation of a normal distribution

- Certain characteristics + sufficient data = Normal distribution

- Certain characteristics + ~~suffici~~ent data = Student's T distribution



fatter tales to accommodate the occurance of values far away from the mean

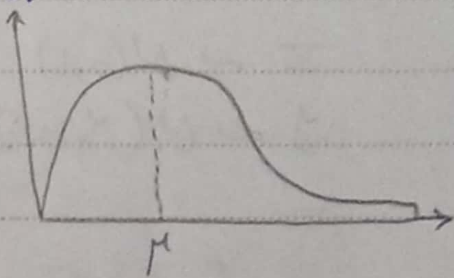
if $k > 2$

- $E(Y) = \mu$
- $Var(Y) = \frac{S^2 * K}{K-2}$

Application : - Frequently used when conducting statistical analysis.
- Hypothesis testing with limited data.
- CDF table (T-table)

Chi_Squared distribution $\chi^2(K)$

- Statistical analysis when doing :
  - Hypothesis testing
  - Computing confidence intervals



- $\sqrt{\chi} \sim t(k)$
- $Y \sim t(k) \rightarrow Y^2 \sim \chi^2(k)$

$E(X) = k$

$V(X) = 2k$

التاريخ : / / الموضوع :

Conditional probability

Ex :- $S = \{1,2,3,4,5,6\}$, $A = \{1,3,5\}$, $B = \{1,2\}$

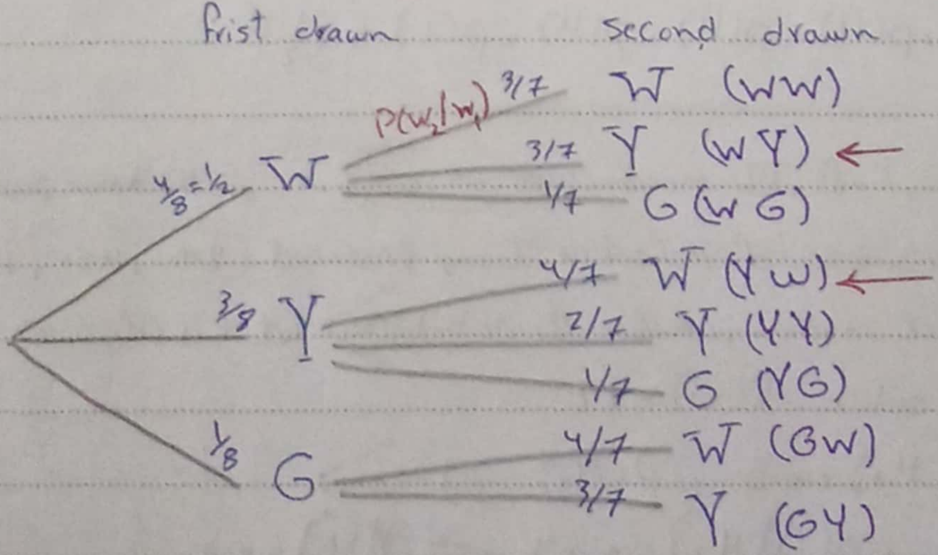
$P(B/A) = \frac{1}{3}$

$$* \; P(E \setminus F) = \frac{P(E \cap F)}{P(F)}$$

Product rule : $\rightarrow$ $P(E \cap F) = P(F) \cdot P(E \setminus F)$

EX 1 : From a Box containing four white (W), three yellow (Y), and one green (G) ball, two ball are drawn one at a time without replacing the first ball before the second is drawn. use a tree diagram to find the probability that one white are one yellow ball are drawn.

يهتش بترتيب الحدثين



Sol :

الحدث الأول الحدث الثاني

$$p(w \cap Y) = P(Y_{1st}) \cdot P(W_{2nd} \mid Y_{1st}) + P(W_{1st}) \cdot P(Y_{2nd} \mid W_{1st})$$

$$= \frac{3}{8} * \frac{4}{7} + \frac{1}{2} * \frac{3}{7} = \frac{3}{7}$$

Independent Events

$$\boxed{p(E|F) = p(E)}$$

$$\boxed{p(E \cap F) = p(E) \cdot p(F)} \rightarrow \text{criterion for independent Events}$$

$$\boxed{p(E_1 \cap E_2 \cap \ldots \cap E_n) = p(E_1) \cdot p(E_2) \cdot \ldots \cdot p(E_n)}$$

Ex :- A fair coin is tossed repeatedly until the first tail appear. what is the probability of getting the first tail at the 5^{th} trial ?

H : Head , $p(H) = \frac{1}{2}$

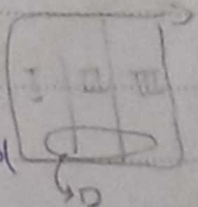
T : Tail , $p(T) = \frac{1}{2}$

$p(\text{1st tail appears in 5th trial}) = p(H_1 \cap H_2 \cap H_3 \cap H_4 \cap T_5)$

$= p(H) \cdot p(H) \cdot p(H) \cdot p(H) \cdot p(T) = (\frac{1}{2})^5$

Ex :- 3 machines , I , II , III manufacture (0.4 , 0.5 , 0.1) of the total production in a plant : The percentage of defective items produced (2% , 4% , 1%) For item chosen at random , what is the probability that it is defective?

D : item is defective $P(D) = ??$



$p(M_1) = 0.4$, $p(M_2) = 0.5$, $p(M_3) = 0.1$

$p(D|M_1) = 0.02$, $p(D|M_2) = 0.04$, $p(D|M_3) = 0.01$

$\therefore p(D) = p(D \cap M_1) + p(D \cap M_2) + p(D \cap M_3)$

$= 0.4 * 0.02 + 0.5 * 0.04 + 0.1 * 0.01$

التاريخ : / / الموضوع :

Bayes' Theorem

$$P(A_j \setminus E) = \frac{P(A_j) \cdot P(E \setminus A_j)}{P(E)}$$

الحد الاحتمال

$$= \frac{P(A_j) \cdot P(E \setminus A_j)}{P(A_1) \cdot P(E \setminus A_1) + P(A_2) \cdot P(E \setminus A_2) \dots}$$