Imp.
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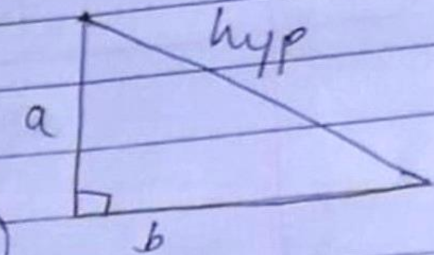
Project Euler #009. Pythagorean Triplets.



Conditions:
(input number ⇒ num.)

A) $a^2 + b^2 = hyp^2$ —— (i)
B) $a + b + hyp = num$ —— (ii).

To find:
for all the pythagorean triplets in range 'num' find the one where ~~that~~ product $a*b*hyp$ is maximum.

Solution:
from condition (ii).

$a + b + hyp = num$
from condition (i) we know ~~$a+b$~~ $hyp^2 = a^2 + b^2$

$a + b + hyp = num$ — (a)
$a + b + \sqrt{a^2 + b^2} = num$

$\sqrt{a^2 + b^2} = num - a - b$
$\sqrt{a^2 + b^2} = num - (a + b)$
Squaring bothsides.
$a^2 + b^2 = (num - (a+b))^2$
$a^2 + b^2 = num^2 - 2(num)(a+b) + (a+b)^2$
$\cancel{a^2} + \cancel{b^2} = num^2 - 2(num)(a+b) + \cancel{a^2} + 2ab + \cancel{b^2}$
$0 = num^2 - 2(num)(a+b) + 2ab.$
$\frac{num^2}{2} = \cancel{2}(num)(a+b) - \cancel{2}ab.$

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$$\frac{num^2}{2} = (num)(a+b) - ab.$$

$$\frac{num^2}{2} = num.(a) + num.(b) - ab.$$

$$num.(b) - a.b = \frac{num^2}{2} - num.(a)$$

$$b[num - a] = (num^2/2) - num.a$$

$$\boxed{b = \frac{(num^2/2) - (num \cdot a)}{(num - a)}}$$

Since we can obtain value of b from a and num provided, the whole problem shifts from a $O(n^2)$ solution to $O(n)$ (or $O(n//2)$) as we check for each 'a' in range of (1 to num−1).

for test case 5