Module 1:-

SHORT ANSWER QUESTIONS:-

1)

Classical Mechanics	Quantum Mechanics.
• It deals with macroscopic particles.	• It deals with microscopic particles.
• Based on newton's law of motion.	• Based on Heisenberg's uncertainity principle & de-Broglie's concept of wave particle duality.
• Based on Maxwell's electromagnetic wave theory.	• Based on plank's quantum theory.
• Any amount of energy may be emitted or absorbed continuously.	• Discrete value of energy are emitted or absorbed.
• The state of the system is given by considering all the forces acting on the particles, their position & velocity.	• It is given the probabilities of finding at various locations in ~~sq~~ space.

2) Quantum Mechanics is science dealing with the behaviour of matter and light ~~to~~ on the atomic and subatomic level. It attempts to describe account for the properties of ~~molecular and~~ molecules and atoms and their constituents - electrons, protons, neutrons and other more ~~esor~~ esoteric particles such as quarks & gluons.

3) de Broglie's Hypothesis:-

Matter is believed to behave both like a particle & a wave at sub-microscopic level.

Not only light but every materialistic particle such as electron, proton

or even heavier objects exhibits wave particle dual nature.
De-Broglie proposed that a moving particle, whatever its nature has waves associated to it. These waves are called "matter waves".

$$\lambda = \frac{h}{mc}$$
$$\Downarrow$$
$$\lambda = \frac{h}{p}$$
$$p = \sqrt{2mE}$$

- Debroglie's wavelength in the form of wavelength $\lambda = \frac{h}{\sqrt{2mE}}$.
- In the ~~from~~ form of KE of material particle

$$E = \frac{p^2}{2m}$$

- When charged particle is carry a charge 'q' and is accelerated by potential difference V, then KE is

$$E = qV$$

Debroglie wavelength associated to particle is $\lambda = \frac{h}{\sqrt{2mqV}}$

Properties of Matter Wave:-

→ Lighter is the particle, greater is the wavelength associated to it.
→ Smaller is the velocity, greater is the wavelength associated to it.
→ It can be shown that matter wave can travel faster than light. i.e., velocity of matter waves can be greater than velocity of light.
→ No single ~~phor~~ phenomenon exhibits particle nature and wave nature simultaneously.

4) De broglie's wavelength in terms of momentum & KE.

$E = mc^2 \longrightarrow$ (Einstien's Relation) $\longrightarrow$ eq (1)
$E = h\nu \longrightarrow$ [For light particle]. $\longrightarrow$ eq (2)
Compare eq (1) & eq (2).

$$mc^2 = h\nu$$
$$mc^2 = \frac{hc}{\lambda} \left[\because \nu = \frac{c}{\lambda}\right].$$

$mc^{\cancel{2}} = \frac{h\cancel{c}}{\lambda} \Rightarrow mc = \frac{h}{\lambda}$ $\lambda = \frac{h}{mc}$

$\lambda = \frac{h}{p}$ (momentum) $[\because p = m \times c]$ (mass, velocity).

$\rightarrow KE = \frac{1}{2}mv^2$.

$E = \frac{m^2v^2}{2m}$ $2mE = m^2v^2$.

$2mE = p^2$ $\boxed{p = \sqrt{2mE}}$ momentum

$\boxed{E = \frac{p^2}{2m}}$ Kinetic energy.

5) $E = h\nu \longrightarrow$ [eq for light particle].
$E = mc^2 \longrightarrow$ [~~Eien~~ Einstien's ~~E~~ Relation].

$E = h\nu$ $E = \frac{hc}{\lambda}$ $mc^{\cancel{2}} = \frac{h\cancel{c}}{\lambda}$ $mc = \frac{h}{\lambda}$ $\lambda = \frac{h}{mc}$ $\lambda = \frac{h}{p}$.

$[\because \nu = \frac{c}{\lambda}]$ $[\because p = m \times c]$.

$E = \frac{1}{2}mv^2$ $E = \frac{m^2v^2}{2m}$ $E = \frac{p^2}{2m}$ $p^2 = E \times 2m$
$p = \sqrt{2mE}$.

$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$

wavelength of matter wave | debroglie wavelength

6) Don't ~~Kw~~ Know.

7) To ~~arrive at~~ arrive at deBroglie's wavelength $\lambda = \frac{h}{mv}$ the two concepts taken into consideration are :-

$E = mc^2 \longrightarrow$ [~~Einstien's~~ Einstein's Relation] $\longrightarrow$ (1) &
$E = h\nu \longrightarrow$ [Equation of Light Particle] $\longrightarrow$ (2).

Solve eq (1) & (2)
& $mc^2 = h\nu$ $[\because \nu = \frac{c}{\lambda}]$.

$mc^{\cancel{2}} = \frac{h\cancel{c}}{\lambda} \Rightarrow \lambda = \frac{h}{mc}$ (plank's constant; mass of particle; velocity of particle) $\Rightarrow \boxed{\lambda = \frac{h}{mv}}$.

8) The different forms of debroglie's wavelength of electron are :-

(a) In terms of KE of material particle then
$E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$. $p^2 = 2mE$
$\Rightarrow p = \sqrt{2mE}$

$\therefore$ De-Broglie's wavelength $\lambda = \frac{h}{\sqrt{2mE}}$.

(b) Charged particle carrying charge 'q' is accelerated by potential diff 'V' then KE

$E = qV$.

Hence, debroglie wavelength associated with this particle is $\lambda = \frac{h}{\sqrt{2mqV}}$

9) Wave function:-

→ The quantity with which Quantum Mechanics is concerned is wave function of the body.

→ Wave function, ψ is a quantity associated with a moving particle. It is a complex quantity.

→ $|\psi|^2$ is proportional to the probability of finding a particle at a particular point at a particular time. It's the probability density.

$$|\psi|^2 = \psi^* \psi$$

→ ψ is the probability amplitude.

10) Normalization:-

First write 9sol" 3rd & 4th point.

→ $|\psi|^2$ is probability density.

The probability of finding the particle within an element of volume dx

$$|\psi|^2 dx.$$

Since the particle will definitely be somewhere, so

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1.$$

A wavefunction that obeys this equation is said to be normalised.

11) Same 9th sol"

12). → lighter the particle, greater is the wavelength associated to it.
→ Smaller the velocity, greater is the wavelength associated to it.
→ When $v=0$, the $\lambda=\infty$, ie wave becomes indeterminate and if $v=\infty$ then $\lambda=0$, this shows that matter waves are generated only when material particles are in motion.

13) The time independent Schrodinger wave equation is given by:-

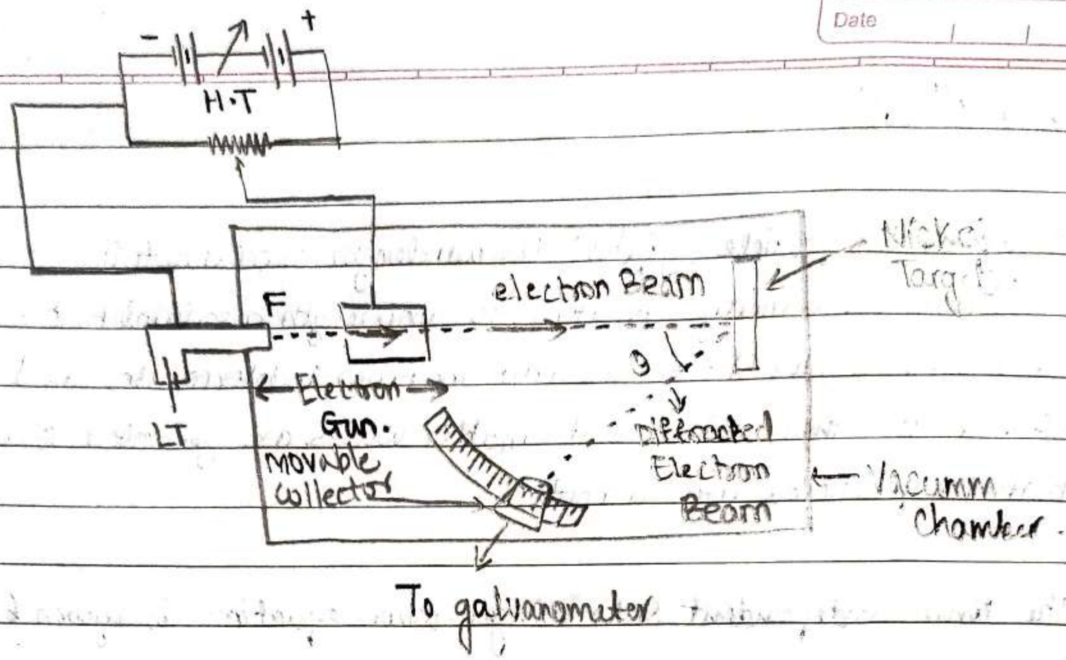
$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0}$$ where $\boxed{E = \frac{1}{2}mv^2 + V}$ → Potential energy

$$\boxed{\hbar = \frac{h}{2\pi}}$$

14) The principle used for the Davison & Germer experiment is to observe diffraction effects of electrons beam to study wave nature of electrons.

15) The apparatus consists of an electron gun G where electrons are produced when the filament of the electron gun is heated to dull, red electrons are emitted due to thermionic emmission.

Now the electrons are accelerated in electric field of known potential difference. These electrons are collimated by suitable slits to obtain a fine beam which is then directed to ~~form~~ fall on large single crystal of nickel, known as target T which is rotated about an angle along the direction of the beam is detected by an electron detector (Faraday cylinder) which is connected to a galvanometer. The faraday cylinder 'C' can move on a circular graduated scale S between 29°C to 90°C to recieve scattered electrons.



16) In the experiment it is observed that a 'bump' begins to appear in the ~~curo~~ curve for 44 volts. It is also observed that:

→ With increasing potential the Bump moves upwards.
→ The bump becomes more prominent in the curve for 54 volts at $\theta = 50^\circ$.
→ At higher potential the bump gradually ~~decreases~~ disappears.

17) Davison Germer experiment obtains a conclusion that electrons exhibit wave nature also, thus supporting ~~to~~ hypothesis given by de-Broglie regarding wave particle duality of matter. Thus electrons were scattered and Bragg's law gave angle of maximum scattering.

18) The expression of wave function of a particle in 3D ~~squre~~ square well box of infinite potential is given by:-

(These expressions are for 1D square well box not 3D square well box)

$$\boxed{\psi = A\sin Kx + B\cos Kx}$$ where $B = 0$

$$A \sin Kx = 0$$

$$K = \frac{n\pi}{L}$$

The expression of energy of particle in 3D square well box of infinite potential is given by

$$\boxed{E_n = \frac{n^2h^2}{8mL^2}}$$ where $n = 1, 2, 3, 4 \ldots$

Page No.
Date

Module 1 :-

→ Long Answer questions.

(1) • The electron theory aims to explain structure and properties of solids through their electronic structures.

• According to this theory a metal can be considered to consist of ion cores having the nucleus and electrons other than valence electrons.

• These valence electrons form an electron gas surround the ion cores and are free to move anywhere within the metal.

The electron theory of solids have been developed in three main stages.

1) Classical free electron theory :- Drude & Lorentz developed this theory ~~not~~ in 1900.
According to this theory metal containing free electrons obey the laws of classical mechanics.

2) Quantum free electron theory :- Sommerfield developed this during 1928. According to this theory the free electrons obey quantum laws.

3) The zone theory :- Bloch stated this theory in 1928. According to this theory the free electrons move in a periodic field provided by the lattice. This theory is called band theory of solids.

2) Q(3) Quantum Mechanics was able to explain :

① photoelectric effect.
② Black body radiation.
③ Compton effect.
④ Emission of line spectra.

The most outstanding development in modern science was the concept of Quantum mechanics in 1925. The new approach was higly successful in explaining about behavior of atoms, molecules & nuclei.

- Photo electric effect experiment result suggest that the energy in light wave is contained in discrete energy packets, which are called energy quanta or photon.

- The wave behaves like a particle. The particle are photons.

particle-like wave behavior
(eg:- Photo electric effect)

wave like particle behavior.
(Eg:- Daivison germer experiment)

wave particle duality.

The momentum of photon $p = \frac{h}{\lambda}$.

the wavelength of photon $\lambda = h/p$.

Light can exhibit both kind of nature i.e., nature of wave & particle. So, light shows wave-particle duality. In some case like interference, diffraction & polarization it behaves as a wave while in other cases such as photoelectric effect & compton effect it behaves as particle (photon).

Page No.
Date

3) **Significance of A Wave function:-**

- The quantity with which quantum mechanics is concerned is called wave function of a body.

- Wave function, ψ is quantity associated with a moving particle. It is a complex quantity.

- $|\psi|^2$ is proportional to the probability of finding a particle at a particular point at a particular time. It is proportionality density.

$$|\psi|^2 = \psi^* \psi$$

ψ is probability amplitude.

4) De broglie proposed that a moving particle no matter what its nature has a wave associated with it. These waves are called "matter waves".

Energy of photon:-

$$E = h\nu \longrightarrow (1)$$

For particle, say photon of mass m,

$$E = mc^2 \longrightarrow (2)$$

$h \rightarrow$ plank's constant
$m \rightarrow$ mass of ~~photon~~ particle
$v \rightarrow$ velocity of particle

$$mc^2 = h\nu$$

$$mc^{\cancel{2}} = \frac{h\cancel{c}}{\lambda}$$

$$mc = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{mc} \longrightarrow (3)$$

Suppose the particle is moving with velocity v, then the wavelength associated with it can be given as

$$\lambda = \frac{h}{mv} \Rightarrow \lambda = \frac{h}{p} \longrightarrow (4) \quad [\because p = m \times v].$$

Page No.

Date

$$E = \frac{1}{2} mv^2 \longrightarrow (5)$$

$$Em = \frac{m^2v^2}{2}$$

$$m^2v^2 = 2Em$$

$$\sqrt{m^2v^2} = \sqrt{2Em}$$

$$mv = \sqrt{2Em}$$

Sub in eq (4).

$$\lambda = \frac{h}{p} \Rightarrow \lambda = \frac{h}{mv} \Rightarrow \lambda = \frac{h}{\sqrt{2mE}}$$

For an electron with KE 'E' & accelerated by potential diff 'V' then

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$$

Substitution of h, m, & e we get.

$$\lambda = \frac{6.625\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.602\times10^{-19}\times V}} = \frac{12.26}{\sqrt{V}} A^\circ$$

6) Energy of particle :-

$$E = h\nu \longrightarrow (1)$$

Einstein's relation :-

$$E = mc^2 \longrightarrow (2)$$

compare eq (1) & eq (2).

$$h\nu = mc^2$$

$$\frac{hc}{\lambda} = mc^2 \qquad \left[\because \nu = \frac{c}{\lambda}\right]$$

$$\frac{h}{\lambda} = mc$$

$$(3) \longleftarrow \lambda = \frac{h}{mc}$$

for a particle with mass 'm' & velocity 'v' eq (3) can be written as $\lambda = \frac{h}{mv}$.

Page No.
Date

7) De-Broglie Waves:-

Not only light but every materialistic particle such as electron, proton or even heavier objects exhibit wave-particle duality.

De-Broglie proposed that ~~no~~ moving particle, whatever its nature has a wave associated with it. These are called "matter waves"

Energy:-

$$E = h\nu \longrightarrow (1)$$

Einstein's equation:-

$$E = mc^2 \longrightarrow (2)$$

solve (1) & (2).

$$h\nu = mc^2$$

$$\frac{hc}{\lambda} = mc^2$$

$$\frac{h}{\lambda} = mc \qquad \lambda = \frac{h}{mc} \qquad \lambda = \frac{h}{mv}$$

plank's constant

mass velocity

$$\lambda = \frac{h}{p}.$$

~~Ene~~ Energy of a moving particle:-

$$E = \frac{1}{2}mv^2$$

$$Em = \frac{1}{2}m^2v^2.$$

$$2mE = m^2v^2$$

$$\sqrt{2mE} = \sqrt{m^2v^2}$$

$$mv = \sqrt{2mE}.$$

$$\lambda = \frac{h}{mv} \Rightarrow \lambda = \frac{h}{\sqrt{2mE}}$$

Page No.
Date

For a particle with Kinetic energy E & potential difference V.

$$\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda = \frac{h}{\sqrt{2meV}}.$$

substituting the value of h, m, e we get :-

$$\lambda = \frac{6.023 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.602 \times 10^{-19} \times V}} \ A^{\circ}.$$

$$\lambda = \frac{12.26}{\sqrt{V}} A^{\circ}.$$

9) Same 7th Answer till $\lambda = \frac{h}{\sqrt{2mE}}$

10) Same 7th Answer till $\lambda = h/p$.

11) → The first experimental evidence of matter wave was given by two american physicists, Davisson & Germer in 1927. The experimental arrangement is shown in the diagram.

→ The apparatus consists of electron gun G where the electrons are produced.

→ When the filament of electron gun is heated to dull red electrons are emitted due to thermionic emission.

→ These electrons are accelerated in electric field of known potential difference.

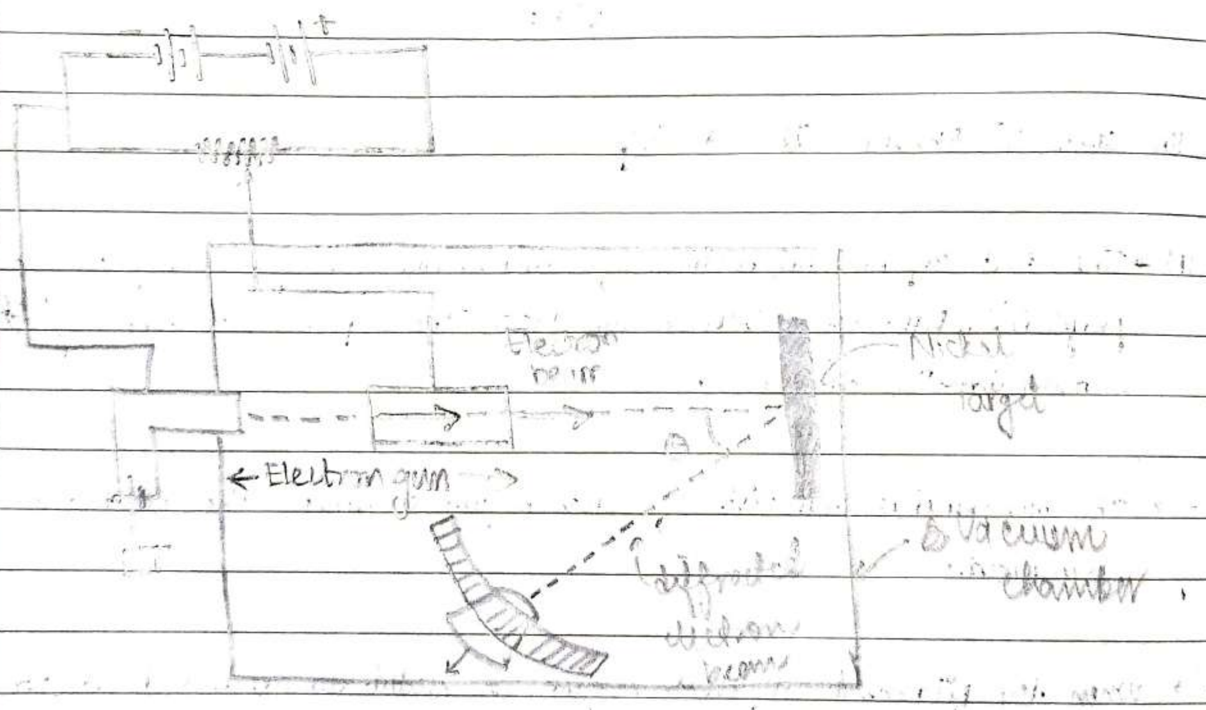
→ These electrons are collimated by suitable slits to obtain a fine beam which is then directed to fall on a large single crystal of nickel, known as target T.

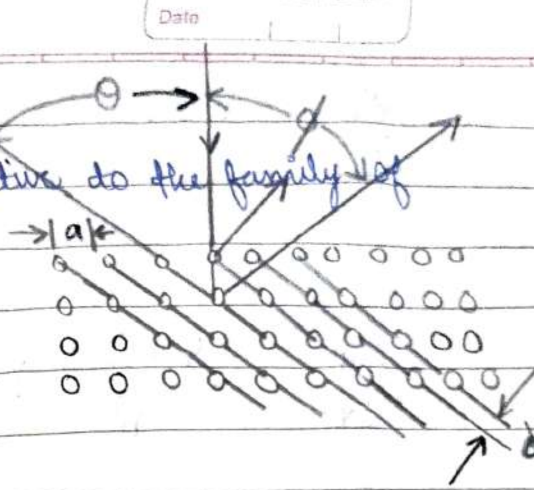
→ These electrons are collimated by suitable slits to obtain a fine beam which is then directed to fall on a large single crystal of nickel known as target T.

→ Using faraday cylinder C as collector, the intensity of scattered electrons is measured in all directions.

→ The faraday cylinder was moved over the circular scale and for a given accelerating voltage V the scattering curve is drawn.

→ For an electron accelerated by 54 volts applied and electrons beam incident normal to the crystal surface, the pronounced scattered direction is found to be 50°.





12) The diffraction angle, $\phi \approx 50^\circ$

The angle of ~~inclination~~ incidence relative to the family of Bragg's plane

$$\theta = \left(\frac{180-50}{2}\right) = 65^\circ.$$

From Bragg's equation

$$\lambda = 2d\sin\theta$$

$$\lambda = 2\times(0.91\text{Å})\times\sin 65 = 1.65\text{Å}$$

which is equal to λ calculated by de-Broglie's hypothesis. It confirms the wavelike nature of electrons.

(According to debroglie, the wavelength associated with an electron accelerated through V volts is

$$\lambda = \frac{12.26}{\sqrt{V}}\text{ Å}$$

Wavelength of 54V electrons

$$\lambda = \frac{12.26}{\sqrt{54}} = 1.67\text{Å}.$$

From X ray analysis we know that nickel ~~target~~ crystal acts as a plane diffraction grating space $d = 0.91\text{Å}$.)

14) If a particle of mass 'm' moving with velocity 'v' is associated with group of waves, let ψ be the wave function of the particle.

Let us consider a simple form of progressive wave represented by the equation.

$$\psi = \psi_0 \sin(\omega t - kx)$$

$$\frac{\partial \psi}{\partial x} = \psi_0 k\cos(\omega t - kx)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\psi_0 k^2 \sin(\omega t - kx) \qquad \frac{\partial^2 \psi}{\partial x^2} = -k^2\psi.$$

$$\frac{\partial^2 \psi}{\partial x^2} = -K^2 \psi . \qquad K = \frac{2\pi}{\lambda} \; ; \; \lambda = \frac{h}{p} \; ; \; P = mv .$$

$$K = \frac{2\pi m v}{h} \qquad K^2 = \frac{4\pi^2 m^2 v^2}{h^2} .$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{-4\pi^2 m^2 v^2}{h^2} \psi .$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{m^2 v^2}{\hbar^2} \psi \; ; \; \hbar = \frac{h}{2\pi} .$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{m^2 v^2}{\hbar^2} \psi = 0 .$$

The total energy E of a particle is the sum of its kinetic energy K and potential energy V.

$$E = K + V$$

$$K = \frac{1}{2} m v^2 \qquad E = \frac{1}{2} m v^2 + V .$$

$$\frac{1}{2} m v^2 = E - V$$

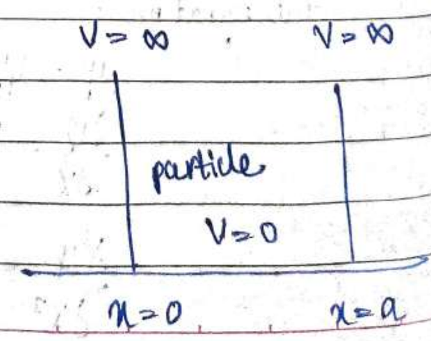
$$m v^2 = 2(E - V)$$

$$m^2 v^2 = 2m(E - V)$$

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E-V)}{\hbar^2} \psi = 0}$$

15) Consider a particle of rest mass m_0 enclosed in a one dimensional box. Boundary conditions for potential

$$V(x) = \begin{cases} 0 & \text{for } 0 < x < a \\ \infty & \text{for } 0 > x > a \end{cases}$$



Boundary conditions for ψ.

$$\psi = \begin{cases} 0 & \text{for } x = 0 \\ 0 & \text{for } x = a. \end{cases}$$

$$\lambda = \frac{h}{p} = \frac{2\pi}{K}.$$

$$K = \frac{p}{\hbar} = \frac{\sqrt{2m_0 E}}{\hbar}$$

Thus particle insider the box schrodinger equation is

$$K^2 = \frac{2m_0 E}{\hbar^2}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m_0}{\hbar^2} E\psi = 0 \longrightarrow (1)$$

$$\frac{\partial^2 \psi}{\partial x^2} + K^2 \psi = 0 \longrightarrow (2) \qquad \boxed{K^2 = \frac{2mE}{\hbar^2}} \longrightarrow (5)$$

~~since~~ since (2) is diff eq one of solution of (2) is

$$\boxed{\psi = A \sin Kx + B\cos Kx} \longrightarrow (3).$$

To find values of A & B we use boundary conditions.

→ 1st Boundary condition:-

$x = 0 \rightarrow \psi = 0$

put in value of x & ψ in (3).

$$0 = A(0) + B(1).$$

$$0 = 0 + B$$

$$\boxed{B = 0} \longrightarrow (4)$$

→ 2nd Boundary Conditions.

$x = a \rightarrow \psi = 0$ & $B = 0 \longrightarrow (4)$

put values in (3)

$$0 = A\sin Ka + 0$$

$$A\sin Ka = 0$$

$$\sin Ka = 0.$$

$$\sin Ka = 0 = \sin n\pi$$

$$K = \frac{n\pi}{a} \qquad K^2 = \frac{n^2\pi^2}{a^2}$$

Now put K^2 in eq (5).

$$\hbar = \frac{h}{2\pi}.$$

$$\frac{n^2\pi^2}{a^2} = \frac{2mE}{\hbar^2} \qquad E = \frac{\hbar^2 n^2 \pi^2}{2ma^2}. \qquad E = \frac{h^2 n^2 \pi^2}{4\pi^2(2ma^2)}.$$

$$E = \frac{h^2 n^2 \cancel{\pi^2}}{8\cancel{\pi^2} ma^2}$$

$$E = \frac{h^2 n^2}{8ma^2}. \qquad E_n = \frac{n^2 h^2}{8ma^2} \qquad n = 1, 2, 3, 4 \ldots$$

17) The conclusions are given by:-

- Energy of a particle inside the box can't be equal to zero. the miniomum energy of a particle obtained for $n=1$

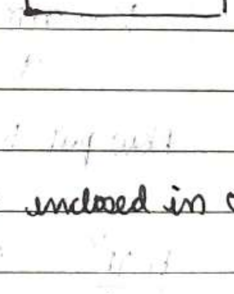
$$E_1 = \frac{h^2}{8m_0L^2} \text{ (zero point energy)}.$$

If $E_1 \longrightarrow 0$
momentum $\longrightarrow 0$ i.e., $\Delta p \longrightarrow 0$.
$\Delta x \longrightarrow \infty$

But $\Delta x_{max} = L$
Since the particle is confined in the box of dimensional L.

- Thus zero value of zero point energy violates the Heisenberg's uncertainly principle and hence zero value is not acceptable.
All the energy values are not possible for a particle in potential well.

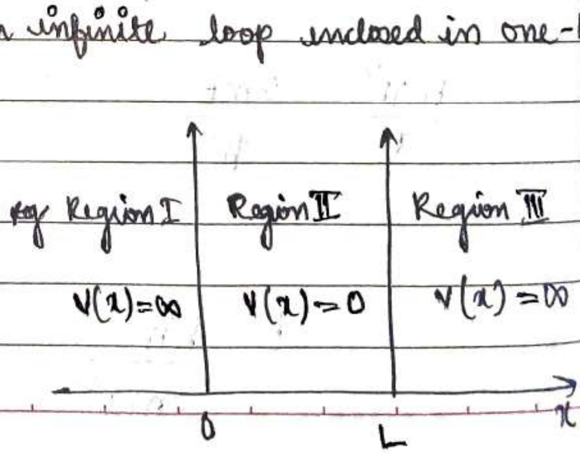
Energy is Quantised.



E_n are eigen values & 'n' is the quantum number.
Energy level (E_n) are not equally spaced.

16) Consider an electron of mass m_0 in an infinite loop enclosed in one-D box. Boundary conditions

$V(x) = 0$ for $L > x > 0$
$V(x) = \infty$ for $x \geq L$ and $x \leq 0$



$$\psi = \begin{cases} 0 & \text{for } x = 0 \\ 0 & \text{for } x = L. \end{cases}$$

Page No.
Date

Thus for a particle inside box schrodinger equation

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m_0 E\psi}{\hbar^2} = 0 \longrightarrow (1)$$

(i) $\because V = 0$ inside

$$\lambda = \frac{h}{p} = \frac{2\pi}{K} = \frac{h}{mv}$$

$$K = \frac{p}{\hbar} = \frac{\sqrt{2m_0E}}{\hbar} \qquad K^2 = \frac{2m_0E}{\hbar^2} \longrightarrow (2)$$

eq (1) will become

$$\frac{\partial^2\psi}{\partial x^2} + K^2\psi = 0 \longrightarrow (iii)$$

General solution of eq (iii)

$$\psi(x) = A\sin Kx + B\cos Kx \longrightarrow (iv)$$

Page No.
Date

Boundary conditions $\psi = 0 \longrightarrow x = 0$

$$\psi(0) = A\sin k(0) + B\cos k(0)$$

$$\psi(0) = 0 + B$$

$$\boxed{0 = B}$$

$$\psi(x) = A\sin kx \longrightarrow (v)$$

Boundary conditions state that $\psi = 0$ when $x = L$

$$\psi(L) = A\sin kL$$

$$0 = A\sin kL$$

$$\sin n\pi = 0 = A\sin kL$$

$$kL = n\pi$$

$$k = \frac{n\pi}{L} \longrightarrow (vi)$$

when $n \neq 0$ i.e., 1, 2, 3. — this gives $\psi = 0$ everywhere.

put value of k in (vi) eq (ii)

$$k^2 = \frac{2m_0E}{\hbar^2} \qquad k^2 = \frac{n^2\pi^2}{L^2}.$$

$$\frac{n^2\pi^2}{L^2} = \frac{2m_0E}{\hbar^2} \qquad E = \frac{\hbar^2n^2\pi^2}{2m_0L^2}$$

$$E = \frac{h^2n^2\cancel{\pi^2}}{4\cancel{\pi^2}(2m_0L^2)} = \frac{hn^2}{8m_0L^2}.$$

$$E = \frac{n^2h^2}{8m_0L^2}.$$

~~It is seen that energy is quantised~~

~~Let find the value of 'ψ_n'~~

$$\cancel{\int_0^a |\psi_n|^2 dx = 1}$$

our new function $\psi_n(x) = A\sin\left(\frac{n\pi x}{L}\right)$

Using Normalization condition

$$\int_{-\infty}^{\infty} |\psi_n(x)|^2 dx = 1.$$

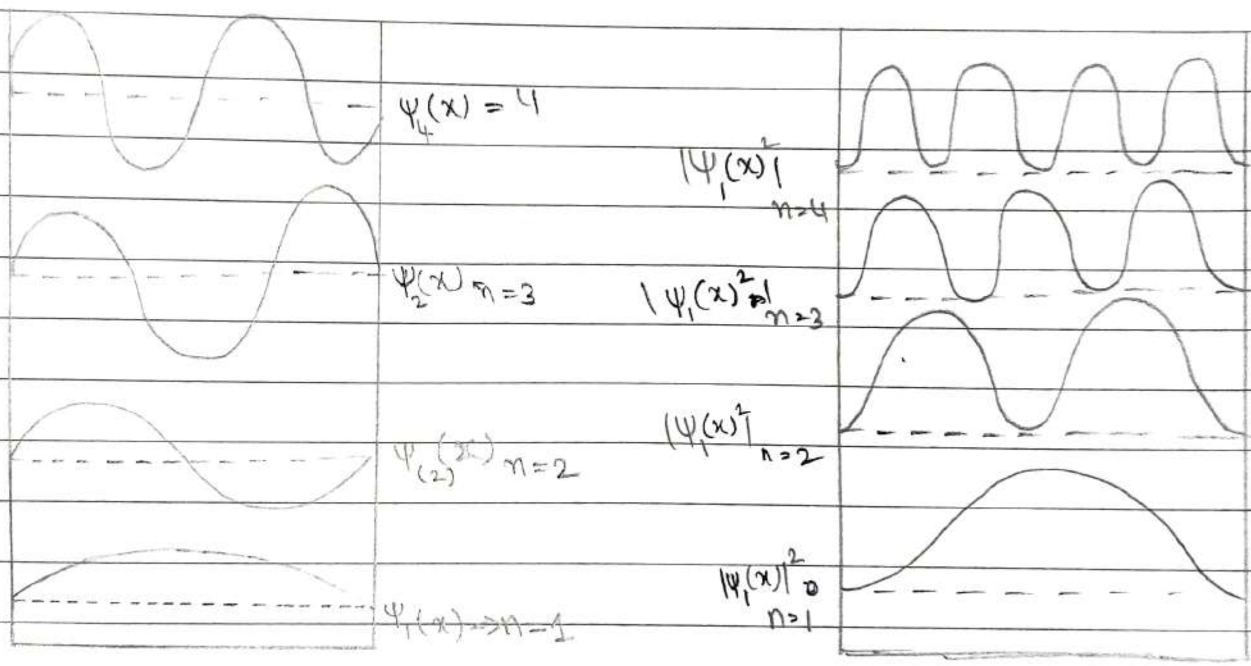
$$A^2 \int_0^L \sin^2 \frac{n\pi x}{L}\, dx = 1$$

$$A^2\left(\frac{L}{2}\right) = 1 \Rightarrow A = \sqrt{\frac{2}{L}}$$

The normalized eigen function of particle are

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

$$\psi_{II} = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \xrightarrow[\text{Born Interpretation}]{\text{Applying the}} |\psi_{II}|^2 = \frac{2}{L}\left(\sin \frac{n\pi x}{L}\right)^2$$



Page No.
Date

Part - C (Analytical Questions)

① $\lambda = \frac{h}{mv}$

$v = \frac{h}{m\lambda} = \frac{6.625\times10^{-34}}{9.1\times10^{-31}\times2.1\times10^{-10}} = 34.67\times10^{5}\ m/s$

Kinetic energy of electron.

$E = \frac{1}{2}mv^2 = \frac{1}{2}\times9.1\times10^{-31}\times34.67\times34.67\times10^{10}$

$= 0.5469\times10^{-17}\ J$

$= \frac{0.5469\times10^{-17}}{1.6\times10^{-19}}\ eV$

$= 34.182\ eV$

② $\lambda = \frac{h}{p}$ $\lambda = \frac{h}{mv}$

$\lambda = \frac{6.625\times10^{-34}}{1.674\times10^{-27}\times\frac{1}{10}\times3\times10^{8}} = 1.31\times10^{-14}\ m.$

③ $\lambda = \frac{12.26}{\sqrt{V}}\ A^\circ = \frac{12.26}{\sqrt{15000}} = \frac{12.26}{122.47} = 0.1\ A^\circ.$

④ $E = \frac{1}{2}mv^2 = 0.025\ eV$

$= 0.025\times1.6\times10^{-19}\ J.$

$v = \left(\frac{2\times0.025\times1.6\times10^{-19}}{1.674\times10^{-27}}\right)^{1/2} = (0.04779\times10^{8})^{1/2}$

$= 0.2186\times10^{4}\ m/s.$

$\lambda = \frac{h}{mv} = \frac{6.625\times10^{-34}}{1.674\times10^{-27}\ kg\times0.2186\times10^{4}} = 0.181\ nm.$

Page No.
Date

⑤ $\lambda = \frac{h}{mv}$

$v = \frac{h}{m\lambda} = \frac{6.625\times10^{-34}}{9.1\times10^{-31}\times1.66\times10^{-10}}$

$= 43.86\times10^{5}\ m/s.$

KE of electron.

$E = \frac{1}{2}mv^2 = \frac{1}{2}\times9.1\times10^{-31}\times43.86\times43.86\times10^{5}\times10^{5}$

$= 4.1927\times10^{-17}\ J.$

$\frac{4.1927\times10^{-17}}{1.6\times10^{-19}}\ eV = 262.04\ eV.$

⑥ $\lambda = \frac{12.26}{\sqrt{V}}\ A^\circ = \frac{12.26}{\sqrt{1600}} = \frac{12.26}{40} = 0.3065\ A^\circ.$

⑦ $E_n = \frac{n^2h^2}{8mL^2}$ = ground state $n=1$

$E_1 = \frac{(6.625\times10^{-34})^2}{8(\textit{9.50}\times10^{-31})(1\times10^{-1})^2}$

$= 6.456\times10^{-16}\ J.$

First excited state $E_2 = 4\times6.625\times10^{-16}\ J$

$= \cancel{25.8\times10^{-16}\ J}$

$= 2.57\times10^{-15}$ $\quad 2.57\times10^{-15}\times10^{-1}\ J$
$\quad \times10^{-1}$

$= 25.7\times10^{-16}\ J.$

8) $E_n = \frac{n^2h^2}{8mL^2} = \frac{(6.625 \times 10^{-34})^2}{8(9.1 \times 10^{-31})(0.1 \times 10^{-9})^2}$

$= 60.307 \times 10^{-19}$ J.

9) $E_n = \frac{n^2h^2}{8mL^2}$

$E_1 = \frac{(6.625 \times 10^{-34})^2}{8(9.1 \times 10^{-31})(10^{-10})^2}$

$= 0.6031 \times 10^{-17}$ J.

$E_2 = 4E_1$

$= 4 \times 0.6031 \times 10^{-17}$ J

$= 2.412 \times 10^{-17}$ Joules

$E_3 = 9 \times E_1$

$= 9 \times 0.6031 \times 10^{-17}$ J

$= 5.428 \times 10^{-17}$ J.

10) $E_n = \frac{n^2h^2}{8mL^2}$ $E_1 = \frac{(6.625 \times 10^{-34})^2}{8(9.1 \times 10^{-31})(4 \times 10^{-10})^2} = 0.346 \times 10^{-18}$ J.

→ MODULE-II Introduction to Solids & SemiConductors:-

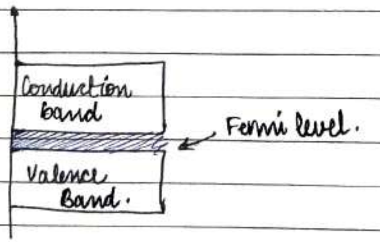
• SHORT ANSWER QUESTIONS:-

1) Bloch theorem states that if the potential V(r) in which the electron moves in periodic with a the periodicity of the lattice, then the solution Ψ(r) is given by

$V(x) = V(x+a)$
'a' periodicity of potential. The schrodiger wave equation of moving electron is
$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\psi = 0 \longrightarrow ①$

the solution of eq ①
$\psi(x) = e^{ikx} u_k(x) \longrightarrow ②$
where $u_k(x) = u_k(x+a) \longrightarrow ③$
Eq ③ represents periodic function & ~~e^{iky}~~ e^{ikx} represents plane wave. The above statement is known as ~~Block~~ Bloch Theorem and eq ② is known as Block function.

2) Metallic solids are composed of metal cations held ~~to fast~~ together by a delocalized "sea" of valence electrons. Because their electrons are mobile, metallic solids are good conductors of heat an electricity.



The conduction bands & the valence band are very close to each other or can be said that they overlap each other.
This helps the incited electrons to move from one orbital to another and hence metals are good conductors of electricity.

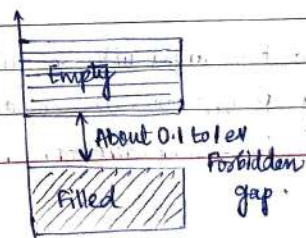
3) Crystalline solids are classified into the following based on band theory:-
→ Solids Conductors
→ Semiconductors
→ Insulators.

• Conductors:- Valence band completely filled & no forbidden energy gap. So there is no gap between valence band & conduction Band. Hence there is easy conduction of electicity.

• Insulators:- Valence Band is completely filled but conduction band is empty. But the gap between the bands is very large which don't allow electrons to cross over. Hence, there is no conduction of electricity.

• SemiConductors:- Semiconductors have moderate gap between Conduction ~~Bon~~ Band & valence band so, the electrons can move from valence band to conduction band but there should be any external energy provided or doping to be done.

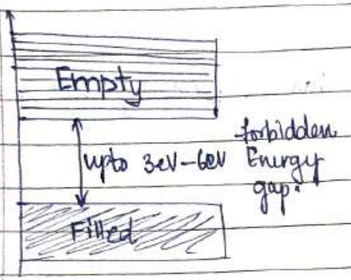
4) Semiconductors are elements that have electrical properties between those of insulators & conductors.



The electrons can only jump ~~from~~ from valence band to conduction band if and only if external energy is supplied like increase in temperature.
Eg:- Silicon & Germanium.

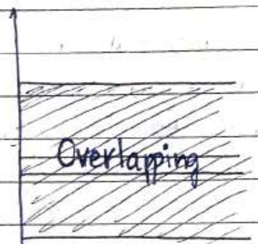
Page No.
Date

5) Insulators are elements or compounds that have high resistivity and are bad conductors of electricity. They have completely filled valence band but an empty conduction band. Eg:- Plastic



• The gap is too wide. Due to this the electrons from valence band can't jump to the conduction band.

6) Conductors are ~~also~~ elements with very low resistivity & high conductivity values.



No forbidden energy gap and valence band conduction band over lap. Many electrons are available for electrical conduction.

7) Explain classification of semiconductors based on variation of conductivity in terms of temperature & doping.

Ans:- • Intrinsic Semiconductors:- A semiconductor in its pure state is known as intrinsic semiconductor.

• Extrinsic Semiconductor:- A semiconductor doped with a suitable impurity to increase its conductivity is called extrinsic semiconductor.

Page No.
Date

Extrinsic semiconductor are of 2 types:-

• n type semiconductors:- doped with pentavalent impurity like As, Sb, Bi, etc. Here electrons work as charge carriers. $n_e \gg n_h$.

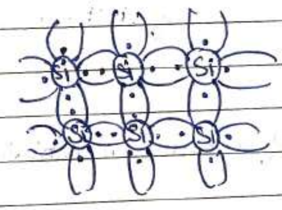
• p type semiconductors:- doped with trivalent impurities like Al, B, etc. the charged holes acts as charge ~~cat~~ carriers. $n_h \gg n_e$.

→ the electrical conductivity of semiconductors increases with increase in temperature because, the no. of electrons for from the valency band can jump into the conduction Band in a semiconductor.

8) ~~Sup~~ Summarise your understanding of Intrinsic semiconductor? Give example.

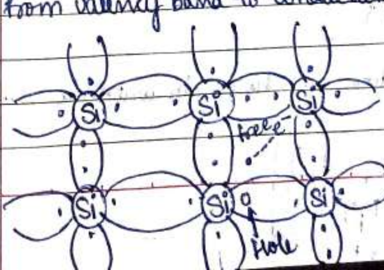
Pure germanium or silicon ~~or~~ are called as intrinsic semiconductors. Each atom ~~possesse~~ possesses four valence electron in the outermost orbital.

At temp = 0 ~~so~~ it is as shown below.



But when (T>0) temp increase the bonds break & become free to move from valency band to conduction band.

Due to this there is flow of electricity.



Eg:- Germanium & Silicon.

Page No.
Date

9) Define the term 'Doping'? Why do you need to dope and intrinsic semiconductor?

Ans:- 'Doping' refers to a process in which an impurity (trivalent or pentavalent) is added to the intrinsic semiconductor.

Doping is required so as to increase the rate & efficiency of current flow in the semiconductor. In N-TYPE semiconductors every pentavalent impurity atom donate one electron in the crystal. & in P-TYPE semiconductor every positively charged hole acts as a charge carrier.
Eg of n-type semiconductor is As, Sb, Bi & Eg of ptype semiconductor is Al, B, etc.

10) What is intrinsic semiconductor? How is it more useful than intrinsic semiconductor?

Ans:- A semiconductor doped with a suitable impurity for better conductivity is called an extrinsic semiconductor.
→ continue 9th Answer.

11) Write an expression of carrier concentration of electrons & holes in intrinsic semiconductors.

Ans The expression of carrier concentration of electrons in intrinsic semiconductors is:-

$$n = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} \exp\left[\frac{E_F - E_C}{kT}\right].$$

The expression of carrier concentration of holes in intrinsic semiconductors is:-

$$p = 2\left[\frac{2\pi m_h^* k_B T}{h^2}\right]^{3/2} \exp\left[\frac{-(E_F - E_C)}{k_B T}\right]$$

Page No.
Date

12) Write an expression for intrinsic carrier concentration in intrinsic semiconductor.

The no of free electrons and holes per unit volume of intrinsic semiconductors is called intrinsic carrier concentration (n_i).
The expression is given as:-

$$n_i = 2\left[\frac{2\pi m^* k_B T}{h^2}\right]^{3/2} e^{\frac{-E_g}{2k_B T}}.$$

13) Give expression of Fermi Level in intrinsic semiconductors.

Ans:- At temp T Kelvin, the electron concentration 'n' is equal to hole concentration 'p' in intrinsic semiconductors.
The expression is given as follows:-

$$E_F = \frac{(E_C + E_V)}{2} + \frac{3}{4} k_B T \ln\left[\frac{m_p^*}{m_e^*}\right] \quad \{At\ T > 0K\}.$$

At T = 0K

$$E_F = \left[\frac{E_C + E_V}{2}\right].$$

14) Define Fermi Energy or Fermi Level.

It is usually referred to the energy difference between the highest & lowest occupied single particle states in a quantum system.

15) Write Fermi Distribution function for electrons in metals.

Ans The expression for fermi distribution function of electrons in metals is given by:-

$$f(E) = \frac{1}{1 + \exp(E - E_F)/k_B T} = \frac{1}{e^{[E - E_F]/kT} + 1}$$

~~16) Show an expression for carrier concentration in P-type semiconductor.~~

~~Ans:- The expression for carrier concentration in P-type semiconductor is given by:-~~

$$= (2N_A)^{1/2}\left[\frac{2\pi m_p^* k_B T}{h^2}\right]^{3/4} e^{\left[\frac{-(E_A-E_V)}{2k_BT}\right]}.$$

At T=0K

$$E_F = \frac{E_V+E_A}{2}$$

At T>0K.

$$E_F = \frac{(E_V+E_A)}{2} - \frac{k_BT}{2}\log_e\left[\frac{N_A}{2\left[\frac{2\pi m_p^* k_BT}{h^2}\right]^{3/2}}\right]$$

17) Write an expression for carrier concentration of holes in n-type semiconductor.

At T=0K;

$$E_F = \frac{E_C+E_D}{2}$$

At T>0K;

$$E_F = \frac{E_C+E_D}{2} + \frac{k_BT}{2}\log_e\left[\frac{N_D}{2\left[\frac{2\pi m_e^* k_BT}{h^2}\right]^{3/2}}\right]$$

16) Show an expression for carrier concentration of P-type semiconductors.

Ans:-

At T=0K;

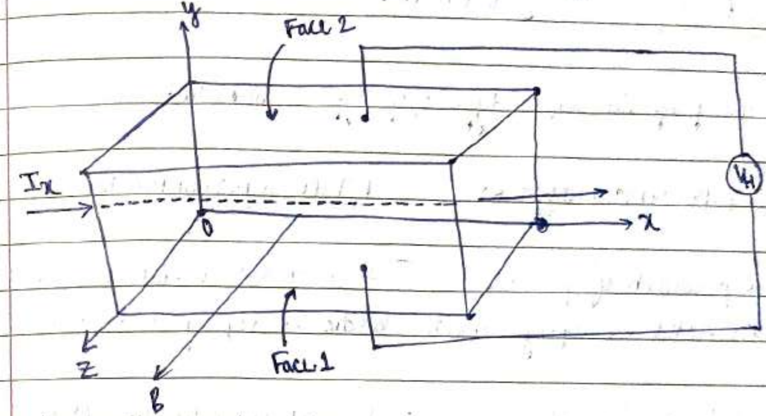
$$E_F = \frac{E_V+E_A}{2}$$

At T>0K;

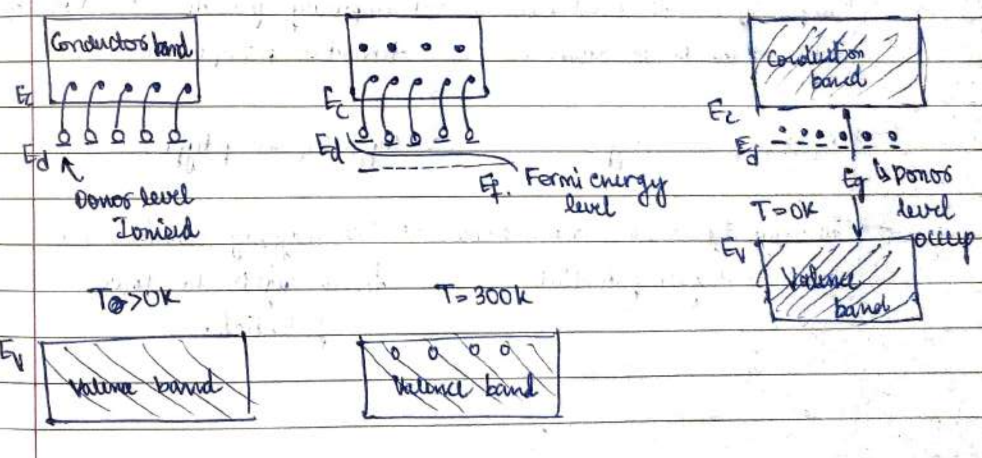
$$E_F = \frac{E_V+E_A}{2} - \frac{k_BT}{2}\log_e\left[\frac{N_A}{2\left[\frac{2\pi m_p^* k_BT}{h^2}\right]^{3/2}}\right]$$

18). Give statement of Hall effect using a proper diagram representing current, magnetic field & Hall voltage?

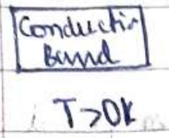
Ans:- If a piece of semiconductor carrying a current 'I' is placed in a transverse or perpendicular magnetic field, then the electric field 'E_H' is generated in perpendicular to both 'I' & 'B'. Hence production of Hall electric field 'E_H' and generation of Hall voltage V_H by applying current 'I' & 'B' in perpendicular directions is called 'Hall effect'.

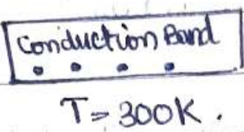


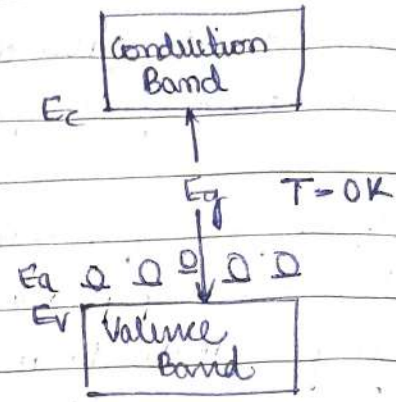
19) N-Type semiconductor:-



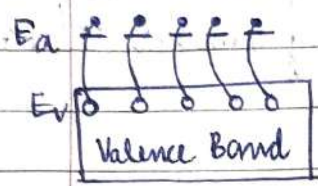
→ P - Type SemiConductor:-

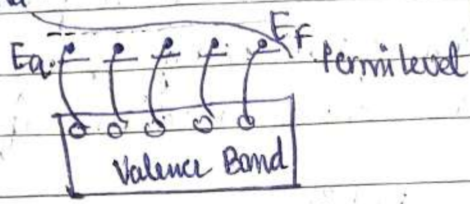






Acceptors have accepted electrons from valence Band.





20) Relate the properties of n-type & p-type semiconductors.

N-TYPE SEMICONDUCTORS	P-TYPE SEMICONDUCTORS.
→ V group elements of periodic table are added as doping element.	→ III group elements of periodic table are added as doping element.
→ Pentavalent ~~grou~~ impurity like Ar, Ph, Bi are added	→ Trivalent impurity like Al, Ga is added.
→ Impurity provide extra electrons & termed as Donor atoms.	→ Impurity provide extra hole & termed as Acceptor Atoms.
→ $n_e >> n_h$ in n type	→ $n_h >> n_e$ in p type.
→ The majority carriers move from lower to higher potential.	→ The majority carriers move from higher to lower potential.

Engineering Physics

Module-2

- Irfan.Mohd

Part-B:

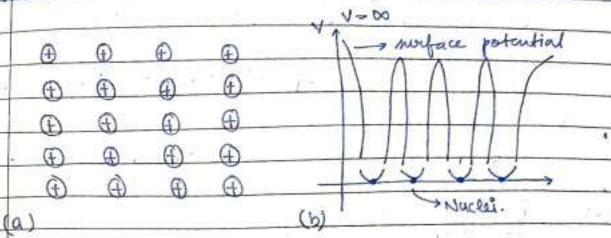
1 | Explain in detail Bloch's theorem and the motion of electron in a periodic potential.

→ Electron In A Periodic Potential :-

Crystalline solid consists of a lattice which is composed of a large no. of ionic cores at regular intervals & the conduction electrons move throughout the lattice.

let us consider the picture of lattice in only one dimension, i.e., only an array of ionic cores along x-axis.

If we ~~plot~~ plot the potential energy of a conduction ~~band~~ electron as a position in the lattice, the variation of potential energy is shown in fig. The potential is ~~maximum~~ minimum at +ve ion sites & ~~min~~ maximum at -ve ion sites.



(a) Periodic positive ion cores inside metallic crystals.
(b) One dimensional periodic potential in crystal.

Periodic potential V(x) may be defined by means of lattice constant 'a' as

$V(x) = V(x+a)$.

For such electrons moving in a periodic potential, Bloch ~~conds~~ considered the solution as

$$\psi_k(x) = \underbrace{\exp(ikx)}_{\text{plane wave}}\, \underbrace{u_k(x)}_{\text{modulation fn}}$$

propagation vector → wave no.

$u_k(x)$ is periodic with periodicity of crystal lattice

i.e., $u_k(x) = u_k(x+a)$.

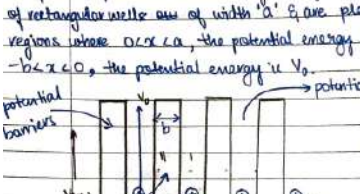
Therefore according to Bloch's Theorem.

the free electron wave is a plane wave modulated by periodic function $u_k(x)$ ~~who~~ which has same periodicity as potential of electron.

2 Show that the energy spectrum of an electron contains a number of allowed energy bands separated by forbidden bands, using Kronig-Penny model.

→ Kronig-Penney Model (Band Theory of Solids).

The periodic potential assumed by Kronig & Penny is shown in the fig. A series of rectangular wells ~~are~~ of width 'a' & are placed at a separate of b in the regions where $0<x<a$, the potential energy is zero and in regions such as $-b<x<0$, the potential energy is V_0.



a → interionic spacing.

One dimensional periodic potential assumed by Kronig & Penny

$$V_x \begin{cases} 0, & 0<x<a. \text{ (potential well } V=0) \\ V_0, & -b<x<0. \text{ (potential barrier } V_0). \end{cases}$$

The dynamic behavior of electrons in the Kronig penny model is represented by the following Schrodinger's eq^n:-

$$\frac{d^2\psi}{dx^2} + \left[\frac{2m}{\hbar^2}\right]E\psi = 0 \quad , \text{ for } 0<x<a \quad \left[\because \hbar = \frac{h}{2\pi}\right].$$

$$\& \ \frac{d^2\psi}{dx^2} + \left[\frac{2m}{\hbar^2}\right](E-V_0)\psi = 0 \quad , \text{ for } -b<x<0 \longrightarrow (1).$$

Lets ~~not~~ assume that the total energy E of the electron under consideration is less than V_0.

Further lets substitute, $\alpha^2 = \frac{2mE}{\hbar^2}$ & $\beta^2 = \frac{2m}{\hbar^2}(V_0 - E) \longrightarrow (2)$.

where α & β are ~~neg~~ real quantities.

$$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \quad \text{for } 0<x<a$$

$$\frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \quad \text{for } -b<x<0 \longrightarrow (3).$$

These eq^n can be solved using Bloch's Theorem. The final solutions of eq(3) is given in the form of the following condition.

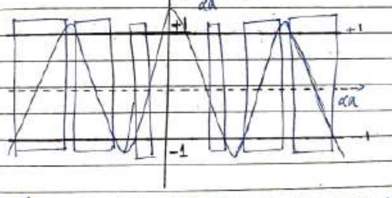
$$\boxed{P\frac{\sin\alpha a}{\alpha a} + \cos\alpha a = \cos ka.} \quad \left[\because \alpha^2 = \frac{2mE}{\hbar^2}\right]$$

where $P = \frac{mbV_0 a}{\hbar^2}$ is scattering power of potential barrier. & V_0 is barrier length.

That means, eq (3) will have a solution only when the condition (4) is satisfied.

Graph of αa versus $\frac{P\sin\alpha a}{\alpha a} + \cos\alpha a$

For best understanding of eq (4), let us plot the condition (4) i.e., LHS versus αa. Since the value of $\cos ka$ on RHS of eq (4) lie b/w +1 & -1, αa (which is measure of energy) can take only those values for which the total left hand side (LHS) value lies b/w -1 to +1. Other values are not allowed. This means energy E is restricted to lie within certain ranges which form the allowed energy bands or zones.



Plot of the left hand side of eq (4) as a function of αa for $P = \frac{3\pi}{2}$.

$\frac{P\sin\alpha a}{\alpha a} + \cos\alpha a = \cos ka$. +1 & -1 permitted value of α

3 Explain the origin of energy band formation in solids.

Origin of Energy Bands formation in Solids :-

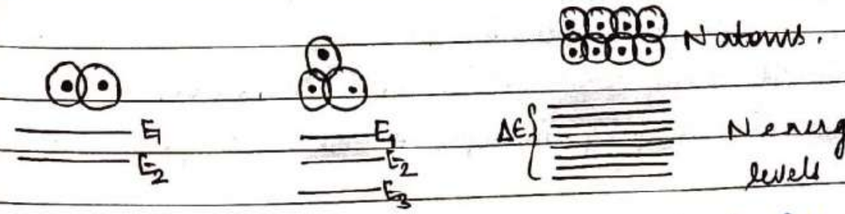
When we consider isolated atom, the electrons are tightly bound & have discrete, sharp energy levels.

When two identical atoms are bought closer the outer most orbits of these atoms overlap & interact.

If more atoms are bought together more levels are formed & for a solid of N-atoms, each of the energy levels of an atom split into N levels of energy.

The level of are so close together that they form an almost continuous band.

The width of this band depends on the degree of ~~open~~ overlap of electrons of adjacent atoms and is largest for outermost atomic electrons.



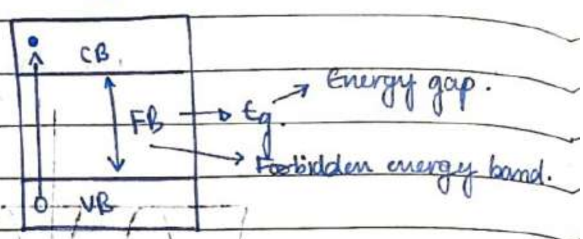
Splitting of energy levels due to interatomic interaction.

The energy band in solids are important to determine many of physical properties of solids. The allowed energy bands

(i) Valence Band.

(ii) Conduction Band.

The band corresponding to the outermost orbit is called conduction band & the next inner band is called ~~forbidden~~ forbidden energy ~~band~~ gap.

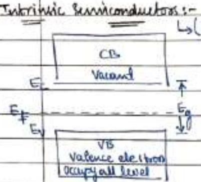


On the basis of band theory, solids can be ~~brea~~ broadly classified into three categories, viz insulators, semiconductors & conductors.

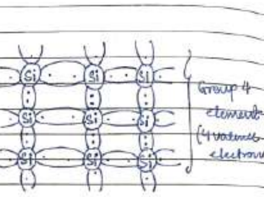
4 Compare the intrinsic and extrinsic semiconductors, indicating on an energy level diagram, the conduction and valence bands, donor and acceptor levels for intrinsic and extrinsic semiconductors.

Basically, semiconductors are of two types :-
1. Intrinsic Semiconductors
2. Extrinsic Semiconductors
(i) n-type semiconductors.
(ii) p-type semiconductors.

① Intrinsic Semiconductors :- ↳ (PURE)

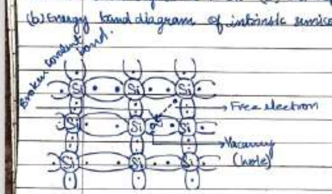


(b)



(a)

Intrinsic silicon crystal at T = 0K (a) 2d representation of silicon crystal & (b) Energy band diagram of intrinsic semiconductor.



(a) $\therefore n_h = n_e$

(b)

Silicon crystal at temp T > 0K.
(a) Due to thermal energy breaking of covalent bonds take place
(b) Energy band representation.

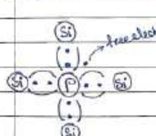
② Extrinsic Semiconductors :-
Electrical conductivity of intrinsic semiconductor is low. It can be increased by adding a small quantity of impurity to an intrinsic semiconductor.

"The process of adding impurities to an intrinsic semiconductor to enhance the electrical conductivity is called Doping".
(i) p-type & (ii) n-type.

(i) N-type semiconductor :- when pentavalent impurity like antimony or phosphorous or arsenic is added to a pure silicon (or germanium) crystal, four of its five valence electrons form covalent bonds with its neighbouring four silicon atoms.

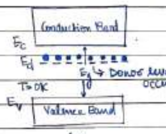
The fifth of each impurities atom is loosely held when its parent atom & can be easily detached by supplying a little amount of energy.

When pentavalent impurities atom like antimony or phosphorous or arsenic is added to pure silicon (or germanium) crystal, four of its five valence electrons form covalent bonds with its neighbouring four silicon atoms.



(a)

Representation of n-type silicon at T = 0K.



(b)

Energy band diagram at T = 0K

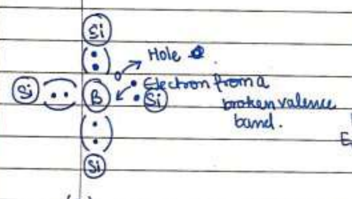
(ii) P-type semiconductors:-

When trivalent impurities like ~~boron~~ Boron, Aluminium or gallium are added to pure Silicon (or germanium ~~or~~ crystal), each impurity atom forms 3 covalent bonds with three of its 4 neighbouring silicon atoms. There is deficiency of electrons to form fourth covalent bond & attain close shell configuration.

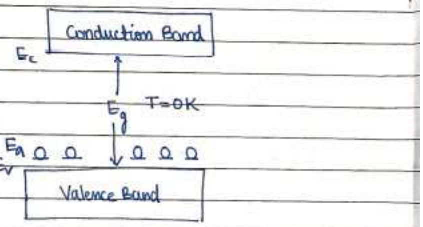
Energy levels are created to the electron defficiencies just above the top of the valence bande. These levels are called acceptor levels.

At room temp, many electrons nearer to the top of the ~~van~~ valence band jump into acceptor level or only ~~free~~ few electrons jump into conduction band, leaving behind holes in the valence band. Therefore the majority charge carriers are holes in valence band & minority charge carriers are electrons in the conduction band.

As there are excess holes, "The semiconductor thus formed after doping trivalent impurities to a pure semiconductor" is called p-type semiconductor & impurities are called acceptor impurities.



(a) Representation of p-type silicon at T=OK



(b) Energy band diagram at T=OK.

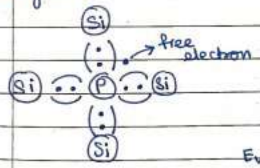
5 Obtain an expression for carrier concentration of n- type semiconductor.

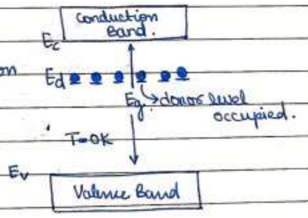
Extrinsic Semi Conductor - n type - carrier concentration :-

When pentavalent impurity atom like antimony, or phosphorous or arsenic is added to the pure silicon (or germanium) crystal, four of its five valence electrons form covalent bonds with its neigbouring four silicon atoms.

The fifth electron of each impurity atom is loosly held ~~when~~ with its parent atom & can be easily detached by supplying a little amount of energy.

When pentavalent impurity atoms like antimony or phosphorous or arsenic is added to a pure silicon (or germanium) crystal, four of its five electrons form covalent bond with its neighbouring four silicon ~~crystals~~ atoms.





Let N_D be the donor concentration (no. of donor atoms per unit volume) & E_D be the donor energy level.

At room temp, donor energy level is filled with ~~one~~ electrons.
As temperature increases, donor atoms get ionized & electrons take transition

from donor energy level to the conduction band. Due to this density of electrons in the conduction band increases. The density of electrons in conduction band is given by

$$n = 2\left[\frac{2\pi m_e^* K_B T}{h^2}\right]^{3/2} e^{\frac{-(E_C - E_F)}{K_B T}} \longrightarrow (1)$$

Density of ionized donor is given by

$$N_d[1 - F(N_D)] = N_D \exp\left[\frac{-(E_F - E_D)}{K_B T}\right]$$

$$= N_D e^{\frac{-(E_F - E_D)}{KBT}} \longrightarrow (2)$$

Fermi-Dirac distribution function.
(probability of an electron occupying an energy level).

At very low temp, when electron hole pairs are not generated due to breaking of covalent bonds, the no. of electrons in conduction bands must be equal to the no. of donor atoms per unit volume.

Therefore carrier concentration in n-type semiconductor is given by,

$$n_{(\text{Extrinsic } n\text{-type})} = \underset{(\text{density of electrons})}{n} = N_d[1 - F(E_D)]$$

$$[n_{(\text{Extrinsic } n\text{-type})}]^2 = n \cdot N_d[1 - F(E_D)]$$

$$= 2\left[\frac{2\pi m_e^* K_B T}{h^2}\right]^{3/2} e^{\left(\frac{E_F - E_C}{K_B T}\right)} \cdot N_D e^{\frac{(E_D - E_F)}{K_B T}}$$

$$n_{(\text{extrinsic } n\text{-type})} = (2N_D)^{1/2}\left[\frac{2\pi m_e^* K_B T}{h^2}\right]^{3/4} e^{\left(\frac{E_D - E_C}{2K_B T}\right)}$$

Thus density of electrons in the conduction band at moderately low temperatures is proportional to square root of donor concentration in n-type semiconductors.

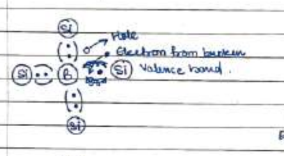
6 Obtain an expression for carrier concentration of p- type semiconductor.

P-type Semiconductors:-

When trivalent impurity atoms like boron or Aluminium or gallium are added to pure silicon (or germanium crystal), each impurity atom forms three covalent bonds with the three of its four neighbouring silicon atoms. There is deficiency of electrons to form fourth covalent bond & attain closed shell configuration.

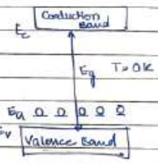
Energy levels are created to the electron deficiencies just above the top of the valence bands. These levels are called acceptor levels.

At around room temp, many electrons nearer to the top of valence band jump into acceptor levels & only few electrons jump to the conduction band, leaving behind holes in the valence band.



(a)

Representation of p-type silicon at T = 0K



(b)

Energy band diagram at T = 0K.

Let N_A be the acceptor concentration (no. of acceptor atoms per unit volume) and E_A be the acceptor energy level.

At room temperature, acceptor energy level is empty. As temperature increases, more acceptor atoms get ionized & electrons take transition from valence band to acceptor energy level leaving holes in valence band.

Density of holes in the valence band is given by

$$p = 2\left[\frac{2\pi m_p^* k_B T}{h^2}\right]^{3/2} e^{-\frac{(E_F - E_V)}{k_B T}} \longrightarrow (1)$$

Density of ionised acceptors is given by:-

$$N_A F(E_A) = N_A \exp\left[\frac{-(E_A - E_F)}{k_B T}\right]$$

$$= N_A e^{(E_F - E_A)/k_B T} \longrightarrow (2).$$

$$F(E) = \exp\left\{\frac{E_F - E}{k_B T}\right\}$$

At very low temperatures, when electron-hole pairs are not generated due to breaking of covalent bonds, the no of holes in valence band must be equal to the no. of acceptor atoms per unit volume.

Therefore carrier concentration in p-type semiconductors is given by

$$P_{(\text{Extrinsic p-type})} = p = N_A F(E_A).$$

$$[P_{(\text{extrinsic p-type})}]^2 = p \cdot N_A F(E_A)$$

$$= 2\left[\frac{2\pi m_p^* k_B T}{h^2}\right]^{3/2} e^{-\frac{(E_F - E_V)}{k_B T}} \cdot N_A \cdot e^{\frac{-(E_A - E_F)}{k_B T}}$$

$$P(\text{extrinsic p-type}) = (2N_A)^{1/2}\left[\frac{2\pi m_p^* k_B T}{h^2}\right]^{3/4} e^{\frac{(E_V - E_A)}{2k_B T}}$$

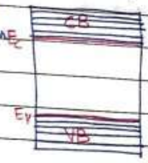
Thus density of holes in valence band at moderately low temperatures is proportional to the square root of acceptor concentration in p-type semiconductors.

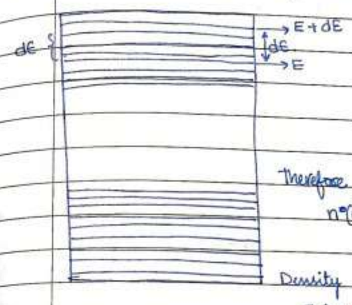
7 Find the mathematical expression for intrinsic carrier concentration and hence show that the Fermi level lies at the middle for an intrinsic semiconductor.

CARRIER CONCENTRATION IN INTRINSIC SEMICONDUCTORS :-

↳ no. of charge carriers present per unit volume of the material.
'n' → represented

Derivation:- No. of free electrons per unit volume of semiconductor having energies E & E+dE in CB is represented by n(E) dE. Obtained by multiplying the density of energy states $Z_c(E)\,d(E)$. [No. of energy states per unit volume] & Fermi-Dirac distribution function for the probability of occupation of electrons $F_c(E)$.





Certain set of energy level are called to be ▷ dE.

Therefore

$$n(E)\,dE = \left[Z_c(E)\,d(E)\right]\left[F_c(E)\right] \longrightarrow (i)$$

Density of energy states is given by

$$Z_c(E)\,d(E) = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}(E)^{1/2}\,dE \longrightarrow (ii)$$

where m_e^* is effective mass of an electron.

electron in periodic potential is accelerated.

↓ mass varies with velocity.

Here $E > E_c$. Since E_c is the min energy of the CB. Hence eq (ii) becomes.

$$Z_c(E)\,d(E) = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}(E-E_c)^{1/2}\,dE \longrightarrow (iii)$$

Fermi Dirac distribution function; where $E_F \rightarrow$ Fermi ~~Energy~~ level.

$$F_c(E) = \frac{1}{1+e^{\frac{E-E_F}{K_BT}}} \longrightarrow (iv)$$

Here $E > E_F$, i.e., $e^{\frac{E-E_F}{K_BT}} \gg 1$. Here '1' can be neglected in eq (iv)

$$F_c(E) = \frac{1}{e^{\frac{E-E_F}{K_BT}}} \qquad F_c(E) = e^{\frac{E_F-E}{K_BT}} \longrightarrow (v).$$

Substituting eq (iii) & eq (v) in eq (i) we get.

Page No.
Date

$$n(E)dE = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}(E-E_c)^{1/2}\,e^{\frac{E_F-E}{K_BT}}\,dE.$$

$$n(E)dE = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}(E-E_c)^{1/2}\,e^{\frac{E_F-E}{K_BT}}\,dE \longrightarrow (vi)$$

Density of electron in entire conduction band is obtained by integrating eq (vi) between limits E_c & ∞.

$$n = \int_{E_c}^{\infty}\frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}(E-E_c)^{1/2}\,e^{\frac{E_F-E}{K_BT}}\,dE.$$

$$n = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}\int_{E_c}^{\infty}(E-E_c)^{1/2}\,e^{\frac{E_F-E}{K_BT}}\,dE \longrightarrow (vii)$$

Let $\varepsilon = E - E_c$

$d\varepsilon = dE$ {E_c is constant}. The limits $\varepsilon = 0$ to $\varepsilon = \infty$.

Hence eq (7) can be written as:

$$n = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}\int_{\varepsilon=0}^{\infty}(\varepsilon)^{1/2}\,e^{\frac{E_F-(\varepsilon+E_c)}{K_BT}}\,d\varepsilon.$$

$$n = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}e^{\frac{(E_F-E_c)}{K_BT}}\int_{\varepsilon=0}^{\infty}(\varepsilon)^{1/2}\,e^{\frac{-\varepsilon}{K_BT}}\,d\varepsilon \longrightarrow (8)$$

In eq (8) but $\int_{\varepsilon=0}^{\infty}(\varepsilon)^{1/2}e^{\frac{-\varepsilon}{K_BT}}d\varepsilon = \frac{\sqrt{\pi}}{2}(K_BT)^{3/2} \longrightarrow (9)$

Sub, (9) in (8) we get.

$$n = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}e^{\frac{(E_F-E_c)}{K_BT}}\frac{\sqrt{\pi}}{2}(K_BT)^{3/2}$$

$$n = \frac{1}{4}\left[\frac{8\pi m_e^* K_BT}{h^2}\right]^{3/2}e^{\frac{(E_F-E_c)}{K_BT}} \qquad n = \frac{1}{4}\cdot\frac{8}{4}\left[\frac{2\pi m_e^* K_BT}{h^2}\right]^{3/2}e^{\frac{E_F-E_c}{K_BT}}$$

electron concentration in Conduction band $\rightarrow$ $n = 2\left[\frac{2\pi m_e^* K_BT}{h^2}\right]e^{\frac{(E_F-E_c)}{K_BT}} \longrightarrow (9)$.

Equal no of holes & electrons.

8 Explain the dependence of Fermi level on carrier-concentration and temperature in extrinsic semiconductors

From donor energy level to the conduction band. Due to this density of electrons in the conduction band increases. The density of electrons in conduction band is given by

$$n = 2\left[\frac{2\pi m_e^* k_B T}{h^2}\right]^{3/2} e^{-\frac{(E_C - E_F)}{k_B T}} \longrightarrow (1)$$

Density of ionized donor is given by

N_d^+

$$N_d[1-F(N_D)] = N_D \exp\left[\frac{-(E_F - E_D)}{k_B T}\right]$$

$$= N_D e^{\frac{-(E_F - E_D)}{k_B T}} \longrightarrow (2)$$

Fermi-Dirac distribution function.
(probability of an electron occupying an energy level).

At very low temp, when electron hole pairs are not generated due to breaking of covalent bonds, the no. of ~~of~~ electrons in conduction bands must be equal to the no. of donor atoms per unit volume.

Therefore carrier concentration in n-type semiconductor is given by.

$$n_{(\text{Extrinsic } n\text{-type})} = \underset{(\text{density of electrons})}{n} = N_d[1-F(E_D)]$$

$$[n_{(\text{Extrinsic } n\text{-type})}]^2 = n \cdot N_d[1-F(E_D)]$$

$$= 2\left[\frac{2\pi m_e^* k_B T}{h^2}\right]^{3/2} e^{\left(\frac{E_F - E_C}{k_B T}\right)} \cdot N_D e^{\frac{(E_D - E_F)}{k_B T}}$$

$$n_{(\text{extrinsic } n\text{-type})} = (2N_D)^{1/2}\left[\frac{2\pi m_e^* k_B T}{h^2}\right]^{3/4} e^{\left(\frac{E_D - E_C}{2k_B T}\right)}$$

Thus density of electrons in the conduction band at moderately low temperatures is proportional to square root of donor concentration in n-type semiconductors.

At very low temp, the no. of electrons in the conduction band must be equal to the no. of donor atoms per unit ~~volume~~ volume.

Hence eq. (1) & (2) we get.

$$2\left[\frac{2\pi m_e^* k_B T}{h^2}\right]^{3/2} e^{\frac{[E_F - E_C]}{k_B T}} = N_D e^{\frac{[E_D - E_F]}{k_B T}}$$

$$\frac{e^{\frac{(E_F - E_C)}{k_B T}}}{e^{\frac{(E_D - E_F)}{k_B T}}} = \frac{N_D}{2\left[\frac{2\pi m_e^* k_B T}{h^2}\right]^{3/2}}$$

$$e^{\frac{2E_F - (E_C + E_D)}{k_B T}} = \frac{N_D}{2\left[\frac{2\pi m_e^* k_B T}{h^2}\right]^{3/2}}$$

Take logarithm on both sides.

$$2E_F - (E_C + E_D) = \log\left[\frac{N_D}{2\left[\frac{2\pi m_e^* k_B T}{h^2}\right]^{3/2}}\right] \times k_B T$$

$$E_F = \frac{(E_C + E_D)}{2} + \frac{k_B T}{2} \log\left[\frac{N_D}{2\left[\frac{2\pi m_e^* k_B T}{h^2}\right]^{3/2}}\right] \quad \{\text{At } T > 0K\} \longrightarrow (3)$$

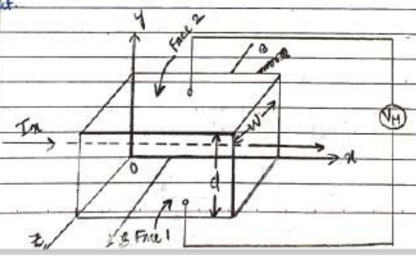
At T = 0K

$E_F = \frac{(E_C + E_D)}{2}$. That is E_F lies b/w E_C & E_D

9 Explain in detail Hall effect and obtain an expression for Hall coefficient.

Hall Effect:-

If a piece of semiconductor carrying a current 'I' is placed in a transverse or perpendicular magnetic field, then an Electrical field 'E_H' is generated in perpendicular to both 'I' & 'B'. Hence production of Hall electric field 'E_H' and generation of Hall voltage V_H by applying current 'I' & 'B' in perpendicular direction is called Hall Effect.



→ Determination of Hall coeff R_H, Hall Voltage V_H.

Assume an N-type semiconductor. Let 'I' be the current passed along X-axis. Let 'B' the magnetic field applied along 'Z' axis. Due to current 'I', due to flow of electrons, the force acting on electrons is in the opposite direction to the direction of conventional current.

Due to magnetic field 'B' there is a downward force act on each electron given by BeV. This makes the electron deviated in downward direction along Z-axis. This causes a negative charge to accumulate on the bottom face.

$F_B = BeV$ ← strength of magnetic field.

'e' → charge of electron; 'V' → velocity of electron.

A potential diff. (emf) is formed from top to bottom of the specimen. The potential diff causes a field E_H to flow along negative Y axis. Due to E_H along negative Y direction a force of eE_H act in upward direction along positive Y-axis.

Hence, $F_E = eE_H$ ← potential difference. (e ← charge of electron)

under equilibrium condition,

Upward force due to E_H = Downward force due to B.

$$F_B = F_E$$

$$eE_H = BeV$$

$$V = \frac{E_H}{B} \rightarrow (1)$$

If J is current density then $J = neV$. (n → carrier concentration.)

$$V = \frac{J}{ne} \rightarrow (2)$$

(V ← drift velocity)

$\boxed{R_H = \frac{1}{ne}}$ → used for application.

Equating (1) & (2) we get:-

$$\frac{E_H}{B} = \frac{J}{ne} \qquad E_H = \frac{JB}{ne}$$

$$E_H = \left[\frac{+1}{ne}\right][BJ]$$

Let Hall coefficient $R_H = \frac{+1}{ne}$ for electrons, $R_H = \frac{+1}{pe}$ for holes

10 | Show the graphical representation of Kronig-Penny model. Explain the conclusions drawn from the graph.

Same as 2nd answer then…..

Date

Conclusion of the graph:-

1) The energy spectrum consists of the alternative regions of the allowed & vacant bands. Forbidden bands imply that the energy levels that lie in this region are not occupied by electrons.

2) The allowed (shaded) bands are narrowed for low values of energy & become broader as energy increases, the unallowed (forbidden) band becoming narrow.

3)(a) For $P=0$ (i.e., on extreme left), the whole energy spectrum is quasi-continuous, that is all allowed bands are joined together forming an almost continuum.

(b) However, the width of a particular allowed band decreases in the value ~~as~~ of P. As $P \to \infty$, the allowed energy band compress into simple energy levels & thus result in line spectrum.

11	Summarize the classification of materials with neat energy band diagrams.

On the basis of band theory, solids can be ~~brea~~ broadly classified into three categories, viz insulators, semiconductors & conductors.

Insulators:-

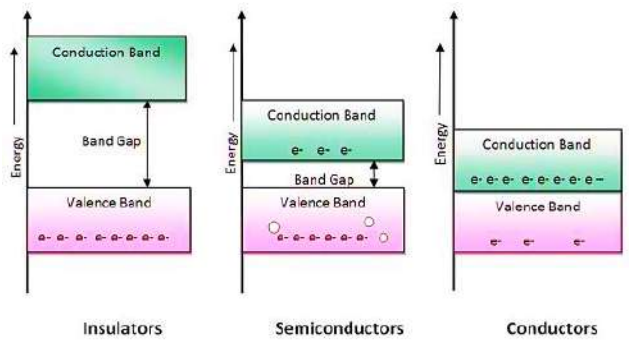
1) In case of insulator, the forbidden energy gap is very wide. Due to ~~at~~ this fact electrons can't jump from valence band to conduction band.
2) They have completely filled valence band & completely empty conduction band.
3) The resistivity of insulator is very high.
4) Insulators are bad conductors of electricity.

Semi Conductors:-

1) In semiconductors, the band gap is very small (0.7eV for germanium and 1.1eV for silicon).
2) At 0K, there are no electrons in the conduction band & valence band is completely filled. As the temp increases, electrons in the valence ~~conduction~~ band jump into conduction band.
3) The resistivity varies from 10^{-4} to 10^{7} ohm meter.
4) They have electrical properties b/w conductors & insulators.

Conductors:-

1) In case of conductors, there is no forbidden gap & valence band conduction band overlap each other.
2) Plenty of free electrons are available for electrical conduction.
3) They possess very low resistivity & high conductivity values.
4) Metals like copper, iron, etc. are best examples of conductors.

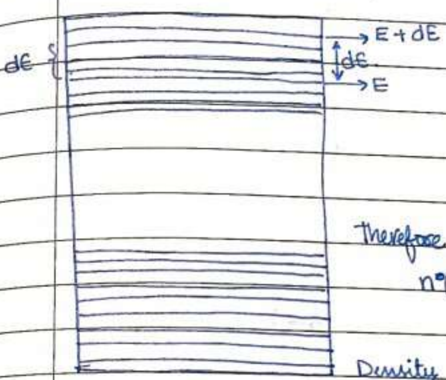


12	Derive an expression for the electron concentration in the conduction band of an intrinsic semiconductor.

CARRIER CONCENTRATION IN INTRINSIC SEMICONDUCTORS :-

↳ no. of charge carriers present per unit volume of the material.
'n' → represented

Derivation:- No. of free electrons per unit volume of semiconductor having energies E & E+dE in CB is represented by $n(E)dE$. Obtained by multiplying the density of energy states $Z_c(E)d(E)$. [No. of energy states per unit volume] & Fermi-Dirac distribution function for the probability of occupation of electrons $F_c(E)$.



Certain set of energy level are called to be ▷ dE.

Therefore

$$n(E)dE = [Z_c(E)d(E)][F_c(E)] \longrightarrow (i)$$

Density of energy states is given by

$$Z_c(E)\,d(E) = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}(E)^{1/2}dE \longrightarrow (ii)$$

where m_e^* is effective mass of an electron.

electron in periodic potential is accelerated. → mass varies with velocity.

Here $E > E_c$. Since E_c is the min energy of the CB. Hence eq (ii) becomes.

$$Z_c(E)d(E) = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}(E-E_c)^{1/2}dE \longrightarrow (iii)$$

substituting eq (iii) & eq (v) in eq (i) we get.

$$n(E)dE = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}(E-E_c)^{1/2}\, e^{\frac{E_F-E}{K_BT}}\, dE.$$

Page No.
Date

$$n(E)dE = \frac{\pi}{2}\left[\frac{8m_e^*}{h}\right]^{3/2}(E-E_c)^{1/2}\, e^{\frac{E_F-E}{K_BT}}\, dE \longrightarrow (vi)$$

Density of electron in entire conduction band is obtained by integrating eq (vi) between limits E_c & ∞.

$$n = \int_{E_c}^{\infty} \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}(E-E_c)^{1/2}\, e^{\frac{E_F-E}{K_BT}}\, dE.$$

$$n = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}\int_{E_c}^{\infty}(E-E_c)^{1/2}\, e^{\frac{E_F-E}{K_BT}}\, dE \longrightarrow (vii)$$

let $\varepsilon = E - E_c$

$d\varepsilon = dE$ {E_c is constant}. The limits $\varepsilon = 0$ to $\varepsilon = \infty$.

Hence eq (7) can be written as:

$$n = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2}\int_{\varepsilon=0}^{\infty}(\varepsilon)^{1/2}\, e^{\frac{E_F-(\varepsilon+E_c)}{K_BT}}\, d\varepsilon.$$

$$n = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2} e^{\frac{(E_F-E_c)}{K_BT}}\int_{\varepsilon=0}^{\infty}(\varepsilon)^{1/2}\, e^{\frac{-\varepsilon}{K_BT}}\, d\varepsilon \longrightarrow (8)$$

In eq (8) but $\int_{\varepsilon=0}^{\infty}(\varepsilon)^{1/2} e^{\frac{-\varepsilon}{K_BT}}\, d\varepsilon = \frac{\sqrt{\pi}}{2}(K_BT)^{3/2} \longrightarrow (9)$

Sub. (9) in (8) we get.

$$n = \frac{\pi}{2}\left[\frac{8m_e^*}{h^2}\right]^{3/2} e^{\left(\frac{E_F-E_c}{K_BT}\right)}\frac{\sqrt{\pi}}{2}(K_BT)^{3/2}$$

$$n = \frac{1}{4}\left[\frac{8\pi m_e^* K_BT}{h^2}\right]^{3/2} e^{\left(\frac{E_F-E_c}{K_BT}\right)} \qquad \therefore\ n = \frac{1}{4}\cdot\frac{8}{4}\left[\frac{2\pi m_e^* K_BT}{h^2}\right]^{3/2} e^{\frac{E_F-E_c}{K_BT}}$$

electron concentration in conduction band

$$n = 2\left[\frac{2\pi m_e^* K_BT}{h^2}\right] e^{\frac{(E_F-E_c)}{K_BT}} \longrightarrow (9).$$

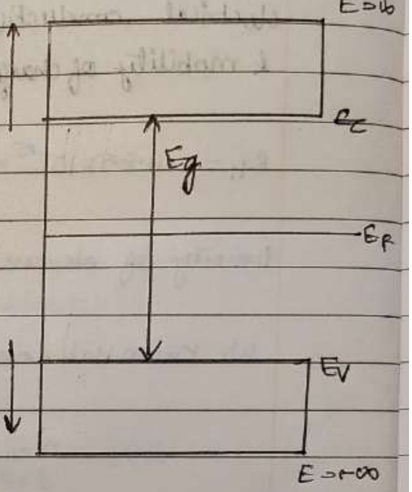
13 Derive an expression for the hole concentration in the valence band of an intrinsic semiconductor.

Hole concentration in intrinsic semiconductor :-

The no. of holes per unit volume having energy in range E & $E+dE$, in valence band of an intrinsic semi-conductor.

$$dP = g(E)\cdot dE\,(1-F(E)) \longrightarrow ①$$

where $1-F(E)$ represents the probability of absence of electron in the particular level E



$$1-F(E) = 1 - \frac{1}{1+\exp\left(\frac{E-E_F}{k_BT}\right)}$$

$$1-F(E) = \frac{1+\exp\left(\frac{E-E_F}{k_BT}\right)-1}{1+\exp\left(\frac{E-E_F}{k_BT}\right)}$$

$$1-F(E) = \frac{\exp\left(\frac{E-E_F}{k_BT}\right)}{1+\exp\left(\frac{E-E_F}{k_BT}\right)}$$

for valence band $1 \gg \exp\left(\frac{E-E_F}{k_BT}\right)$

So exponential term can be neglected in the denominator of above equation

$$[1-F(E)] = \exp\left[\frac{E-E_F}{k_BT}\right] \longrightarrow ②$$

$$dp = \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2} E^{1/2} \exp\left(\frac{E-E_F}{k_BT}\right) dE$$

$\cancel{dp = \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2} E}$ For valence band the energy level $(E_V-E)^{1/2}$

$$dp = \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2} (E_V-E)^{1/2}\cdot \exp\left(\frac{E-E_F}{k_BT}\right) dE.$$

To get total no. of holes in the valence band. We have to integrate the above eq^n. between the limits bottom of the valence band to the top of valence band

$$P = \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2} \int_{E_V-\infty}^{E_V} (E_V-E)^{1/2}\cdot \exp\left(\frac{E-E_F}{k_BT}\right) dE$$

$$P = \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2} \int_{-\infty} (E_V-E)^{1/2} \exp\left(\frac{E-E_F+E_V-E_V}{k_BT}\right) dE$$

$$P = \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2}\int_{-\infty}^{E_V}(E_V-E)^{1/2}\exp\left(\frac{E-E_V}{K_BT}\right)\exp\left(\frac{E_V-E_F}{K_BT}\right)dE$$

Page No.
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$$P = \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2}\exp\left(\frac{E_V-E_F}{K_BT}\right)\int_{-\infty}^{E_V}(E_V-E)^{1/2}\exp\left(\frac{E-E_V}{K_BT}\right)dE$$

$$P = \pi/2\left(\frac{8m_h^*}{h^2}\right)^{3/2}\exp\left(\frac{E_V-E_F}{K_BT}\right)\int_{-\infty}^{E_V}(E_V-E)^{1/2}\exp\left(\frac{-(E_V-E)}{K_BT}\right)dE$$

Let $\frac{E_V-E}{K_BT} = x$

lower limit

$E = -\infty$ then $x = \infty$

$E_V - E = (K_BT)(x)$

upper limit

$E = E_V$ then $x = 0$

$-dE = K_BT(dx)$

$dE = -K_BT(dx)$

$$P = \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2}\exp\left(\frac{E_V-E_F}{K_BT}\right)\int_{\infty}^{0}(K_BT\cdot x)^{1/2}\exp(-x)(-K_BT)dx$$

$$P = \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2}\exp\left(\frac{E_V-E_F}{K_BT}\right)\int_{0}^{\infty}(K_BT)^{3/2}x^{1/2}\exp(-x)dx$$

$$P = \frac{\pi}{2}\left(\frac{8m_h^*K_BT}{h^2}\right)^{3/2}\exp\left(\frac{E_V-E_F}{K_BT}\right)\int_{0}^{\infty}x^{1/2}\exp(-x)dx$$

$$P = \frac{\pi}{2}\left(\frac{8m_h^*K_BT}{h^2}\right)^{3/2}\exp\left(\frac{E_V-E_F}{K_BT}\right)\frac{\sqrt{\pi}}{2} \qquad \because \pi\sqrt{\pi} = \pi^{3/2}$$

$$P = \frac{\pi^{3/4}}{4}\left(\frac{8m_h^*K_BT}{h^2}\right)^{3/2}\exp\left(\frac{E_V-E_F}{K_BT}\right).$$

$$P = \frac{8}{4}\left(\frac{2\pi m_h^*K_BT}{h^2}\right)^{3/2}\cdot\exp\left(\frac{E_V-E_F}{K_BT}\right)$$

$$P = 2\left(\frac{2\pi m_h^*K_BT}{h^2}\right)^{3/2}\exp\left(\frac{-(E_F-E_V)}{K_BT}\right)$$

$P = N_V\exp\left(\frac{-(E_F-E_V)}{K_BT}\right)$ where $N_V = 2\left(\frac{2\pi m_h^*K_BT}{h^2}\right)^{3/2}$

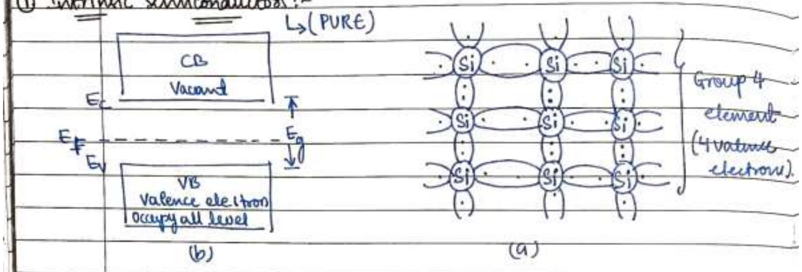
N_V = No. of holes. (unit volume in valence band)

14 What is an intrinsic semiconductor? Explain why an intrinsic semiconductor behaves as an insulator at 0K. Give 2D representations of the crystal of Silicon at T = 0K and T > 0K.

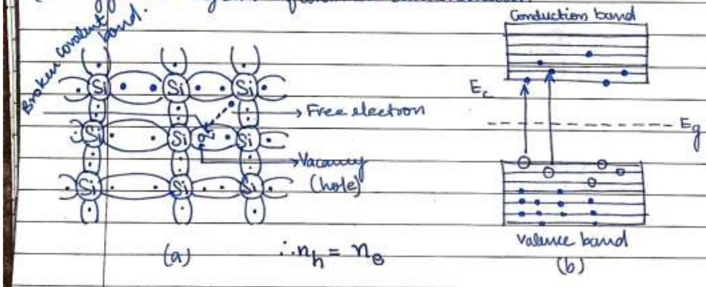
Semiconductors that are chemically pure, in other words, free from impurities are termed as intrinsic semiconductors. The number of holes and electrons is therefore determined by the properties of the material itself instead of the impurities. In intrinsic semiconductors, the number of excited electrons is equal to the number of holes; $n = p$. They are also termed as undoped semiconductors or i-type semiconductors. Silicon and germanium are examples of i-type semiconductors. These elements belong to the IVth Group of the periodic table and their atomic numbers are 14 and 32 respectively.

Intrinsic semiconductors behave as insulators at 0K as there is no external energy provided for the electrons to jump into the conduction band and therefore there is no chance of conduction to take place.

① Intrinsic Semiconductor:- ↳(PURE)



Intrinsic silicon crystal at T=0K (a) 2d representation of silicon crystal & (b) Energy band diagram of intrinsic semiconductor.



Silicon crystal at temp T>0K.

(a) Due to thermal energy breaking of covalent bonds take place

(b) Energy band representation.

15 Summarize what an extrinsic semiconductor is and distinguish between n-type and p-type semiconductors.

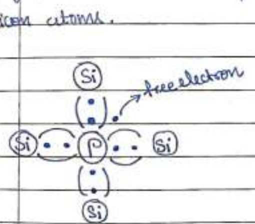
Extrinsic semiconductors are semiconductors that are doped with specific impurities. The impurity modifies the electrical properties of the semiconductor and makes it more suitable for electronic devices such as diodes and transistors.

While adding impurities, a small amount of suitable impurity is added to pure material, increasing its conductivity by many times. Extrinsic semiconductors are also called *impurity semiconductors* or *doped semiconductors.*

(i) N-type semiconductor :- when pentavalent impurity like antimony or phosphorous or arsenic is added to a pure silicon (or germanium) crystal, four of ~~the~~ its five valence electrons form covalent bonds with its neighbouring ~~for~~ four silicon atoms.

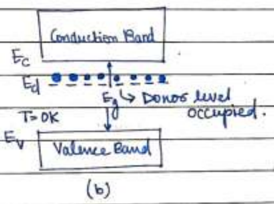
The ~~fit~~ fifth of each impurities atom is loosely held when its parent atom & can be easily detached by supplying a little amount of energy.

When pentavalent impurities atom like antimony or phosphorous or arsenic ~~os~~ is added to pure silicon (or germanium) ~~eg~~ crystal, four of its five valence electrons form covalent bonds with its neighbouring four silicon atoms.



(a)

Representation of n-type silicon at $T = 0K$.



(b)

Energy band diagram at $T = 0K$

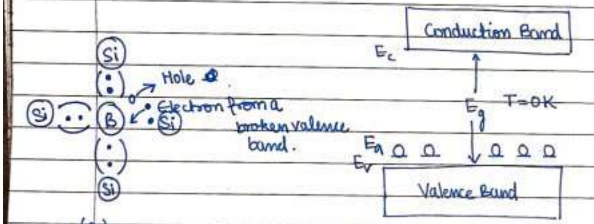
(ii) P-type semiconductors:-

When trivalent impurities like ~~boro~~ Boron, Aluminium or gallium are added to pure silicon (or germanium ~~cr~~ crystal), each impurity atom forms 3 covalent bonds with three of its 4 neighbouring silicon atoms. There is deficiency of electrons to form fourth covalent bond & attain close shell configuration.

Energy levels are created to the electron defficiencies just above the top of the valence bands. These levels are called acceptor levels.

At room temp, many electrons nearer to the top of the ~~vam~~ valence band jump into acceptor level or only ~~fe~~ few electrons jump into conduction band, leaving behind holes in the valence band. Therefore the majority charge carriers are holes in valence band & minority charge carriers are electrons in the conduction band.

As there are excess holes, "The semiconductor thus formed after doping trivalent impurities to a pure semiconductor" is called p-type semiconductor & impurities are called acceptor impurities.



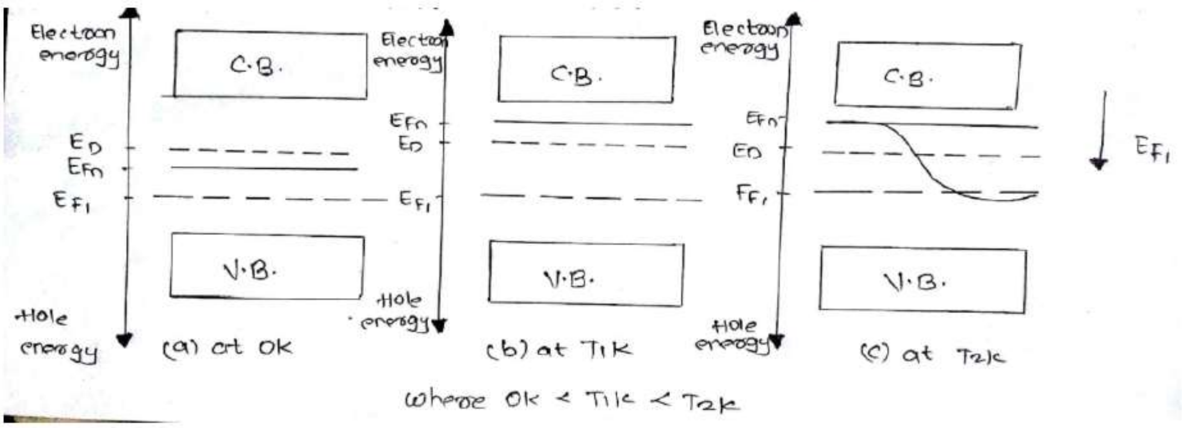
(a) Representation of p type silicon at T=0K

(b) Energy band diagram at T=0K.

16 | Explain the variation of Fermi-level position with temperature in n-type semiconductor with neat diagram.

IN n-TYPE SEMICONDUCTOR.

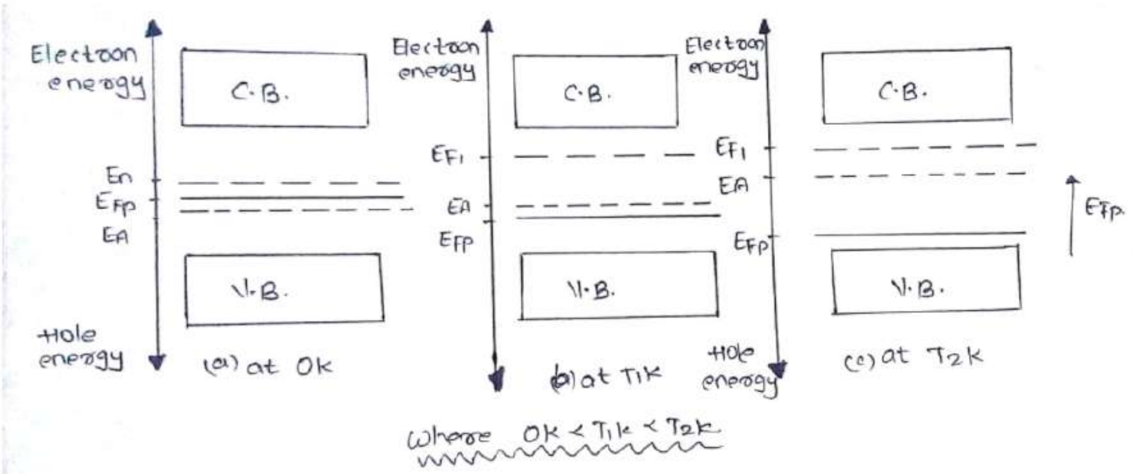
- At 0K the fermi level E_{Fn} lies between the conduction band and the donor level.
- As temperature increases more and more electrons shift to the conduction band leaving behind equal number of holes in the valence band. These electron hole pairs are intrinsic carriers.
- With the increase in temperature the intrinsic carriers dominate the donors.
- To maintain the balance of the carrier density on both sides the fermi level E_{Fn} gradually shifts downwards.
- Finally at high temperature when the donor density is almost negligible E_Fn is very close to E_{Fi}.



17 Summarize the variation of Fermi-level position with temperature in p-type semiconductor with neat diagram.

IN p-TYPE SEMICONDUCTOR.

- At 0K the fermi level E_{Fp} in a p-type semiconductor lies between the acceptor level and the valence band.
- With the increase in temperature more and more holes are created in the valence band as equal number of electrons move to the conduction band.
- As temperature increases the intrinsic holes dominate the acceptor holes.
- Hence the number of intrinsic carriers in the conduction band and in the valence band become nearly equal at high temperature.
- The fermi level E_{Fp} gradually shifts upwards to maintain the balance of carrier density above and below it.
- At high temperature when the acceptor density become insignificant as compared to the intrinsic density, E_{Fp} is positioned very close to the intrinsic fermi level E_{Fi} but little below it.



Did not find the 18th and 19th solution…..

20	Infer the uses of Hall effect in determining various properties of a semiconductor

→ **Application of Hall effect:-**

① For determination of type of given semiconductor.
② For N type, $R_H = -ve$; p type $R_H = +ve$.

② To determine carrier conc 'n' & 'p'; that is $n = p = \frac{1}{R_H e}$

③ ~~Determine~~ Determination of mobility of charge carriers (μ)

$$\sigma = ne\mu$$

$$\mu = \left[\frac{1}{ne}\right]\sigma = R_H \sigma$$

$$\mu = \left[\frac{V_H b}{BI}\right]\sigma$$

conductivity.

4) Measurement of magnetic flux density & Hall voltage.

5) To determine the sign of charge carriers, whether the conductivity is due to electrons or holes.

Part-C:

1 Find carrier concentration of an intrinsic semiconductor of band gap 0.7eV at 300K. [Given that the effective mass of electron = effective mass of hole = rest mass of electron].

Given data :

$$E_g = 0.7\ eV = 0.7\times1.6\times10^{-19}\ \text{Joules}$$

$$T = 300\ K$$

$$m_0 = 9.1\times10^{-31}\ \text{kg}$$

$$K_B = 1.38\times10^{-23}\ \text{J/K}$$

$$\text{Formula } n_i = 2\left[\frac{2\pi m_0 K_B T}{h^2}\right]^{\frac{3}{2}} \exp\left(\frac{-E_g}{2k_B T}\right)$$

$$n_i = 2\left[\frac{2\times3.14\times9.10\times10^{-31}\times1.38\times10^{-23}\ X\ 300}{(6.625\times10^{-34})^2}\right] \times \exp\left(\frac{-0.7\times1.6\times10^{-19}}{2\times1.38\times10^{-23}\times300}\right)$$

$$n_i = 33.49\times10^{18}/m^3$$

2 | What temperature would the E_F is shifted by 15% from middle of forbidden gap (E_g)? Given E_g =1.2eV, effective mass of holes is 5 times that of electrons.

Given data: Energy gap, $E_g = 1.2 \text{ eV} = 1.2 \times 1.6 \times 10^{-19}$ J

At temperature T, $m_h^*/m_e^* = 5$

We know that the Fermi energy is

$$E_F = \frac{E_C + E_V}{2} + \frac{3kT}{4} \ln\left(\frac{m_h^*}{m_e^*}\right)$$

For 0 K, E_F lies between the band gap. Hence, E_F is given by

$$E_F = E_V + 0.6 \text{ eV}$$

$$(E_V + 0.6)\,\text{eV} = \frac{E_C + E_V}{2} \quad (\because T = 0)$$

Let Fermi level shift by 15% at temperature T, i.e., 0.18 eV

i.e., $$(E_V + 0.78)\,\text{eV} = \frac{E_C + E_V}{2} + \frac{3kT}{4} \ln 5$$

Substracting the above equation, we get

$$0.18\,\text{eV} = \frac{3kT}{4} \ln 5$$

Simplying, we get

$$T = \frac{4 \times 0.288 \times 10^{-19}}{3 \times 1.38 \times 10^{-23} \times \ln 5}$$

$$= \frac{1.152 \times 10^{-19}}{6.663 \times 10^{-23}} = 1729\,\text{K}$$

Therefore, the temperature at which Fermi level is shifted 15% is 1729K

3 For silicon semiconductor with bandgap 1.12 eV, determine the position of the Fermi level at 300 K if $m_e^* = 0.12\ m_o$ and $m_h^* = 0.28\ m_o$.

Given data:

E_g=1.12 = 1.12 x 1.6 x 10^{-19} J; T = 300K; m_e^* = 0.12m_o & m_p^* = 0.28 m_o

$$E_F = \frac{E_C + E_V}{2} + \frac{3}{4} K_B T \ln\left(\frac{mh}{me^*}\right)$$

$$\frac{E_C+E_V}{2} = E_F \quad \text{(or)} \quad E_C\text{-}E_V = E_g$$

$$E_F = \frac{E_C+E_V}{2} \quad \text{(or)} \quad \frac{E_C+E_V}{2} = \frac{E_g}{2}$$

$$E_F = \frac{1.12\ X\ 1.6\ X\ 10^{-19}}{2} + \frac{3}{4} \times 1.38 \times 10^{-23} \times 300\ X \ln 5$$

$$E_F = 0.576 \text{ev}.$$

4 Calculate Hall voltage developed across the width of the slab of a metallic slab carrying a current of 30A is subjected to a magnetic field of 1.75T. The magnetic field is perpendicular to the plane of the slab and to the current. The thickness of the slab is 0.35 cm. The concentration of free electrons in the metal is 6.55 x 10^{28} electrons/m^3.

Given data:

Current, I = 30 A
Magnetic field, B = 1.75 T
Thickness of the slab, t = 0.35 x 10^{-2}
Concentration of electrons, n = 6.55 x 10^{28} /m^3

The Hall voltage is given by

$$V_H = \frac{R_H BI}{d}$$

$$V_H = \frac{BI}{ned}$$

Substituting the value, we get

$$V_H = \frac{1.75 \text{ x } 30}{6.55 \text{ x } 10^{28} x \text{ } 1.6 \text{ x } 10^{-19} \text{ x } 0.35 \text{ x } 10^{-2}}$$

$$= 14.313 \text{ x } 10^{-7} \text{ volt}$$

The Hall voltage is $1.431 \text{ x } 10^{-6}$ V or $1.431\ \mu V$.

5	Calculate the resistivity if the intrinsic carrier density at room temperature in Ge is $2.37 \text{ x } 10^{19} /m^3$ and the electron and hole mobilities are 0.38 and 0.18 $m^2 V^{-1} s^{-1}$ respectively.

$$\text{Carrier density, } n = 2.37 \text{ x } 10^{19} / m^3$$

$$\textit{Electron mobility, } \mu_e = 0.38 \text{ m}^2 V^{-1} s^{-1}$$

$$\text{Hole } \textit{mobility, } \mu_e = 0.18 \text{ m}^2 V^{-1} s^{-1}$$

$$\textit{Conductivity, } \sigma_i = n_i e(\mu_e + \mu_e) \textit{ and}$$

$$\text{Resistivity, } \rho = \frac{1}{\sigma}$$

$$\textit{Conductivity, } \sigma_i = n_i e(\mu_e + \mu_e)$$

$$= 2.37 \text{ x } 10^{19} \text{ x } 1.6 \text{ x } 10^{-19} \text{ x } (0.38 + 0.18)$$

$$= 2.1235 \Omega^{-1} m^{-1}$$

$$\text{Resistivity, } \rho = \frac{1}{\sigma} = \frac{1}{2.1235} = 0.471 \Omega\text{m}$$

6	Calculate the intrinsic charge carrier concentration for Ge at 27° C (For Ge, atomic weight =72.6, Density=5400 kg/ml, Band gap = 0.70 eV)

Given data : $E_g = 0.7\,eV = 0.7\times1.6\times10^{-19}$ Joules

$$T = 300\text{ K}$$

$$m_o = 9.1\times10^{-31}\text{ kg}$$

$$K_B = 1.38\times10^{-23}\text{ J/K}$$

$$\text{Formula } n_i = 2\left[\frac{2\pi m_o K_B T}{h^2}\right]^{\frac{3}{2}} \exp\left(\frac{-E_g}{2k_B T}\right)$$

$$n_i = 2\left[\frac{2\times3.14\times9.10\times10^{-31}\times1.38\times10^{-23}\,X\,300}{(6.625\times10^{-34})^2}\right]\times\exp\left(\frac{-0.7\times1.6\times10^{-19}}{2\times1.38\times10^{-23}\times300}\right)$$

$$n_i = 33.49\times10^{18}/m^3$$

7 Find the conductivity of intrinsic Silicon at 300K. It is given that n_i at 300 K in silicon is $1.5 \times 10^{16}\,/m^3$ and the mobilities of electrons and holes in silicon are 0.13 m^2/V-s and 0.05 m^2/V-s respectively.

Solution :

Given :

$$n_i = 1.5\times10^{16}\text{ m}^{-3}$$

$$\mu_e = 0.13\text{ m}^2\text{ V}^{-1}\text{ s}^{-1}$$

$$\mu_h = 0.05\text{ m}^2\text{ V}^{-1}\text{ s}^{-1}$$

Conductivity $\sigma = n_i\, e\,(\mu_e + \mu_h)$

$$\sigma = 1.5\times10^{16}\times1.6\times10^{-19}\,(0.13+0.05)$$

Conductivity $\sigma = 4.32\times10^{-4}\,\Omega^{-1}\text{m}^{-1}$

8 Calculate the density and mobility of charge carriers for a semiconductor, the Hall coefficient is -6.85 x 10^{-5} m^3/Coloumb, and electrical conductivity is 250 m^{-1} Ω^{-1}.

Given data: $R_H = -6.85 \times 10^{-5} m^3/C$

σ=$250 m^{-1}\Omega^{-1}$

Density of elements 'n' =no of atoms/unit volume=?

Mobility $\mu = ?$

We know Hall coefficient $R_H = \frac{-1}{ne}$

$$n=\frac{-1}{R_H\, e}$$

$$n = \frac{1}{6.85\times10^{-5}\times1.6\times10^{-19}}$$

$$n = 9.124\times10^{22} \text{atoms/m}^3$$

Conductivity $\sigma = ne\mu$

Mobility $\mu = \frac{\sigma}{ne}$

$$\mu = \frac{250\times1}{9.124\times10^{22}\times1.6\times10^{-19}}$$

$$\mu = 17.125\times10^{-3} m^2 v^{-1} s^{-1}$$

9 Find μ for a specimen whose R_H is 3.66 x 10-4 m^3 C^{-1}. Its resistivity is 8.93 x 10^{-3} Ω m.

Given data: \` $R_H = 3.66\times10^{-4}m^3c^{-1}$

$$\sigma = \frac{1}{\rho} = \frac{1}{8.93\times10^{-3}}\,\Omega^{-1}m^{-1}$$

Formula:- $R_H = \frac{-1}{ne}$; Density 'n' $= \frac{-1}{R_H\,e} = \frac{1}{3.66\times10^{-4}m^3c^{-1}}$.

n=1.708 atoms/m^3

Mobility μ=?

But σ=neμ ; $\mu = \frac{\sigma}{ne} = \frac{1}{\rho ne}$

$$\mu = \frac{1}{8.93\times10^{-3}} \times \left(\frac{1}{ne}\right) = \frac{R_H}{8.93\times10^{-3}}$$

$$= \frac{3.66\times10^{-3}}{8.93\times10^{-3}} = 0.041 m^2v^{-1}s^{-1}$$

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MODULE-4

OPTICAL FIBRES

-IRFAN.MOHD

PART-A:

11	What is an optical fiber? Explain its construction and principle with a neat diagram.

- Optical Fibre is a flexible, transparent fiber made of extruded glass (silica) or plastic, slightly thicker than a human hair.

- It can function as a waveguide, or “light pipe”, to transmit light between the two ends of the fiber.

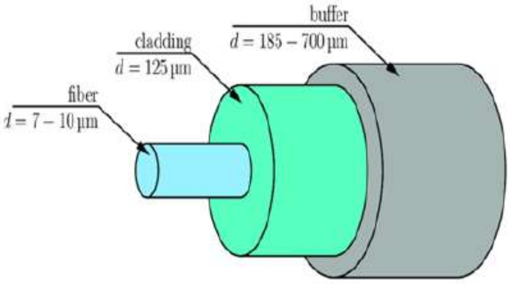
- Power over Fiber (POF) optic cables can also work to deliver an electric current for low-power electric devices.

- **Structure of an optical fiber consists of three parts.**

The core

The cladding and

The coating (or buffer or outer jacket).



Optical Fibre Structure

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12 | Findan expression for angle of acceptance of an optical fiber in terms of refractive indices of core and cladding

- For light rays to propagate through the optical fibre, by total internal reflection, they must be incident on the fibre core within the angle θ_o

called the acceptance angle.

Applying Snell's law at B,

$$n_1 \sin(90^o - \theta_1) = n_2 \sin 90^o$$

$$n_1 \cos \theta_1 = n_2$$

$$\cos \theta_1 = n_2/n_1$$

or $\sin\theta_1 = (1-\cos^2\theta_1)^{1/2}$

$$=\{1-(n_2^2/n_1^2)\}^{1/2} \text{........ (1)}$$

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Applying Snell's law at O,

$$n_o \sin\theta_0 = n_1 \sin\theta_1$$

$$\text{or } \sin\theta_0 = (n_1/n_0) \sin\theta_1 \text{ ....(2)}$$

Substituting eq. (1) in eq. (2),

$$\text{Sin } \theta_0 = (n_1/n_0)(1 - n_2^2/n_1^2)^{1/2}$$

$$= \frac{(n_1^2 - n_2^2)^{1/2}}{n_0} \text{ .... (3)}$$

As the fibre is in air, $n_0 = 1$

Therefore, eq. (3) becomes

$$\text{Sin } \theta_0 = (n_1^2/n_2^2)^{1/2}$$

$$\theta_0 = \sin^{-1}(n_1^2/n_2^2)^{1/2} \text{ ....(4)}$$

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Eq. (4) is the equation for Acceptance angle.

13	Explain Numerical aperture and derive an expression for numerical aperture of an optical fiber.

- Light gathering capacity of the fiber is expressed in terms of maximum acceptance angle and is termed as “Numerical Aperture”.
- Light gathering capacity is proportional to the acceptance angle θ_o.
- So, numerical aperture can be represented by the sine of the acceptance angle of the fibre

i.e., $NA = \sin \theta_o$.

According to the definition of Numerical aperture (NA),

$$NA = \sin\theta_0 = (n_1^2 - n_2^2)^{1/2} \qquad \rightarrow (1)$$

Let 'Δ,' the fractional change in the refractive index, be the ratio between the difference in the refractive indices of core and cladding material to the refractive index of core material respectively.

$$\text{i.e.,}\ \Delta = \frac{n_1 - n_2}{n_1} \qquad \rightarrow (2)$$

$$\text{or}\quad \Delta n_1 = n_1 - n_2 \qquad \rightarrow (3)$$

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Eq. (1) can be written as,

$$NA = (n_1^2 - n_2^2)^{1/2}$$

$$= \{(n_1 - n_2)(n_1 + n_2)\}^{1/2} \qquad \rightarrow (4)$$

Substituting eq. (3) in eq. (4),

$$NA = \{(\Delta n_1)(n_1 + n_2)\}^{1/2}$$

$$\text{As } n_1 \approx n_2,\ n_1 + n_2 = 2n_1$$

And therefore, Numerical Aperture $= (2n_1^2\Delta)^{½}$

$$= n_1 (2\Delta)^{1/2} \qquad \rightarrow (5)$$

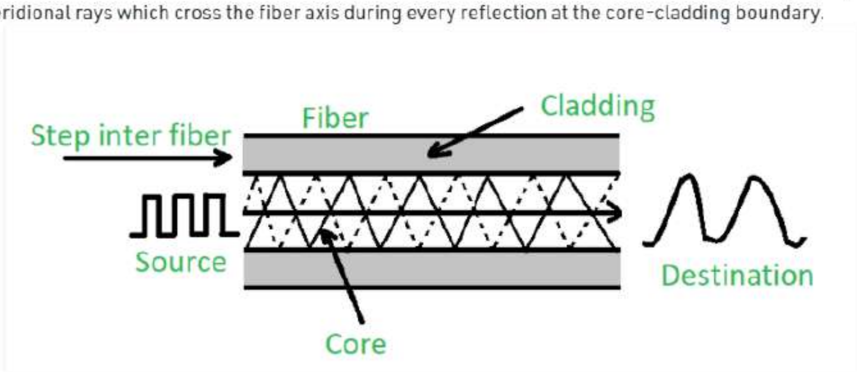
From equation (5) it is seen that numerical aperture depends only on the refractive indices of core and cladding materials and it is independent on the fiber dimensions

14	Explain in detail, different types of optical fibers based on refractive index profile of core medium.

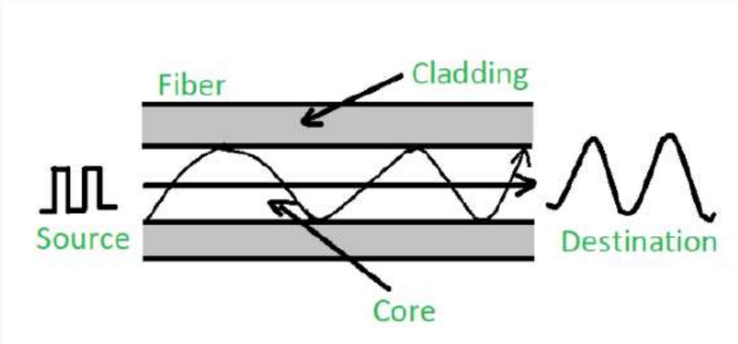
2. On the basis of Refractive Index:

It is also classified into 2 types:

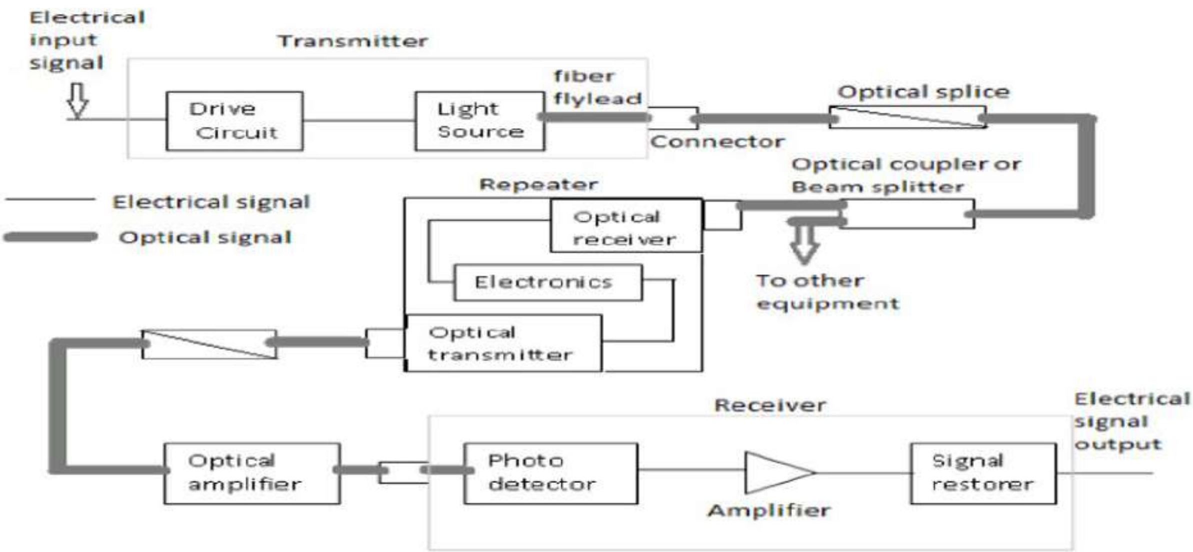
- **(a). Step-index optical fiber:**
  The refractive index of core is constant. The refractive index of the cladding is also constant. The rays of light propagate through it in the form of meridional rays which cross the fiber axis during every reflection at the core-cladding boundary.



- **(b). Graded index optical fiber:**
  In this type of fiber, the core has a non-uniform refractive index that gradually decreases from the center towards the core-cladding interface. The cladding has a uniform refractive index. The light rays propagate through it in the form of skew rays or helical rays. it is not cross the fiber axis at any time.



15	Draw the block diagram of fiber optic communication system and explain the functions of each block in the system.



•The optical fiber consists of three main elements:

1.Transmitter: An electric signal is applied to the optical transmitter. The optical transmitter consists of driver circuit, light source and fiber flylead.

- ❖Driver circuit drives the light source.
- ❖Light source converts electrical signal to optical signal.
- ❖Fiber flylead is used to connect optical signal to optical fiber.

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2.Transmission channel: It consists of a cable that provides mechanical and environmental protection to the optical fibers contained inside. Each optical fiber acts as an individual channel.

•Optical splice is used to permanently join two individual optical fibers.

•Optical connector is for temporary non-fixed joints between two individual optical fibers.

•Optical coupler or splitter provides signal to other devices.

•Repeater converts the optical signal into electrical signal using optical receiver and passes it to electronic circuit where it is reshaped and amplified as it gets attenuated and distorted with increasing distance because of scattering, absorption and dispersion in waveguides, and this signal is then again converted into optical signal by the optical transmitter.

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3. Receiver: Optical signal is applied to the optical receiver. It consists of photo detector, amplifier and signal restorer.

❖Photo detector converts the optical signal to electrical signal.

❖Signal restorers and amplifiers are used to improve signal to noise ratio of the signal as there are chances of noise to be introduced in the signal due to the use of photo detectors.

16	Explain the advantages of optical fibers in communication.

Advantages of Optical Fiber Cable

Bandwidth

Fiber optic cables have a much greater bandwidth than metal cables. The amount of information that can be transmitted per unit time of fiber over other transmission media is its most significant advantage.

Low Power Loss

An optical fiber offers low power loss, which allows for longer transmission distances. In comparison to copper, in a network, the longest recommended copper distance is 100m while with fiber, it is 2km.

Interference

Fiber optic cables are immune to electromagnetic interference. It can also be run in electrically noisy environments without concern as electrical noise will not affect fiber.

Size

In comparison to copper, a fiber optic cable has nearly 4.5 times as much capacity as the wire cable has and a cross sectional area that is 30 times less.

Weight

Fiber optic cables are much thinner and lighter than metal wires. They also occupy less space with cables of the same information capacity. Lighter weight makes fiber easier to install.

Security

Optical fibers are difficult to tap. As they do not radiate electromagnetic energy, emissions cannot be intercepted. As physically tapping the fiber takes great skill to do undetected, fiber is the most secure medium available for carrying sensitive data.

Flexibility

An optical fiber has greater tensile strength than copper or steel fibers of the same diameter. It is flexible, bends easily and resists most corrosive elements that attack copper cable.

Cost

The raw materials for glass are plentiful, unlike copper. This means glass can be made more cheaply than copper.

17	Explain in detail, different types of optical fibers based on mode propagation

Single mode fiber
1. In single mode fiber there is only one path for ray propagation
2. A single mode step index fiber has less core diameter (< 10 μm) and the difference between the reflective indices of core and cladding is very small.
3. In single mode fibers, there is no dispersion.
4. The band width is about 50 MHz for multimode step index fiber where as it is more than 1000 MHz km in case of single mode step index fiber.
5. NA of multimode step index fiber is more where as in single mode step index fibers, it is very less.
6. Launching of light into single mode fibers is difficult.
7. Fabrication cost is very high.

Multimode fiber
1. In multimode fiber, large number of paths is available for light ray propagation.
2. Multi mode fibers, large number of paths are available for light ray propagation.
3. There is signal distortion and dispersion takes place in multimode fibers.
4. The band width of the fiber lies in between 200 MHz km to 600 MHz km even though theoretically it has an infinite bandwidth.
5. NA of graded index fibers is less.
6. Launching of light into multimode fibers is easy.
7. Fabrication cost is less

18 Explain about different types attenuations in optical fibers

Step index fibers and graded index fiber -transmission of signals in them:

Based on the variation of refractive index of core, optical fibers are divided into: (1) step index and (2) graded index fibers. Again based on the mode of propagation, all these fibers are divided into: (1) single mode and (2) multimode fibers. In all optical fibers, the refractive index of cladding material is uniform. Now, we will see the construction, refractive index of core and cladding with radial distance of fiber, ray propagation and applications of above optical fibers.

i **Step index fiber:** The refractive index is uniform throughout the core of this fiber. As we go radially in this fiber, the refractive index undergoes a step change at the core-cladding interface. Based on the mode of propagation of light rays, step index fibers are of 2 types: a) single mode step index fiber & b) multimode step index fibers. Mode means, the number of paths available for light propagation of fiber. We describes the different types of fiber below

a) **Single mode step index fiber:** The core diameter of this fiber is about 8 to 10µm and outer diameter of cladding is 60 to 70 µm. There is only one path for ray

10

propagation. So, it is called **single mode fiber**. The cross sectional view, refractive index profile and ray propagation are shown in fig. (i). In this fiber, the transmission of light is by successive total internal reflections i.e. it is a reflective type fiber. Nearly 80% of the fibers manufactured today in the world are single mode fibers. So, they are extensively used.

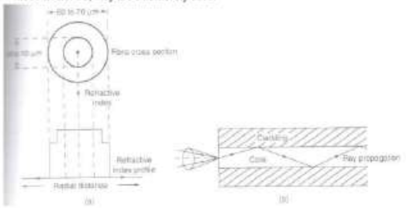


Fig (i).Single mode step index fiber ;(a) Cross sectional view and refractive index profile ;(b) Ray propagation

Multimode step index fiber: The construction of multimode step index fiber is similar to single mode step index fiber except that its core and cladding diameters are much larger to have many paths for light propagation. The core diameter of this fiber varies from 50 to 200 µm and the outer diameter of cladding varies from 100 to 250 µm. The cross-sectional view, refractive index profile and ray propagations are shown in fig 2. Light propagation in this fiber is by multiple total internal reflections i.e it is a reflective type fiber.

b) **Transmission of signal in step index fiber:** Generally the signal is transmitted through the fiber in digital form i.e. in the form of 1"s and 0"s. The propagation of pulses through the multimode fiber is shown in fig (i)(b). The pulse which travels along path 1(straight) will reach first at the other end of fiber. Next the pulse that travels along with path 2(zig-zag) reaches the other end. Hence, the pulsed signal received at the other end is broadened. This is known as intermodal dispersion. This imposes limitation on the separation between pulses and reduces the transmission rate and capacity. To overcome this problem, graded index fibers are used.

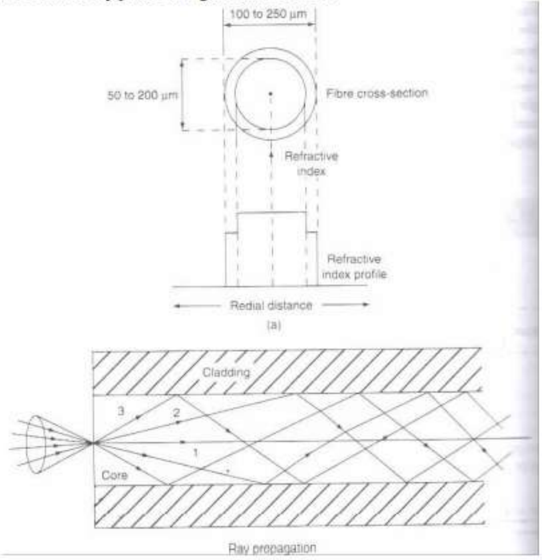
2) **Graded index fiber:** In this fiber, the refractive index decreases continuously from center radially to the surface of the core. The refractive index is maximum at the center and minimum at the surface of core. This fiber can be single mode or multimode fiber. The cross sectional view,

10

refractive index profile and ray propagation of multimode graded index fiber are shown in fig .(ii)(a). The diameter of core varies from 50 to 200μm and outer diameter of cladding varies from 100 to 250 μm.

The refractive index profile is circularly symmetric. As refractive index changes continuously radially in core, light rays suffer continuous refraction in core. The propagation of light ray is not due to total internal reflection but by refraction as shown in fig. (ii)(b). in graded index fiber, light rays travel at different speed in different paths of the fiber. Near the surface of the core, the refractive index is lower, so rays near the outer surface travel faster than the rays travel at the center. Because of this, all the rays arrive at the receiving end of the fiber approximately at the same time. This fiber is costly. .

Transmission of signal graded index fiber: In multimode graded index fiber, large number of paths is available for light ray propagation. To discuss about inter modal dispersion, we consider ray path 1 along the axis of fiber.



19	Compare Step-Index and Graded-index optical fiber and write the differences between them.

Differences between step index fiber and graded index fibers:

Step index fiber	Graded index fiber
1.The refractive index of core Is uniform and step or abrupt Change in refractive index Takes place at the interface of core and cladding in step index fibers	1.The refractive index of core is non uniform, the refractive index of core decreases Para-Felicity from the axis of the fiber to its surface.
2. The light rays propagate in zigzag manner inside the core. The rays travel in the fiber as meridional rays they cross the fiber axis for every reflection.	2. Light rays propagate in the form of skew rays or helical rays. They will not cross the fiber axis

20	Summarize different applications of optical fibers

Internet

Fibre optic cables transmit large amounts of data at very high speeds. This technology is therefore widely used in internet cables. As compared to traditional copper wires, fibre optic cables are less bulky, lighter, more flexible, and carry more data.

Computer Networking

Networking between computers in a single building or across nearby structures is made easier and faster with the use of fibre optic cables. Users can see a marked decrease in the time it takes to transfer files and information across networks

Surgery and Dentistry

Fibre optic cables are widely used in the fields of medicine and research. Optical communication is an important part of non-intrusive surgical methods, popularly known as endoscopy. In such applications, a minute, bright light is used to light up the surgery area within the body, making it possible to reduce the number and size of incisions made. Fibre optics are also used in microscopy and biomedical research.

Automotive Industry

Fibre optic cables play an important role in the lighting and safety features of present-day automobiles. They are widely used in lighting, both in the interior and exterior of vehicles. Because of its ability to conserve space and provide superior lighting, fibre optics are used in more vehicles every day. Also, fibre optic cables can transmit signals between different parts of the vehicle at lightning speed. This makes them invaluable in the use of safety applications such as traction control and airbags.

Telephone

Calling telephones within or outside the country has never been so easy. With the use of fibre optic communication, you can connect faster and have clear conversations without any lag on either side.

Lighting and Decorations

The use of fibre optics in the area of decorative illumination has also grown over the years. Fibre optic cables provide an easy, economical and attractive solution to lighting projects. As a result, they are widely used in lighting decorations and illuminated Christmas trees.

Mechanical Inspections

Fibre optic cables are widely used in the inspection of hard-to-reach places. Some such applications are on-site inspections for engineers and also inspection of pipes for plumbers.

Cable Television

The use of fibre optic cables in the transmission of cable signals has grown explosively over the years. These cables are ideal for transmitting signals for high definition televisions because they have greater bandwidth and speed. Also, fibre optic cables are cheaper as compared to the same quantity of copper wire.

Military and Space Applications

With the high level of data security required in military and aerospace applications, fibre optic cables offer the ideal solution for data transmission in these areas.

PART-C:

5 Calculate the refractive indices of core &cladding of an optical fiber with a numerical aperture of 0.33 and their fractional differences of refractive indices being 0.02.

Solution:

Given: Numerical aperture, NA = 0.33

Fractional refractive index change, $\Delta = 0.02$

We have to find, refractive index of core, $n_1 = ?$

refractive index of cladding, $n_2 = ?$

Formula,

$$NA = n_1\sqrt{2\Delta} \Rightarrow n_1 = \frac{NA}{\sqrt{2\Delta}} = \frac{0.33}{\sqrt{2\times 0.02}} = \frac{0.33}{0.2} = 1.65$$

$$NA = \sqrt{n_1^2 - n_2^2} \Rightarrow n_2^2 = n_1^2 - NA^2$$

$$n_2 = \sqrt{n_1^2 - NA^2} = \sqrt{1.65^2 - 0.33^2} = \sqrt{2.7225 - 0.1089} = 1.54$$

6 A step index fiber has a numerical aperture of 0.16 and core refractive index of 1.45. Calculate the acceptance angle of the fiber and refractive index of the cladding.

Solution:

Given: Numerical Aperture, NA = 0.16,

Refractive index of core, $n_1 = 1.45$

We have to find, Acceptance angle, $i_{max} = ?$

Refractive index of cladding, $n_2 = ?$

Formula, Acceptance angle, $i_{max} = \sin^{-1}(NA) = \sin^{-1}(0.16) = 9.206^o$

$$n_2 = \sqrt{n_1^2 - NA^2} = \sqrt{1.45^2 - 0.16^2} = \sqrt{2.1025 - 0.0256} = 1.44$$

7 The refractive indices of core and cladding materials of a step index fiber are 1.48 and 1.45 respectively. Calculate i) Numerical aperture ii) Acceptance angle.

Solution:

Given: Refractive index of core=1.48

Refractive index of the cladding=1.45

We have to find, Numerical Aperture, NA = ?

and Acceptance angle, i_{max} = ?

Numerical aperture, $NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.48^2 - 1.45^2} = 0.296$

Acceptance angle, $i_{max} = \sin^{-1}(NA) = \sin^{-1}(0.2965) = 17^{o}\ 15'$

8 An optical fiber has a numerical aperture of 0.02 and a cladding refractive index of 1.59. Find the acceptance angle for the fiber in water which has a refractive index of 1.33.

Solution:

Given: Numerical Aperture, NA = 0.2,
Refractive index of cladding, n_2 = 1.59,
Refractive index of water, n_o = 1.33

We have to find acceptance angle, i_{max} = ?

Numerical aperture, $NA = \sqrt{n_1^2 - n_2^2} = 0.2$

In water,

$$NA = \sin i_{max} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} = \frac{0.2}{1.33} = 0.1504$$

$$i_{max} = 8.65^{o}$$

9 Calculate the fractional index change for a given optical fiber if the refractive indices of the core and the cladding are 1.563 and 1.498 respectively.

Solution:

Given: Refractive index of core, n_1 = 1.563

Refractive index of cladding, n_2 = 1.498

We have to find, fractional index change, Δ= ?

Formula, $$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.563 - 1.498}{1.563} = 0.0416$$

MODULE-4

INTERFERENCE

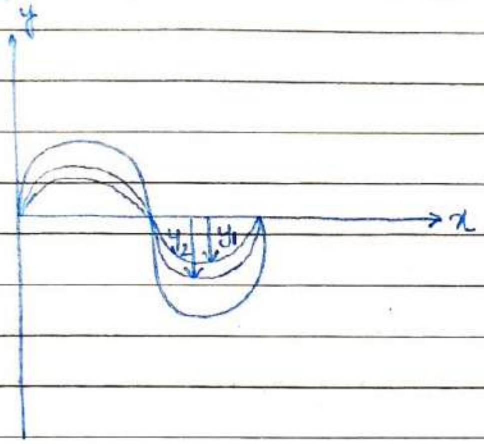
-IRFAN.MOHD

PART-A:

1 Explain the principle of superposition of waves in case of two or more waves travelling simultaneously in a medium?

• Principle of superposition of wave:-

According to superposition of wave, the resultant displacement of the no. of waves in a medium at a particular point is the vector sum of the individual displacement produced by each of the waves at that point.



Mathematically,

$$\underbrace{y(x,t) = y_1(x,t) \pm y_2(x,t)}_{\text{Algebraic sum}}$$

+ → when in phase

- → when not in phase.

2 What is the concept of interference of light and different types of this phenomenon

- Interference:-

Interference is based on principle of superposition of waves.

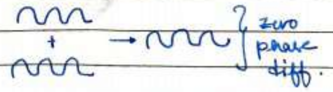
The phenomenon of formation of max intensity at some points and min intensity at some points when two (or) more waves of eq. frequency having constant phase diff arrives at a point simultaneously, superimpose with each other is known as interference.

Interference is divided into two types:-

- Constructive.
- Destructive.

- Constructive Interference:- In same phase & the resultant amplitude is equal to the sum of individual amplitudes resulting in max. intensity of light. This is known as constructive interference.

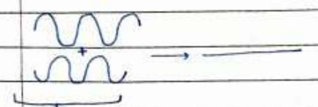
$$I = I_1 + I_2$$



Constructive Interference

- Destructive Interference:-

Superimposing of two waves in opp. phase, the amplitude of resultant is equal to the diff in amplitude of individual waves, resulting in min intensity of waves, this is destructive interference.



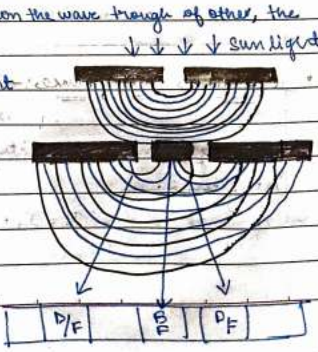
(destructive interference)

3 How did Young explain the formation of fringes in his experiment?

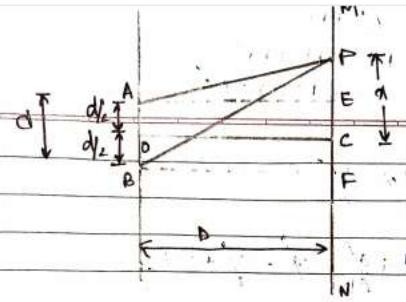
Spherical waves coming from source S_0 are divides into two parts by division of amplitude at once they pass through S_1 & S_2. Thus these waves superimpose each other. At the points where wave crest (or trough) of one wave falls on the wave crest (trough) of other, the resultant amplitude is maximum. ($I = A^2$).

At the points, where the wave crest of one fall on the wave trough of other, the resultant intensity is ~~maximum~~ minimum.

In this way large no. of bright & dark fringes are formed on screen.



4 Monochromatic light from a narrow-slit fall on two parallel slits and the interference fringes are obtained on a screen. Show this experiment with a sketch



Bright → $\Delta = 0, \lambda, 2\lambda, 3\lambda \ldots$

Dark → $\Delta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2} \ldots$

$\Delta = BP - AP$.

Light from a narrow slit illuminated by a monochromatic source, So is allowed to fall on two narrow slits A & B placed very close to each other. Since A & B are equidistant from S, light waves from S reach A & B in phase. So, A & B act as coherent sources.

$\triangle^{le}$ AEP,

$AP^2 = AE^2 + EP^2$

$AP^2 = D^2 + \left(x - \frac{d}{2}\right)^2$

$\triangle^{le}$ BFP,

$BP^2 = BF^2 + FP^2$

$= D^2 + \left(x + \frac{d}{2}\right)^2$

$BP^2 - AP^2 = \cancel{D^2} + \left(x + \frac{d}{2}\right)^2 - \cancel{D^2} - \left(x - \frac{d}{2}\right)^2$

$(BP+AP)(BP-AP) = \cancel{x^2} + 2x\frac{d}{2} + \cancel{\frac{d^2}{4}} - \cancel{x^2} - \cancel{\frac{d^2}{4}} + 2d\frac{x}{2}$

$= 4dx.$

$\Delta = \frac{xd}{D}$

Therefore bright fringe (maxima):

$\frac{xd}{D} = n\lambda$

$x = \frac{n\lambda D}{d} \longrightarrow (i)$, $n = 1, 2, 3, 4 \ldots$

For dark fringe:-

$\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$

$x = (2n+1)\frac{\lambda D}{2d} \longrightarrow (ii)$ where $n = 1, 2, 3, 4 \ldots$

5 What are coherent sources that are used for the phenomenon of interference?

Two sources are said to be coherent when the waves emitted from them have the same frequency and constant phase difference.

Interference from such waves happen all the time, the randomly phased light waves constantly produce bright and dark fringes at every point. But, we cannot see them since they occur randomly. A point that has a dark fringe at one moment may have a bright fringe at the next moment. This cancels out the effect of the interference effect, and we see only an average brightness value. The interference is not said to be sustained since we cannot observe it.

Characteristics of Coherent Sources

Coherent sources have the following characteristics:

1. The waves generated have a constant phase difference
2. The waves are of a single frequency

Coherent Source Example

- Laser light is an example of coherent source of light. The light emitted by the laser light has the same frequency and phase.
- Sound waves are another example of coherent sources. The electrical signals from the sound waves travel with the same frequency and phase.

6 Show the condition for constructive and destructive interference in terms of path difference and phase difference

ntensity of wave after interference can be expressed as

$\mathbf{I} = \mathbf{I_1} + \mathbf{I_2} + 2\sqrt{\mathbf{I_1 I_2}}\cos\phi$ where ϕ is the phase difference.

For $\mathbf{I} = \mathbf{I_{max}} = \mathbf{I_1} + \mathbf{I_2} + 2\sqrt{\mathbf{I_1 I_2}}$, interference is constructive interference

$\Rightarrow \cos\phi = 1 \Rightarrow \phi(\mathbf{PhaseDifference}) = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi.....2n\pi.$

$\Rightarrow \delta(\mathbf{PathDifference}) = 0, \pm\lambda, \pm 2\lambda, \pm 3\lambda, \pm 4\lambda...... \pm \lambda.\ (\because \delta = (\frac{\phi}{2\pi})\lambda)$

For $\mathbf{I} = \mathbf{I_{min}} = \mathbf{I_1} + \mathbf{I_2} - 2\sqrt{\mathbf{I_1 I_2}}$, interference is destructive interference

$\Rightarrow \cos\phi = -1 \Rightarrow \phi = \pi, \pm 3\pi, \pm 5\pi, \pm 7\pi.....(2n+1)\pi.$

$\Rightarrow \delta(\mathbf{PathDifference}) = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}, \pm\frac{7\lambda}{2}...... \pm \frac{(2n+1)\lambda}{2}.$

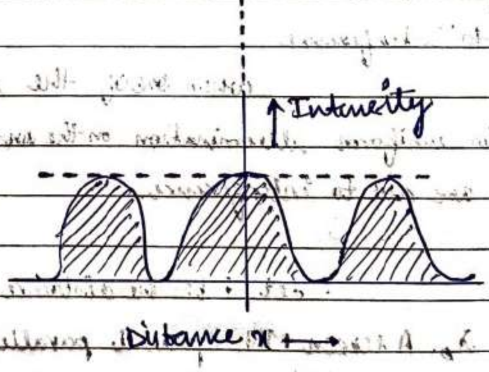
7 | Define fringe width. Write the expression of fringe width.

So Fringe Width, Y is defined as distance b/w two consecutive bright fringes or dark fringes.

$$Y = x_n - x_{n-1}$$

$$= (2n+1)\frac{\lambda D}{2d} - \{2(n-1)-1\}$$

$$Y = \frac{\lambda D}{d}$$



$$Y = x_n - x_{n-1}$$

$$= \frac{n\lambda D}{d} - (n-1)\frac{\lambda d D}{d}$$

So $\boxed{Y = \frac{\lambda D}{d}}$

$$Y = \frac{\lambda D}{d}.$$

Same for both bright fringe & dark fringe.

8 Write the expression for distance between two i) consecutive maxima and ii) two consecutive minima in Youngs experiment.

Suppose S_1A is the perpendiculr from S_1 to S_2P .

Suppose,

$D = OB$ = seperation between slits and the screen,

d = seperation between th slits

and $D >> d.$

Under the above approximation S_1P and S_2P are nearly parallel and hence S_1A is very nearly perpendicular to S_1P , S_2P

and OP . AS S_1S_2 is perpendicular to OB and S_1A is perpendicular to $$OP$, we have

$\angle S_2S_1A = \angle POB = \theta$

This is a small angle as $D >> d.$

The path difference is

$\triangle x = PS_2 - PS_1 \approx PS_2 - PA$

= S_2A = $d\sin\theta \approx d\tan\theta$

= $d \times \frac{y}{D}$

The centers of the bright fringes are obtained atdistances y from the point B, where

$\triangle x = d \times \frac{y}{D}$ = nλ.

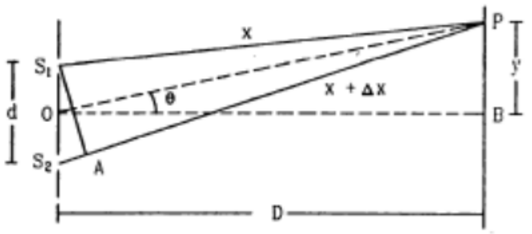
or, $y = \frac{nD\lambda}{d}$ i.e.,

at y = 0, $\pm\frac{D\lambda}{d}, \pm\frac{2D\lambda}{d}, \pm\frac{3D\lambda}{d}$, ...etcSimilarly we can show for dark fringes $y = \pm\frac{D\lambda}{2d}$, $y = \pm\frac{3D\lambda}{2d}$, $y = \pm\frac{5D\lambda}{2d}$,.......etc

the width fringe is therefore ,

$\omega = \frac{D\lambda}{d}$

(a)



9 Outline the conditions to maintain a permanent and stationary interference pattern.

When waves come together they can interfere constructively or destructively. To set up a stable and clear interference pattern, two conditions must be met:

1. The sources of the waves must be ***coherent***, which means they emit identical waves with a constant phase difference.
2. The waves should be ***monochromatic*** - they should be of a single wavelength.

11 Explain in brief the formation of Newton rings in two sentences.

The rings of Newton's are formed as **a result of interference which is between the light waves that are reflected from the top and bottom surfaces of the air film formed between the lens and glass sheet**. ... An air film which is of varying thickness is formed between the lens and the sheet of glass.

12 Explain the function of the 45° inclined glass plate and why Newton rings are circular in shape.

It turns the light rays coming from an extended source to ninety degrees and so the rays fall normally on the plano convex lens. 17-Feb-2015

13	Give the expression for diameter of dark ring and bright ring.

→ Bright Fringe:-

$2t = \lambda(n \pm 1/2)$ subeq ④

$\Rightarrow 2\left(\frac{r^2}{2R}\right) = \lambda(n \pm 1/2)$

$r^2 = \lambda R(n \pm 1/2)$

$D = 2r \Rightarrow r = \frac{D}{2}$.

$D_n^2 = 4\lambda R(n \pm 1/2) \longrightarrow ⑤$

→ Dark fringe:-

$2t = n\lambda$ sub eq ④

$\Rightarrow \cancel{2}\left(\frac{r^2}{\cancel{2}R}\right) = n\lambda$

$r^2 = n\lambda R$

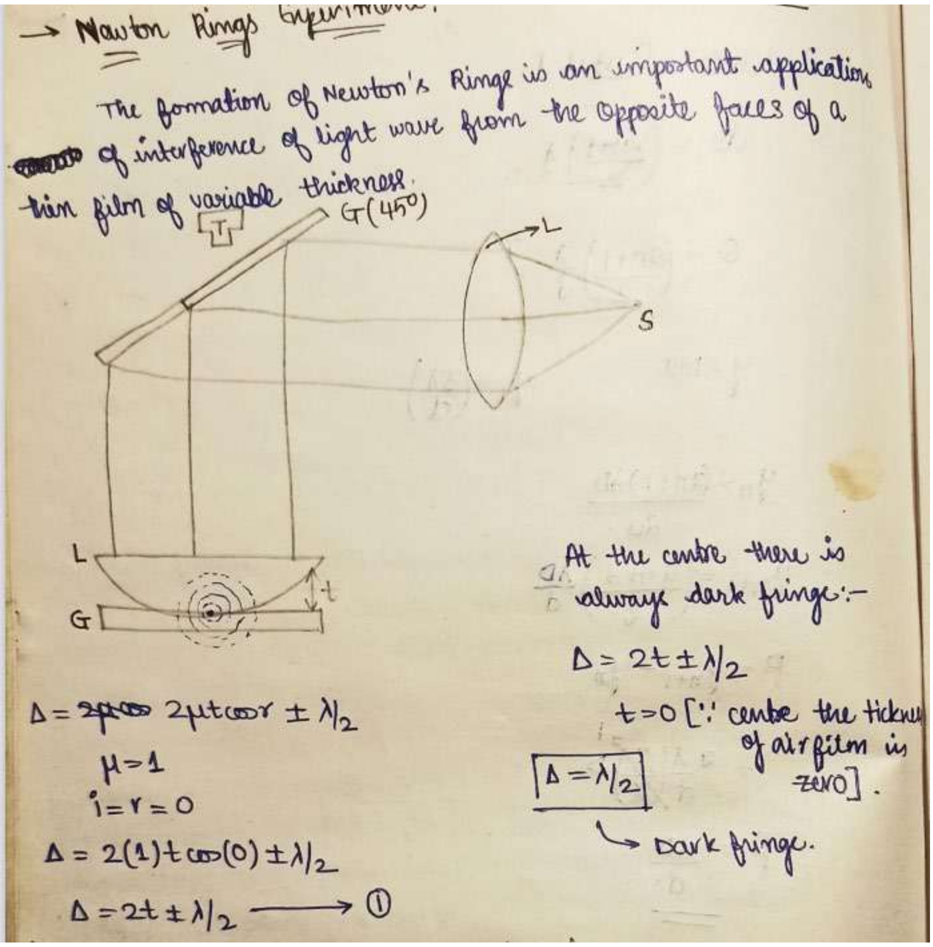
$D = 2r$

$r = \frac{D}{2}$

$\boxed{D_n^2 = 4n\lambda R} \longrightarrow ⑥$

$D_m^2 = 4m\lambda R \longrightarrow ⑦$

14 Explain why the center of the ring is dark.



→ Newton Rings Experiment:

The formation of Newton's Rings is an important application of interference of light wave from the opposite faces of a thin film of variable thickness.

$\Delta = 2\mu t \cos r \pm \lambda/2$

$\mu = 1$

$i = r = 0$

$\Delta = 2(1)t\cos(0) \pm \lambda/2$

$\Delta = 2t \pm \lambda/2 \longrightarrow (1)$

At the centre there is always dark fringe:-

$\Delta = 2t \pm \lambda/2$

$t = 0$ [∵ centre the thickness of air film is zero].

$\boxed{\Delta = \lambda/2}$

↳ Dark fringe.

16 Compare the important phenomena of interference and diffraction exhibited by light.

Answer: The difference between interference and diffraction of light is important to understand in Physics. The basic difference occurs is that diffraction occurs when waves encounter an obstacle while interference occurs when two waves meet each other.

A light undergoes that passes through the edges of opaque bodies or through narrow openings and in which the rays appear to be deflected is diffraction, while interference can be seen in two sound waves meeting each other and makes it hard to distinguish between the two.

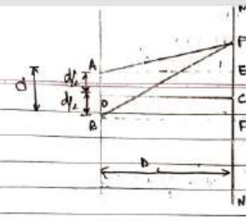
PART-B:

1 State principle of superposition of waves in case of two or more waves travelling simultaneously in a medium.

A defining characteristic of all waves is superposition, which describes the behaviour of overlapping waves. The superposition principle states that when two or more waves overlap in space, the resultant disturbance is equal to the algebraic sum of the individual disturbances. (This is sometimes violated for large disturbances; *see below* Nonlinear interactions.) This simple underlying behaviour leads to a number of effects that are collectively called interference phenomena.

There are two extreme limits to interference effects. In constructive interference the crests of two waves coincide, and the waves are said to be in phase with each other. Their superposition results in a reinforcement of the disturbance; the amplitude of the resulting combined wave is the sum of the individual amplitudes. Conversely, in destructive interference the crest of one wave coincides with the valley of a second wave, and they are said to be out of phase. The amplitude of the combined wave equals the difference between the amplitudes of the individual waves. In the special case where those individual amplitudes are equal, the destructive interference is complete, and the net disturbance to the medium is zero.

2 | Monochromatic light from a narrow-slit fall on two parallel slits and the interference fringes are obtained on a screen. Sketch and explain this Young's double slit experiment



Bright → $\Delta = 0, \lambda, 2\lambda, 3\lambda \ldots$

Dark → $\Delta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2} \ldots$

$\Delta = BP - AP$

Young's Double ~~Pt~~ Slit Experiment :-

In 1801 Thomas young provided the 1st ~~one~~ experimental evidence for the wave theory of light from double slit interference experiment.

Young allowed monochromatic light of wavelength λ to pass a through a pin hole S_0 on first screen following S_1 & S_2 at some distance on 2nd screen.

Two pin holes are separated by distance 'd' & act as virtual coherent sources.

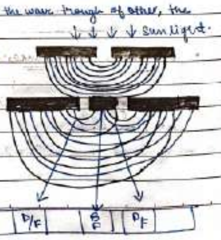
Screen is kept at a distance of 'D' from these two coherent source.

$\boxed{D >>> d.}$

Spherical waves coming from source So are divided into two parts by division of amplitude at once they pass through S_1 & S_2. Then these waves superimpose each other. At the points where wave crest (or trough) of one ~~wave~~ a wave falls on the wave crest (trough) of other, the ~~re~~ resultant amplitude is maximum ($I = A^2$).

At the points, where the wave crest of one fall on the wave trough of other, the resultant intensity is ~~maximum~~ minimum.

In this way large no. of bright & dark fringes are formed on screen.



Light from a narrow slit illuminated by a monochromatic source, So is allowed to fall on two narrow slits A & B placed very close to each other. Since A & B are equidistant from S, light waves from S reach A & B in phase. So, A & B act as coherent sources.

According to Huygen's principle wavelets from A & B spread out and overlapping take place on the right side of AB, when a screen MN is placed at a distance of about 1 meter, equally spaced alternate bright & dark fringes appear on the screen. These are called interference fringes or bands. Using an eyepiece, the fringe can be seen directly. At a point C on the screen, waves from A & B travel equal distance and arrive in phase. These two waves constructively interfere & bright fringe is observed at 'C'. This is called "central bright fringe".

When one of the slits is covered, the fringes disappear and there is uniform illumination on the screen. This shows clearly that the bands are due to interference.

When one of the slit is covered the fringes disappear & there is uniform illumination on the ~~screen~~ screen. This shows clearly that the bands are due to interference.

Let d be the distance between two coherent sources A & B of wavelength λ. A screen MN is placed parallel to AB at a distance of D from the coherent sources. 'O' is midpoint of AB. 'C' is point on screen equidistant from A & B. P is a point on screen ~~equidistant~~ at distance x from the C. Waves from A & B meet at P in phase or out of phase depending on path diff b/w two waves.

Δ^{le} AEP,

$AP^2 = AE^2 + EP^2$

$AP^2 = D^2 + \left(x - \frac{d}{2}\right)^2$

Δ^{le} BFP.

$BP^2 = BF^2 + FP^2$

$= D^2 + \left(x + \frac{d}{2}\right)^2$

Page No.

Date

$BP^2 - AP^2 = \cancel{D^2} + \left(x+\frac{d}{2}\right)^2 - \cancel{D^2} - \left(x - \frac{d}{2}\right)^2$

$(BP+AP)(BP-AP) = \cancel{x^2} + 2x\frac{d}{2} + \cancel{\frac{d^2}{4}} - \cancel{x^2} - \cancel{\frac{d^2}{4}} + 2d\frac{x}{2}$

$= 4dx.$

$\Delta = \frac{xd}{D}$

Therefore bright fringe (maxima):

$\frac{xd}{D} = n\lambda$

$x = \frac{n\lambda D}{d} \longrightarrow (i)$, $n = 1, 2, 3, 4 \ldots$

For dark fringe:-

$\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$

$x = (2n+1)\frac{\lambda D}{2d} \longrightarrow (ii)$ where $n = 1, 2, 3, 4 \ldots$

3 | Give the analytical treatment of interference of light and hence obtain the condition for maximum and minimum intensity by using Young's double slit experiment.

- **Interference:-**

Interference is based on principle of superposition of waves.

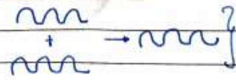
The phenomenon of formation of max intensity at some points and min intensity at some points when two (or) more waves of eq. frequency having constant phase diff arrive at a point simultaneously, superimpose with each other is known as interference.

Interference is divided into two types:-

- Constructive.
- Destructive.

- **Constructive Interference:-** In same phase & the resultant amplitude is equal to the sum of individual amplitudes resulting in max. intensity of light. This is known as constructive interference.

$I = I_1 + I_2$

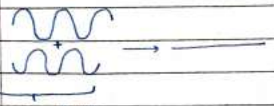


zero phase diff.

Constructive Interference

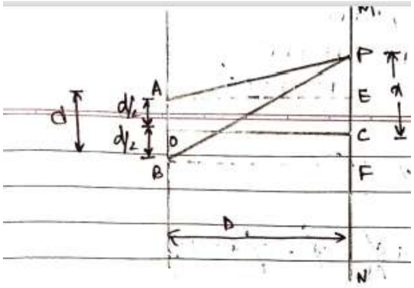
- **Destructive Interference:-**

Superimposing of two waves in opp. phase, the amplitude of resultant is equal to the diff in amplitude of individual waves, resulting in min intensity of waves, this is destructive interference.



(destructive interference)

Const. phase diff.



BRIGHT FRINGE IS MAXIMUM INTENSITY AND DARK FRINGE IS MINIMUM INTENSITY!

Δ^{le} AEP.

$AP^2 = AE^2 + EP^2$

$AP^2 = D^2 + \left(x - \frac{d}{2}\right)^2$

Δ^{le} BFP.

$BP^2 = BF^2 + FP^2$

$= D^2 + \left(x + \frac{d}{2}\right)^2$

Page No.

Date

$BP^2 - AP^2 = \cancel{D^2} + \left(x + \frac{d}{2}\right)^2 - \cancel{D^2} - \left(x - \frac{d}{2}\right)^2$

$(BP + AP)(BP - AP) = \cancel{x^2} + 2x\frac{d}{2} + \cancel{\frac{d^2}{4}} - \cancel{x^2} - \cancel{\frac{d^2}{4}} + \frac{2dx}{2}$

$= 4dx.$

$\Delta = \frac{xd}{D}$ Therefore bright fringe (maxima):

$\frac{xd}{D} = n\lambda$

$x = \frac{n\lambda D}{d} \longrightarrow (i)$, $n = 1, 2, 3, 4 \ldots$

For dark fringe, -

$\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$

$x = (2n+1)\frac{\lambda D}{2d} \longrightarrow (ii)$ where $n = 1, 2, 3, 4 \ldots$

4 | Findan expression for fringe width in interference pattern and show that fringe width of both bright and dark fringes is equal.

$\Delta = \frac{xd}{D}$

Therefore bright fringe (maxima):

$$\frac{xd}{D} = n\lambda$$

$$x = \frac{n\lambda D}{d} \longrightarrow (i) \qquad n = 1, 2, 3, 4 \ldots$$

For dark fringe :-

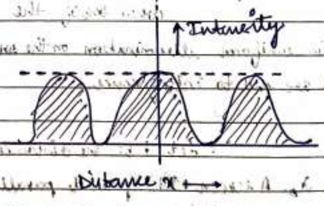
$$\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$$

$$x = (2n+1)\frac{\lambda D}{2d} \longrightarrow (ii) \qquad \text{where } n = 1, 2, 3, 4 \ldots$$

So Fringe width, Y is defined as distance b/w two consecutive bright fringes or dark fringes

$$Y = x_n - x_{n-1}$$

$$= (2n+1)\frac{\lambda D}{2d} - \{2(n-1)-1\}$$



$$Y = \frac{\lambda D}{d}$$

$$Y = x_n - x_{n-1}$$

$$= \frac{n\lambda D}{d} - (n-1)\frac{\lambda D}{d}$$

$$Y = \frac{\lambda D}{d}.$$

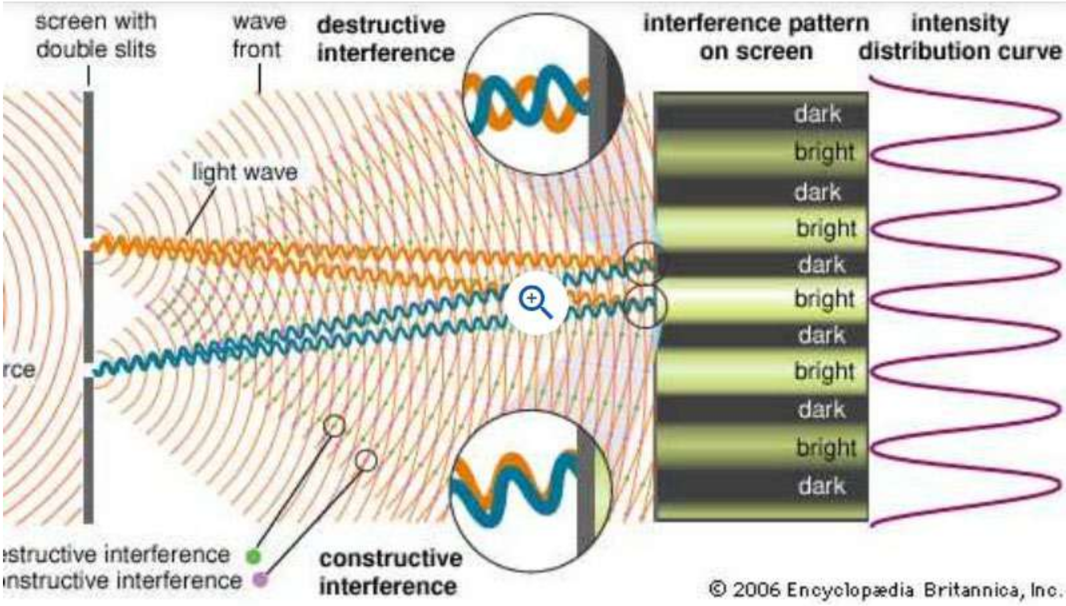
So $\boxed{Y = \frac{\lambda D}{d}}$

same for both bright fringe & dark fringe.

6 | Explain how the intensity and amplitude of waves lead to constructive and destructive interference in Young's experiment

SAME AS 2ND ANSWER.

7 Explain how the energy distribution looks like in Young' s experiment taking the help of analytical treatment



Young's double-slit experiment

When monochromatic light passing through two narrow slits illuminates a distant screen, a characteristic pattern of bright and dark fringes is observed. This interference pattern is caused by the superposition of overlapping light waves originating from the two slits. Regions of constructive interference, corresponding to bright fringes, are produced when the path difference from the two slits to the fringe is an integral number of wavelengths of the light. Destructive interference and dark fringes are produced when the path difference is a half-integral number of wavelengths.

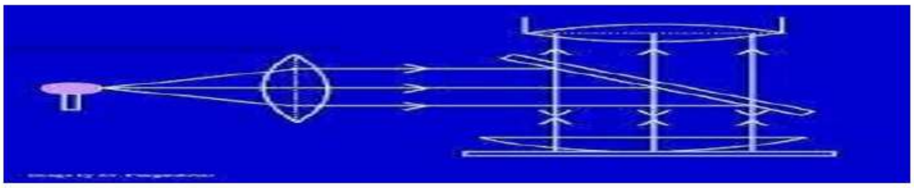
8 Summarize the conditions to observe sustained interference and to observe good contrast in the interreference pattern

To obtain well defined interference patterns, the intensity at points corresponding to destructive interference must be zero, while intensity at the point corresponding to constructive interference must be maximum. To accomplish this the following conditions must be satisfied.

- The two interfering sources must be coherent, that is, they must keep a constant phase difference.
- The two interfering sources must emit the light of the same wavelength and time period. This condition can be achieved by using a monochromatic common original source, that is, the common source emits light of a single wavelength.
- The amplitudes or intensities of the interfering waves must be equal or very nearly equal so that the minimum intensity would be zero.
- The separation between the two coherent sources must be as small as possible so that the width (D ?/2 *d*) of the fringes is large and are separately visible.
- The two sources must be narrow or they must be extremely small. A broad source is equivalent to a large number of fine sources. Each pair of fine sources will give its own pattern. The fringes of different interference patterns will overlap.
- The distance between the two coherent sources and the screen must be as large as possible so that the width of fringes (D ? /2 *d*) is large and are separately visible.
- The two interference waves must be propagated along the same direction so that their vibrations are along a common line.

9 Describe the experimental arrangement and formation of Newton's rings.

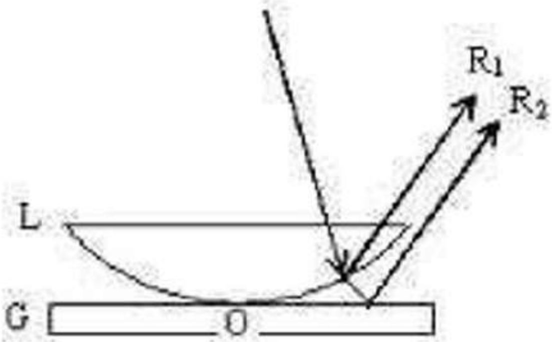
EXPERIMENTAL SET UP



The cross wire of the microscope is fixed at a particular dark ring, say, n^{th} and reading is noted on microscope. The microscope is moved to diametrically opposite side and the cross wire is fixed on the same ring and reading is noted. The difference between the two readings will be the diameter of that particular ring(n^{th}) . This procedure is repeated for another ring ,say,$(n+m)^{th}$. Thus diameter of $(n+m)^{th}$ ring is obtained The radius of curvature R of the lower surface of the lens is measured by a spherometer and by putting the values in equation (3), wavelength of light is determined.

FORMATION OF NEWTON'S RINGS

A thin air film of increasing thickness in all direction from one point can be easily obtained by placing a plano-convex lens of large radius of curvature on a plane glass plate.



10 Discuss the theory in the Newton's rings by reflected light and arrive at the expression for diameter of dark and bright rings.

SAME AS 9TH ANSWER.

Conditions for Bright fringe & Dark fringe:-

Bright Fringe:-

$2t \pm \lambda/2 = n\lambda$

$2t = n\lambda \pm \lambda/2$

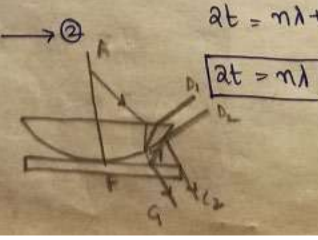
$\boxed{2t = \lambda(n \pm \frac{1}{2})} \longrightarrow ②$

Dark fringe:-

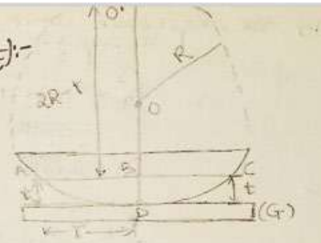
$2t \pm \lambda/2 = (n + \frac{1}{2})\lambda$

$2t = n\lambda + \frac{\lambda}{2} \pm \lambda/2$

$\boxed{2t = n\lambda} \longrightarrow ③$



Thickness of the air film (t):-



$AB \times BC = BD \times BO'$

$r \times r = t(2R - t)$

$r^2 = 2Rt - t^2$ ← compared to R, t is very small & t^2 is very very small so neglecting t^2 terms.

$r^2 = 2Rt$ $\boxed{t = \frac{r^2}{2R}}$ ← for a particular ring.

for $r_n \rightarrow \boxed{t = \frac{r_n^2}{2R}} \longrightarrow ④$

n → no of rings.

26th Feb 2021

→ Bright Fringe:-

$2t = \lambda(n \pm 1/2)$ sub eq ④

$\Rightarrow 2\left(\frac{r^2}{2R}\right) = \lambda(n \pm 1/2)$

$r^2 = \lambda R(n \pm 1/2)$

$D = 2r \Rightarrow r = \frac{D}{2}$

$D_n^2 = 4\lambda R(n \pm 1/2) \longrightarrow ⑤$

→ Dark fringe:-

$2t = n\lambda$ sub eq ④

$\Rightarrow 2\left(\frac{r^2}{2R}\right) = n\lambda$

$r^2 = n\lambda R$

$D = 2r$

$r = \frac{D}{2}$

$D_n^2 = 4n\lambda R \longrightarrow ⑥$

$D_m^2 = 4m\lambda R \longrightarrow ⑦$

11 Expalin how Newton rings experiment b can e useful to determine the wavelength of the monochromatic light source used?

Conditions for Bright fringe & Dark fringe:-

Bright fringe:-

$2t \pm \lambda/2 = n\lambda$

$2t = n\lambda \pm \lambda/2$

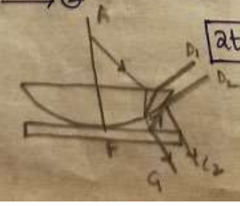
$\boxed{2t = \lambda(n \pm \frac{1}{2})} \longrightarrow (2)$

Dark fringe:-

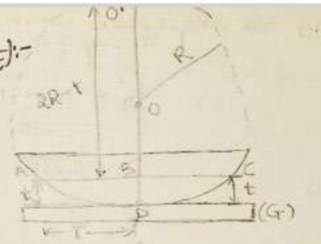
$2t \pm \lambda/2 = (n + \frac{1}{2})\lambda$

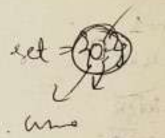
$2t = n\lambda + \frac{\lambda}{2} \pm \lambda/2$

$\boxed{2t = n\lambda} \longrightarrow (3)$



Thickness of the air film (t):-





$AB \times BC = BD \times BO'$

$r \times r = t(2R - t)$

$r^2 = 2Rt - t^2$ ← compared to R, t is very small & t^2 is very very small so neglecting t^2 terms.

$r^2 = 2Rt$ $\quad \boxed{t = \frac{r^2}{2R}}$ ← for a particular ring.

for r_n → $\boxed{t = \frac{r_n^2}{2R}} \longrightarrow (4)$

no of rings.

26th Feb 2021

→ Bright Fringe:-

$2t = \lambda(n \pm 1/2)$ sub eq (4)

$\Rightarrow 2\left(\frac{r^2}{2R}\right) = \lambda(n \pm 1/2)$

$r^2 = \lambda R(n \pm 1/2)$

$D = 2r \Rightarrow r = \frac{D}{2}$.

$D_n^2 = 4\lambda R(n \pm 1/2) \longrightarrow (5)$

→ Dark fringe:-

$2t = n\lambda$ sub eq (4)

$\Rightarrow 2\left(\frac{r^2}{2R}\right) = n\lambda$

$r^2 = n\lambda R$

$D = 2r$

$r = \frac{D}{2}$

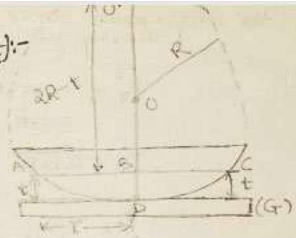
$D_n^2 = 4n\lambda R \longrightarrow (6)$

$D_m^2 = 4m\lambda R \longrightarrow (7)$

The Newton's ring experiment can be also used to find the wavelength of a monochromatic light. In this case, the radius of curvature of the convex surface of the given lens is supplied or is determined otherwise. By employing sodium light whose mean wavelength is 5893Å, R can be determined from Eqn.(3), as in the present experiment. Then the same equation can be used to find the wavelength λ of any other given monochromatic light.

12 | Extend your understanding of Newton rings experiment to determine the refractive index of any given liquid.

→ Thickness of the air film (t):-



$AB \times BC = BD \times BO'$

$r \times r = t(2R-t)$

$r^2 = 2Rt - t^2$ ← compared to R, t is very small & t^2 is very very small so neglecting t^2 terms.

$r^2 = 2Rt$ $\quad \boxed{t = \frac{r^2}{2R}}$ ← for a particular ring.

for r_n → $\boxed{t = \frac{r_n^2}{2R}}$ ⟶ (4)

no of rings.

26th Feb 2021

→ Bright Fringe:-

$2t = \lambda(n \pm 1/2)$ sub eq (4)

$\Rightarrow 2\left(\frac{r^2}{2R}\right) = \lambda(n \pm 1/2)$

$r^2 = \lambda R(n \pm 1/2)$

$D = 2r \Rightarrow r = \frac{D}{2}$.

$D_n^2 = 4\lambda R(n \pm 1/2)$ ⟶ (5)

→ Dark fringe:-

$2t = n\lambda$ sub eq (4)

$\Rightarrow 2\left(\frac{r^2}{2R}\right) = n\lambda$

$r^2 = n\lambda R$

$D = 2r$

$r = \frac{D}{2}$

$\boxed{D_n^2 = 4n\lambda R}$ ⟶ (6)

$D_m^2 = 4m\lambda R$ ⟶ (7)

eq (7) − eq (6): $D_m^2 - D_n^2 = 4m\lambda R - 4n\lambda R$
$= 4\lambda R(m-n)$

$\boxed{R = \frac{D_m^2 - D_n^2}{4\lambda(m-n)}}$ ⟶ (8).

Using the eq we can find radius of planoconvex lens when wavelength of monochromatic source is known.

similarly we can find wavelength if Radius of curvature is known using eq (9).

can also find RI of the medium.

$\boxed{\lambda = \frac{D_m^2 - D_n^2}{4R(m-n)}}$ ⟶ (9).

PART-C:

1 Two slits separated by a distance of 0.2 mm are illuminated by a monochromatic light of wavelength 550 nm. Calculate the fringe width on a screen at distance of 1 m from the slits.

1) $d = 0.2\,mm = 0.2 \times 10^{-3}\,m$

$\lambda = 550\,nm = 550 \times 10^{-9}\,m$

$D = 1\,m$

fringe width $= \frac{\lambda D}{d} = \frac{550 \times 10^{-9} \times 1}{0.2 \times 10^{-3}} = 2.75 \times 10^{-3}\,m.$

2 Two coherent sources of monochromatic light of wavelength 6000 A° produce an interference pattern on a screen kept at distance of 1 m from them. The distance between two consecutive bright fringes on the screen is 0.5 mm. Find the distance between the two coherent sources

2) $\lambda = 6000\,A^{\circ} = 6000 \times 10^{-10}\,m.$

$D = 1\,m$

$\beta = 0.5\,mm = 5 \times 10^{-4}\,m$

$d = ?$

$\beta = \frac{\lambda D}{d} \Rightarrow d = \frac{\lambda D}{\beta} = \frac{6 \times 10^{-7} \times 1\,m}{5 \times 10^{-4}}$

$d = 1.2 \times 10^{-3}\,m$ (or) $1.2\,mm.$

3 In a Newton's rings experiment, the diameter of 15^{th} ring was found to be 0.59 cm and that of 5^{th} ring is 0.336 cm. If the radius of curvature of lens is 100 cm, find the wavelength of the light.

3) $\lambda = \dfrac{D_{n+m}^2 - D_n^2}{4mR}$.

15^{th} ring $\rightarrow D_{n+m} = 0.59\,cm = 5.9 \times 10^{-3}\,m$

diameter of 5^{th} ring $= D_n = 0.336\,cm = 3.36 \times 10^{-3}\,m$

$m = 15 - 5 = 10$

$R = 100\,cm = 1\,m$

wavelength, $\lambda = [(5.9 \times 10^{-3})^2 - (3.36 \times 10^{-3})^2] / 4 \times 10 \times 1$.

$= \{(34.81 - 11.2896)/40\} \times 10^{-6} = 23.5204/40 \times 10^{-6}$

$= 0.5880 \times 10^{-6}\,m$

$= 5880\,A^{\circ}$.

4 Newton's rings are observed in the reflected light of wavelength 5900A°. The diameter of tenth dark ring is 0.5 cm. Find the radius of curvature of the lens used.

4). Diameter of n^{th} dark ring.

$D_n = 2\sqrt{n\lambda R}$

$R = \dfrac{D_n^2}{4n\lambda}$

$n = 10$; $D_{10} = 0.005\,m$.

$\lambda = 5 \times 10^{-7}\,m$. $(5000 \times 10^{-10}\,m)$.

$R = \dfrac{(0.005\,m)^2}{4 \times 10 \times 5 \times 10^{-7}\,m} = \underline{\underline{1.25\,m}}$

5 | Newton's rings formed by sodium light between a flat glass plate and a convex lens are viewed normally. What will be the order of the dark ring which will have double the diameter of that of the 40^{th} dark ring.

5) radius of n^{th} ring

$$r_n^2 = nR\lambda.$$

So diameter

$$d_n = \sqrt{4nR\lambda}.$$

So diameter is twice as diameter of 40^{th} dark ring then.

$$n = n_{40} \times \left(\frac{d}{d_{40}}\right)^2 = 40 \times 2^2 = 40 \times 4 = 160$$

So, 160^{th} order dark ring has the diameter twice as big as diameter of 40^{th} dark ring.

6 | The diameter of 9^{th} dark ring in Newtons rings experiment is 0.29 cm. What is the diameter of 16^{th} dark ring when λ =6000 A^o

6) $D_9 = 0.29cm$

$\lambda = 6000A^{\circ}$

$D_{16} = ?$

$$D_9^2 = 4 \times 9 \times \lambda R.$$

$$(0.29)^2 = 4 \times 9 \times 6000 \times 10^{-8} \times R$$

$$0.0841 = 216 \times 10^3 \times 10^{-8} \times R$$

$$\frac{0.841}{216 \times 10^{-5}} = R \Rightarrow R = 38.93cm.$$

$$D_{16}^2 = 4 \times 16 \times \lambda \times R$$

$$D_{16} = \sqrt{64 \times 6000 \times 10^{-8} \times 38.93} = 0.38cm$$

ENGINEERING PHYSICS

MODULE-5(OSCILLATIONS)

-IRFAN.MOHD(20951A0488@iare.ac.in)

PART-A:

1	When can we call a body in motion to execute a periodic motion? Define it.

A wave in which a uniform series of crests and troughs follow one after the other in regular succession. By contrast, the wave produced by applying a pulse to a stretched string does not follow regular, repeated patterns.

2	When can we call a body executing periodic motion to be simple harmonic.

simple harmonic motion is a special type of periodic motion where the restoring force on the moving object is directly proportional to the object's displacement magnitude and acts towards the object's equilibrium position.

3 | Define amplitude of a body executing simple harmonic motion.

t is the maximum displacement of the particle from the mean position.

4 | Define time period of a body executing simple harmonic motion

Time period :- Time taken to complete one cycle or revolution.

Angular velocity $= \omega = \dfrac{\text{Angle described in one revolution}}{\text{time taken to complete one revolution}}$

Page No.

Date

$\omega = \dfrac{2\pi}{T}$ $\quad T = \dfrac{2\pi}{\omega}$ $\quad$ But acceleration $a = -\omega^2 x$

In terms of magnitude, $\omega = \sqrt{\dfrac{A}{x}}$

Time period $T = \dfrac{2\pi}{\sqrt{\dfrac{A}{x}}}$ $\quad$ therefore $T = 2\pi\sqrt{\dfrac{x}{A}} = 2\pi\sqrt{\dfrac{\text{Displacement}}{\text{Acceleration}}}$.

5 | Define phase of a body executing simple harmonic motion

The phase of a vibrating particle at any instant is the state of the vibrating (or) oscillating particle regarding its displacement and direction of vibration at that particular instant.

The expression, position of a particle as a function of time.

$$x = A \sin (\omega t + \Phi)$$

where $(\omega t + \Phi)$ is the phase of the particle, the phase angle at time $t = 0$ is known as the initial phase.

6 | Showthe expression for the velocity of a body executing simple harmonic motion.

→ Velocity :-

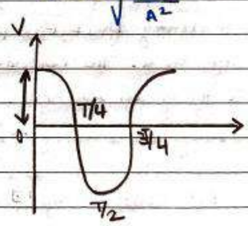
$x = A\sin\omega t$ $\quad v = \frac{dx}{dt} = A\omega\cos\omega t$

$v = A\omega\sqrt{1-\sin^2\omega t}$ $\quad v = A\omega\sqrt{1 - \frac{x^2}{A^2}}$ $\quad v = A\omega\sqrt{\frac{A^2 - x^2}{A^2}}$

$v = \omega\sqrt{A^2 - x^2}$.

If $x = 0$, $v = A\omega$

If $x = A$; $v = 0$.



Time	0	T/4	T/2	3T/4	T
x	0	A	0	−A	0
v	Aω	0	−Aω	0	Aω

7 Define frequency of a body executing simple harmonic motion.

The number of oscillations per second is defined as the frequency.

Frequency = 1/T and, angular frequency $\omega = 2\pi f = 2\pi/T$

8 Define the characteristics of simple harmonic motion

Characteristic of SHM:-

1. The motion of the body is periodic.
2. Repeated back & forth movement over the same path about an equilibrium position, such as mass on spring or pendulum.
3. When body is displaced from eq. position, a restoring force must act on the body in opp. direction to bring it back to mean position.
4. Restoring force mag depends only on displacement, such as in Hooks law $F \propto x$
5. The system must have inertia (mass)
6. Can ~~be~~ be represented by a ~~single~~ simple sine or cosine function.

9 Distinguish between free and forced oscillation.

Free vs **Forced Oscillations**

Comparison Chart

Free Oscillations	Forced Oscillations
Free oscillations are oscillations in which a body or system oscillates with its own natural frequency without being acted upon by an external force.	Forced oscillations are oscillations in which the body oscillates with a frequency other than its natural frequency under the influence of an external periodic force.
They occur due to the elastic forces and inertia of the system.	They occur due to the action of a periodic force applied externally.
Free oscillations diminish gradually due to the resisting forces called damping forces.	Forced oscillations persist as long as the body is acted upon by an external force.
The frequency depends on the mass and elasticity of the body.	The frequency is equal to the frequency of periodic force applied to the body.
For example, when you push a swing just once, it oscillates at its own natural frequency, so it acts as a free oscillator.	If you push the swing each time it slows down, it will continue to swing because external force is applied to it, so it becomes a forced oscillator.

10 Explain the two forces which act on a damped harmonic oscillator.

Damping force is subjected to two forces;

1. Restoring force $\propto$ displacement
$= -kx$.

2. Frictional force $\propto$ velocity
$= -r\frac{dx}{dt}$. where r is frictional force per unit velocity (damping constant)

$F = ma = \frac{d^2x}{dt^2}(m)$.

$m\frac{d^2x}{dt^2} = -kx - r\frac{dx}{dt}$.

$m\frac{d^2x}{dt^2} + kx + r\frac{dx}{dt} = 0 \Rightarrow \frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$

Take $\frac{r}{m}$ as $2b$ & $\omega^2 = \frac{k}{m}$

$$\boxed{\frac{d^2x}{dt^2} + \underbrace{2b\frac{dx}{dt}}_{\text{damping factor}} + \underbrace{\omega^2 x}_{\text{spring factor}} = 0}$$

Page No.
Date

13 Compare the three different types of damped oscillations?

SHM is defined as the motion in which the restoring force is directly proportional to the displacement of body from its mean position. The direction of restoring force is always towards the mean position.

Oscillatory motion in which the acceleration of the particle at any position is directly proportional to the displacement from mean position. Its a special case of oscillatory motion.

Date

Damped Harmonic Motion:-

For a body executing vibrations, amplitude of vibration keeps on decreasing slowly due to the resistive frictional force and vibration will die. Such motion damped by frictional force is called Damped Oscillation.

It is important in realistic oscillatory system as air resistance, friction, and intermolecular forces are the resistive frictional force.

Damping forces are often due to the ~~forces~~ motion of an oscillatory system through a fluid like air or water, where interaction b/w molecules of fluid is imp.

Eg:- yo-yo, clock pendulum, guitar string.

→ FORCED HARMONIC MOTION:-

When we displace a system, say a simple pendulum, from its equilibrium position & then release it, it oscillates with a natural frequency ω & ~~then~~ these oscillations are free oscillations.

But all free oscillations eventually die out due to the ever present damping force in the surrounding. ~~Howeve~~ However an external agency can maintain these oscillations.

14 What are the forces which act on a body executing forced vibrations?

Force acting on body

1. Restoring force $\propto$ displacement

$= -kx$ (k is force const. defined as force per unit distance).

2. Frictional force $\propto$ velocity

$= -r\frac{dx}{dt}$ $r \rightarrow$ frictional force per unit velocity (damping constant)

magnitude of force

3. External periodic force $= F\sin pt$ $\rightarrow$ driving frequency

PART-B:

1 Derive the equation of a motion of a Simple mechanical harmonic oscillator.

Derivation of Force Law :-

Assume the spring is weightless & surface is frictionless.

Let restoring force F & displacement of the block from its equillibrium position be x. The restoring force is directly proportional to displacement from the mean position.

$$\therefore F = -Kx \quad \text{......} (1).$$

$K \rightarrow$ force constant (or) spring constant.

N/m $\rightarrow$ SI unit. dyne/cm $\rightarrow$ CGS unit.

Defined as restoring force per unit oscillation.

(-ve) sign indicates that restoring force & displacement are always in opposite direction.

Similarity b/w electrical & mechanical harmonic oscillation

$$m \cdot \frac{d^2x}{dt^2} = -Kx \qquad L\frac{d^2q}{dt^2} = -\frac{q}{C}$$

q takes place of x

L takes place of m

$\frac{1}{C}$ takes place of C.

But according to Newton's 2nd Law of motion;

$$F = m\frac{d^2x}{dt^2} \longrightarrow (2).$$

Page No.

Date

Compare eq (1) & (2) we get

$$m \cdot \frac{d^2x}{dt^2} = -Kx \qquad \frac{d^2x}{dt^2} = \frac{-Kx}{m} \qquad \boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$$

$$\omega^2 = \frac{K}{m} \rightarrow \text{constant}$$

The 1st diff eqn for simple harmonic motion

2 | Summarize the concept of simple harmonic motion and its characteristics.

SHM is defined as the motion in which the restoring force is directly proportional to the displacement of body from its mean position. The direction of restoring force is always towards the mean position.

Oscillatory motion in which the acceleration of the particle at any position is directly proportional to the displacement from mean position. Its a special case of oscillatory motion.

Characteristic of SHM:-

1. The motion of the body is periodic.
2. Repeated back & forth movement over the same path about an equilibrium position, such as mass on spring or pendulum.
3. When body is displaced from eq. position, a restoring force must act on the body in opp. direction to bring it back to mean position.
4. Restoring force mag depends only on displacement, such as in Hooks law $F \propto x$
5. The system must have inertia (mass)
6. Can be, be represented by a ~~singl~~ simple sine or cosine function.

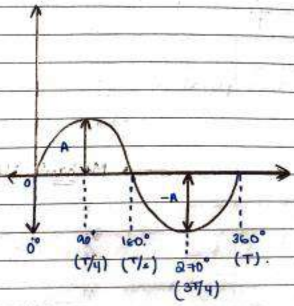
3 Explain the terms i) displacement ii) velocity iii) Acceleration of a simple harmonic oscillator along with their expressions

- **Displacement**:- The distance of the particle along the direction of motion from equilibrium position at a given instance of time is called displacement.

$x = A\sin\omega t$

- x → displacement in meters (m)
- A → Amplitude in (m)
- ω → Angular frequency in radians per second (rad/s).
- t → time in seconds (s).

Angle	0°	90°	180°	270°	360°
Time	0	T/4	T/2	3T/4	T
x	0	A	0	-A	0



→ **Velocity**:-

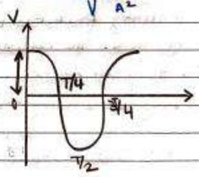
$x = A\sin\omega t \qquad v = \frac{dx}{dt} = A\omega\cos\omega t$

$v = A\omega\sqrt{1-\sin^2\omega t} \qquad v = A\omega\sqrt{1-\frac{x^2}{A^2}} \qquad v = A\omega\sqrt{\frac{A^2-x^2}{A^2}}$

$v = \omega\sqrt{A^2-x^2}$.

If $x = 0$, $v = A\omega$

If $x = A$; $v = 0$.



Time	0	T/4	T/2	3T/4	T
x	0	A	0	-A	0
v	Aω	0	-Aω	0	Aω

→ **Acceleration**:- $x = A\sin\omega t$

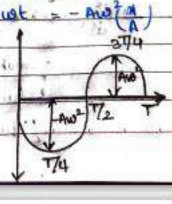
$v = A\omega\cos\omega t$

Acceleration, $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = A\omega(-\sin\omega t) = -A\omega^2\sin\omega t = -A\omega^2\left(\frac{x}{A}\right)$

$a = -\omega^2 x$.

If $x = 0$, $a = 0$

If $x = A$, $a = -\omega^2 A$



t	0	T/4	T/2	3T/4	T
x	0	A	0	-A	0
a	0	-Aω²	0	Aω²	0

4 Show an expression for the potential energy of a simple harmonic oscillator

For SHM,
acceleration , $a = -\omega^2 y$
$F = ma = -m\omega^2 y$
now, work , $W = F.dy \cos 180°$ { because displacement and acceleration are in opposite direction so, cos180° taken }
$W = \int m\omega^2 y.dy = m\omega^2 y^2/2$
use standard form of SHM , $y = A\sin(\omega t \pm \Phi)$
$W = m\omega^2 A^2/2 \sin^2(\omega t \pm \Phi)$
We know, Potential energy is work done stored in system .
so, $P.E = W = m\omega^2 A^2/2 \sin^2(\omega t \pm \Phi)$

5 Find an expression for the kinetic energy of a simple harmonic oscillator

For SHM,
acceleration , $a = -\omega^2 y$
$F = ma = -m\omega^2 y$
now, work , $W = F.dy \cos 180°$ { because displacement and acceleration are in opposite direction so, cos180° taken }
$W = \int m\omega^2 y.dy = m\omega^2 y^2/2$
use standard form of SHM , $y = A\sin(\omega t \pm \Phi)$
$W = m\omega^2 A^2/2 \sin^2(\omega t \pm \Phi)$
We know, Potential energy is work done stored in system .
so, $P.E = W = m\omega^2 A^2/2 \sin^2(\omega t \pm \Phi)$
again, Kinetic energy , $K.E = 1/2mv^2$, here v is velocity
we know, $v = \omega A\cos(\omega t \pm \Phi)$
so, $K.E = m\omega^2 A^2/2\cos^2(\omega t \pm \Phi)$

6 Summarize how the energy of body executing simple harmonic motion is proportional to the square of the frequency and square of the amplitude.

DID NOT FIND THE SOLUTION, IF FOUND PLZ LET ME KNOW.

7 Show the differential equation of a damped harmonic oscillator and illustrate the conditions of different types of damping

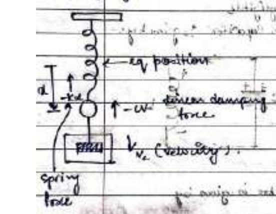
Damped Harmonic Motion:-

For a body executing Vibrations, amplitude of vibration keeps on decreasing slowly due to the resistive frictional force and vibration will die. Such motion damped by frictional force is called Damped Oscillation.

It is important to realistic oscillatory system as air resistance, friction, and intermolecular forces are the resistive frictional force.

Damping forces are often due to the motion of an oscillating system through a fluid like air or water, where interaction b/w molecules of fluid is imp.

eg:- yo-yo, clock pendulum, guitar string.



An example of damped motion is mass connected to a spring. If mass is immersed in a liquid. Fluid exerts frictional force on mass damping its motion.

Damping force is subjected to two forces;

1. Restoring force ∝ displacement
$= -kx$.

2. Frictional force ∝ velocity
$= -\gamma \frac{dx}{dt}$ where γ is frictional force per unit velocity (damping constant)

$F = ma = \frac{d^2x}{dt^2}(m)$.

$m\frac{d^2x}{dt^2} = -kx - \gamma\frac{dx}{dt}$

$m\frac{d^2x}{dt^2} + kx + \gamma\frac{dx}{dt} = 0 \Rightarrow \frac{d^2x}{dt^2} + \frac{\gamma}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$

Take $\frac{\gamma}{m}$ as $2b$ & $\omega^2 = \frac{k}{m}$

$$\boxed{\underbrace{\frac{d^2x}{dt^2} + 2b\frac{dx}{dt}}_{\text{damping factor}} + \underbrace{\omega^2 x}_{\text{spring factor}} = 0}$$

Solution of 2nd order diff eq^n is given by : $x = Ae^{\alpha t}$, where A & α are constants

diff of BS:-

$\frac{dx}{dt} = A\alpha e^{\alpha t}$, $\frac{d^2x}{dt^2} = A\alpha^2 e^{\alpha t}$

(Sub the values in eq^n

$A\alpha^2 e^{\alpha t} + 2b A\alpha e^{\alpha t} + \omega^2 A e^{\alpha t} = 0$

$Ae^{\alpha t}[\alpha^2 + 2b\alpha + \omega^2] = 0$

since $Ae^{\alpha t} \neq 0$; $[\alpha^2 + 2b\alpha + \omega^2] = 0$,

$\alpha = \frac{-2b \pm \sqrt{4b^2 - 4\omega^2}}{2}$

$\alpha = \frac{-2b \pm 2\sqrt{b^2-\omega^2}}{2}$

$\boxed{\alpha = -b \pm \sqrt{b^2-\omega^2}}$

The general sol^n can be given as .

$x = A_1 \exp[-b + \sqrt{b^2-\omega^2}]t + A_2 \exp[-b - \sqrt{b^2-\omega^2}]t$

constants.

Depending upon the relative values of b & w, there are 3 possible cases.

Case 1: Underdamped.

If $b^2 < w^2$, the system oscillate & amplitude gradually decreases to zero.

then $\sqrt{b^2-w^2} = \sqrt{-1}\sqrt{w^2-b^2} = i\sqrt{w^2-b^2}$.

where $\beta = \sqrt{w^2-b^2}$.

Substitute in solution,

$$x = A_1 \exp\{-b+i\beta\}t + A_2 \exp\{-b-i\beta\}t$$

$$x = e^{-bt}\{A_1(\cos\beta t + i\sin\beta t) + A_2(\cos\beta t - i\sin\beta t)\}$$

$$x = e^{-bt}\{(A_1+A_2)\cos\beta t + (A_1-A_2)i\sin\beta t\}$$

Let $A_1 + A_2 = a\sin\phi$; $(A_1-A_2)i = a\cos\phi$

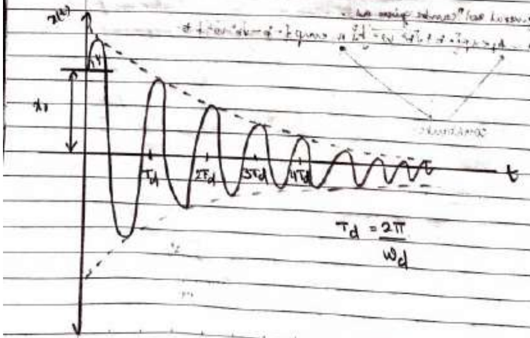
$$x = e^{-bt}\{a\sin\phi\cos\beta t + a\cos\phi\sin\beta t\} \qquad x = ae^{-bt}\sin(\beta t \pm \phi)$$

Since $\sin(\beta+\phi)$ varies b/w -1 to 1, amplitude decreases exponentially b/w ae^{-bt} & $-ae^{-bt}$ due to damping factor e^{-bt}.

Time period $T = \frac{2\pi}{\sqrt{w^2-b^2}}$, which is greater than $T = \frac{2\pi}{w}$.

Therefore amplitude decreases gradually.

Eg:- motion of pendulum in air, LCR circuit, electrical oscillator.



Case II :- Critically damped.

If $b^2 = w^2$,

$\sqrt{b^2-w^2} = 0$, but $\sqrt{b^2-w^2} = h \rightarrow 0$

$$x = A_1\exp\{-b+h\}t + A_2\exp\{-b-h\}t$$

$$x = e^{-bt}[A_1e^{ht} + A_2e^{-ht}]$$

$$x = e^{-bt}[A_1(1+ht+\ldots) + A_2(1-ht+\ldots)]$$

$$x = e^{-bt}[A_1(1+ht) + A_2(1-ht)]$$ {neglecting higher power as they are very small}

$$x = e^{-bt}[(A_1+A_2) + ht(A_1-A_2)]$$

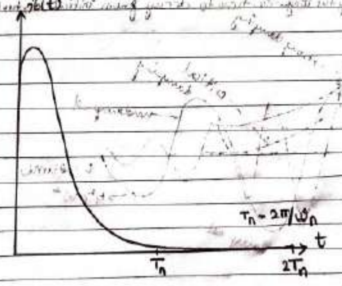
$$x = e^{-bt}[p+qt]$$

$p = (A_1+A_2)$; $q = h(A_1-A_2)$; $e^{-bt} \rightarrow$ damping factor.

The eq shows that displacement ~~tries to f~~ first increases due to factor $(p+qt)$ but at the same time reversal occurs due to factor e^{-bt} & displacement reduces to zero as 't' increases. body comes back to eq position more quickly. This type of motion is called critically damped.

Eg:- In pointer instrument like voltmeter & ammeter, the pointer moves to the correct position & comes back to rest without oscillating.

Critical Damping is defined as condition in which the damping of oscillator result in it returning quickly & as possible to its equilibrium position.



Critical damping is represented by curve in figure.

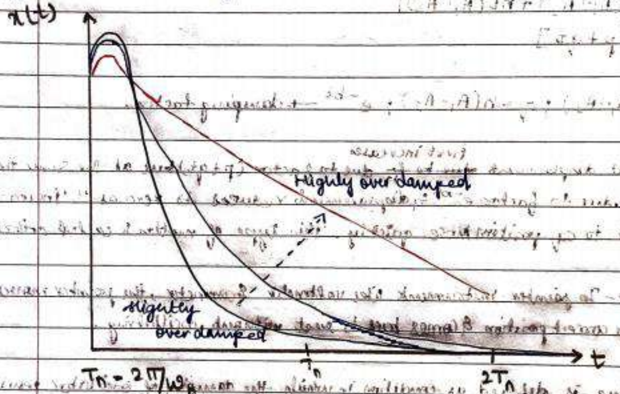
Case III :- Over damping :-

If $b^2 > \omega^2$, then $\sqrt{b^2-\omega^2}$ is real & less than b.

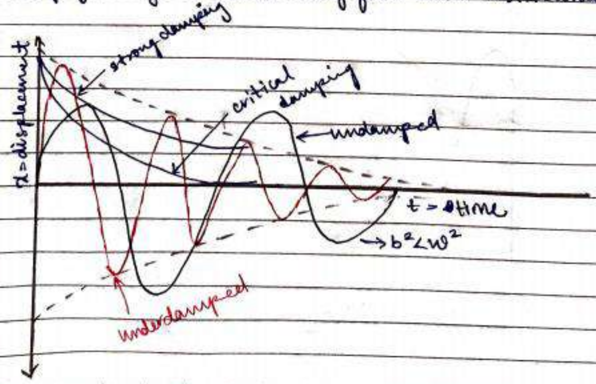
therefore $-b+\sqrt{b^2-\omega^2}$ and $-b-\sqrt{b^2-\omega^2}$ will be negative.

It means displacement consists of two terms & both die off exponentially to zero without performing any oscillation. This type is called overdamped or deathbeat damping.

eg:- Pendulum moving in thick oil.



The motion of overdamped system is non periodic, regardless of initial condition. The larger the damping the longer the time to decay from initial disturbance.



8	Illustrate the motion of damped oscillator for the cases of light damping, heavy damping and critical damping with a neat sketch of the graph

SAME AS 7Th answer.

9 Discuss the oscillations and amplitude variation with respect to forcing frequency in case of forced damped oscillator.

Depending upon the relative values of p & ω, there are three cases.

case(i) :- If $p << \omega$ (Driving frequency is low),
As $b \to 0$, $A = \frac{f}{\omega^2}$ and $\theta = \tan^{-1}(0) = 0$

Amplitude of vibration depends on magnitude of Force F & force constant K.
Displacement & applied force are in phase.

case(ii) :- $\omega = p$ (Resonance).

$A = \frac{f}{2bp} = \frac{f}{\gamma\omega}$ Amplitude depends on damping factor γ, For small damping amplitude is large.
$\theta = \tan^{-1}(\infty) = \pi/2$. phase diff is $\pi/2$.

case(iii) :- $p >> \omega$
then $A = \frac{f}{p^2}$ $\theta = \tan^{-1}(-0) = \pi$
At high applied frequencies, amplitude decreases
Phase diff is π.

16 Relate the displacement and frequency of a particle executing simple harmonic motion

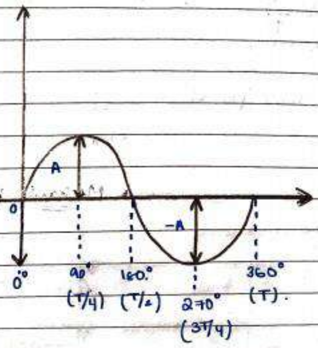
→ Frequency :- Frequency of SHM is no. of oscillations that a particle performs per unit time. The SI unit of frequency is Hertz or rps (rotation per second)

• Displacement :- The distance of the particle along the direction of motion from equilibrium position at a given instance of time is called displacement

$x = A\sin\omega t$.

Amplitude in (m)
time in seconds (s).
displacement in meters (m)
Angular frequency in radians per second (rad/s).

Angle	0°	90°	180°	270°	360°
Time	0	T/4	T/2	3T/4	T
x	0	A	0	-A	0



18 Explain the terms: (i) Periodic motion (ii) Oscillatory motion (iii) Damped and undamped oscillations (iv) Forced oscillations

A wave in which a uniform series of crests and troughs follow one after the other in regular succession. By contrast, the wave produced by applying a pulse to a stretched string does not follow regular, repeated patterns.

Oscillatory motion is defined as the to and fro motion of an object from its mean position. The ideal condition is that the object can be in oscillatory motion forever in the absence of friction but in the real world, this is not possible and the object has to settle into equilibrium.

To describe mechanical oscillation, the term vibration is used which is found in a swinging pendulum. Likewise, the beating of the human heart is an example of oscillation in dynamic systems.

Damping is the process of restraining or controlling the oscillatory motion, such as mechanical vibrations, by the dissipation of energy.

UNDAMPED IS THE OPPOSITE OF DAMPED. (ASAT)

When a body oscillates by being influenced by an external periodic force, it is called forced oscillation. Here, the amplitude of oscillation, experiences damping but remains constant due to the external energy supplied to the system.

PART-C:

1 A particle executes a S.H.M of period 10 seconds and amplitude of 1.5 meter. Calculate its maximum acceleration and velocity.

Solution :-

Data :-

$T = 10$ Sec

$a = 1.5$ m

$a_{max} = ?$

$V_{max} = ?$

Formulae :- $V_{max} = a\omega$

$a_{max} = \omega^2 a$

$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{10} = 0.628$ rad/sec

$V_{max} = a\omega = 1.5 \times 0.628 = 0.942$ m/s

$a_{max} = \omega^2 a = (0.628)^2 \times 1.5 = 0.591$ m/s^2

2 A body executing S.H.M has its velocity 16cm/s when passing through its centre mean position. If it goes 1 cm either side of mean position, calculate its time period.

Solution :-

Data :- $V_{max} = 16\ cm/s$

$a = 1\ cm$

$T = ?$

Formulae :- $V_{max} = a\omega$

$T = \frac{2\pi}{\omega}$

$\omega = \frac{V_{max}}{a} = \frac{16}{1} = 16\ rad/sec$

$T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{16} = 0.3925\ Sec$

3 A body of mass 5 gm is subjected to an elastic force of 40 dyne/cm, and a frictional force of 5 dyne-sec/cm. If it is displaced through 2 cm and then released. Find whether the resulting motion is oscillatory or not? Also find the time period if it is oscillatory.

Solution :-

Data :- $m = 5g$

$k = 40$ dyne/cm

$b = 5$ dyne-sec/cm

$x = 2cm$

$T = ?$

Formulae :- $\frac{d^2x}{dt^2} + 2r\frac{dx}{dt} + \omega^2 x = 0$ (Expression for damped oscillation)

$2r = \frac{b}{m}$ or $r = \frac{b}{2m}$

$\omega^2 = \frac{k}{m}$

$T = \frac{2\pi}{\sqrt{\omega^2 - r^2}}$

If $r^2 < \omega_0^2 \Rightarrow$ oscillatory motion

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(20)

$r = \frac{b}{2m} = \frac{5}{2 \times 5} = 0.5$

$r^2 = 0.25$

$\omega^2 = \frac{k}{m} = \frac{40}{5} = 8$

Since $r^2 < \omega^2$ the motion is oscillatory.

$T = \frac{2\pi}{\sqrt{\omega^2 - r^2}} = \frac{2 \times 3.14}{\sqrt{8^2 - 0.25}} = \frac{6.28}{\sqrt{7.75}} = \underline{2.25 \text{ sec}}$

4 A 0.5 kg mass suspended from a linear spring of force constant 1000 N/m has a damping coefficient 0.05 Ns/m. An external force $F = F_0\, Sin\,(pt)$ is applied, where $F_0 = 25$N and p is twice the natural frequency of the system, then calculate (i) Amplitude of resulting motion (ii) Phase shift of displacement with request to driving force.

Solution :-

Data :-
$m = 0.5$ kg
$k = 1000$ N/m
$b = 0.05$ Ns/m
$F = F_0 \sin pt$
$F_0 = 25$ N
$p = 2\omega_0$
$A = ?$
$\phi = ?$

Formulae :-

$$A = \frac{f_0}{\sqrt{(\omega_0^2 - p^2)^2 + 4r^2p^2}}$$

$$f_0 = \frac{F_0}{m}$$

$$\omega_0^2 = \frac{k}{m}$$

$$r = \frac{b}{2m}$$

$$\phi = \tan^{-1}\left(\frac{2rp}{\omega^2 - p^2}\right)$$

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$$f_0 = \frac{F_0}{m} = \frac{25}{0.5} = 50 \text{ N/kg}$$

$$\omega_0^2 = \frac{k}{m} = \frac{1000}{0.5} = 2000/\text{sec}^2$$

$$r = \frac{b}{2m} = \frac{0.05}{2 \times 0.5} = 0.05/\text{sec}$$

$$A = \frac{50}{\sqrt{(2000 - 8000)^2 + 4 \times (0.05)^2 \times 8000}} = \underline{0.0083\text{ m}}$$

$$\phi = \tan^{-1}\left(\frac{2rp}{\omega_0^2 - p^2}\right) = \tan^{-1}\left(\frac{2r \times 2\omega_0}{\omega_0^2 - 4\omega_0^2}\right)$$

$$\phi = \tan^{-1}\left(\frac{-4r}{3\omega_0}\right) = \tan^{-1}\left(\frac{-4 \times 0.05}{3 \times \sqrt{2000}}\right) = \underline{0.085}$$

9 A body executes S.H.M. such that its velocity at the mean position is 1m/s and acceleration at one of the extremities is 1.57 m/s^2. Calculate the time period of vibration.

$$\frac{a_{\max}}{v_{\max}} = \frac{A \bullet^2}{A \bullet} = \frac{1.57}{1} \Rightarrow \bullet = 1.57 rad$$

$\therefore$Time period $T = \frac{2 \bullet}{1.57} = \frac{2(3.14)}{1.57} = 4s.$

but $A \bullet = 1$, i.e., $A(1.57) = 1$ or $A = \frac{1}{1.57}$

$\therefore$Amplitude $A = 0.637m.$

****