Polar Coordinates

Comparing Polar Coordinates and Cartesian Coordinates

n	Error (Polar)	Error (Cartesian)
10	7.2×10^{-6}	2.0×10^{-4}
100	7.4×10^{-10}	7.2×10^{-6}
500	1.2×10^{-12}	6.4×10^{-7}
1000	7.5×10^{-14}	2.3×10^{-7}
3000	9.2×10^{-16}	4.4×10^{-8}
5000	3.1×10^{-16}	2.0×10^{-8}
10000	1.9×10^{-16}	7.2×10^{-9}

Legendre and Gauss

Legendre Polynomials

- General Formula: $P_n(x) = \frac{1}{2^n n!} \frac{\mathrm{d}^n}{\mathrm{d}x^n} \left(x^2 - 1\right)^n$.
- First few terms: $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = x^2 - \frac{1}{3}$.
- Suppose we have a n^{th} order Legendre polynomial $P_n(x)$, it then has roots $x_1, \ldots, x_n$, which are symmetric. The weight of symmetric roots can be calculated by

$$c_i = \int_{-1}^{1} \prod_{\substack{j=1 \\ j \neq i}}^{n} \frac{x - x_i}{x_i - x_j} dx.$$

First Few Terms

n	Roots $r_{n,i}$	Coefficients $c_{n,i}$
2	-0.577350269189625 7	1.000000000000000
	0.577350269189625 7	1.000000000000000
3	0.000000000000000	0.888888888888888
	-0.774596669241483 4	0.555555555555556
	0.774596669241483 4	0.555555555555556
4	-0.339981043584856 3	0.652145154862546 1
	0.339981043584856 3	0.652145154862546 1
	-0.861136311594052 6	0.347854845137453 8
	0.861136311594052 6	0.347854845137453 8

Gaussian Double v.s Simpson's Double

Comparison Between Simpson and Gaussian Integral Method

n	error(Gaussian)	error(Simpson)
1	0.037682072451781	0.108669726772769
2	0.001525047658392	0.002503112733404
3	0.000049493897754	0.039809322528766
4	0.000001511841346	0.000202140564106
5	0.000000045271619	0.023412864485344
6	0.000000001345410	0.000042754312220
7	0.000000000039849	0.016514363451853
8	0.000000000001178	0.000013899605129
9	0.000000000000035	0.012743272142899
10	0.000000000000001	0.000005769449753

Gaussian Triple

Error (Gaussian Triple Integral)

n	error(Gaussian)
1	0.16666666666666674068
2	0.00000000000000022204
3	0.00000000000000022204
4	0.00000000000000066613
5	0.00000000000000000000
6	0.00000000000000022204
7	0.00000000000000088818
8	0.00000000000000022204
9	0.00000000000000000000
10	0.00000000000000022204