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2017

MCM/ICM

Summary Sheet

(Your team's summary should be included as the first page of your electronic submission.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

Modeling Attenuation of HF Radio Waves from Physical Principles

When modeling a physical phenomenon like the interaction of electromagnetic radiation with the earth's seas or atmosphere, physicists are confronted with a tug-of-war between *knowing* and *understanding* should a model sacrifice tangible physical significance to optimize its fit to empirical data, or should physical grounding be prioritized in hopes that *understanding* can lead to future intuitions about a problem? We take a strong stance on this matter and assert that understanding is the way to go. Our paper exemplifies this line of thought.

Starting from basic physical principles, we enlist a ground up approach in which electromagnetic interactions are analyzed in detail before analytical expressions are derived to specifically model the propagation and reflection of radio waves. Committing to physical rigor poses some formidable challenges, as the analytical expressions for the complicated systems we consider are not tame. Nonetheless we stay the course!

To model the impact of seawater turbulence on radio wave transmittance, we first establish a metric for the turbulence of a seawater flow based on turbid kinetic energy. Using a corrected linear wave theory, we can then correlate this energy to an approximate wave amplitude and wavelength starting with Bernoulli's equations. This is a crucial corollary because it allows us to parameterize both seawater permittivity and electromagnetic scattering indirectly as functions of kinetic energy. Our models for permittivity and scattering are also understood via physical explanation. We outline the complicated electro-chemical process that underlies permittivity and give both statistical and deterministic descriptions of the scattering of radio waves off rough water surfaces.

Thus, the turbulence model runs as follows: a turbulent kinetic energy and radio-wavelength are chosen--for which a particular electric permittivity and conductivity of seawater correspond--and the water wave amplitude and wavelength are calculated. The permittivity and conductivity then define a reflection coefficient via Fresnel's equations and a scattering parameter is calculated based on the amplitude and wavelength of the water waves. With the reflection coefficient and scattering distribution in hand, a net attenuation can be calculated for the ricocheting radio wave. Our calculations for beam attenuation due to incidence on turbulent seawater are in general agreement with empirical data. We calculated attenuations of 2 decibels for moderately turbulent flows.

A detailed model was also developed for the ionosphere from atomic principles, which elegantly describe the interactions between incident angle, frequency, reflection, and attenuation of the radio wave.

"And it is entirely possible that future generations will look back, from the vantage point of a more sophisticated theory, and wonder how we could have been so gullible."
-David J. Griffiths.

Modeling the Attenuation of HF Radio Waves from Physical Principles

Team 87932

Synopsis

Since it was first discovered that high frequency (HF) radio waves can bounce extreme distances around the earth, there have been countless efforts to model the process. Thus far, the interaction with the ionosphere has been much better described than the interaction with the ocean surface. While the ionosphere can be described accurately with the classical laws of deterministic electromagnetic theory, the chaotic nature of the ocean surface has made a similar analysis challenging. We have built an improved model of the ocean surface that accounts for both geometric and electrochemical properties.

We propose two different models for the geometry of the surface. The first uses a statistical approach. If a surface is rough, then the path length from an emitter to receiver is different depending on where a ray hits the surface. This causes destructive interference, resulting in a decreased reflection coefficient. To model this, we assume that the height of the ocean is caused by many uncorrelated factors, and thus follows a normal distribution. This allows us to determine that the distribution of phase changes also follows a normal distribution. From this we can determine that the reflection coefficient is reduced (from that of a mirror) by an exponential decay that goes as the average of the variation of the surface height.

This model fits experimental data well for relatively smooth surfaces, but for very rough surfaces we modify it with a modified Bessel function of 0th order, so that it agrees well with measurements from a wider range of surface types.

The second model we propose is deterministic model that is based off smoothing out the original surface with a kernel and then applying geometrical optics. The benefit of this method is that it works on arbitrary surfaces that do not necessarily have a normal height distribution. We need to average the original ocean surface with a half-wavelength sized kernel because of the Abbe diffraction limit: the 10-100m long radio waves can't "see" any of the corrugations on the ocean surface smaller than 5-50m. Once we apply the averaging kernel, it is possible to apply geometric optics and Fresnel equations to determine the reflectance coefficient of that one patch, and find the direction θ_r of any reflected beam coming from direction θ_i. For any one reflected ray, we can find the component of the ray that actually goes in the "correct" direction (i.e. θ_i, the one that it would if the surface were a mirror) by scaling the reflectance by $\cos(\theta_r - \theta_i)$. If we take the average of the effective reflectances of the patches, we get the effective reflectance of the entire system.

We also modeled the electrochemical properties of the ocean using a statistical model parameterized by energy, which allows us to describe how turbulence affects the varying index of refraction of the seawater. This was important because the changing index of refraction modifies the reflection coefficients described by the Fresnel equations.

Contents

1 Nomenclature

Symbol	Description
ϵ	Electric permittivity of a material
μ	Magnetic Permeability of a material
n	Index of refraction of a material, may or may not be complex
R	Net reflectance of an EM wave interacting with a material
σ	Conductivity of a material
ω	Angular frequency of an EM wave
θ_i	Angle of incidence with respect to vertical
θ_t	Angle of transmission with respect to vertical
λ	Wavelength of electromagnetic radiation
$*$	Represents the complex conjugate of a number
$\Delta h, \sigma_h$	Average standard deviation of the mean height of a rough surface
SNR	Signal to Noise Ratio

Table 1: All important variables

2 Introduction

2.1 The Problem

The modeling of ultra-long-distance radio communication can be broken up into three main components:

1. Transmission through the atmosphere
2. Reflection off the ionosphere
3. Reflection off the ocean/ground

Once all three components are described, it is a relatively simple matter to combine them in order to determine a description of how a beam will travel, how much the signal is attenuated, and thus the maximum communication range.
Here is how we have interpreted each part of the problem, in terms of the inputs to each section and our methods of modeling them:

1. Transmission through the atmosphere: Given the distance traveled by a beam of radio energy, we can find how much its strength was attenuated over that distance. Atmospheric transmission is extremely simple, since the atmosphere is fairly unresponsive to radio frequencies, and we can completely ignore it except for the geometric attenuation as the energy of the beam spreads out in a cone.

2. Reflection off the ionosphere: Given the frequency and direction of an incoming beam of radio energy, as well as the current state of the ionosphere (e.g. location and thickness of the different layers) we can determine the location where the beam will exit the ionosphere and how much its strength was attenuated. We model this using the electron density of the ionosphere, which determines its plasma frequency and thus its index of refraction. Using the index of refraction and Snell's law of refraction, we can find how the beam

slowly bends back down towards the earth. We can then actually compute the location of the exiting beam by modeling the beam as being reflected off some imaginary mirror far above the ionosphere.

3. Reflection off the ocean/ground: This is the most difficult of the components. Given the frequency and direction of an incoming beam of radio energy, as well as the geometry and material of the surface, we can find what fraction of the energy is reflected back at the same angle (As we argue later, we only care about the energy reflected back at the same angle). We model the geometry using several different models. First, we deal with the simple case of a flat ocean, using Fresnel's equations and the complex index of refraction of seawater to determine a reflection coefficient. Then, we will build off of this with a statistical model based off of the assumption that the angle of the normal vectors of the ocean surface follow a Gaussian distribution, which causes the amount of light to reflect in the correct direction to also follow a Gaussian. Finally, we refine this further with a deterministic model that allows the geometry of any ocean surface to be taken into account.

2.2 Assumptions

Throughout this paper we assume various simplifications

- Losses due to atmospheric factors such as rain, fog, clouds, lightning are negligible. These effects only become significant in much higher frequencies. Quantum Mechanics assures us that this will not be the case when radio waves are considered [1].
- The magnetic permeability of all relevant mediums are approximately μ_o. This is in correspondence with the classical literature that the magnetic permeability of most normal substances is most nearly equal to the magnetic permeability of free space [2].
- We treat In the statistical model of scattering, we assume that the ocean is a complex system, consisting of many factors coupled together in highly sophisticated ways, each contributing its part to a complex system. We can then treat this plethora of variables as random variables, and apply the Central Limit theorem.
- We assume that the usual conductivity of sea is $5\left(\frac{S}{m}\right)$, and the real part of the relative dielectric permittivity of sea is 80. We are justified in assuming this because they are in correspondence with the experimentally found values. [3].
- We assume that ϵ and σ are primarily functions of temperature. Temperature of sea water dictates (for the most part) these two quantities.
- We assume that the radio waves hit the earth at only glancing angles smaller than 45 degrees. This is because, as will be explained later, if this angle is large, the radio waves will escape from the ionosphere.
- We assume that the radio waves hitting the sea surface a unpolarized. This is justified because, even if the radio waves were polarized in the first place, with the reflection from the ionosphere, they are bound to become depolarized.

3 Electromagnetic Theory Basics

3.1 Waves

Electromagnetic radiation is a perturbation of a pair of electric and magnetic field that travels through space. For this problem, we only care about the electric field portion. The $\vec{E}$ at some location $\vec{x}$ and time t can be described mathematically as [2]

$$\vec{E} = \vec{E}_o \cos(\vec{k} \cdot \vec{x} - \omega t) \tag{1}$$

Here, $\vec{E}_0$ is the amplitude of the wave, as well as the direction that the field points. $\vec{k}$ is called the *propagation vector*, and $|\vec{k}|$ is called the *propagation number*. $\vec{k}$ points in the direction that the wave travels and $|\vec{k}|$ defines the wavelength λ of the wave as $|\vec{k}| = k = \frac{2\pi}{\lambda}$. Finally, ω is called the *angular frequency* of the wave, and defines the temporal frequency f of the wave as $\omega = 2\pi f$.

3.2 Index of Refraction

The speed v of a wave is simply $v = f\lambda = \frac{\omega}{k}$. In a vacuum, the speed of light is c, but in other media it can be different. When the speed changes, the wavelength also changes, but the frequency stays the same. The *index of refraction* of a material, denoted n, describes the speed of light in that media [2]:

$$n \equiv \frac{v}{c} \tag{2}$$

In materials that are conductors, the wave attenuates exponentially as it travels through the material. This happens because the charges in the conductor are able to vibrate along with the incident wave, causing their own electric field which cancels out the incident wave. [1, 2]The *skin depth* of a conductor describes how far a wave will penetrate into it, and is defined as the distance at which the amplitude has attenuated by a factor of $\frac{1}{e}$. In a conductor, when the wave attenuates, the index of refraction gains an imaginary term and becomes [2]:

$$n \equiv \frac{v}{c} + i\kappa \tag{3}$$

3.3 Permittivity and Permeability

The permittivity ϵ of a material is a measure of how much that material responds to an applied electric field [2]. ϵ_0 is the permittivity of a vacuum, and is a constant throughout the universe. We can also describe the permittivity of a material in relation to a vacuum using the relative permittivity ϵ_r as

$$\epsilon_r \equiv \frac{\epsilon}{\epsilon_0} \tag{4}$$

In most materials ϵ_r is less than 1.
The permeability μ of a material is the analog to permittivity in relation to magnetic fields. In most materials $\mu \approx \mu_0$, and so we make this approximation throughout this paper.
Using the permittivity and permeability, we can also describe the velocity of the wave as [2]

$$v = \frac{1}{\sqrt{\mu\epsilon}} \tag{5}$$

3.4 Power and Intensity

The power (energy per unit time) P of a wave is determined by the amplitude of the wave as [4]

$$P = E_0^2 \tag{6}$$

The Intensity I of a wave describes the flux of power through a unit area A [4]

$$I = \frac{P}{A} \tag{7}$$

4 Model

We have broken our model into three parts: Interactions with the atmosphere, reflections off the ionosphere, and reflections off the earth's surface.

4.1 Atmospheric Interactions

What will happen to the radio beam as it propagates through the atmosphere? Actually, not much. Air has an electric permittivity, magnetic permeability, index of refraction, and conductivity that are all pretty much the same as a vacuum. What this means is that the wave propagates steadily without any attenuation from the matter. Also, the wavelengths that we are working with don't interact with the water vapor and other particulates in the air [1].
The one thing we do have to deal with in this mode is the dissipation of the emitter's initial energy across space. This happens because we need to assume that the transmitting antenna is emanating spherical waves outwards in all directions. If our antenna acted like a laser, and directed its 100 Watts of energy in one collimated beam, then no matter where you looked along the beam the total amount of power flux through a cross section would be the same. However, because we are assuming that our transmitting antenna does not know where the receiving ship is (or there could be multiple ships at the same time), the antenna must be designed to radiate in all directions. Thus, if our transmitting antenna has a power of P_0, then out at a distance r away from it that initial energy is spread out over a sphere of radius r. Thus, the intensity I of the radiated energy at that point would be [2]

$$I = \frac{P_0}{4\pi r^2} \tag{8}$$

This is called the *inverse square law*.

4.2 Ionospheric Interactions

4.2.1 Overview

The ionosphere is a region of the upper atmosphere characterized by ionization, full of many free electrons and positively charged atoms. A profile of the ionization density can be seen in Figure 1. The ionosphere is caused by the intersection of two things: high amounts of radiation from the sun, and low pressure. Throughout the atmosphere high energy (UV, soft X-rays) photons from the sun constantly strike gas molecules and knock electrons off. Low in the atmosphere, where the pressure is higher, the electrons and ions quickly recombine to become neutral, but at the low pressures of the ionosphere these recombinations are less likely, so the electrons and ions can exist for longer in a plasma [5].

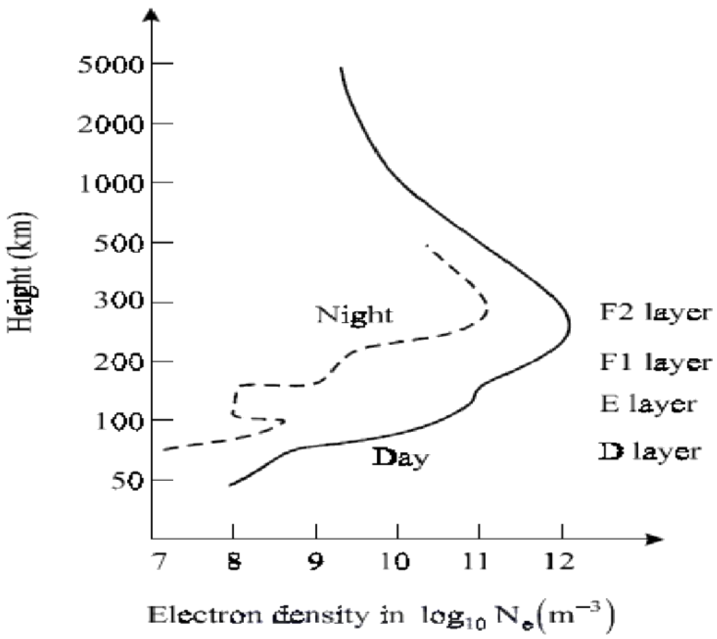

Figure 1: Profile of the electron density of the ionosphere. At night, attenuation from the lower D Layer is not present, and waves are able to bounce off of the higher F1 and F2 layers instead of the lower E layer.

4.2.2 Radio Wave Interactions

Because of the free charges, the ionosphere acts like a conductor and interacts with radio waves. All of the electrons act like tiny charged oscillators and begin to vibrate when bathed in the varying $\vec{E}$ of the radio wave (the positive ions, with their larger mass, don't vibrate appreciably) [2]. The vibrating electrons re-emit the radiation in all directions at the same frequency[1]. more dense the plasma, the better the conductivity, and the more frequent the collisions. If the conductivity were perfect, then it would be as if there were a giant metal mirror in the sky, and all of the up-going radiation would be reflected back downwards. However, because the conductivity is not infinite, and instead slowly increases as you move higher, the ionosphere acts like a "gradual mirror," slowly bending the up-going radiation back down in a gentle curve. Let's do this more quantitatively. The motion of an electron in an electric field $\vec{E} = \vec{E_o} \cos \omega t$ can be modeled using [5]

$$m_e \frac{\partial^2 \vec{r}}{\partial t^2} = -e\vec{E} = -e\vec{E_o} \cos \omega t \tag{9}$$

where $\vec{r}$ is the position vector of an electron, m_e is the mass of an electron, and e is the charge of an electron. We are ignoring any collisions with other electrons. This has the solution

$$\vec{r} = \frac{e\vec{E}}{m_e \omega^2} \tag{10}$$

Thus the dipole moment cause by one electron is $e\vec{r}$, and thus the Polarization $\vec{P}$ of the plasma, which is the dipole moment per unit volume, is [2]

$$\vec{P} = Ne\vec{r} = \frac{e^2\vec{E}N}{m_e\omega^2} \tag{11}$$

where N is the electron density, the number of electrons per unit volume. Next we will find the relative permittivity ϵ_r as [2]

$$\epsilon_r = 1 - \frac{\vec{P}}{\epsilon_0\vec{E}} = 1 - \frac{e^2N}{m_e\epsilon_0\omega^2} \tag{12}$$

If instead of angular frequency we use actual frequency $f = \frac{\omega}{2\pi}$, and define a constant called the *plasma frequency* ω_0 such that [5]

$$\omega_0^2 = \frac{e^2N}{m_e\epsilon_0} \approx 3183N\ s^{-2}m^{-3} \tag{13}$$

then we can rewrite the relative permittivity as [5]

$$\epsilon_r = 1 - \frac{\omega_0^2}{\omega^2} \approx 1 - \frac{81N}{f^2}\ s^{-2}m^{-3} \tag{14}$$

Once we have the relative permittivity, we can find the index of refraction $n = \frac{v}{c} = \sqrt{\epsilon_r\mu_r} \approx \sqrt{\epsilon_r}$ of the plasma.

$$n = \sqrt{1 - \frac{81N}{f^2}\ s^{-2}m^{-3}} \tag{15}$$

The index of refraction is useful because we can define how the radio wave will bend when entering this layer using Snell's law [1]. As illustrated in Figure 2, Snell's law states:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{16}$$

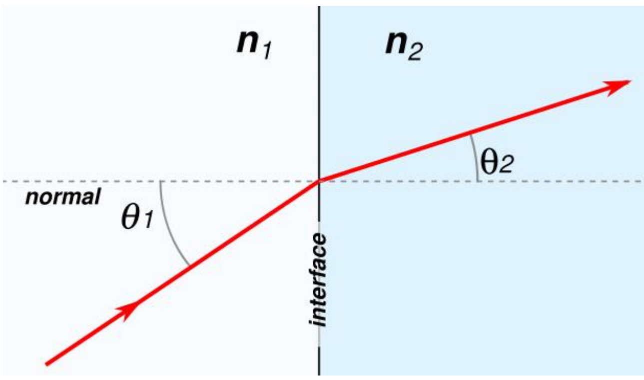

Figure 2: Snell's law says that at the interface of two materials $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Let's interpret equation (15) a bit [5]:

- $f^2 > 81N\ s^{-2}m^{-3}$, frequency is moderately high compared to electron density: n is real, but less than one. If a radio wave hits this layer from below, where the $n_{air} = 1$, then following Snell's law the light will be refracted back downward.

- $f^2 \gg 81N\ s^{-2}m^{-3}$, frequency is extremely high compared to electron density: This means that $n \approx 1$, so the high frequency light is hardly bent at all, and just passes through this layer of the ionosphere.
- $f^2 < 81N\ s^{-2}m^{-3}$, frequency is low compared to electron density: This means n is imaginary, so the wave attenuates exponentially inside the plasma (it is still there, just perfectly canceled by the secondary wave! Remember, we ignored collisions), and is reflected back off of the surface.

Using Snell's law and equation (15) we can determine the critical angle θ_c at which we can expect a certain frequency to reflect back through the ionosphere.
As our wave travels upwards through the ionosphere from air ($n_0 = 1$), it encounters many layers of different density plasma until it reaches the layer with the smallest index of refraction n_{min} at the kth layer:

$$n_0 \sin\theta_0 = n_1 \sin\theta_1 = \cdots = n_{min} \sin\theta_k \tag{17}$$

If the beam reflects back down, we need $\theta_k = \frac{\pi}{2}$. We know $n_0 = 1$, and so we can solve for θ_0:

$$\sin\theta_c = \sin\theta_0 > n_{min} \tag{18}$$

which implies (since n_{min} corresponds to N_{max})

$$\sin^2\theta_c = 1 - \cos^2\theta_c > 1 - \frac{81N_{max}}{f^2}\ s^{-2}m^{-3} \tag{19}$$

$$\cos^2\theta_c < \frac{81N_{max}}{f^2}\ s^{-2}m^{-3} \tag{20}$$

If our beam is shot straight up, and is incident normal to the ionosphere with $\theta_c = 0$, then we can find the critical frequency [5]:

$$f_c = 9\sqrt{N_{max}}\ ms^{-1} \tag{21}$$

This typically results in a f_c of between 40 MH and 25 MHz [5]. If we are dealing with a beam that is incident to the ionosphere at some general angle θ, then the maximum usable frequency is [5]

$$f_{mu} = \frac{9\sqrt{N_{max}}\ ms^{-1}}{\cos\theta} = f_c \sec\theta \tag{22}$$

Any radio wave emitted above f_{mu} at angle θ will pass through the ionosphere and be lost.

4.2.3 Attenuation

As the radio waves oscillate the electrons in the ionosphere, if the electrons collide with other particles, their energy will be lost as heat before they can re-radiate the wave. This will attenuate the signal. Only in the lower, denser D layer do these electron collisions significantly occur. If we shoot our radio beam at a low angle, it will travel through a longer distance of slab, and will thus be attenuated more. If we use a frequency near or below the plasma frequency of the D layer, the beam will interact more with the D layer and be attenuated less. Finally, if we only need to perform communication at night, we won't have to worry about the D layer at all, since it dissipates a few hours after sunset.

4.3 Ocean Reflections

4.3.1 Calculating Reflection Coefficients off a smooth surface.

The theory of radio wave interaction with sea is guided by Maxwell's Equations. The purpose of this section is to quantify, in the end, how much a reflected radio wave at a particular point in sea is attenuated, with respect to an incident radio wave. To do this, we start from the concept of refractive index. Since sea water is conducting, it has a complex relative permittivity, given by [3]:

$$\epsilon_r = \epsilon + i\frac{\sigma}{\omega\epsilon_o} \tag{23}$$

For sea water, the real part of relative permittivity usually equals 80 [3].
Thus:

$$n = \sqrt{\frac{\epsilon_o\mu_0}{\epsilon\mu}} = \sqrt{\frac{\epsilon}{\epsilon_o}} = \sqrt{\epsilon_r} \tag{24}$$

So let's consider the simplest case of radio wave striking a perfectly smooth point on the surface. When we apply Maxwell's equations to an EM wave hitting the boundaries of surfaces, applying boundary conditions of continuity, we can get the amplitude reflection coefficients of the electromagnetic wave. These are given by Fresnel's equations[1].

$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i cos(\theta_i) - n_t cos(\theta_t)}{n_i cos(\theta_i) + n_t cos(\theta_t)} \tag{25}$$

$$r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t cos(\theta_i) - n_i cos(\theta_t)}{n_i cos(\theta_t) + n_t cos(\theta_i)} \tag{26}$$

These equations determine ratio of the amplitudes of the parallel and perpendicular components of the incident and reflected electromagnetic waves. Notice that n is complex here.
Also important is the quantity Reflectance, that measure the ratio of the reflected power (per unit area) to that of incident power (per unit area).Therefore: [1]

$$R = \frac{P_{reflected}}{P_{incident}}. \tag{27}$$

But power is directly proportional to the amplitude-squared of the Electric field of the Electromagnetic wave. Therefore, we have, in term of the parallel and perpendicular components:[1]

$$R_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)^2_\perp \tag{28}$$

$$R_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)^2_\parallel \tag{29}$$

For an unpolarized wave, half of reflectance of the radio wave is perpendicular to the plane of incidence, whereas the other half is parallel to the plane of incidence.[1]

$$R = \frac{1}{2}(R_\parallel + R_\perp). \tag{30}$$

The point of all of this math is this: the amplitude reflection coefficients $r_\parallel$ and $r_\perp$ are dependent on the complex index of refraction, which is dependent on the permittivity, which is dependent on conductivity. So here is the final relationship we were seeking. Changing conductivity and permittivity change index of refraction of seawater, therefore affecting the reflection coefficients of the radio wave hitting the sea waves. In its full form, the perpendicular component of Reflectance, for example, can be written in terms of permittivity. This is done by plugging equation and equation 23 into equation 24.Then plugging in equation 24 into equation 25. Finally, we substitute equation 25 into 28.

$$R_\perp = |(r_\perp)|^2 = \left(\frac{(\epsilon + i\frac{\sigma}{\omega\epsilon_o})^{\frac{1}{2}}.cos(\theta_i) - n_i cos(\theta_t)}{n_i cos(\theta_t) + (\epsilon + i\frac{\sigma}{\omega\epsilon_o})^{\frac{1}{2}}.cos(\theta_i)}\right)\left(\frac{(\epsilon + i\frac{\sigma}{\omega\epsilon_o})^{\frac{1}{2}}.cos(\theta_i) - n_i cos(\theta_t)}{n_i cos(\theta_t) + (\epsilon + i\frac{\sigma}{\omega\epsilon_o})^{\frac{1}{2}}.cos(\theta_i)}\right)^* \quad (31)$$

The faint of the heart can take courage. We will enlighten the reader by harnessing the power of numerical computing, plotting $R_\perp$ and $R_\|$ with varying incidence angles and permittivity. Note that $\cos(\theta_t)$ was calculated using Snell's law [2]. So let's start with the simplest of all the cases. Consider a calm ocean and model it as being perfectly smooth. The complex index of refraction can be calculated from the equations and numbers discussed above. We find, for a 30MHz radio wave, usually $n = 39.2 + 38.2i$. The figure below shows the Reflectance vs. Incidence angles at this n.

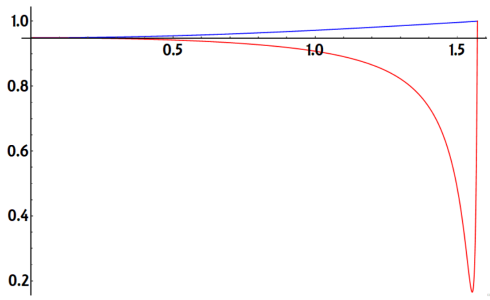

Figure 3: $R_\perp$**(blue) and** $R_\|$ **(red) vs. angles of incidence of a 30MHz radio wave**

4.3.2 Quantifying Turbulence

Seawater turbulence is a chaotic physical phenomena that requires statistical interpretations. The purpose of this section is to develope a metric by which we can measure the turbidity of ocean surfaces. Because of the unpredictable topography of turbulent water, this is most reliably done by looking at the kinetic energy of flows. We separate kinetic energy into two parts: average flow energy and trubid flow energy. The kinetic energy dK_e of an area of seawater (in 2D) da can be given by

$$dK_e = \frac{1}{2}\rho(u^2 + w^2)da \quad (32)$$

Where ρ is the surface density of the fluid and u and w are the Cartesian components (x and z) of fluid velocity at da. If we let $u = \bar{u} + \langle u \rangle$ (as well as for w), where $\bar{u}$ is the deviation of u from the mean velocity due to turbulence, we can split our expression for kinetic energy into turbid and non-turbid parts. Thus, we can define an infinitesimal for turbid kinetic energy dK_t which can be integrated to find the total turbid energy.

$$dK_t = \frac{1}{2}\rho(\bar{u}^2 + \bar{w}^2)da \implies K_t = \frac{\rho}{2}\iint_S(\bar{u}^2 + \bar{w}^2)dxdz \quad (33)$$

This expression defines turbid kinetic energy, which is a good start, but not enough. To useful in our model, we must correlate this energy to some topographical approximation of the surface so it can be applied to our scattering predictions. First we look at the Turbulence Intensity, I_u of a flow:

$$I_u = \frac{\bar{u}}{\langle u \rangle} \tag{34}$$

This quantity is simply a ratio of the turbulent velocity fluctuations to the average velocity of the fluid. Because there is good empirical data for this quantity, we can use it to deduce the energy of the mean flow. If we approximate the mean flow to be a linear standing wave, we can use our value for the kinetic energy of the mean flow (K_m) to solve for the amplitude of such a wave. First lets get an expression for the velocity field of a standing wave, ignoring time.

$$\vec{v}(x, z) = \nabla \Phi \tag{35}$$

$$\Phi = -\frac{a\omega}{k} e^{kz} sin(kx) \tag{36}$$

(Reference) Where a is the wave amplitude, ω is the oscillation frequency, and k is the wave number. This is the velocity vector field of a standing linear wave assuming infinite water depth. It is defined by the scalar veloctiy function Φ which can be derived from Bernouli's equation. The kinetic energy of this wave can be found the same way as above. Setting the kinetic energy equal to the mean kinetic energy, K_m, we have

$$K_m = \frac{\rho}{2} \iint_S |\vec{v}(x, z)|^2 dxdz = \frac{\rho}{2} \iint_S (\nabla \Phi \cdot \nabla \Phi) dxdz \tag{37}$$

Plugging in for Φ, taking the gradients, and simplifying, we have

$$K_m = \rho \iint_S a^2 \omega^2 e^{4kz} (cos^2(kx) + sin^2(kx)) dxdy \tag{38}$$

$$K_m = \rho \frac{a^2 \omega^2}{2k} L(e^{2kD} - 1) \implies \frac{k}{a^2} = \frac{\omega^2 \rho}{4K_m} L(e^{2kD} - 1) \tag{39}$$

Where L and D are the length and depth over which we have integrated. Now we have an expression for the mean kinetic energy in terms of amplitude a and wave number k. Making the stipulation that $\frac{a}{\lambda} \leq \frac{1}{7}$, as is necessary with any linear wave model (Reference), we have a system of equations with which can be solved for a and k. Before we solve this, we would be amiss not to add a correction to the amplitude of this wave due to the additional energy caused by turbulence. What follows is an subtle physics argument. Consider a small area of oscillating seawater da just under the surface. It is driven by a wind force and oscillates according to both a gravitational and bouyant restoring force. Because its ocilations are transverse, it alternates between having high kinetic energy and high potential energy. If an simple oscillating body has an average verticle kinetic energy $\langle K_z \rangle$ then according to the Hamiltionain, it must have an equal average potential energy $\langle T \rangle$. We take this simple case to be approximately true for the turbulent case and add a correction to the wave amplitude a which accounts for the additional potential energy. Then total turbulent energy (sum of kinetic and Potential) can be related to the amplitude correction a_c.

$$E_{tot} = 2K_z = \frac{\rho g a_c^2}{2} \implies a_c = \sqrt{\frac{4K_z}{\rho g}} \tag{40}$$

Notice here we have used turbulent energy to solve for an additional potential energy correction, but not actually double accounted for kinetic energy. Finally, we have a transcendental for the water wave number k which can be solved numerically and and expression for the amplitude a in terms of the solution to this transcendental.

$$k^3 = \frac{4\pi^2\omega^2\rho}{196K_m}L(e^{2kD} - 1)] \tag{41}$$

$$a = \frac{2\pi}{7k} + \sqrt{\frac{4K_z}{\rho g}} \tag{42}$$

4.3.3 Modeling the Permittivity of Seawater

On average, turbulence will decrease the electric permittivity, magnetic permeability, and conductivity of seawater. This results in a reduced reflection coefficient. Water's permittivity decreases with salination because of a shielding polarization effect. Polar water molecules in the vicinity of charged Na^+ and Cl^- will orient themselves such that their net electric field partially cancels the coulomb field of the ions. Below, the solid lines are the field produced by the ions while the dotted lines are the canceling field caused by the water molecules.

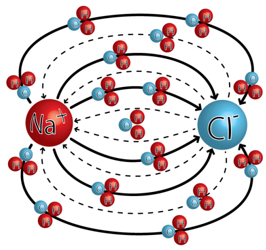

Figure 4: **Polar H_2O molecules polarizing between a dipole formed by Na^+ and Cl^- ions. https://chemistry.stackexchange.com/questions/16434/salt-concentration-and-electrical-permittivity-of-water**

In calm seawater, the water molecules can align less readily to an external electric field than in fresh water because they remain preoccupied with the dipole field caused by the ions. Despite this, calm seawater is still somewhat polarizable (and hence permittive) because the individual dipoles formed by the weak pairing of Na^+ and Cl^- ions can be partially aligned to an external electric field. When turbulent energy is added to seawater, these microstructures break down resulting in another decrease in electric permittivity. The kinetic entropy of the system supersedes the coulomb attraction between ions so that they can no longer experience a torque as a paired dipole. Whats left is a random soup of positively and negatively charged ions with the local hydrogen atoms aligned to the closet ions field.

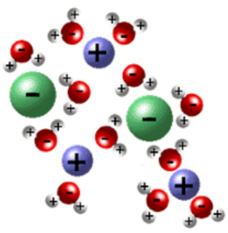

Figure 5: **Polar H_2O molecules polarizing radially around charged ions http://butane.chem.uiuc.edu/pshapley/genchem1/l21/1.html**

In this configuration, the water molecules are still occupied with their local ion electric fields and there exists no opportunity to align weakly paired ion dipoles to the field. Hence the permittivity decreases. The microstructure changes in seawater that effect permittivity are predominantly governed by temperature. We can determine changes to the permittivity of turbulent seawater by correlating turbulent energy to a change in temperature which governs permittivity. For this we use the specific heat of seawater C_{sw}.

$$T = \frac{K_e}{A\rho C_{sw}} \implies C_{sw} = 3850Jkg^{-1}K^{-1} \tag{43}$$

Where A is the total turbulent area of sea in the x,z plain that we are concerned with, K_e is the total turbulent plus mean kinetic energy, and ρ is the 2D density. We can then use this temperature to calculate permittivity. The permittivity of seawater, or rather the dielectric constant (related to ϵ by $\epsilon_r = \frac{\epsilon}{\epsilon_o}$), can be calculated using the Debye equaiton(Reference).

$$\epsilon_r(\omega) = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1+\omega^2\tau^2} + i\frac{\omega\tau(\epsilon_s - \epsilon_\infty)}{1+\omega^2\tau^2} \tag{44}$$

Where ϵ_r is the complex dielectric constant, ϵ_s is the static permittivity, ϵ_∞ is the permittivity at an infinite driving frequency, ω is the angular frequency of an incident wave, and τ is the relaxation time of the medium. In seawater, ϵ_s and τ are functions of temperature which, given only small changes in T are approximately linear.

$$\epsilon_s(T) \approx 87.8 - 0.38T, \qquad \tau(T) \approx 0.031 + 0.0086T \tag{45}$$

Given equations 44 and 45, we can formulate an expression for the dielectric constant $\epsilon_r(\omega, T)$. This will allow us to calculate the reflection coefficients of a turbulent sea surface. Setting $\epsilon_\infty = 4.23m^{-3}kg^{-1}s^4A^2$ (Reference), and plugging in, we have

$$\epsilon_r(\omega, T) = 4.23 + \frac{83.57 - 0.38T}{1+\omega^2(0.031+0.0086T)^2} + i\frac{\omega(0.031+0.0086T)(83.57-0.38T)}{1+\omega^2(0.031+0.0086T)^2} \tag{46}$$

In addition we can approximate the change in conductivity as a function of temperature by the following.

$$\sigma(T) = 0.01T + 4.15 \tag{47}$$

4.3.4 Complication: Scattering

The purpose of this section is to generalize our previous model and consider the interaction of radio waves when they hit a rough surface. We find in the end that over a turbulent ocean, the reflected radio waves are attenuated more as opposed to the radio waves hitting a calm ocean. We first characterize what do we mean by a rough surface. Since radio waves have wavelengths ranging from 10-100 m, they cannot "see" small corrugations present on the surface. But then we may ask the question, how small is small for radio waves to "see" a corrugated surface? This is quantified with the help of Rayleigh and Fraunhofer Criterion. In general, the rougher the surface, the more diffuse the scattering. We classify different roughness in terms of different types of reflections from a surface[6]

(a) **Specular:** The Electromagnetic radiation hitting the surface is smooth, and is reflected as if the surface is a mirror.

(b) **Quasi-Specular:** If an EM wave hits a slightly rough surface, then we will have a mixture of diffuse scattering as well as some specular spike.

(c) **Diffuse:** If an EM wave hits a very rough surface, then most of the scattering is diffuse, and the radiation scatters in every direction. The three situations are shown below.

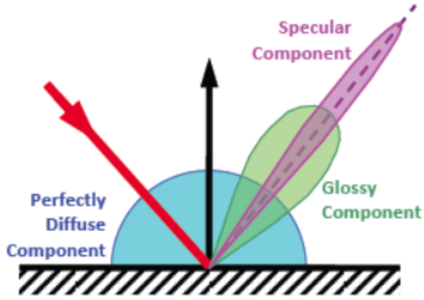

Figure 6: Different types of reflections, depending on the roughness of the surface, with respect to the EM wave. *http://www.virial.com/reflection-models.html*

The question now is when will perfect specular reflection take place? let's answer this question by considering an arbitrary surface below:

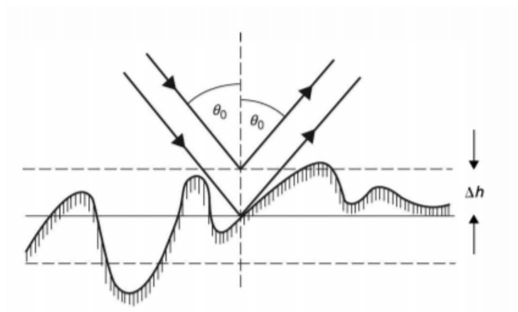

Figure 7: Rayleigh Criterion, [7]

One ray is hitting the reference plane, while the other ray is hitting the plane that is Δh above the reference plane. It can be seen that, after hitting and getting specularly scattered, the path difference between the two rays is $2\Delta X = \Delta h cos(\theta_o)$. Now we attempt to calculate the phase difference between them. The concept of phase difference is that, when one wave travels some fraction of the wavelength more than the other wavelength, then there will be a difference between their phases. Therefore, we can write the path difference as [7]:

$$\Delta X = \frac{\lambda \Delta \phi}{2\pi} \tag{48}$$

Where $\Delta\phi$ is the phase difference. Solving for this phase difference, we have [7]:

$$\Delta\phi = \frac{4\pi \Delta h cos(\theta_o)}{\lambda} \tag{49}$$

Now, suppose h is the root mean square (r.m.s) value of height across this arbitrary surface. This then implies that $\Delta\phi$ is also the r.m.s variation in the phase of the scattered rays. Following convention, a surface is defined to be smooth if $\Delta\phi$ is less than $\frac{\pi}{8}$. This leads to the following selection criterion, known as Fraunhofer Criterion [7]:

$$\Delta\phi = \frac{4\pi\Delta h cos(\theta_o)}{\lambda} < \frac{\pi}{8} \implies \Delta h < \frac{\lambda}{32 \cdot cos(\theta_o)} \tag{50}$$

If this condition is met, then we can assume specular reflection. So let's compute some numbers to get an idea of what kinds of reflections must exist when radio waves hit a turbulent ocean. Let's compute an extreme case. Suppose the r.m.s variation in our height of the waves is 0.5m. Let's also suppose that the radio waves hit the surface of the ocean at near grazing incidence angle, so that $cos(\theta_i) \approx 1$. Finally, let's suppose that we are working with a radio wave of 30Mhz. Then:

$$\frac{\lambda}{32 \cdot cos(\theta_o)} = \frac{10}{32} \approx 0.3m. \tag{51}$$

Our average variation in the height in this case is not less than 0.3m. This means that the radio waves hitting the turbulent sea surface are not all specularly reflected. We must generalize our model so that it incorporates quasi-specular and diffuse scattering.

4.3.5 Calculating reflections from rough surfaces: A statistical approach.

As shown in the previous section, there will be times when specular reflection might not happen when radio waves strike a turbulent sea surface. We are interested in the specular component of reflection. This component arises from coherent reflection in the plane of incidence, calculated through Fresnel equations. Now when a EM wave hits the rough surface, its specular component has been reduced by some factor, as in some part of it has been distributed to the diffuse scattering component. Therefore, we can write the specular component as [8]:

$$R_{eff} = \rho R \tag{52}$$

Where ρ is some reduction factor. We need to find this function ρ, for if we can do it, we will know the reflection coefficient of the EM-wave in the specular direction when it hits this rough surface. Consider the function $h(\bar{r})$, the surface height at some point $\bar{r}(x, y)$. Let's also consider the distribution of these specific heights, P(h). The most remarkable thing is that every point on the sea surface height is the result of a large number of local events, each factor (wind, constantly changing permittivity, conductivity, height, waveform, temperature, salinity etc), contributing toward a cumulative result. All of these factors are coupled together in some highly sophisticated manner, such that we can treat them (for the most part) as random variables.[9]. Now, we use the remarkable result from statistics: the **Central Limit Theorem**, which states that "in most situations, when independent random variables are added, their properly normalized sum tends toward a normal distribution" [10]. Therefore, arguing it statistically, we say that our sea surface height distribution function is distributed normally over some large local area of the turbulent sea where the radio waves are hitting it. Mathematically [9]:

$$P(h) = \frac{1}{\sigma_h\sqrt{2\pi}} e^{-\frac{(h-h_o)^2}{2\sigma_h^2}} \tag{53}$$

Where h_o is assumed to be the mean height of the random rough sea surface and σ is the average standard deviation of the mean height. From this height distribution function, it is easy to derive the reduction function: it must also be distributed normally. Henceforth, using equation 50 and making use of equation 53, our reduction function is given by [8]:

$$\rho = e^{-\frac{\Delta\phi^2}{2}} \tag{54}$$

Therefore, we can write Reflection coefficient from a random rough surface as [8]:

$$R_{eff} = e^{-\frac{\Delta\phi^2}{2}} \cdot R \tag{55}$$

Where R, as usual, is the coefficient of reflectance as if the wave interacts with a perfectly smooth surface. It turns out that this model predicts the reflectance nicely, but sways away a bit when we have very rough surfaces. In order to quantify the reflectance from a very rough surface, we must apply some correction to our normal function. This is because, the rougher the surface becomes, the greater the diffuse component of reflection and lesser its specular component. We have to make a modification [8]:

$$R_{eff} = \frac{\Delta\phi^2}{2} \cdot I_0 \cdot e^{-\frac{\Delta\phi^2}{2}} \cdot R \tag{56}$$

To a good approximation, a nicer way this function could be written as is [8]:

$$R_{eff} \approx \frac{R}{\sqrt{(1.6\Delta\phi^2) - 2 + (\sqrt{(1.6\Delta\phi^2)^2 - 3.5\Delta\phi^2 + 9})}} \tag{57}$$

Where I_0 is the modified Bessel function of zero order. Enough with the equations. Let's illustrate our results graphically.

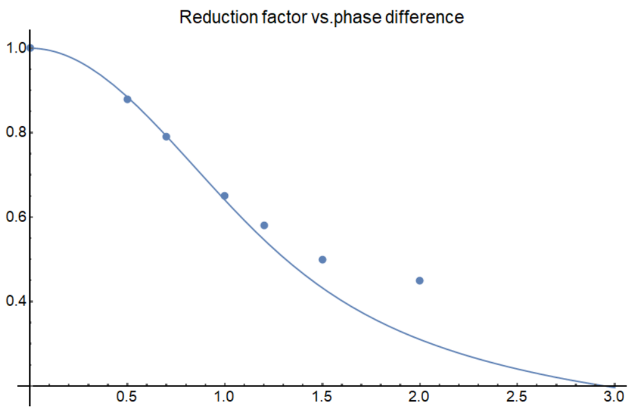

Figure 8: Reduction factor vs. phase difference. Rayleigh's criterion ends at $\Delta\phi \approx 1.57$.

The graph above illustrates some important aspects of reflection from a rough surface. When the phase difference between the reflected waves becomes bigger and bigger, the surface moves farther away from the Rayleigh criterion. As a result, the component of specular scattering decreases–and in fact it decreases in a normal distribution fashion, due to our assumption that the height distribution of rough sea is distributed normally. Of course, when the phase difference is zero the reduction factor equals 1. This means perfect specular reflection from a flat surface, whose reflectance coefficient is simply the Fresnel's coefficient.
The dotted plot shows the real experimental data that was performed.[8]. This shows that our model starts deviating when the phase difference is around 1 radians. It shows that as the phase difference increases, reaching up to 3 radians, the reduction function goes to zero. The percentage of specular reflection is zero. However, we know from experiment that this is not true. The specular reflection percentage does not decrease by that much when the surface

becomes more and more rough. A better model would take this thing into account, by further modifications to the exponential function. Note also that this graph assumes that R remains constant in $R_{eff} = \rho R$. We are trying to show the general trend of roughness vs. reduction factor. We know that as roughness increases, permittivity and conductivity of sea starts to decrease and, therefore, R decreases also.

4.3.6 Average Tangent Plane Model

We developed another model that can find the geometrical reflection coefficient for any surface (e.g. not limited by the normal distribution of elevation requirement). It is based off smoothing out the original surface using an averaging function, so that geometrical optics can be applied. Because of the large wavelength of the radio waves, we can't perform simple ray tracing on the raw ocean's surface to see where the reflected light goes. This comes from diffraction causing the limit of resolving power of a light wave to be approximately half its wavelength [4]. This means that our 10-100m wavelength radio waves can only "see" things on the order of 5-50m or greater.

Therefore, if we apply a moving average over the ocean's surface, computing the average height in a window with a width of half a wavelength, then we can model the radio wave bouncing off this surface using the laws of geometrical optics, where the reflected wave bounces off at an angle equal to the incident angle.

So, here is our algorithm: integrate over the smoothed surface. At each little patch of surface dS, calculate the normal of that surface. Then, using Fresnel's Equations we can find the reflectance R of that little patch, and using the Law of Reflection we can find the direction of the reflected wave θ_r incident from angle θ_i. Not many of these little patches are going to reflect light in the perfectly right direction $\theta_{desired} = -\theta_i$, back up to the ionosphere at the same angle they came down. However, what we really care about is the intensity of energy being radiated in that direction $\theta_{desired}$, so if we define a coefficient $K = \cos(\theta_{desired} - \theta_r)$, then the effective reflectance from a single patch is KR. Thus, the effective reflectance is the average reflectance of all the little patches on the surface.

4.3.7 Attenuation over turbulent seas vs. calm seas (both ρ and R are changing)

We are now in a position to attack the question "how much does a radio wave, on average, attenuate on its reflection over a calm ocean as opposed to a turbulent ocean?". So consider this scenario: a huge wavefront of a radio wave hits a local patch of a violent ocean. What is the reflectance of the specular component of this wave? Let's crunch in some numbers for this. Assume $(\theta_i = 60, \Delta h = 0.5m, \lambda = 15m, \sigma = 2\frac{S}{m}, Re(\epsilon_r) = 80)$. From this, we can calculate our Reflection coefficient, R. Using equations 23 and 24 we calculate the index of refraction of this turbulent sea:

$$n = \sqrt{80 + \frac{2i}{8.85 \cdot 10^{-12} \cdot 2\pi \cdot 30 \cdot 10^6}} = 25.3 + 23.7i. \tag{58}$$

And from equation **??**, we have:

$$\Delta\phi = \frac{16\pi \cdot 0.5m \cdot cos(60)}{15m} \approx 0.84. \tag{59}$$

Also, from 57, we have:

$$R_{eff} \approx \frac{R}{\sqrt{(1.6(0.84)^2) - 2 + (\sqrt{(1.6(0.84)^2)^2 - 3.5(0.84)^2 + 9})}} \approx 0.72 \cdot R. \tag{60}$$

Finally, invoking equation 31 and 30, we calculate R for the turbulent ocean with decreased conductivity. We find R_{eff} to be ≈ 0.65. Therefore, given such parameters, about 35 percent of power is lost per one reflection. This corresponds to about 2db loss per reflection. This fits well within the range of real data of average loss from very, very rough surfaces, which is around 3db [11]. Note, however, that this is one specific case of the loss. As we know, it depends on many parameters $(\theta_i, \Delta h, \lambda, \sigma, \epsilon_r)$. Similarly, for a calm smooth sea, we calculate R to be ≈ 0.94, ($\Delta\phi = 0$, $\sigma = 5\frac{S}{m}$, all other parameters same) corresponding to about a 0.2 db loss.

$$R_{eff} = \langle K_{patch} R_{patch} \rangle \tag{61}$$

5 Results

Now that we have the physics under our belt, we can compute how many hops can our radio wave make before we reach our 10 db signal to noise ratio. Before we do this, first notice that signal to noise ratio is the ratio of desired signal level (strength) to the background noise in the system [12]. Mathematically:

$$SNR_{db} = 10 \cdot log_{10}\left(\frac{P_{Signal}}{P_{noise}}\right) = 10(log_{10}(P_{signal}) - log_{10}(P_{noise})) = 10. \tag{62}$$

$$\implies P_{Signal} = P_{noise}. \tag{63}$$

Consider the figure below for the propagation of radio waves through the atmosphere:

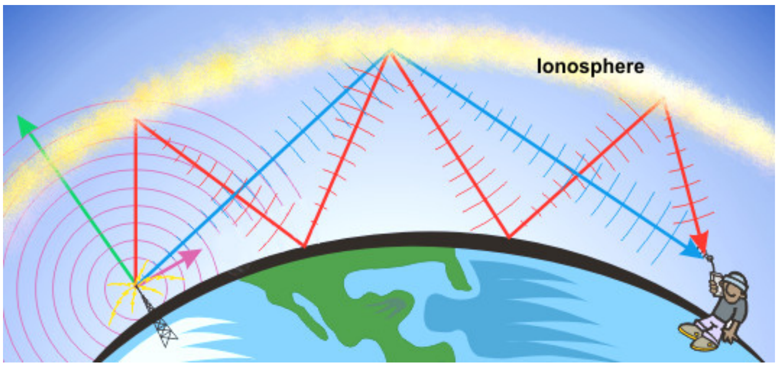

Figure 9: Radio Wave hopping, http://planetfacts.org/ionosphere/

Instead of thinking of the wave bouncing between the two shells for a total distance d, getting attenuated at each bounce by some factor R_i, we can imagine the beam simply traveling a distance d from a point source, passing through many filters, each of which reduces the strength by a factor R_i. The net intensity must be the product of each filtering and, since these waves traverse over the surface of a sphere, we must divide by $4\pi d^2$. Hence:

$$I_{final} = \frac{P_0}{4\pi d^2} \cdot R_0 R_1 R_2 ... R_n = I_{noise} \tag{64}$$

5.1 Number of hops: Calm Ocean vs. Turbulent Ocean

Let's compare how far can a radio wave go through a calm ocean first. To make our result more illuminating, let's do this by putting in actual numbers for the parameters. From section (4.3.7), we have that $R_{eff} = 0.94$ for a calm sea, and $R_{eff} = 0.65$.

$$I_{noise} = \frac{100W}{4\pi(d_0 \cdot n)^2}(0.94)^n \cdot (R_{ion})^{n+1} = \frac{100W}{4\pi(d_0 \cdot r)^2}(0.65)^r \cdot (R_{ion})^{r+1}. \tag{65}$$

Solving this equation will give us how much more hops a radio wave will make through a calm ocean vs a turbid one. We do, however, need to know how much the wave attenuates over the ionosphere, as well as the background noise.

5.2 Smooth Terrain vs. Mountainous terrain

This result can also be generalized to radio wave interaction on land. Only the Coefficient of Reflectance of the bounce from the ground will change, governed by $R_{eff} = \rho R$, and everything else remains the same. The rougher the ground is, the greater our reduction factor, ρ. R here, however, will remain constant, since the conductivity and permittivity of ground is constant with respect to time. But do note one thing: the index of refraction of ground will differ significantly from that of sea, so R will be different, for same incidence angles.

5.3 Multihop Mode Switching

If a ship is traveling across the ocean away from the transmitting antenna, it eventually will have to switch modes, or the number of hops the beam takes on the way from the transmitter. This is because there is a minimum and maximum distance that a certain frequency wave can travel in a single hop.
At any frequency the maximum single hop distance is determined by the geometry of the earth and ionosphere: The furthest you could hope to send a signal would be if you transmitted it perfectly horizontally. The beam would travel until it hit the ionosphere as it curves down below the horizon. Assuming ideal ionosphere conditions, when it hits the F layer at 300km, geometry tells us that the beam will have traveled 17.2 degrees around the earth. It will collide with the F layer at a glancing angle of 17.2 degrees, and then bounce back, again traveling 17.2 degrees around the earth. Thus the radio wave traveled 34.4 degrees around the earth, or about 3,800 km. This means that we move further away, we have to change our angle and the number of hops.
The minimum single hop distance is determined by the frequency of the wave, using Equation 66.

$$\cos\theta - max = \frac{9\sqrt{N_{max}}\; ms^{-1}}{f} \tag{66}$$

Any beam leaving the transmitter at an angle steeper than this will be lost through the atmosphere, and this caps the distance for a single hop.

5.4 Sensitivity Analysis

Our model is highly deterministic, one input depends on another based on concrete laws of Physics. Changing the inputs will not make our output chaotic. As an example, let's tweak some parameters and see its effect on the output. Consider the graph below for the net reflectance vs incidence angles, when conductivity, temperature, and permittivity of sea is changed. What happens is that the Reflectance curve simply shifts up or down. Same is the case when we tweak the roughness of a surface, or consider radio interactions in the ionosphere. Our model is robust to changes in the parameters (inputs.)

6 Conclusions: Strengths and weaknesses

One of the greatest strengths of our model is that it is strongly based on the concrete principles of Physics. We try our best to explain each interaction and each travel of radio waves using the most fundamental laws of Physics, in the hope to to fit a deterministic model. From using well known Maxwell's equation to Fresnel's coefficients, Snell's law and geometry to Central Limit Theorem, Turbulence to Physical Chemistry, we have tried our best to incorporate each and every parameter (and their interaction with each other) that plays its part in the attenuation of radio waves.

Another strength of our model is that we build it up from a simple one. For example, when considering the interaction of radio waves with the sea surface, we start off with the simplest case of specular reflection off a smooth surface. Then, we classify different reflections, and go a step further, generalizing our model to incorporate reflections from rough surfaces, subject to constant change in their fundamental quantities, such as σ, ϵ and heights.

Our model also has some considerable weaknesses. We did not fully illuminate our results using a more graphical approach. An important improvement would be to graph the results of one variable with respect to another, keeping all other variables constant, then compare results with actual experimentally verified values. This will be more enlightening than a hodgepodge of equations. A more numerical, and less theoretical approach will be more entertaining. Our model lacks error analysis. If not pressed for time, we would have incorporated this into our analysis, giving us a better estimate of how much variance in the traveling of radio waves can we expect. A more theoretical physics and a microscopic approach to the problem is good, but it doesn't really solve the problem. A better model would be a nice mix of both microscopic and macroscopic aspects of the problem–and be more computational rather than theoretical. We were not able to find any data on how much does the radio wave attenuate on average over the ionosphere, nor could we find out any empirical data on background noise. This has prevented us from actually computing how much more the radio wave through a turbulent ocean will go as opposed to a non turbid ocean.

7 Appendix

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