Applied Graph Theory (Jan'25-May'25)

Problem Sheet

1. For $d \geq 1$, let $V = \{0,1\}^d$, the set of all binary sequences of length d. Form a graph $G = (V, E)$ by defining E to be the set of all edges (x, y) where $x, y \in V$ and x and y differ in only one coordinate, that is, for some j, $1 \leq j \leq d$, $x_j \neq y_j$ and $x_i = y_i$ for every $i \neq j$. Determine the average degree, number of edges, radius and diameter, girth, circumference and connectivity of this graph.

2. Does there exists a function $f : \mathcal{N} \to \mathcal{N}$ such that $diam(G) \leq circ(G)$ for every graph having at least one cycle ? $circ(G)$ is the length of a longest cycle in G and $diam(G)$ is the maximum length of a shortest path between two vertices in G. By length, we mean the number of edges.

3. Show that every 2-connected graph contains a cycle.

4. Show that any tree T has at least $\Delta(T)$ leaves.

5. Show that every graph which has a cut-edge has a cut-vertex. Is the converse true ? Justify.

6. Let A be a set whose size is kn. Prove that any two partitions $\{B_1, \ldots, B_n\}$ and $\{C_1, \ldots, C_n\}$ of A into k-element subsets admits a common SDR (System of Distinct Representatives).

7. For each of the following conditions, construct infinite families $\{G_n : n \geq 1\}$ of graphs (if they exist) satisfying the conditions : $k(G)$ denotes vertex connectivity, $\lambda(G)$ denotes edge-connectivity, $\delta(G)$ denotes minimum degree. $rad(G)$ denotes radius of G.

    (a) $k(G)$ is bounded by a constant, but $\lambda(G)$ can become arbitrarily large.

    (b) $\lambda(G)$ is bounded by a constant, but $\delta(G)$ can become arbitrarily large.

    (c) $rad(G)$ is bounded by a constant, but $diam(G)$ can become arbitrarily large.