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% CONTINUOUS TIME FOURIER TRANSFORM OF ONES-SIDED EXPONENTIAL SIGNAL

%1)
help fourier;
help ifourier;

%2)
syms t;
a=1;
f=exp(-a*t)*heaviside(t);
T=-5:0.01:5;
y1=subs(f,T);
subplot(121);
plot(T,y1);

F=fourier(f);
y2=subs(F,T);
subplot(122);
plot(T,y2);

%3)
disp(F);
% w is the default variable in which is return by the function "fourier"

%4)
W=-10:10;
subplot(121);
y3=abs(subs(F,W));
plot(W,y3,'linewidth',2);
subplot(122);
y4=angle(subs(F,W));
plot(W,y4,'linewidth',2);

%5)
x1=exp(-1*t)*heaviside(t);
x2=exp(-4*t)*heaviside(t);
T=-5:0.01:5;
subplot(321);
y5=subs(x1,T);
plot(T,y5,'linewidth',2,'color','b');
subplot(322);
y6=subs(x2,T);
plot(T,y6,'linewidth',2,'color','r');

F1=fourier(x1);
F2=fourier(x2);

subplot(323);
plot(T,subs(F1,T),'linewidth',2,'color','b');
subplot(324);
plot(T,subs(F2,T),'linewidth',2,'color','r');
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subplot(325);
y7=abs(subs(F1,T));
y8=abs(subs(F2,T));
plot(T,y7,'linewidth',2,'color','b'); hold ON;
plot(T,y8,'linewidth',2,'color','r'); hold OFF;

subplot(326);
y9=angle(subs(F1,T));
y10=angle(subs(F2,T));
plot(T,y9,'linewidth',2,'color','b'); hold ON;
plot(T,y10,'LineWidth',2,'color','r'); hold OFF;

% UNDERSTANDING AN IDEAL LOW PASS FILTER

%1)
wc=20;
A=2;
syms w;
LPF=A*(heaviside(w+wc)-heaviside(w-wc));
lpf=ifourier(LPf);

T=-10:0.01:10;
T(1001)=0.01; %This is done in order to resolve the zero division error.
subplot(211);
plot(T,subs(LPf,T),'linewidth',2);
subplot(212);
plot(T,subs(lpf,T),'linewidth',2);

%2)
lim=limit(lpf,0);
disp(lim);

%3
%{
lpf(t) is non-causal because for a causal signal at t<0 it should have
value=0. But as we can see lpf(t) has non-zero value at t<0.
%}

% AMPLITUDE MODULATION

%1)
syms t;
m=exp(-0.5*abs(t-1))+exp(-0.5*abs(t+1));
T=-5:0.01:5;
F=fourier(m);
subplot(121);
plot(T,subs(m,T),'linewidth',2);
subplot(122);
plot(T,subs(F,T),'linewidth',2);

%2)
w0=20;
s=m*cos(w0*t);
W=-40:0.1:40;

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T=-10:0.1:10;
Fs=fourier(s);
subplot(121);
plot(T,subs(s,T),'linewidth',2);
subplot(122);
plot(W,subs(Fs,W),'linewidth',2);

%3)
%{
From the property of DTFT we can say that if  $y[n]=\text{convolution}(x[n],h[n])$ 
Their DTFT follow
 $Y(W)=X(W)*H(W)$ 
This is the multiplicative proeprty and can be observed in the plot.
%}

%4)
d=s*cos(w0*t);
Dw=fourier(d);
W=-60:0.1:60;
subplot(121);
plot(T,subs(d,T),'linewidth',2);
subplot(122);
plot(W,subs(Dw,W),'linewidth',2);

%5)
W=-40:0.1:40;
R=Dw*LPF;
subplot(111);
plot(W,subs(R,W),'linewidth',2);

%6)
T=-5:0.01:5;
r=ifourier(R);
plot(T,subs(r,T),'linewidth',2);

%7)
subplot(121);
plot(T,subs(m,T),'linewidth',2);
subplot(122);
plot(T,subs(r,T),'linewidth',2);

%8)
%{
We can observe that  $r(t)$  has breaks at its peak. Also, it is smoother than
 $m(t)$ . Both  $r(t)$  and  $m(t)$  have the same amplitude as expected.
%}

% THE DISCRETE-TIME FOURIER TRANSFORM

%1)
% Function named DTFT_Analysis is made below.

% DTFT OF A RECTANGULAR PULSE

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%1)
N=3;
n=-2*N:2*N;
x=(abs(n)<=N);

%2)
subplot(111);
stem(n,x,'linewidth',2);

%3)
syms w;
X=DTFT_Analysis(x,n,w);
disp(X);

%4)
W=-2*pi:0.01:2*pi;
subX=subs(X,W);
flipX=subs(X,-W);
if(flipX==subX)
    disp("X(e^jw) is even over the range");
else
    disp("No, X(e^jw) is not even over the range");
end

if(subX==real(subX))
    disp("X is real.");
else
    disp("X is not real.");
end

%{
From looking at x[n] we can see that it is symmetric about the y-axis and
hence X is even.
Also, X is real, as displayed.
%}

%5)
plot(W,subX);
xticks(-2*pi:pi:2*pi);

%6)
W1=-4*pi:0.01:4*pi;
plot(W1,subs(X,W1));
xticks(-4*pi:pi:4*pi);

%{
We can observe from the graph that X is periodic at a period of 6.28 i.e.
2*pi.
%}

%7)
syms w;
N1=6;
n1=-2*N1:2*N1;

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x1=(abs(n1)<=N1);
X1=DTFT_Analysis(x1,n1,w);
disp(X);
disp(X1);
W1=-2*pi:0.01:2*pi;
subX1=subs(X1,W1);
subplot(121);
plot(W,subX);
xticks(-2*pi:pi:2*pi);
subplot(122);
plot(W1,subX1);
xticks(-2*pi:pi:2*pi);
%{
From the plotted graph and the values displayed we can see that apart the
extra terms in X1. X1(N=6) has a more wiggly nature, more oscillating and a
higher amplitude.
%}

%8)
N=6;
n=-2*N:2*N;
y=(abs(n)<=N).*(2*(mod(n,2)==0)-1);

%9)
subplot(111);
stem(n,y,'linewidth',2);
xticks(-2*N:N/2:2*N);

%10)
syms w;
Y1=DTFT_Analysis(y,n,w);

%11)
subY1=subs(Y1,W1);
subplot(211);
plot(W1,subX1);
xticks(-2*pi:pi/2:2*pi);
subplot(212);
plot(W1,subY1);
xticks(-2*pi:pi/2:2*pi);
%{
From this we observe that the peaks of the two DTFT occur at alternating
locations.
max of one is the min of other and vice-versa.
%}

% TIME EXPANSION AND INVERSE DTFT

%1)
% Nothing to display or explain.

%2)
k=3;
N=3;

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xk=x;
n=-2*N:2*N;
for i=1:length(x)
    if(rem(n(i),k)==0)
        xk(i)=xf(n(i)/k,N);
    else
        xk(i)=0;
    end
end

%3)
x3k=xk;
subplot(111);
stem(n,x3k,'linewidth',2);

%4)
syms w;
X3=DTFT_Analysis(x3k,n,w);
W=-2*pi:0.01:2*pi;
subX3=subs(X3,W);
plot(W,subX3);
xticks(-2*pi:pi:2*pi);
%{
We can see that the frequency is 3 times of that of x.
hence it is indeed.  $X(e^{3jw})$ 
%}

%5)
% Function Inverse_DTFT is made below

%6)
N=3;
n=-2*N:2*N;
syms w;
invX=Inverse_DTFT(X,n,w);
stem(n,invX,'linewidth',2);
%{
From the plot(stem) we can see that the inverse of X is the same as x
defined initially.
%}

% DISCRETE FOURIER TRANSFORM (DFT)

%1)
% N_DFT function is made below

%2)
N=8;
n=0:20;
x=(n>=0).*(n<=3)+(n>=4).*(n<=7).*(-n);
subplot(131);
stem(n,x,'linewidth',2);
Xcap=N_DFT(x,N);
subplot(132);

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stem(0:(N-1),Xcap,'linewidth',2);
subplot(133);
stem(0:(2*N-1),N_DFT(x,N*2),'linewidth',2);

%3)
Xfft=fft(x,(N*2));
subplot(111);
stem(Xfft,'linewidth',2);

% RELATIONSHIP BETWEEN DFT DTFT

%1)
syms w;
n=0:(N-1);
X=DTFT_Analysis(x,n,w);
W=n*2*pi/N;
Y=subs(X,W);

%2)
% Plotting Real Parts
subplot(121);
stem(real(Xcap),'linewidth',2,'color','b'); hold ON;
stem(real(Y),'linewidth',1,'color','r'); hold OFF;
% Plotting Imaginary Parts
subplot(122);
stem(n,imag(Xcap),'linewidth',2,'color','b');hold ON;
stem(n,imag(Y),'linewidth',1,'color','r'); hold OFF;
% From the graph we see that both the imaginary and real parts are
% overlapping.

% INVERSE DFT

%1)
xinv=IDFT(Xcap,N);

%2)
subplot(121);
stem(n,xinv,'LineWidth',2);
subplot(122);
stem(n,ifft(Xcap,N),'linewidth',2);

% FUNCTIONS

function X=DTFT_Analysis(x,t,w)
    INF=10;
    len=length(t);
    X=0;
    for i=1:len
        X=X+x(i)*exp(-(1i)*w*t(i));
    end
end

function x=xf(n,N)
    x=abs(n)<=N;

```

```

end

function x=Inverse_DTFT(X,n,w)
    X1=X*exp((1i)*w*n);
    x=int(X1,0,2*pi)/(2*pi);
end

function Xcap=N_DFT(x,N)
    y=zeros(1,N);
    for i=1:min(N,length(x))
        y(i)=x(i);
    end
    Xcap=zeros(1,N);
    for k=0:N-1
        for n=0:N-1
            Xcap(k+1)=Xcap(k+1)+y(n+1)*exp(-1*(1i)*k*(2*pi/N)*n);
        end
    end
end

function x=IDFT(Xcap,N)
    x=zeros(N);
    for n=0:N-1
        for k=0:N-1
            x(n+1)=x(n+1)+Xcap(k+1)*exp((1i)*k*n*(2*pi)/N)/N;
        end
    end
end

--- help for sym/fourier ---

FOURIER Fourier integral transform.

F = FOURIER(f) is the Fourier transform of the symbolic expression
or function f with default independent variable x. If f does not
contain x, then the default variable is determined by SYMVAR.
By default, the result F is a function of w. If f = f(w), then F
is returned as a function of the variable v, F = F(v).

By definition,  $F(w) = c \int (f(x) \exp(s \cdot i \cdot w \cdot x), x, -\infty, \infty)$ .

You can set the parameters c,s to any numeric or symbolic values
by setting the preference SYMPREF('FourierParameters',[c,s]).
By default, the values are c = 1 and s = -1.

F = FOURIER(f,v) returns F as a function of the variable v
instead of the default variable w:
 $F(v) = c \int (f(x) \exp(s \cdot i \cdot v \cdot x), x, -\infty, \infty)$ .

F = FOURIER(f,u,v) treats f as a function of the variable u instead
of the default variable x:
 $F(v) = c \int (f(u) \exp(s \cdot i \cdot v \cdot u), u, -\infty, \infty)$ .

Examples:
syms t v w x f(x)

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```

fourier(1/t)    returns  -pi*sign(w)*1i
fourier(exp(-x^2),x,t)  returns  pi^(1/2)*exp(-t^2/4)
fourier(exp(-t)*heaviside(t),v)  returns  1/(1+v*1i)
fourier(diff(f(x)),x,w)  returns  w*fourier(f(x),x,w)*1i

```

See also *SYM/FOURIER*, *SYM/LAPLACE*, *SYM/HTRANS*, *SYM/ZTRANS*, *SUBS*, *SYMPREF*.

Documentation for *sym/fourier*
doc sym/fourier

--- help for *sym/ifourier* ---

IFOURIER Inverse Fourier integral transform.

$f = \text{IFOURIER}(F)$ is the inverse Fourier transform of the symbolic expression or function F with default independent variable w . If F does not contain w , then the default variable is determined by *SYMPREF*. By default, the result f is a function of x . If $F = F(x)$, then f is returned as a function of the variable t , $f = f(t)$.

By definition,

$$f(x) = \text{abs}(s)/(2\pi c) * \int(F(w)*\exp(-s*i*w*x),w,-\text{inf},\text{inf}).$$

You can set the parameters c, s to any numeric or symbolic values by setting the preference *SYMPREF*('FourierParameters',[c, s]). By default, the values are $c = 1$ and $s = -1$.

$f = \text{IFOURIER}(F,u)$ returns f as a function of the variable u instead of the default variable x :

$$f(u) = \text{abs}(s)/(2\pi c) * \int(F(w)*\exp(-s*i*w*u),w,-\text{inf},\text{inf}).$$

$f = \text{IFOURIER}(F,v,u)$ treats F as a function of the variable v instead of the default variable w :

$$f(u) = \text{abs}(s)/(2\pi c) * \int(F(v)*\exp(-s*i*v*u),v,-\text{inf},\text{inf}).$$

Examples:

```

syms t u v w f(x)
ifourier(w*exp(-3*w)*heaviside(w))  returns  1/(2*pi*(-3+x*1i)^2)
ifourier(1/(1 + w^2),u)  returns  exp(-abs(u))/2
ifourier(v/(1 + w^2),v,u)  returns  -(dirac(1,u)*1i)/(w^2+1)
ifourier(fourier(f(x),x,w),w,x)  returns  f(x)

```

See also *SYM/FOURIER*, *SYM/LAPLACE*, *SYM/HTRANS*, *SYM/ZTRANS*, *SUBS*, *SYMPREF*.

Documentation for *sym/ifourier*
doc sym/ifourier

$1/(1 + w*1i)$

$40/\pi$

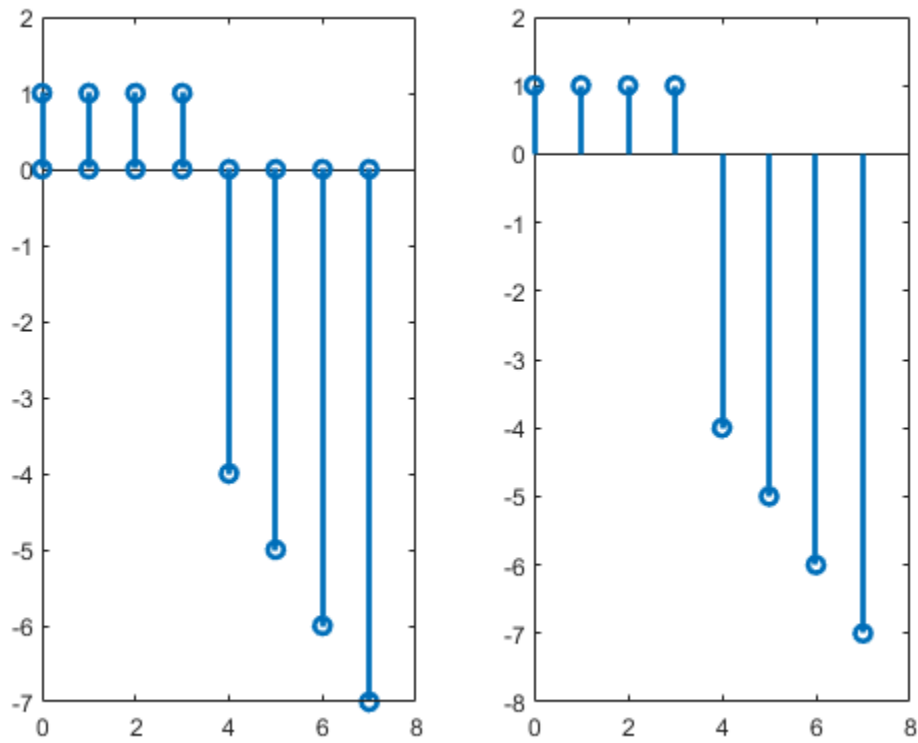
$\exp(-w*1i) + \exp(w*1i) + \exp(-w*2i) + \exp(w*2i) + \exp(-w*3i) + \exp(w*3i) + 1$

$X(e^{jw})$ is even over the range

X is real.

$$\exp(-w*1i) + \exp(w*1i) + \exp(-w*2i) + \exp(w*2i) + \exp(-w*3i) + \exp(w*3i) + 1$$

$$\exp(-w*1i) + \exp(w*1i) + \exp(-w*2i) + \exp(w*2i) + \exp(-w*3i) + \exp(w*3i) + \exp(-w*4i) + \exp(w*4i) + \exp(-w*5i) + \exp(w*5i) + \exp(-w*6i) + \exp(w*6i) + 1$$



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