Stable Matching Problem

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Stable Matching Algorithm

- Each male candidate has a preference list of females.
- Each female candidate has a preference list of males.
- A man a is paired with a woman y, and a woman x is paired with a man b, where the man a prefers the woman x over y, and the woman also prefers the man a over b. In such a condition, pair (a, x) is called as an unstable unmatched pair.
- A(M) -> Y (F)
- B(M) -> X (F)
- A -> X > Y
- X-> A > B
- A man proposes a women in the order of decreasing preference, whereas a woman says “may be” in the order of increasing preference

Stable Matching Algorithm

- Input: preference list of males and females.
- Output: A stable matching (No unstable unmatched pair exists at the termination of the algorithm)

- X : A>B>C>D A: Z>X>Y>W
- Y: A>C>B>D B: Y>W>X>Z
- Z: C>D>A>B C: W>X>Y>Z
- W: C>B>A>D D: X>Y>Z>W

Stable Matching Algorithm: Men propose women

- X : A>B>C>D A: Z>X>Y>W
- Y: A>C>B>D B: Y>W>X>Z
- Z: C>D>A>B C: W>X>Y>Z
- W: C>B>A>D D: X>Y>Z>W
- X proposes A. A says maybe
- Y proposes A. A rejects Y
- Z proposes C. C says maybe
- W proposes C. C says maybe and rejects Z
- X remains idle
- Y proposes C. C rejects Y
- Z proposes D. D says maybe
- W remains idle
- X remains idle
- Y proposes B. B says maybe
- Z remains idle
- W remains idle
- X remains idle
- Y remains idle
- Z remains idle
- W remains idle
- **X->A**
- **Y->B**
- **Z->D**
- **W->C**

Proof of Correctness

- Prove that at the end of the stable matching algorithm, there is no unstable unmatched pair.
- Proof: There is an unstable unmatched pair (X, A), where X is matched to B and A is matched to Y and
- X: A > B
- A: X > Y
- Objective: To prove such situation is not possible. To match X to B over A, the only possibility is X never proposes A or X proposes A but gets rejected in favour of B.
- Case 1: “X never proposes A” is not possible since the proposal order is in order of decreasing preferences.
- Case 2: “X proposes A but gets rejected in favour of B” is not possible because A likes X more than Y