Experiment–6: Continuous Time Fourier Transform & Discrete Time Fourier Transform

Signals and Systems Lab(EC2P002)

School of Electrical Sciences, IIT Bhubaneswar

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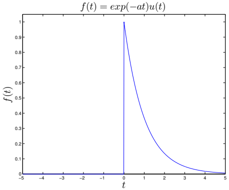
Agenda of the Experiment

In this lab session, we will learn about:

1. Continuous-time Fourier Transform
2. Understanding Scaling Property
3. Understanding Low Pass Filter
4. Understanding Amplitude Modulation

Continuous Time Fourier Transform of One-sided Exponential Signal

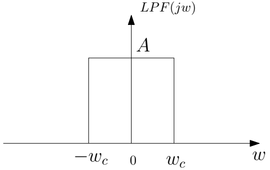
Consider a single sided exponential signal $f(t)$ given below.



1. Use *help fourier* and *help ifourier* to understand the built-in MATLAB commands.
2. Using the *fourier()* command find the Fourier transform of the signal $f(t)$ for $a = 1$.
3. Observe that you are getting an output in terms of syms w.
4. Plot the spectrum for $w \in [-10 \;\; 10]$. Observe that the spectrum is complex. Hence, plot both magnitude and phase spectrum. Verify that magnitude spectrum is even and phase spectrum is odd. [To obtain better plot override the heaviside function value at t=0 to 1]
5. Choose two signals $x_1(t)$ and $x_2(t)$ with $a = 1$ and 4 respectively. Plot the signal and also corresponding Fourier Transform (both magnitude and phase) side by side. The figure should have six subplots. Verify the scaling property learnt in the class.

Understanding an Ideal Low pass filter

Consider a signal $lpf(t)$ having the Fourier transform $LPF(jw)$ as given below. The spectrum $LPF(jw)$ is that of an ideal low pass filter.



1. Using *ifourier()*, obtain the time domain representation of the signal $lpf(t)$ for $w_c = 20$ and $A = 2$.

2. Plot the spectrum and the time domain signal $lpf(t)$. Here you might encounter a division by zero error. Identify the location of this and evaluate the value of the signal at this point using the Limit function in MATLAB.

3. Comment on the causality of the signal $lpf(t)$. Also observe that $lpf(t)$ is a signal with infinite support making it impossible to practically realize.

Amplitude Modulation

Consider a signal with low frequency component. If we want to effectively broadcast this signal we need very large antennas. To avoid this, we shift this signal to higher frequencies, transmit and receive the high frequency signal and shift the signal back to low frequency to obtain the original signal. For more information you may read about modulation.

1. Consider a message signal $m(t) = e^{-0.5|t-1|} + e^{-0.5|t+1|}$. Obtain the Fourier transform $M(jw)$ and plot both signal and transform side by side.

2. Consider the signal $s(t) = m(t)\cos(w_o t)$ with $w_o = 20$. Plot $s(t)$ and its spectrum $S(jw)$ side by side. While plotting $S(jw)$ plot for $w \in [-40 \;\; 40]$.

3. Observe the spectrum $S(jw)$. Explain the spectrum in your report using the multiplication property of Fourier Transform.

The signal $s(t)$ is transmitted. However typically w_o will correspond to frequencies in kHz. Also, it is referred to as the amplitude modulation because the amplitude of the signal $s(t)$ varies according to the signal $m(t)$. This is FYI.

4. At the receiver, obtain the signal $d(t) = s(t)cos(w_o t)$. Use the same w_o. Plot the spectrum $D(jw)$ of $d(t)$ for $w \in [-60 \;\; 60]$. Can you identify the spectrum of the original signal $m(t)$ in the spectrum of $d(t)$.
5. We learnt about ideal low pass filter earlier. Use the spectrum $LPF(jw)$ with $w_c = 20$ and multiply with $D(jw)$. Call this product spectrum $R(jw)$. Plot $R(jw)$ for $w \in [-40 \;\; 40]$.
6. Using inverse Fourier transform, obtain the reconstructed signal $r(t)$.
7. Plot the original signal $m(t)$ and the reconstructed signal $r(t)$ side by side. Again at certain points you might need to evaluate the signal $r(t)$ using limits.
8. Compare $m(t)$ and $r(t)$ and note down your observations.

A version of this with more signal processing was actually used by All India Radio Akashvani. FYI

Agenda of the Experiment

In this session, we will learn about:

1. The Discrete-Time Fourier Transform (DTFT) and the inverse DTFT
2. The time-expansion property of the DTFT
3. The Discrete Fourier Transform (DFT), its relation to the DTFT, and evaluating the DFT using MATLAB's inbuilt fast-Fourier transform function.

The Discrete-Time Fourier Transform

In class, we learned the Discrete-Time Fourier Transform (DTFT) of a discrete-time signal $x[n]$ and the Inverse DTFT to be

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\mathrm{e}^{-j\omega n}$$

and

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})\mathrm{e}^{j\omega n}\,\mathrm{d}\omega,$$

respectively.

1. Write a function named *DTFT_Analysis()* that takes three inputs: a vector $x[n]$, a vector n containing the time indexes of $x[n]$, and the symbolic variable ω. The output is the symbolic expression $X(\mathrm{e}^{j\omega})$.

DTFT of a Rectangular Pulse

1. Define the signal $x[n]$ to be

$$x[n] = \begin{cases} 1, & |n| \leq N_1 \\ 0, & |n| > N_1 \end{cases}$$

   for some integer N_1.

2. Set $N_1 = 3$. Give a stem plot of $x[n]$ for the time range $n = -2N_1 : 1 : 2N_1$.
3. Use the *DTFT_Analysis()* function to find the DTFT $X(\mathrm{e}^{j\omega})$, which should be a symbolic expression in terms of ω.
4. Use the *subs()* function to evaluate $X(\mathrm{e}^{j\omega})$ over the frequency range $\omega = -2\pi : 0.01 : 2\pi$. Is $X(\mathrm{e}^{j\omega})$ real over this range? Is it an even function of ω? Is your observation consistent with what you would expect by looking at $x[n]$?
5. Plot $X(\mathrm{e}^{j\omega})$ vs. ω for $\omega = -2\pi : 0.01 : 2\pi$. Use *xticks()* to show the frequency values that are integer multiples of π. (I.e., use *xticks(-2*pi : pi : 2*pi)* below the *plot* command.)
6. Verify that $X(\mathrm{e}^{j\omega})$ is periodic in ω.

DTFT of a Quickly Varying Signal

7. Repeat steps 1–6 for $N_1 = 6$. What changes do you observe in $X(e^{j\omega})$ relative to the $N_1 = 3$ case?
8. Define the signal $y[n]$

$$y[n] = \begin{cases} (-1)^n, & |n| \leq N_1 \\ 0, & |n| > N_1 \end{cases}$$

9. Set $N_1 = 6$. Give a stem plot of $y[n]$.
10. Using the *DTFT_Analysis()* function, evaluate and plot the DTFT $Y(e^{j\omega})$.
11. Compare the plot of $Y(e^{j\omega})$ with that of $X(e^{j\omega})$ for $N_1 = 6$. In particular, notice where the peaks of the two DTFTs occur. What do the locations of the peaks tell you about the frequency contents of the signal, and does it tally with the characteristics of the two time domain signals as observed from the stem plots. (Recall: ω values close to even (odd) multiples of π correspond to slowly (fast) varying signals.)

Time Expansion and Inverse DTFT

1. Define the time-expansion of the signal $x[n]$ by

$$x_k[n] = \begin{cases} x[n/k], & \text{if } n \text{ is a multiple of } k \\ 0, & \text{otherwise,} \end{cases}$$

   where k is a positive integer. In class, we learned that the DTFT of $x_k[n]$ is given by $X(\mathrm{e}^{jk\omega})$, where $X(\mathrm{e}^{j\omega})$ is the DTFT of $x[n]$.

2. Consider $x[n]$ as defined earlier and use $N_1 = 3$.
3. Determine and stem plot $x_3[n]$.
4. Evaluate and plot the DTFT of $x_3[n]$, and verify whether it is indeed equal to $X(\mathrm{e}^{j3\omega})$.
5. Write a function *Inverse_DTFT()* that takes three input parameters, the symbolic expression for $X(\mathrm{e}^{j\omega})$, a vector of time indexes n, and the symbol ω. The output is a vector of the $x[n]$ values obtained using the inverse DTFT equation.
6. Determine the inverse DTFT of $X(\mathrm{e}^{j\omega})$ using your *Inverse_DTFT()* function and verify whether it is indeed the same as $x[n]$.

The Discrete Fourier Transform (DFT)

Consider a discrete-time signal $x[n]$ that is zero outside the interval $0 \leq n \leq N_1 - 1$. Suppose $N_1 \leq N$. The N-point DFT of $x[n]$ is defined as

$$\hat{X}[k] = \sum_{n=0}^{N-1} x[n] \mathrm{e}^{-jk(2\pi/N)n}, \qquad k = 0, 1, \ldots, N-1.$$

1. Write a function *DFT()* that takes as its input a sequence $x[n]$ and an integer N, and returns the N-point DFT $\hat{X}$. (Note that if $x[n]$ has length smaller than N, then you will need to perform *zero-padding* by adding zeros to the tail of $x[n]$ such that the length becomes N.)
2. Define a signal $x[n]$ such that

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 3 \\ -n, & 4 \leq n \leq 7 \\ 0, & \text{otherwise.} \end{cases}$$

   and determine its 8-point and 16-point DFTs.
3. The inbuilt MATLAB function *fft()* also computes the DFT of an input sequence by utilizing a computationally efficient algorithm called *Fast Fourier Transform*. Perform step 2 again, this time using *fft()*, and verify whether the output is the same as that of your *DFT()* function.

Relationship Between DFT and DTFT

The DFT $\hat{X}[k]$ can be viewed as the samples of the DTFT $X(\mathrm{e}^{j\omega})$, taken at frequency intervals of $2\pi/N$. That is,

$$\hat{X}[k] = X(\mathrm{e}^{j(2\pi k/N)}) \qquad k = 0, 1, \ldots, N-1.$$

1. Use your *DTFT_Analysis()* function to obtain the DTFT of $x[n]$ defined on the previous page. Then obtain the sequence $Y[k] = X(\mathrm{e}^{j(2\pi k/N)})$ by evaluating $X(\mathrm{e}^{j\omega})$ at $\omega = 0 : 2\pi/N : 2(N-1)\pi/N$.
2. Verify that $Y[k]$ is indeed equal to $\hat{X}[k]$. For this purpose, first draw a stem plot of the real parts of $\hat{X}[k]$ and $Y[k]$ (both on the same graph, use different stem markers for each). In another graph, give stem plots of the imaginary parts of $\hat{X}[k]$ and $Y[k]$.

Inverse DFT

The inverse DFT (IDFT) is given by

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} \hat{X}(k)\mathrm{e}^{jk(2\pi/N)n} \qquad n = 0, 1, \ldots, N-1.$$

MATLAB's inbuilt function *ifft()* can be used to evaluate the IDFT.

1. Take the 8-point DFT output of $x[n]$ and use *ifft()* to reconstruct $x[n]$.
2. Give a stem plot of the ifft() output.