Assignment-X

Ans) **Greedy is used.**

Here greedy is used instead of DP primarily because this is an optimization sequencing problem. Similar, to the scheduling problem where the brute force approach generates all permutations and then checking the optimal solution. This problem also has the same brute-force solution. This is the basic motivation behind using greedy instead of DP.

Observations:

1. The minimum extra space required is always at least equal to the first element of the sequence. Thus, since the current space of the first element is always included in the minimum extra space required, we will want to minimize the current space of the first room.
2. The space in the newly refurbished room can always be used by the next room, except for the last room, since, there are no more rooms left to be refurbished. Thus we would want to put the last room to be refurbished with the minimum possible new space so as to reduce the wasted amount of available space.

Order of Sorting:

1. First we group together all the rooms which have new space > current space, arrange them in ascending order of current space and put them in the front of the array.
   If 2 elements have the same current space we order them in decreasing order of new space.
2. After the first group we group all those elements which have new space = current space and sort them in any order.
3. Finally we have the third group for which new space < current, we order them in decreasing order of new space.
   If 2 elements have the same new space, we order them in decreasing order of current space

Time Complexity – O(NlogN)
Time Complexity Analysis –
Only 2 for loops traversing up to n elements have been used and sorting requires NlogN time.

Space Complexity – O(N)
Space Complexity Analysis –
Only 1 new array of pairs of size n has been used.

Proof of Greedy:

1. Let us assume we didn't do the 3 categories of grouping as mentioned, this would as randomly chose some other order. This will not lead to an optimal solution because the grouping in part 1 leads to a constant increase in current space available for the remaining set of guests whose rooms are yet to be refurbished. Thus if we randomly put another room, since the max space is still not available this will lead to an increase in extra space needed. Because if the same room had been refurbished at the end of Part 1, as a result of more available space, then extra space needed to accommodate the current room will be less, hence it will be a more optimal answer.
2. In Group 1 it is arranged in ascending order of current space because as a result of Observation 1, the first element has to have a minimal current space.
   Also, as there is a constant increase in available space for new guests, those will higher space required are to be accommodated later so that less amount of extra needed space has to be used.
3. In Group 2 order does not matter because the available space remains the same after the refurbishing of the current room as new space = current space so there is no net increase in available space, hence we can place those in any order.
4. In Group 3 we order them in decreasing order of new space as a result of Observation 2.
   Also, sorting them in the given order ensures that the decrease in the available space is gradual. Since the available space is gradually decreasing, for those elements with equal new space in Group 3 we order them in decreasing order of current space so that more available space can be allocated to the room requiring a higher space.

This completes the Proof.