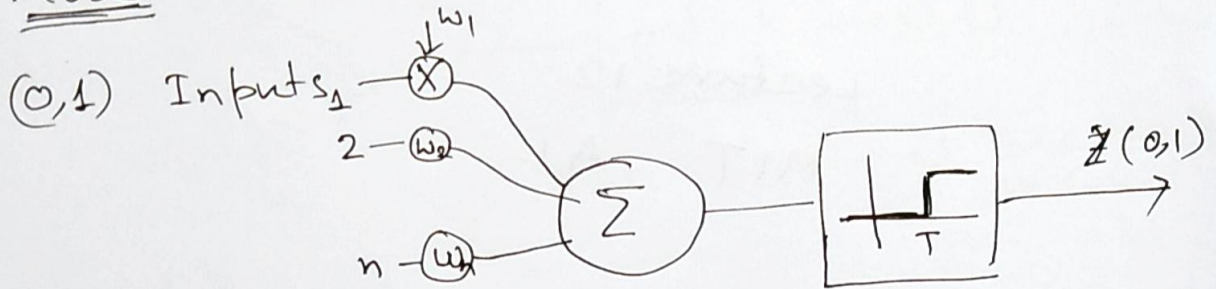


1. Synaptic weight
2. ~~can~~ cumulative effect of stimulus
3. All or none
- ⋮
- Na^+ , K^+ , gradients, refractory periods etc.

Model

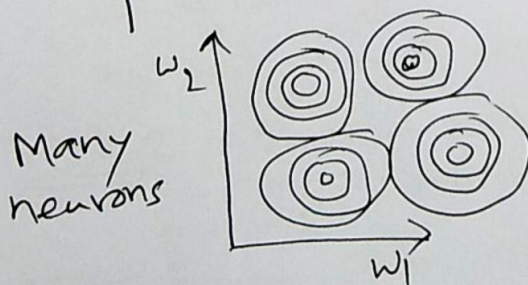
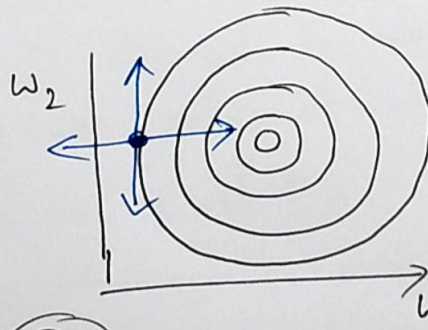
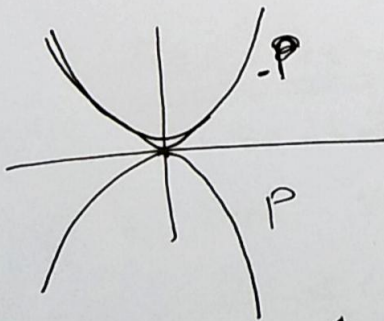


$$\bar{z} = f(\bar{x}, \bar{w}) \quad \text{desired output } \bar{d} = g(\bar{x})$$

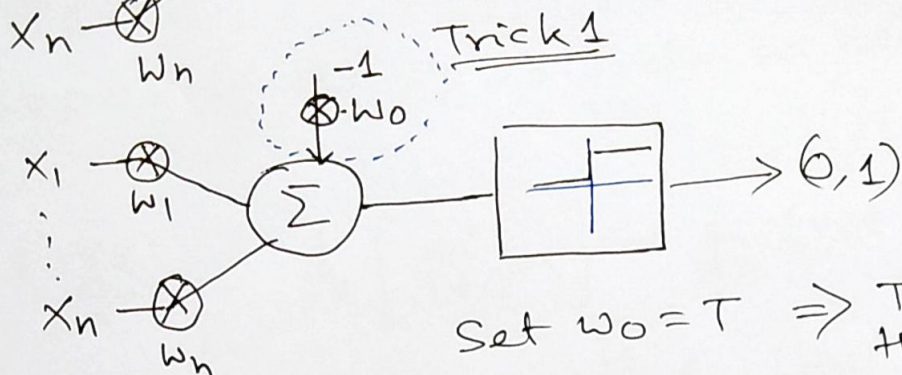
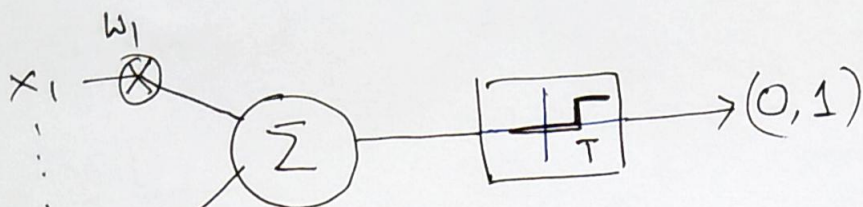
Idea: Want $f(\bar{x}, \dots)$ in alignment with $g(\bar{x})$
 ↓
 adjust \bar{w}

Need Measure of how good/bad we are doing

$$P(\bar{d}, \bar{z}) = -\frac{1}{2} \|\bar{d} - \bar{z}\|^2 \Leftarrow \text{it turns out to be a mathematically convenient metric}$$

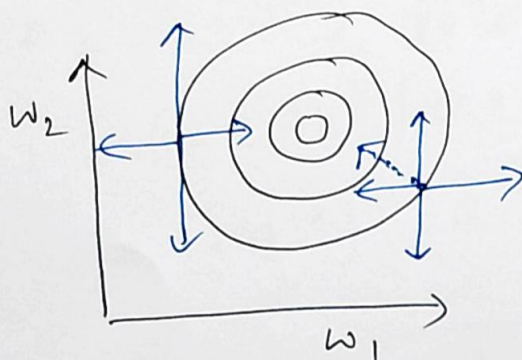


What about threshold T ?



Set $w_0 = T \Rightarrow$ This gives the bias.

Trick 2



$$\Delta W = r \left(\frac{\partial P}{\partial x} i + \frac{\partial P}{\partial y} j \right)$$

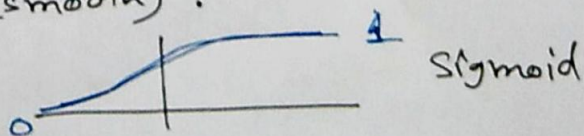
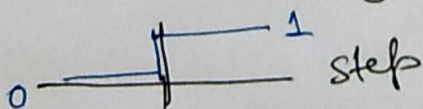
↙ rate constant.

Requirement: P needs to be smooth fn. so that we can calculate derivatives.

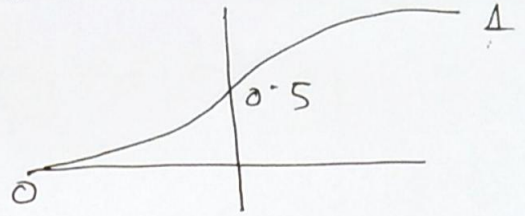
But P is a fn. of \bar{d} & \bar{z} $P(\bar{d}, \bar{z})$.

\bar{z} is coming through a step fn. (discontinuous) so, we cannot use gradient ascent.

Trick 2: Instead of using step fn., we can use a sigmoid (smooth).



Trick 3 Sigmoid :



$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

choosing the sigmoid fn. as activation fn.

$$\frac{d\sigma(x)}{dx} = \frac{d}{dx} \left[\frac{1}{1 + e^{-x}} \right] = \frac{d}{dx} (1 + e^{-x})^{-1}$$

$$= -(1 + e^{-x})^{-2} \frac{d}{dx} [1 + e^{-x}]$$

$$= -(1 + e^{-x})^{-2} \left[0 + \frac{d}{dx} (e^{-x}) \right]$$

$$= -(1 + e^{-x})^{-2} \cdot e^{-x} (-1)$$

$$= \frac{e^{-x}}{(1 + e^{-x})(1 + e^{-x})}$$

$$= \frac{(1 + e^{-x}) - 1}{(1 + e^{-x})(1 + e^{-x})}$$

$$= \frac{1}{(1 + e^{-x})} - \frac{1}{(1 + e^{-x})^2}$$

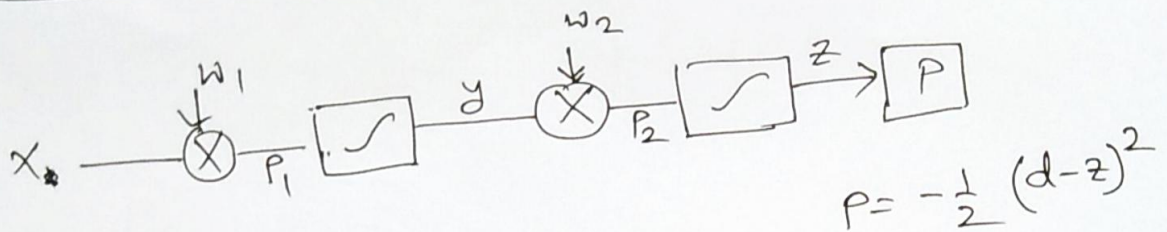
$$= \frac{1}{(1 + e^{-x})} \left[1 - \frac{1}{1 + e^{-x}} \right]$$

$$= \frac{1}{1 + e^{-x}} \left[\frac{1 + e^{-x} - 1}{1 + e^{-x}} \right]$$

$$= \sigma(x) (1 - \sigma(x))$$

★ Derivative of the output w.r.t. input is expressed in terms of output alone!!!

Computing
calculating
derivatives
are trivial!!



Now $\Delta W = \left(\frac{\partial P}{\partial w_1} i + \frac{\partial P}{\partial w_2} j \right) \delta$

$$\frac{\partial P}{\partial w_2} = \frac{\partial P}{\partial z} \cdot \frac{\partial z}{\partial w_2} = (d-z) \cdot \frac{\partial z}{\partial w_2}$$

$$= (d-z) \cdot \frac{\partial z}{\partial p_2} \cdot \frac{\partial p_2}{\partial w_2}$$

$$= (d-z) \frac{\partial z}{\partial p_2} \cdot y$$

$$\frac{\partial P}{\partial w_1} = \frac{\partial P}{\partial z} \cdot \frac{\partial z}{\partial w_1} = (d-z) \cdot \frac{\partial z}{\partial p_2} \cdot \frac{\partial p_2}{\partial w_1}$$

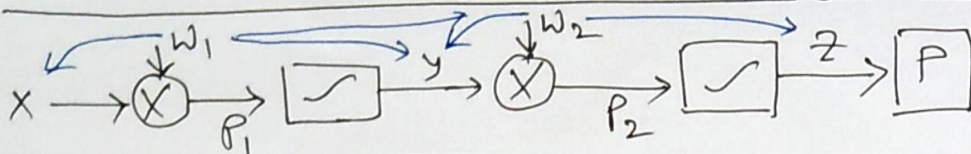
$$= (d-z) \frac{\partial z}{\partial p_2} \cdot \frac{\partial p_2}{\partial y} \cdot \frac{\partial y}{\partial w_1}$$

$$= (d-z) \frac{\partial z}{\partial p_2} \cdot w_2 \cdot \frac{\partial y}{\partial p_1} \cdot \frac{\partial p_1}{\partial w_1}$$

$$= (d-z) \frac{\partial z}{\partial p_2} \cdot w_2 \cdot \frac{\partial y}{\partial p_1} \cdot x$$

$$\frac{\partial P}{\partial w_2} = (d-z) \cdot z \cdot (1-z) \cdot y$$

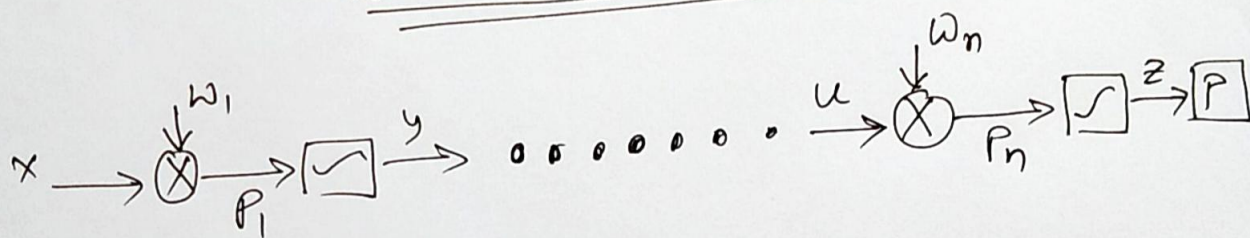
$$\frac{\partial P}{\partial w_1} = (d-z) \cdot z \cdot (1-z) \cdot w_2 \cdot y \cdot (1-y) \cdot x$$



$$\frac{\partial P}{\partial w_2} = \underbrace{(d-z) \cdot z \cdot (1-z) \cdot y}_{\downarrow}$$

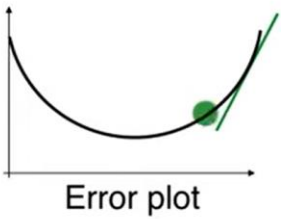
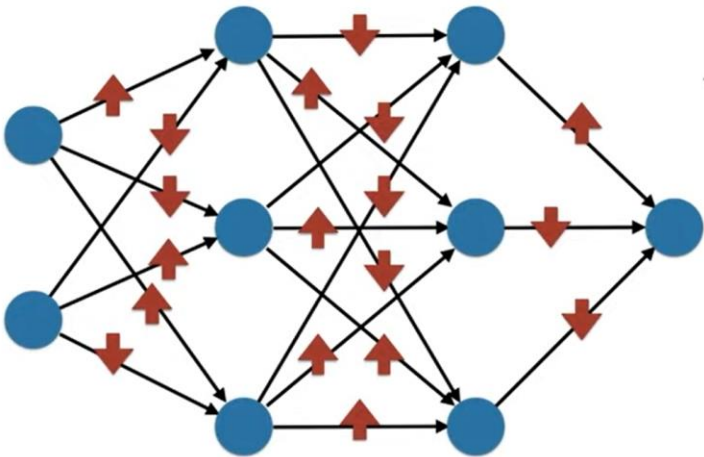
$$\frac{\partial P}{\partial w_1} = \underbrace{(d-z) \cdot z \cdot (1-z)}_{\downarrow} \cdot y \cdot (1-y) \cdot x$$

What about a complex network?



$$\frac{\partial P}{\partial w_m} = \left(\frac{\partial P}{\partial w_{m-1}} \right) \cdot (1 - \text{output}) \cdot \text{input}$$

Backpropagation



Prediction

Error