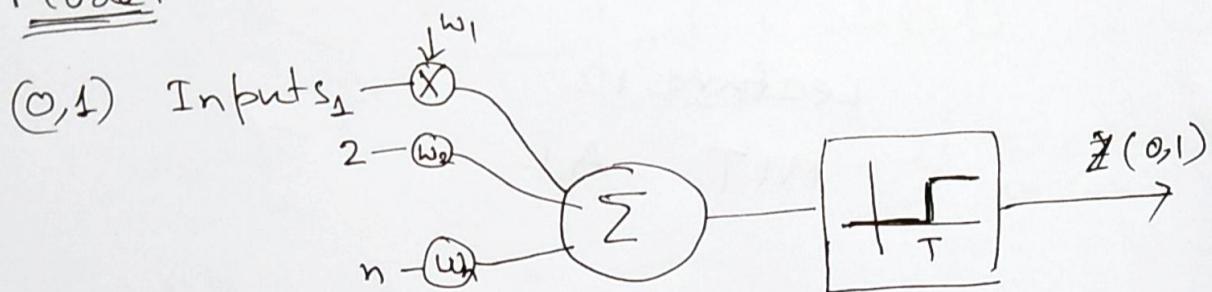


1. Synaptic weight
2. ~~cumulative effect~~ of stimulus
3. All or none
- ⋮
Na⁺, K⁺, gradients, refractory periods etc.

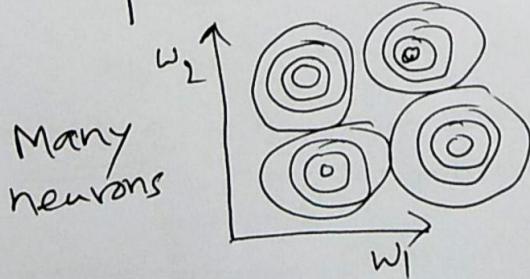
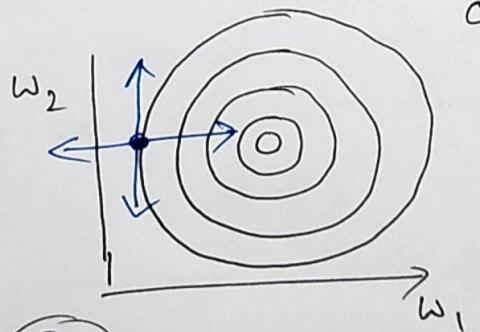
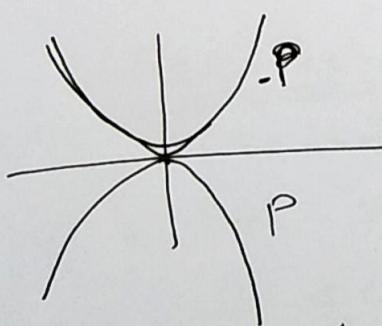
Model

$$\bar{z} = f(\bar{x}, \bar{w}) \quad \text{desired output } \bar{d} = g(\bar{x})$$

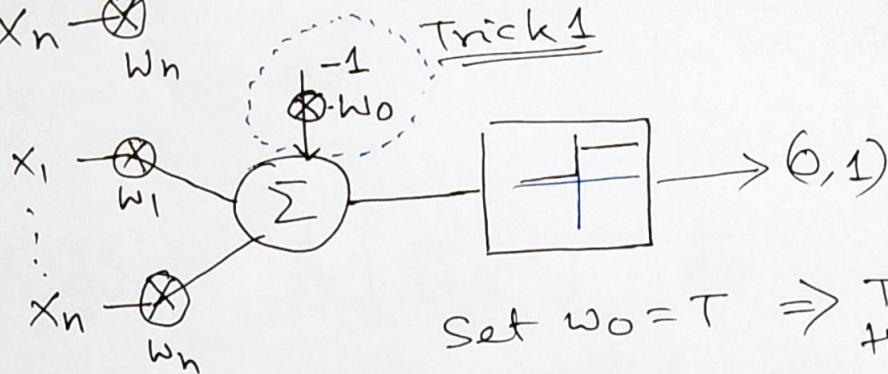
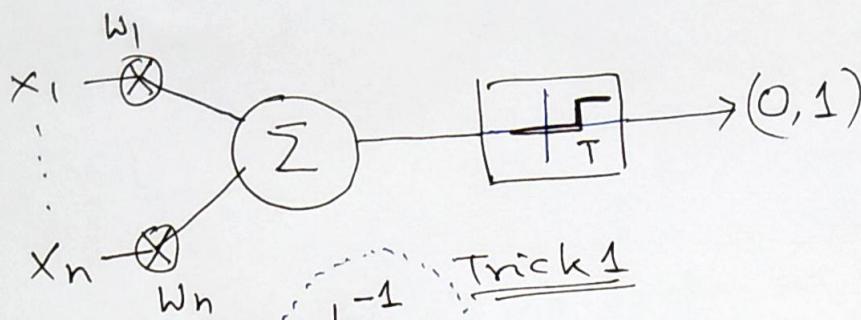
Idea: Want $f(\bar{x}, \dots)$ in alignment with $g(\bar{x})$

↓
adjust \bar{w}

Need Measure of how good/bad we are doing

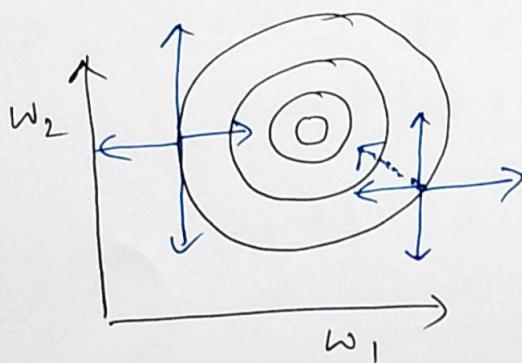
$$P(\bar{d}, \bar{z}) = -\frac{1}{2} \|\bar{d} - \bar{z}\|^2 \leftarrow \text{It turns out to be a mathematically convenient metric}$$


What about threshold T?



Set $w_0 = T \Rightarrow$ This gives the bias.

Trick 2



$$\Delta w = \gamma \left(\frac{\partial P}{\partial x} i + \frac{\partial P}{\partial y} j \right)$$

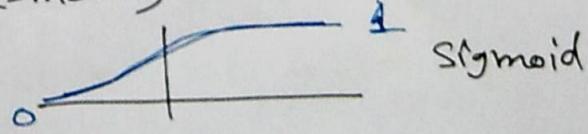
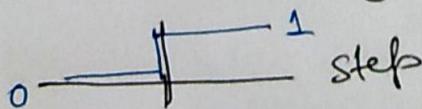
↓ rate constant.

Requirement: P needs to be smooth fn. so that we can calculate derivatives.

But P is a fn. of \bar{d} & \bar{z} $P(\bar{d}, \bar{z})$.

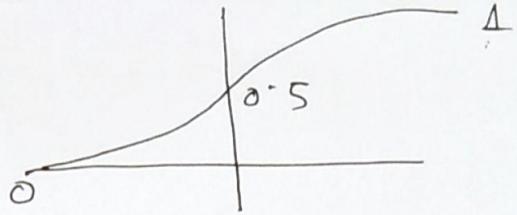
\bar{z} is coming through a step fn. (discontinuous) so, we cannot use gradient ascent.

Trick 2: Instead of using switchfn., we can use a sigmoid (smooth).



Trick³ Sigmoid :

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$



choosing the sigmoid fn. as activation fn.

$$\frac{d\sigma(x)}{dx} = \frac{d}{dx} \left[\frac{1}{1+e^{-x}} \right] = \frac{d}{dx} (1+e^{-x})^{-1}$$

$$= - (1+e^{-x})^{-2} \frac{d}{dx} [1+e^{-x}]$$

$$= - (1+e^{-x})^{-2} \left[0 + \frac{d}{dx} (e^{-x}) \right]$$

$$= - (1+e^{-x})^{-2} \cdot e^{-x} (-1)$$

$$= \frac{e^{-x}}{(1+e^{-x})(1+e^{-x})}$$

$$= \frac{(1+e^{-x}) - 1}{(1+e^{-x})(1+e^{-x})}$$

$$= \frac{1}{(1+e^{-x})} - \frac{1}{(1+e^{-x})^2}$$

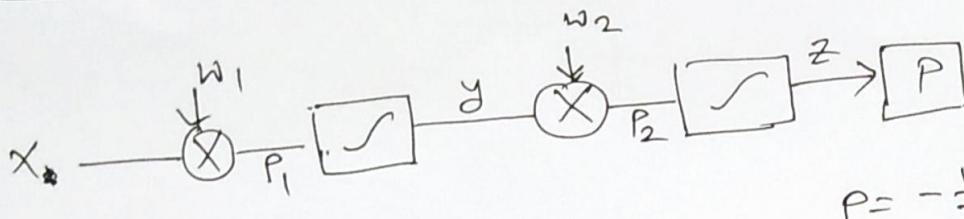
$$= \frac{1}{(1+e^{-x})} \left[1 - \frac{1}{1+e^{-x}} \right]$$

$$= \frac{1}{1+e^{-x}}$$

$$= \sigma(x) (1-\sigma(x))$$

~~Computing
Sigmoid
derivatives
are trivial!!~~

★ Derivative of the out put w.r.t. input is expressed in terms of output alone!!!



$$P = -\frac{1}{2} (d-z)^2$$

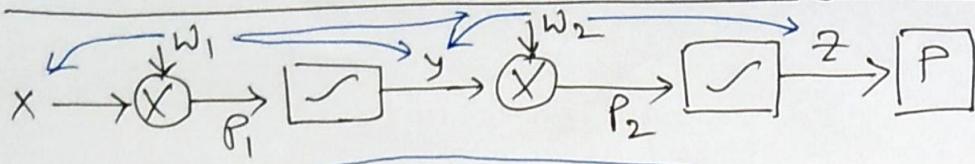
Now $\Delta W = \left(\frac{\partial P}{\partial w_1} i + \frac{\partial P}{\partial w_2} j \right)$

$$\begin{aligned} \frac{\partial P}{\partial w_2} &= \frac{\partial P}{\partial z} \cdot \frac{\partial z}{\partial w_2} = (d-z) \cdot \frac{\partial z}{\partial w_2} \\ &= (d-z) \cdot \frac{\partial z}{\partial P_2} \cdot \frac{\partial P_2}{\partial w_2} \\ &= (d-z) \frac{\partial z}{\partial P_2} \cdot y \end{aligned}$$

$$\begin{aligned} \frac{\partial P}{\partial w_1} &= \frac{\partial P}{\partial z} \cdot \frac{\partial z}{\partial w_1} = (d-z) \frac{\partial z}{\partial P_2} \cdot \frac{\partial P_2}{\partial w_1} \\ &= (d-z) \frac{\partial z}{\partial P_2} \cdot \frac{\partial P_2}{\partial y} \cdot \frac{\partial y}{\partial w_1} \\ &= (d-z) \frac{\partial z}{\partial P_2} \cdot w_2 \cdot \frac{\partial y}{\partial P_1} \cdot \frac{\partial P_1}{\partial w_1} \\ &= (d-z) \frac{\partial z}{\partial P_2} \cdot w_2 \cdot \frac{\partial y}{\partial P_1} \cdot x \end{aligned}$$

$$\frac{\partial P}{\partial w_2} = (d-z) \cdot z \cdot (1-z) \cdot y$$

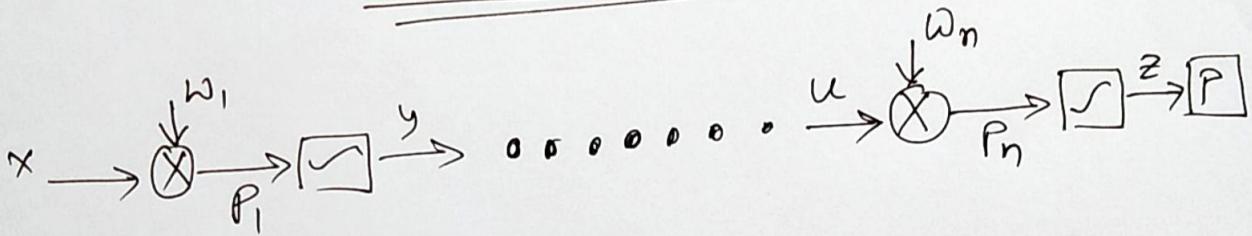
$$\frac{\partial P}{\partial w_1} = (d-z) \cdot z \cdot (1-z) \cdot w_2 \cdot y \cdot (1-y) \cdot x$$



$$\frac{\partial P}{\partial w_2} = \boxed{(d-z) \cdot z \cdot (1-z) \cdot y}$$

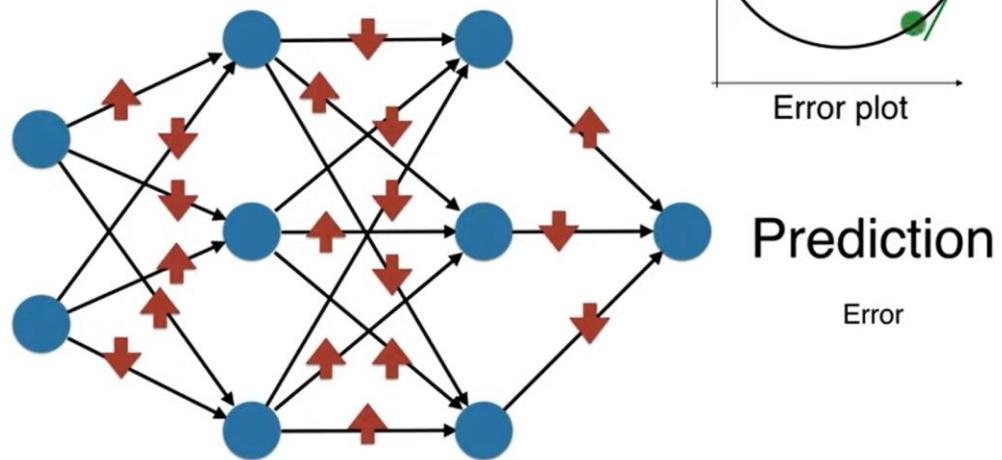
$$\frac{\partial P}{\partial w_1} = \boxed{(d-z) \cdot z \cdot (1-z) \cdot y} (1-y) x$$

what about a complex network?



$$\frac{\partial P}{\partial w_m} = \left(\frac{\partial P}{\partial w_{m-1}} \right) \cdot (1 - \text{output}) \cdot \text{input}$$

Backpropagation



Prediction

Error