# Doubt Clearing Session

Comprehensive Course on Linear Algebra

Alka Singh • Lesson 16 • Sept 20, 2021

Ordered Basis $= \{ \underline{v_1}, \underline{v_2}, \underline{v_3}, \underline{\ldots}, \underline{v_n} \}$

# CO-ORDINATE VECTOR

let $V$ be a F.D.V.S. over the field $F$. let $B$ be a basis of $V$. ($B = \{v_1, v_2, \ldots, v_n\}$) then any $v \in V$;

$$v = c_1 v_1 + c_2 v_2 + \cdots + c_n v_n$$

$$[v]_B = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \end{bmatrix}$$

(eg)

$B = \{ (1,2), (2,-1) \}$

$v = (2,3)$ , $v_1 = (-1,2)$

$\Longrightarrow$

$(x,y) = c_1(1,2) + c_2(2,-1)$

$(x,y) = (c_1 + 2c_2 \,,\, 2c_1 - c_2)$

$c_1 + 2c_2 = x$

$2c_1 - c_2 = y \longrightarrow c_2 = 2c_1 - y$

$= \frac{2x + 4y - 5y}{5}$

$5c_1 = 2y + x$

$c_1 = \frac{x + 2y}{5}$

$c_2 = \frac{2x - y}{5}$

$$\boxed{(x,y) = \left(\frac{x+2y}{5}\right)(1,2) + \left(\frac{2x-y}{5}\right)(2,-1)}$$

$$\Rightarrow \quad v = (2,3)$$

$$v = (2,3) = \frac{8}{5}(1,2) + \frac{1}{5}(2,-1)$$

$$[v]_B = \begin{bmatrix} 8/5 \\ 1/5 \end{bmatrix}$$

$$(-1,2) = v_1 = (3/5)(1,2) + (-4/5)(2,-1)$$

$$[v_1]_B = \begin{bmatrix} 3/5 \\ -4/5 \end{bmatrix}$$

(eg)  $P_4[x]$    dim $P_4[x] = 5$

$B = \{1, x, x^2, \underline{x}^3, \underline{x}^4\}$

$p(x) = 3 + 2x^3 + \underline{9x}^4$

$p(x) = 3\cdot 1 + 0\cdot x + 0\cdot x^2 + 2\cdot x^3 + 9\cdot x^4$

$$[p(x)]_B = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 2 \\ 9 \end{bmatrix}_{\underline{5\times 1}}$$

$\Rightarrow \mathbb{C}(\mathbb{R})$

$$\mathbb{C} = \{ x + iy \mid x, y \in \mathbb{R} \}$$

$$x + iy = x \cdot 1 + y \cdot i$$

$$= c_1 v_1 + c_2 v_2$$

$$c_1 = x \in \mathbb{R}$$

$$c_2 = y \in \mathbb{R}$$

$B = \{ 1, i \}$ $\quad$ $\dim(\mathbb{C}(\mathbb{R})) = 2$

$\Rightarrow \quad \mathbb{C}^2(\mathbb{R})$

dim $V = 4$

$B \subseteq \mathbb{C}^2$

$\mathbb{C}^2 = \{ (x_1 + iy_1, x_2 + iy_2) \mid x_1, x_2, y_1, y_2 \in \mathbb{R} \}$

$x_1 = 1 \qquad (1, 0)$

$y_1 = 1 \qquad (i, 0)$

$B = \{ (1, 0) (i, 0), (0, 1), (0, i) \}$

$x_2 = 1 \qquad (0, 1)$

$y_2 = 1 \qquad (0, i)$

(Note)

$$\dim(\mathbb{C}(\mathbb{R})) = 2$$

$$\dim(\mathbb{C}^2(\mathbb{R})) = 4$$

$$\vdots$$

$$\boxed{\dim(\mathbb{C}^n(\mathbb{R})) = 2n}$$

$$\dim\left(\mathbb{C}^n(\mathbb{R})\right) = 2n$$

$$\dim\left(\mathbb{C}^n(\mathbb{C})\right) = n$$

$\mathbb{C}(\mathbb{C})$ $\dim(\mathbb{C}(\mathbb{C})) = 1$

$B = \{ 1 \}$

$$\underset{\text{vector}}{(x+iy)} = \underbrace{(x+iy)}_{\text{scalar} \in \text{Field}} \cdot 1$$

linear sum

linear sum

$$\underbrace{W_1 + W_2}_{} = \{ w \mid \exists\, w_1 \in W_1 \text{ and } w_2 \in W_2 \text{ s.t. } w = w_1 + w_2 \}$$

# DIRECT SUM

Let $W_1$ and $W_2$ be two sub-spaces of vector space V. $W_1 + W_2$ is

is called direct sum of V if every member of V can be uniquely expressed in terms of $W_1$ and $W_2$.

Then direct sum is represented by $W_1 \oplus W_2$

THEOREM let $W_1$ and $W_2$ be two sub-spaces of $V$

Then $W_1 + W_2$ is said to be the direct

sum of $V$ iff $W_1 \cap W_2 = \{0\}$

(eg) $W_1 = \{ (0, y, z) \mid y, z \in \mathbb{R} \}$

$W_2 = \{ (x, y, 0) \mid x, y \in \mathbb{R} \}$

$W_1$ and $W_2$ does not form direct sum.

Step (1) $V = \underset{\downarrow \atop W_1}{W_1} + \underset{\downarrow \atop W_2}{W_2}$ linear sum

Step (2)

$W_1 \cap W_2 = \{0\}$

$W_1 \cap W_2 = \{ (0, y, 0) \mid y \in \mathbb{R} \}$

$\neq \{ (0, 0, 0) \}$

$W_1 = \{ (0, y, z) \mid y, z \in \mathbb{R} \}$ $W_2 = \{ (x, y, 0) \mid x, y \in \mathbb{R} \}$

$(1, -2, 1) = (0, -2, 1) + (1, 0, 0)$

$= (0, -1, 1) + (1, -1, 0)$

$= (0, -3, 1) + (1, 1, 0)$

$\vdots$

does not forms a direct sum.

$\infty$ly many ways of expressing $v = w_1 + w_2$ and $w_1 \in W_1$ and $w_2 \in W_2$

$W_1 = \{ (0, y, z) \mid y, z \in \mathbb{R} \}$  $W_3 = \{ (x, 0, 0) \mid x \in \mathbb{R} \}$

$$(x, y, z) = (x, 0, 0) + (0, y, z)$$ (linear sum)

$W_1 \cap W_2 = \{0\}$

$W_1$ and $W_2$ forms direct sum.

$$\boxed{W_1 \oplus W_2 = V = \mathbb{R}^3}$$

$(x, 0, 0) + (0, y, z)$

$(x, y, z) \in \mathbb{R}^3$

$(x, y, z) = (x, 0, 0) + (0, y, z)$

$$\boxed{\dim(W_1 \cup W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)}$$

# Linear Transfor.

▲ 1 • Asked by Lakshmi

Ma'am ye question

18. If $W_1$ and $W_2$ are subspaces of $\mathbb{R}^4$ and $\{(1,0,0,0), (1,1,0,0), (1,1,1,0)\}$, $\{(0,0,0,1), (0,0,1,1), (0,1,1,1)\}$ are bases of $W_1$ and $W_2$ respectively. Find a basis of $W_1 \cap W_2$.

Sol:- $W_1 \cap W_2 = \{$

$W_1 = \text{span}\{(1,0,0,0)\ (1,1,0,0)\ (1,1,1,0)\}$

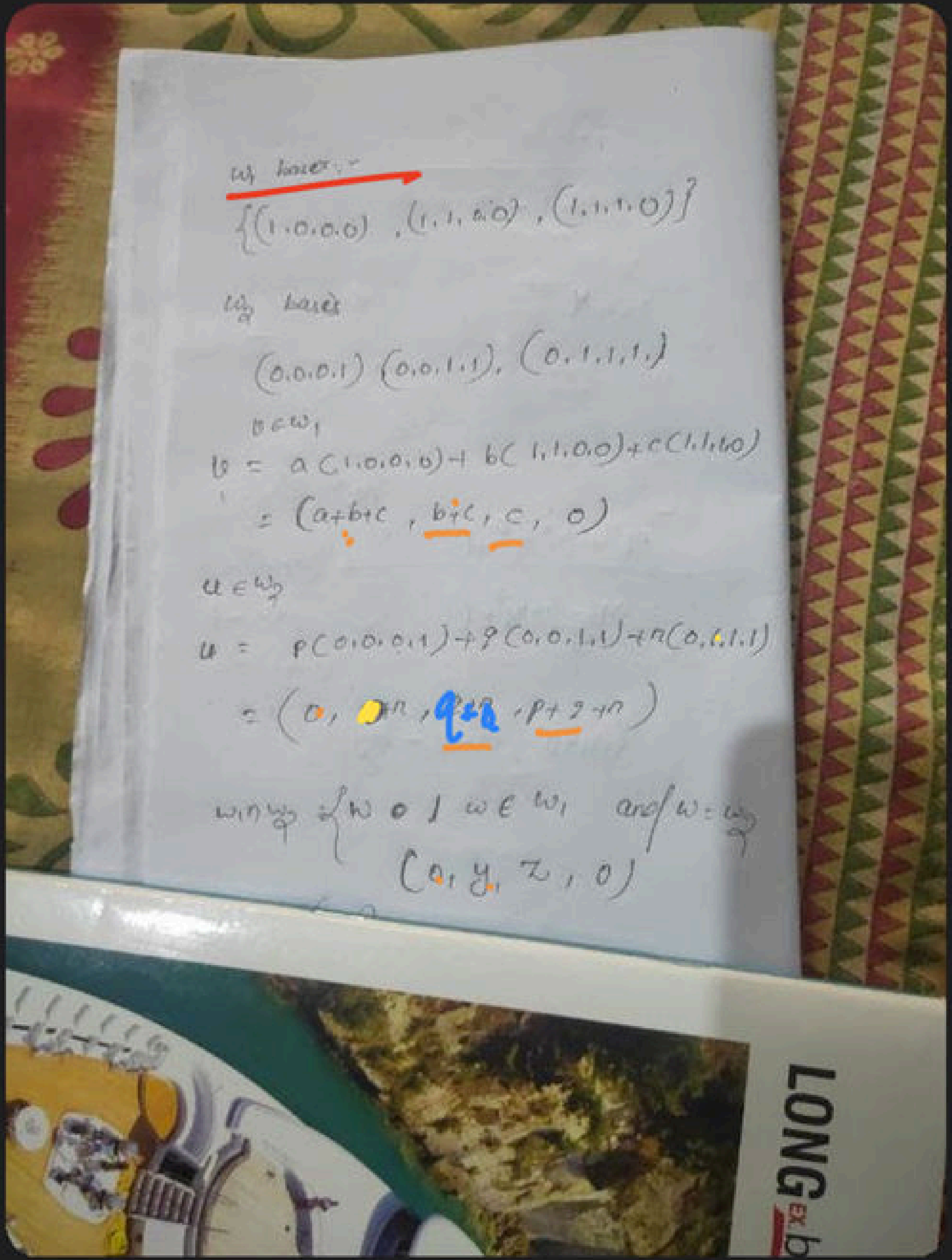
$W_2 = \text{span}\{(0,0,0,1)\ (0,0,1,1),\ (0,1,1,1)\}$

$W_1 \cap W_2 : \text{span}\{$

unacadémy

1 • Asked by Prasant

Esse karsakte



$W_1$ bases:-

$\{(1,0,0,0), (1,1,0,0), (1,1,1,0)\}$

$W_2$ bases

$(0,0,0,1)\ (0,0,1,1), (0,1,1,1)$

$v \in W_1$

$v = a(1,0,0,0) + b(1,1,0,0) + c(1,1,1,0)$

$= (a+b+c, b+c, c, 0)$

$u \in W_2$

$u = p(0,0,0,1) + q(0,0,1,1) + r(0,1,1,1)$

$= (0, r, q+r, p+q+r)$

$W_1 \cap W_2 \{w \circ \mid w \in W_1$ and $w \in W_2$

$(x, y, z, 0)$

$v \in W_1 \cap W_2$

$v \in W_1$

$v \in W_2$

$v = (c_1 v_1 + c_2 v_2 + c_3 v_3) \Rightarrow (\alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3)$

$1 (\quad) + 0 (\quad)$

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