Doubt Clearing Session

Comprehensive Course on Linear Algebra

Alka Singh • Lesson 8 • Sept 12, 2021

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1 • Asked by Lakshmi

Please help me with this doubt

4.88. Show that (a) If $S \subseteq T$, then $\text{span}(S) \subseteq \text{span}(T)$. (b) $\text{span}[\text{span}(S)] = \text{span}(S)$

Linear Dependence and Linear Independence

4.89. Determine whether the following vectors in $\mathbf{R}^4$ are linearly dependent or independent:
(a) $(1, 2, -3, 1)$, $(3, 7, 1, -2)$, $(1, 3, 7, -4)$; (b) $(1, 3, 1, -2)$, $(2, 5, -1, 3)$, ...

4.90. Determine whether the following polynomials u, v, w in $\mathbf{P}(t)$ are linearly dependent or independent:
(a) $u = t^3 - 4t^2 + 3t + 3$, $v = t^3 + 2t^2 + 4t - 1$, $w = 2t^3 - t^2 - 3t + 5$;
(b) $u = t^3 - 5t^2 - 2t + 3$, $v = t^3 - 4t^2 - 3t + 4$, $w = 2t^3 - 17t^2 - 7t + 9$

4.91. Show that the following functions f, g, h are linearly independent:
(a) $f(t) = e^t$, $g(t) = \sin t$, $h(t) = t^2$; (b) $f(t) = e^t$, $g(t) = e^{2t}$, $h(t) = t$

4.92. Show that $u = (a, b)$ and $v = (c, d)$ in K^2 are linearly dependent if and only if $ad - bc = 0$.

4.93. Suppose u, v, w are linearly independent vectors. Prove that S is linearly independent, where:
(a) $S = \{u + v - 2w,\ u - v - w,\ u + w\}$; (b) $S = \{u + v - 3w,\ u + 3v - w, \ldots\}$

4.94. Suppose $\{u_1, \ldots, u_r, w_1, \ldots, w_s\}$ is a linearly independent subset of V. Show that

$$\text{span}(u_i) \cap \text{span}(w_j) = \{0\}$$

4.95. Suppose $v_1, v_2, \ldots, v_n$ are linearly independent. Prove that S is linearly independent where

$$c_1 e^t + c_2 \sin t + c_3 t^2 = 0 \quad \text{——— } (1)$$

$$c_1 e^t + c_2 \cos t + c_3 2t = 0 \quad \text{——— } (2)$$

$$c_1 e^t - c_2 \sin t + 2 c_3 = 0$$

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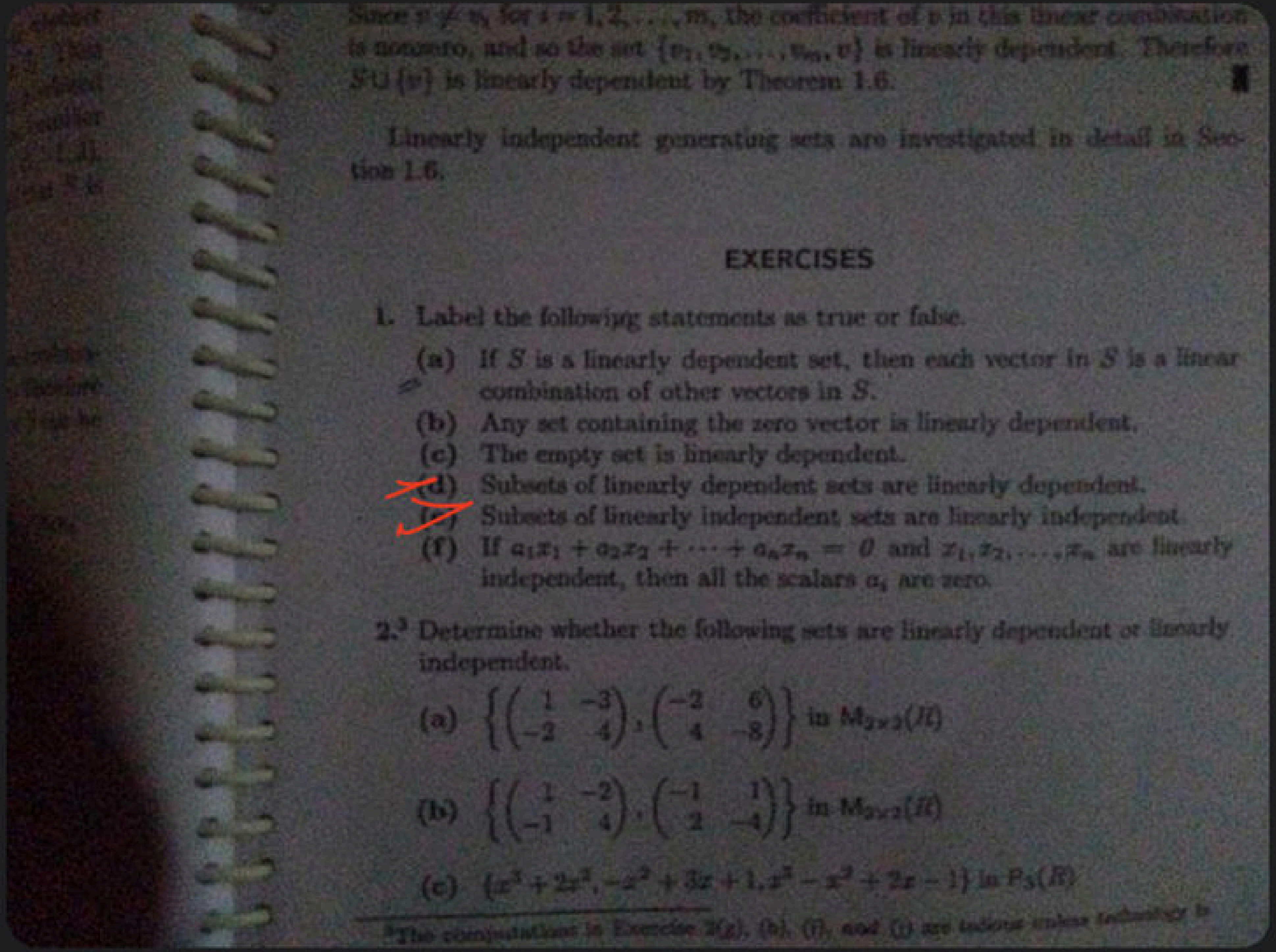
2 • Asked by Rahul

D,E true or false explain



Since $v \neq v_i$ for $i = 1, 2, \ldots, m$, the coefficient of v in this linear combination is nonzero, and so the set $\{v_1, v_2, \ldots, v_m, v\}$ is linearly dependent. Therefore $S \cup \{v\}$ is linearly dependent by Theorem 1.6. ∎

Linearly independent generating sets are investigated in detail in Section 1.6.

EXERCISES

1. Label the following statements as true or false.
   (a) If S is a linearly dependent set, then each vector in S is a linear combination of other vectors in S.
   (b) Any set containing the zero vector is linearly dependent.
   (c) The empty set is linearly dependent.
   (d) Subsets of linearly dependent sets are linearly dependent.
   (e) Subsets of linearly independent sets are linearly independent.
   (f) If $a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0$ and $x_1, x_2, \ldots, x_n$ are linearly independent, then all the scalars a_i are zero.

2.[3] Determine whether the following sets are linearly dependent or linearly independent.
   (a) $\left\{\begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix}, \begin{pmatrix} -2 & 6 \\ 4 & -8 \end{pmatrix}\right\}$ in $M_{2\times 2}(R)$
   (b) $\left\{\begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ 2 & -4 \end{pmatrix}\right\}$ in $M_{2\times 2}(R)$
   (c) $\{x^3 + 2x^2, -x^2 + 3x + 1, x^3 - x^2 + 2x - 1\}$ in $P_3(R)$

[3]The computations in Exercise 2(g), (h), (i), and (j) are tedious unless technology is

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▲ 1 • Asked by Vidhi

Mam in the def every v ka lc likha h toh mam hum yeh kyu nhi bol sakte ki s generates v . Pls bata do confusion ho rha h

Reducing this system to echelon form and solving by back-substitution again yields the solution ... $z = -2$. Thus (again) $v = 3p_1 + p_2 - 2p_3$.

set of all possible linear com

Spanning Sets

Let V be a vector space over K. Vectors $u_1, u_2, \ldots, u_m$ in V are said to *span* V or to form a *spanning set* of V if every v in V is a linear combination of the vectors $u_1, u_2, \ldots, u_m$, that is, if there exist scalars $a_1, a_2, \ldots, a_m$ in K such that

$$v = a_1u_1 + a_2u_2 + \ldots + a_mu_m$$

The following remarks follow directly from the definition.

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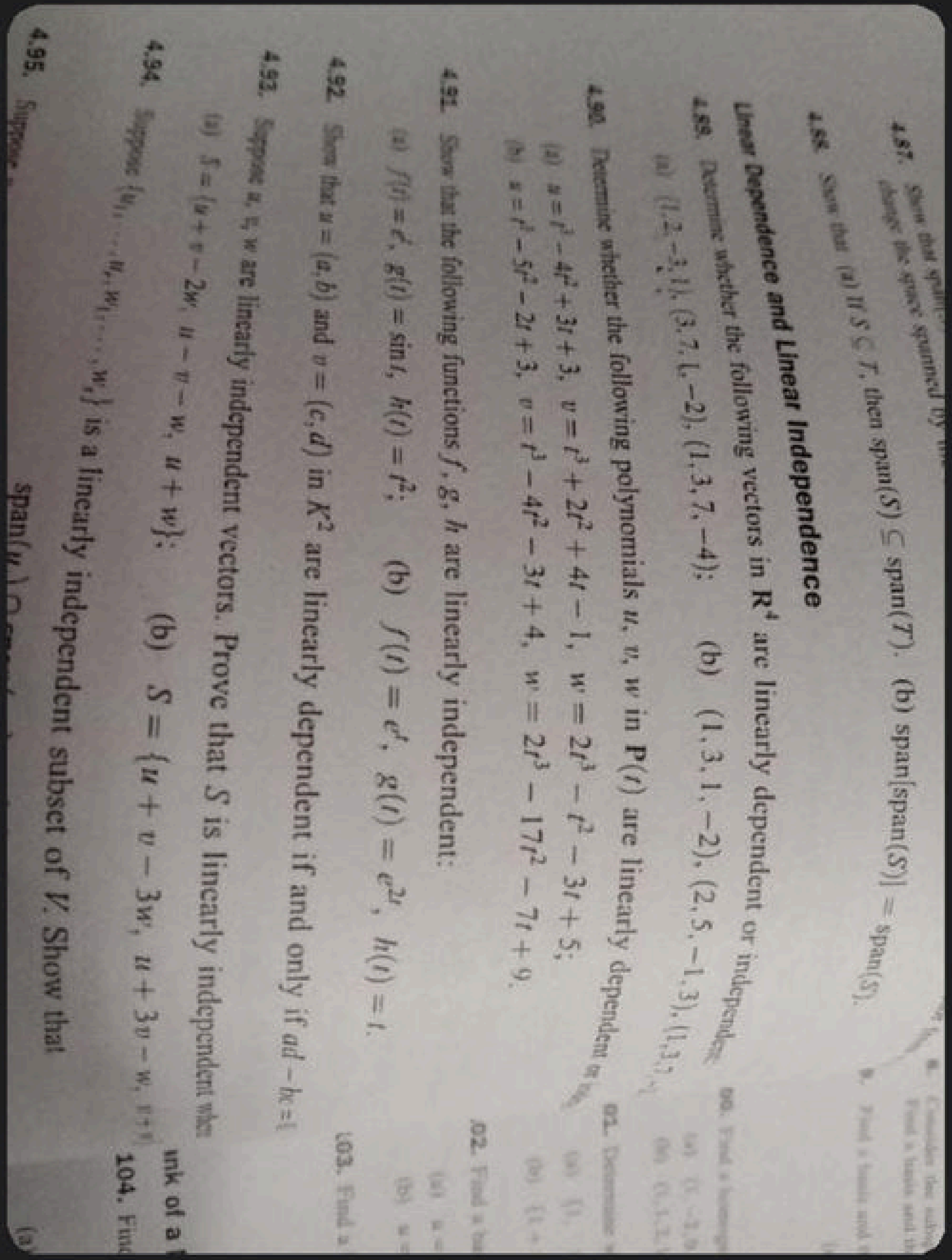
▲ 2 • Asked by Lakshmi

Ma'am 4.91 wala question??



4.87. Show that span(S) ... change the space spanned by the ...

4.88. Show that (a) If $S \subseteq T$, then $\text{span}(S) \subseteq \text{span}(T)$. (b) $\text{span}[\text{span}(S)] = \text{span}(S)$.

Linear Dependence and Linear Independence

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4.94. Suppose $\{u_1, \ldots, u_r, w_1, \ldots, w_s\}$ is a linearly independent subset of V. Show that

$\text{span}(u_i) \cap \ldots$

4.95. Suppose ...

$S = \{ v_1, v_2, \ldots, v_n \} \rightarrow LD$

$v_i = c_1 v_1 + \ldots + c_n v_n$

THEOREM: Let V be a V.S. over the field F. Let S be a non-empty subset of V. S is LD iff atleast one vector of S can be written as L.C. of others.

Proof: Let $S = \{v_1, v_2, v_3, \ldots, v_n\}$ be a non-empty subset of V.

$1 \implies 2$

Suppose S is LD.

$\exists\, c_i \neq 0$ s.t. $c_1 v_1 + c_2 v_2 + \cdots + c_n v_n = 0$

$c_1 v_1 + c_2 v_2 + \cdots + c_{j-1} v_{j-1} + c_j v_j + c_{j+1} v_{j+1} + \cdots + c_n v_n = 0$

W.L.O.G., we assume $c_j \neq 0$

$$c_j v_j = -c_1 v_1 - c_2 v_2 - \cdots - c_{j-1} v_{j-1} - c_{j+1} v_{j+1} - \cdots - c_n v_n$$

$$v_j = -\frac{c_1}{c_j} v_1 - \frac{c_2}{c_j} v_2 - \cdots - \frac{c_{j-1}}{c_j} v_j - \frac{c_{j+1}}{c_j} v_{j+1} - \cdots - \frac{c_n}{c_j} v_n$$

$$v_j = k_1 v_1 + k_2 v_2 + \cdots + k_{j-1} v_{j-1} + k_{j+1} v_{j+1} + \cdots + k_n v_n \quad \text{--- } (1)$$

$$K_i = \frac{-c_i}{c_j} \qquad j = 1, 2 \ldots n \qquad i \neq j$$

(1) represents that atleast one vector can be written as L.C of remainings.

$2 \Rightarrow 1$

Atleast one vector can be written as L.C. of others.

Claim : S is LD.

$$v_i = c_1 v_1 + c_2 v_2 + \cdots + c_{i-1} v_{i-1} + c_{i+1} v_{i+1} + \cdots + c_n v_n$$

$$c_1 v_1 + c_2 v_2 + \cdots + c_{i-1} v_{i-1} + (-1) v_i + c_{i+1} v_{i+1} + \cdots + c_n v_n = 0$$

$$\Downarrow$$

$\exists\ c_i = -1\ (\neq 0)$ st

$$c_1 v_1 + c_2 v_2 + \cdots + c_n v_n = 0.$$

$\{v_1, v_2, v_3, \ldots v_n\}$ is LD.

(eg) $S = \{ (1,0), (2,1), (0,1) \}$

$$(2,1) = 2\cdot(1,0) + 1\cdot(0,1)$$

S is LD

(eg) $\{ (2,0), (-3,2) \}$

LI

(eg)

$S = \{ (1,1) (2,2) \}$ — LD

$$(1,1) = \frac{1}{2}\cdot(2,2)$$

or

$$(2,2) = 2\cdot(1,1)$$

(1) $S = \phi$ is a LI set.

(2) $S = \{ v \}$

- Suppose $v \neq 0$

$c \cdot v = 0$

$\Rightarrow c = 0$

S is LI

- Suppose $v = 0$

$c \cdot v = 0$

$c \cdot 0 = 0$

$0 = 0$

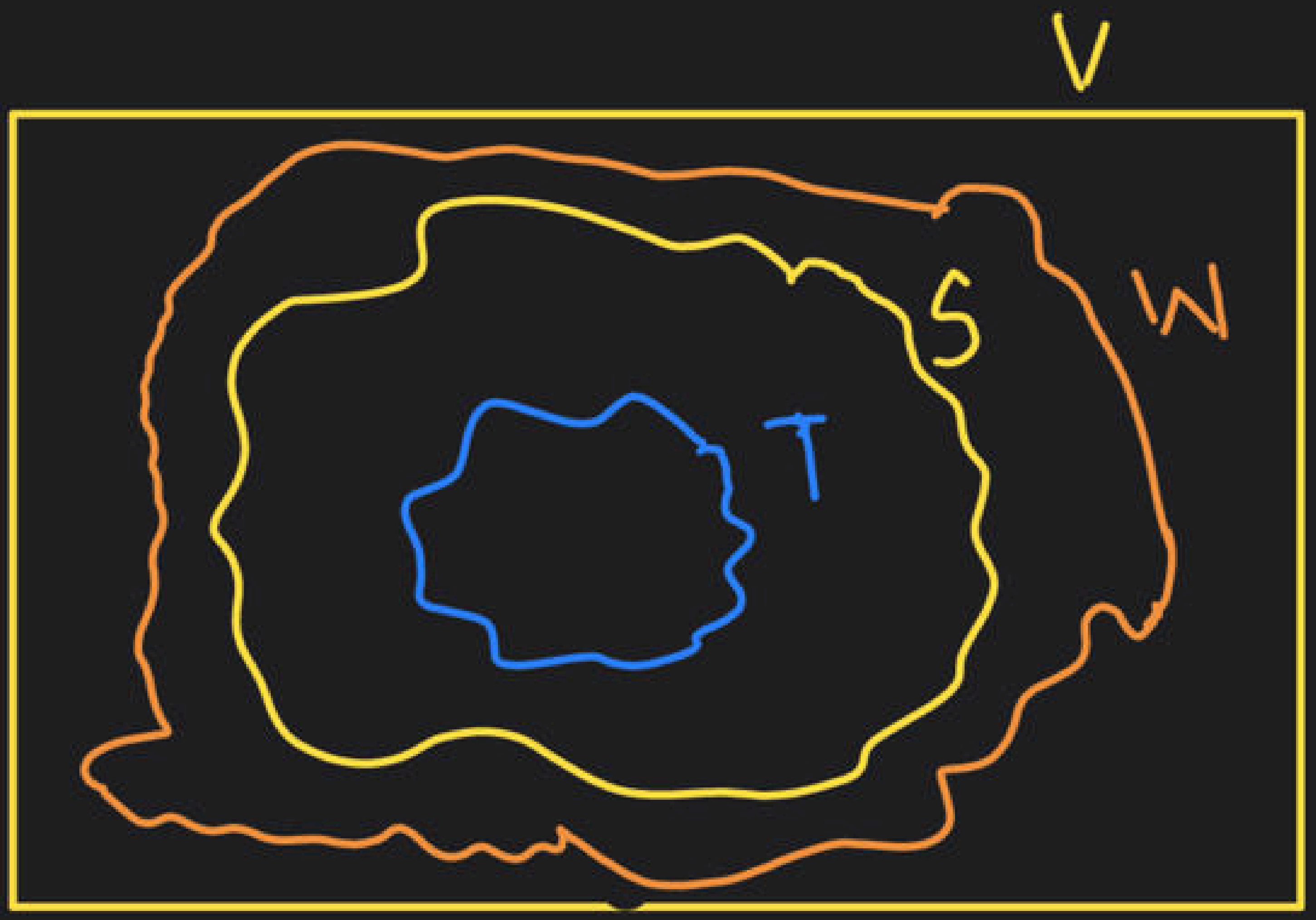
$\exists\, c \neq 0$ s.t. $c v = 0$

S is LD.

$\Rightarrow$ every non-zero singleton set is LI.

$\Rightarrow$ singleton zero set is LD.

(3) If S is linearly dependent set in V then subset of S may or may not be LD in V



(eg) $S = \{ (1,0) \ (1,1) (2,2) \}$

S is LD set.

$(2,2) = 2 \cdot (1,1) + 0 \cdot (1,0)$

$T_1 = \{ (1,1) (2,2) \}$ is LD

$T_1 \subset S$

$T_2 = \{ (1,0) \}$ $T_2 \subset S$

is LI.

(4) If S is a LD set in V then any superset of S is always a LD set in V.

Suppose $S = \{v_1, v_2, \ldots, v_n\}$ is LD

$W = \{v_1, v_2, \ldots v_n, v_{n+1}, v_{n+2}, \ldots v_m\}$

$S \subset W$

$$\underbrace{\underline{c_1 v_1 + c_2 v_2 + \cdots + c_n v_n} + \underline{c_{n+1}} v_{n+1} + \cdots + \underline{c_m} v_m = 0}_{\text{L.C. of vectors in } S.}$$

$\Rightarrow W$ is LD.

(5) If S is a LI set in V then any subset of S is always LI set in V.

- every possible subset of a LI set is always LI..

(6) If S is a LI set in V then any superset of S may or may not be LI.

$S = \{ (-1,0) \}$ is LI

W_1, W_2 s.t. $S \subseteq W_1$ and $S \subseteq W_2$

$S \subset W_1$ $W_1 = \{ (-1,0), (0,2) \} \rightarrow$ LI

$S \subset W_2$ $W_2 = \{ (-1,0) \; (0,1), (2,2) \}$

$\downarrow$

LD

$(2,2) = -2(-1,0) + 2(0,1)$

LI

$S \rightarrow LI$

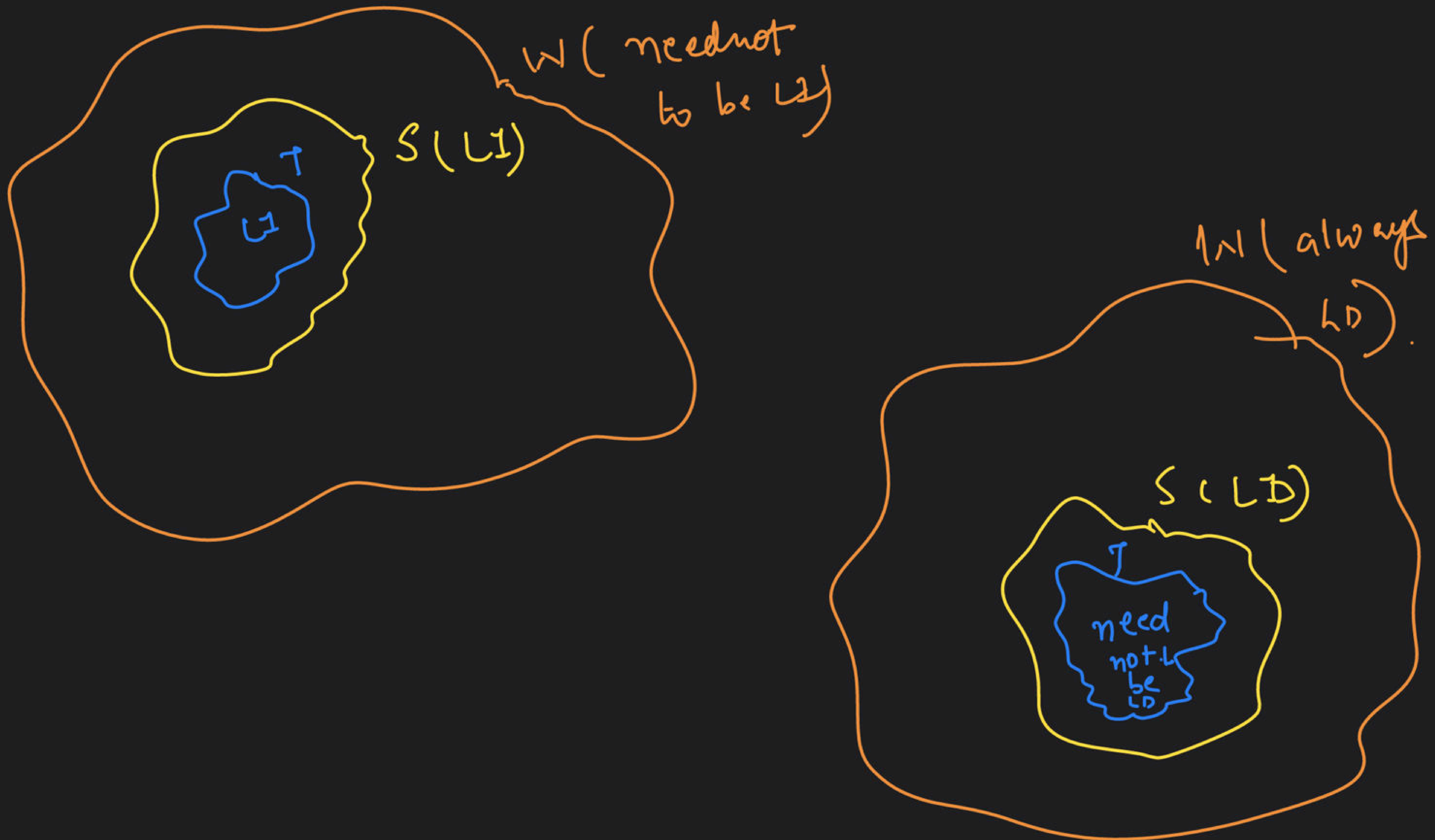
subset — always LI

superset — may or may not be LI

$S \rightarrow LD$

subset — may or may not be LD

superset — always LD.


W (neednot to be LI)
S (LI)
T
LI
W (always LD).
S (LD)
T
need not be LD

Spanning set

+

Direct sum.

+ ques

Basis + Dim.