exact & IF 2

↑ + 1

Ordinary Diffal eqn : (O D E) (15 marks)

1. Diffal eqn of Ist order and Ist Degree — 7 , Also order and degree of O D E — 8
2. Linear Diffal eqn with constant coefficient — 1 ‖ 3
3. Homogenus L.D.E with constant coefficient — 2 ‖ + 1
4. Orthogonal Trajectory — 4 — 2
5. variation of parameter — 3 — 1
6. Linear non-homo. Diffal eqn of IInd order — 5 — 2
7. Wronskian — 6 — 2 + 1

(1) ordinary diff^tal eq^n :- A diff^tal eq^n which contains one dependent and one independent variable is called ODE.

Ex: (i) $\frac{dy}{dx} + y = x$ (2) $\frac{d^2y}{dx^2} + y = x^3$

(iii) $\frac{dz}{dx} + z = x$

(2) Order of a DE :- The order of a DE is the highest order derivative appearing in the given DE.

Ex: (i) $\frac{d^2y}{dx^2} + y = x$ order = 2

(ii) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + y = x^2$, order = 2

(iii) $\frac{dy}{dx} + y = \frac{d^3y}{dx^3}$ order = 3

Note: The order of a DE is always exists and is a unique +ve integer

(3) **Degree of a DE:-** The highest power of highest order derivative is called degree of a DE provided it is free from radical & fraction.

Ex: (i) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + y = x + \sin x$ order = 2, Degree = 1

(ii) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/3} = a \cdot \left(\frac{d^2y}{dx^2}\right)$

ii: $\left(1+\left(\frac{dy}{dx}\right)^2\right) = a^3\left(\frac{d^2y}{dx^2}\right)^3$ order = 2, degree = 3

(iii) $\left(\frac{d^2y}{dx^2}\right)^5 = \left(\frac{d^2y}{dx^2}\right)^7$.

order = 2
Degree = 7

Note: ① For degree of a DE: The diff^tal eq^n must be polynomial in its diff^tal coefficients (or in its derivative)

② The degree of a diff^tal eq^n may or may not be exists.

Ex: ① $\frac{dy}{dx} + y = \sin\left(\frac{dy}{dx}\right)$

order $= 1$

But degree is not defined ($\because$ it is not a polynomial in its differential coefficient)

(ii) $$\frac{d^2y}{dx^2} + y = e^{\frac{d^2y}{dx^2}}$$

order $= 2$

But degree is not defined ($\because$ it is not a polynomial in its differential coefficient)

(iii) $$\frac{d^2y}{dx^2} + y = e^{\frac{dy}{dx}} = \left(1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \dots \right).$$

order $= 2$, degree $= 1$

④ Linear diff^tal eq^n :-

For linear diff^tal eq^n

* (i) The dependent variable and its derivative does not multiply with each other.

(ii) The degree of dependent variable & all its derivative should be 1.

Ex: (i) $y\dfrac{dy}{dx} + y = x \longrightarrow$ non-linear.

(ii) $x\dfrac{dy}{dx} + y = x^2 \longrightarrow$ linear

(iii) $\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = x^2 \longrightarrow$ linear

(iv) $\frac{dy}{dx} + y^2 = x^2$ non-linear

Linear diffal eqⁿ with constant coefficients

A diffal eqⁿ is of the form

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + a_n y = X$$

i.e. $$(a_0 D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_n) y = X \quad \left(D = \frac{d}{dx}\right) \qquad (1)$$

where $a_0, a_1, a_2 \, -- \, a_n$ are all constants & X is a funⁿ of only x or constant is called LDE with constant coefficients.

The required solⁿ is

$$y = C \cdot F + PI \quad (= y_c + y_p)$$

CF → Complementary funⁿ

PI → Particular Integral

If $X = 0$

Then eqⁿ (1) becomes

$$(a_0 D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_n) y = 0 \quad \text{--- (2)}$$

which is called homogeneous L.D.E with constant coefficient.

∴ The required soln is

$$y = CF.$$

Let $y = e^{mx}$ be the soln of eqn (2)

∴ Eqn (2) becomes

$$(a_0 m^n + a_1 m^{n-1} + a_2 m^{n-2} + \cdots + a_n) e^{mx} = 0$$

$$\Rightarrow a_0 m^n + a_1 m^{n-1} + a_2 m^{n-2} + \cdots + a_n = 0 \quad \text{--- (3)}$$

which is called Auxiliary eqn or $(\because e^{mx} \neq 0)$

Char. eqn (For AE Replace D by m)

$$(D - m) y = 0$$

$$\frac{dy}{dx} - my = 0$$

$$\frac{dy}{y} = m\,dx$$

$$\log y = mx + \log c$$

$$\log y - \log c = mx$$

$$\log (y/c) = mx$$

$$\boxed{y = c e^{mx}}$$

Case I: Roots are real & distinct:

Let $m = m_1, m_2$ (say)

Then $y = CF = c_1 e^{m_1 x} + c_2 e^{m_2 x}$.

$D(e^{mx})$
$= m e^{mx}$
$D^2(e^{mx}) = m^2 e^{mx}$
$-\ -\ \_\_\_$
$-\ \_\_\_$
$D^n(e^{mx}) = m^n e^{mx}$

NOTE: Principle of superposition:

(i) If y_1, y_2 be two solns of a homo. L.D.E with constant coefficient, then their linear combination $c_1 y_1 + c_2 y_2$ is also a soln of the above diff^{tal} eq^n

(ii) If $y_1, y_2 - - - y_n$ be n solns of a homo. L.D.E with constant coefficient, then their linear combination $c_1 y_1 + c_2 y_2 + \cdots + c_n y_n$ is also a soln of the above diff^{tal} eq^n.

Let $m = m_1, m_2, m_3$ (say)

Then $y = CF = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$

$\therefore$ In general

Let $m = m_1, m_2, m_3 \ldots m_n$

Then $y = CF = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \ldots + c_n e^{m_n x}$.

Ex: Solve $(D^2 - 5D + 6) y = 0 \quad \left(D = \frac{d}{dx}\right)$

Soln: The A E is

$$m^2 - 5m + 6 = 0$$

$$\Rightarrow (m-2)(m-3) = 0$$

$$\Rightarrow m = 2, 3$$

$\therefore \quad y = CF = c_1 e^{2x} + c_2 e^{3x}$

which is required soln.

Ex:- If $y = ae^{2x} + be^{3x}$, find a diffl eqn (Form a DE)

soln. $\big((D-2)(D-3)\big)y = 0$

$$(D^2 - 5D + 6)\, y = 0$$

i.e. $$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

Note:- All the constant involve in C.F., Particular integral does not contain any arbitrary constant.