Doubt Clearing Session

Comprehensive Course on Linear Algebra

Alka Singh • Lesson 20 • Sept 24, 2021

(eg) $T: \mathbb{R}^3 \to \mathbb{R}^4$

$T(x, y, z) = (x - y + z, 2x - y + z, x - y, z)$

(2 min)

(a) $N(T)$

(b) $\eta(T)$

(c) $R(T)$

(d) $\ell(T)$

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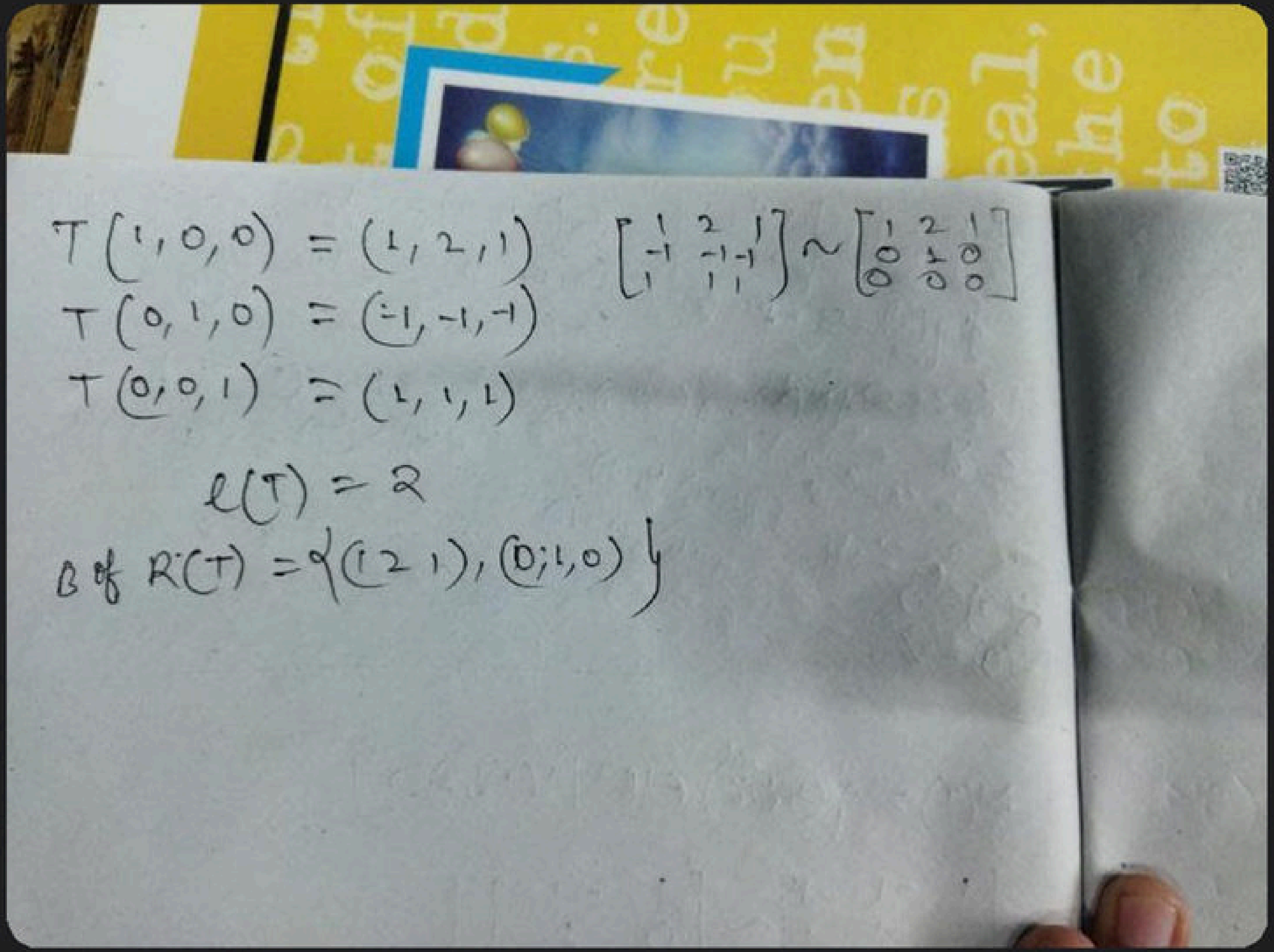
▲ 1 • Asked by Bhanu

Please help me with this doubt



$T(1,0,0) = (1,2,1)$ $\quad \begin{bmatrix} 1 & 2 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$T(0,1,0) = (-1,-1,-1)$

$T(0,0,1) = (1,1,1)$

$\rho(T) = 2$

B of $R(T) = \{(1\ 2\ 1), (0,1,0)\}$

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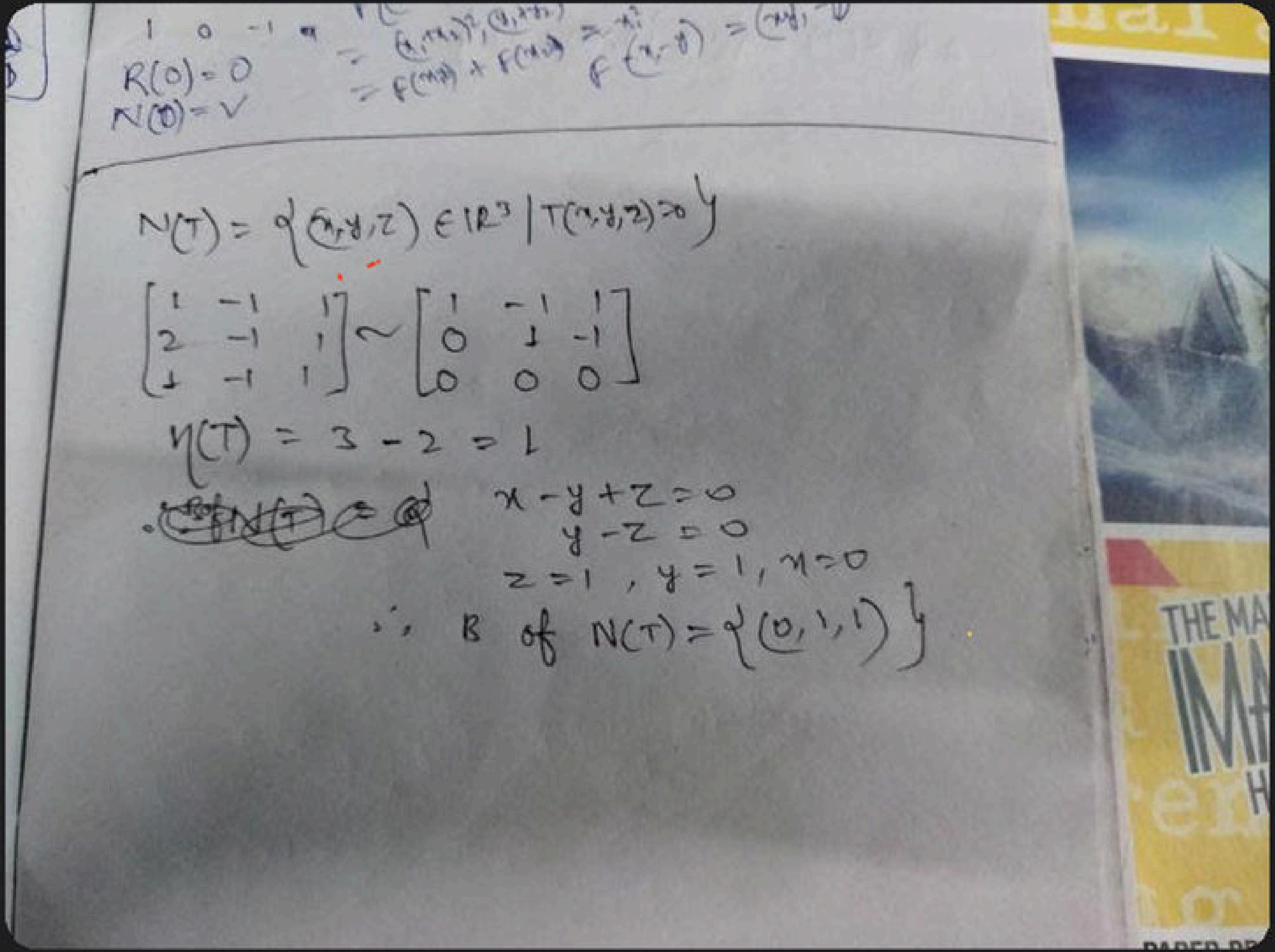


$R(0)=0$

$N(0)=V$

$N(T)=\{(x,y,z)\in\mathbb{R}^3 \mid T(x,y,z)=0\}$

$$\begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 1\\ 1 & -1 & 1\end{bmatrix} \sim \begin{bmatrix}1 & -1 & 1\\ 0 & 1 & -1\\ 0 & 0 & 0\end{bmatrix}$$

$\eta(T) = 3-2 = 1$

$x-y+z=0$

$y-z=0$

$z=1,\ y=1,\ x=0$

$\therefore$ B of $N(T)=\{(0,1,1)\}$

$$T(x, y, z) = (x - y + z,\ 2x - y + z,\ x - y,\ z)$$

$$T(1,0,0) = (1, 2, 1, 0)$$

$$T(0,1,0) = (-1, -1, -1, 0)$$

$$T(0,0,1) = (1, 1, 0, 1)$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ -1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\rho(T) = 3 \qquad B = \{(1,2,1,0), (0,1,0,0), (0,0,-1,0)\}$$

$$N(T) = \{(x,y,z) \in \mathbb{R}^3 \mid T(x,y,z) = (0,0,0,0)\}$$

$$(x-y+z,\ 2x-y+z,\ x-y,\ z) = (0,0,0,0)$$

$$(x,y,z) = (0,0,0)$$

$z = 0$

$x - y = 0$

$2x - y + z = 0$

$x - y + z = 0$

$2x - y = 0$

$\| -$

$x = 0$

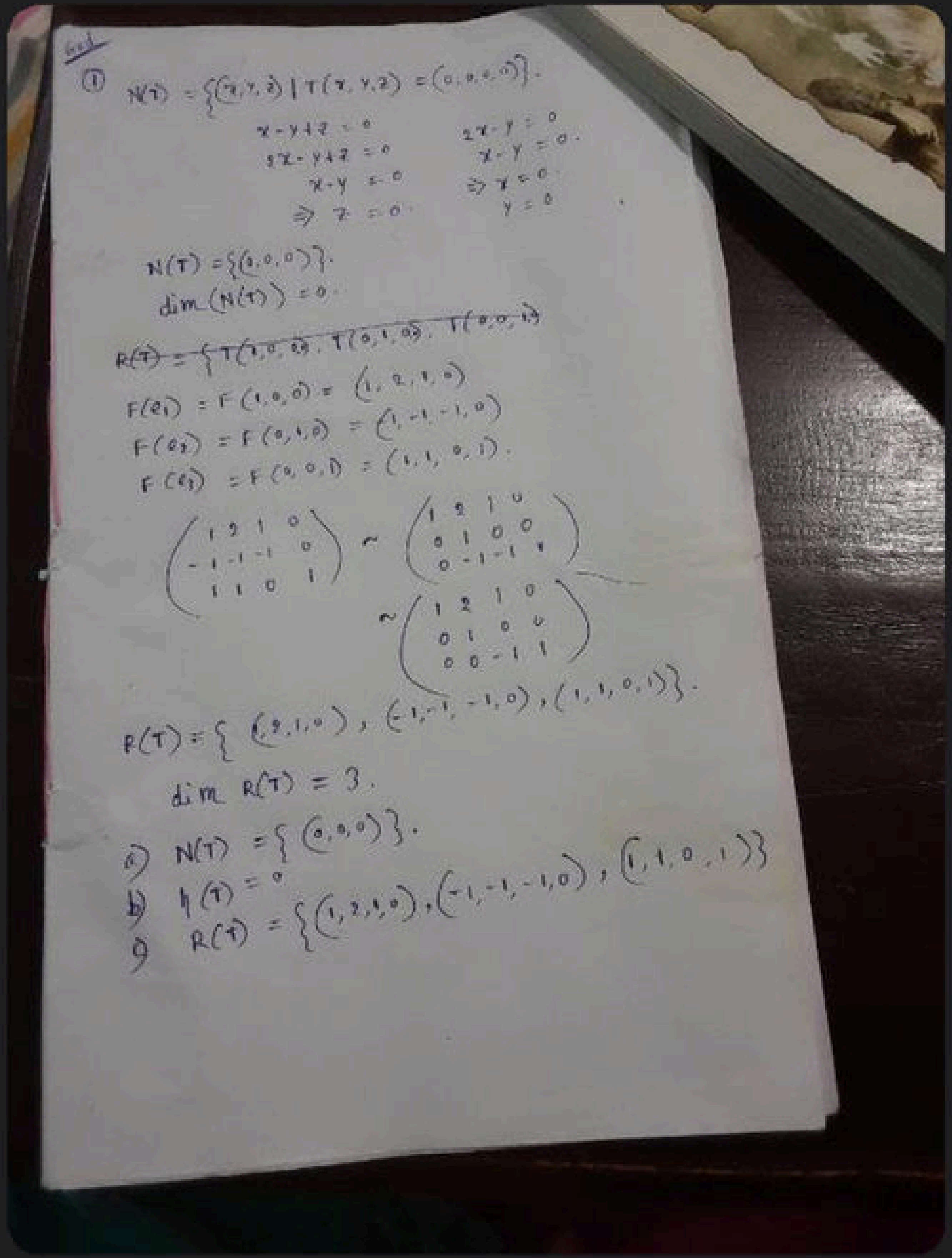
$y = 0.$

$$N(\gamma) = \{ (0,0,0) \}$$

$$\eta(\gamma) = 0$$

▲ 1 • Asked by Snehasmita

Please help me with this doubt



Sol

(1) $N(T) = \{(x,y,z) \mid T(x,y,z) = (0,0,0,0)\}$.

$$x - y + z = 0 \qquad 2x - y = 0$$
$$3x - y + z = 0 \qquad x - y = 0$$
$$x - y = 0 \qquad \Rightarrow x = 0$$
$$\Rightarrow z = 0 \qquad y = 0$$

$N(T) = \{(0,0,0)\}$.

$\dim(N(T)) = 0$.

~~$R(T) = \{T(1,0,0), T(0,1,0), T(0,0,1)\}$~~

$F(e_1) = F(1,0,0) = (1,2,1,0)$

$F(e_2) = F(0,1,0) = (-1,-1,-1,0)$

$F(e_3) = F(0,0,1) = (1,1,0,1)$.

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ -1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$R(T) = \{(1,2,1,0), (-1,-1,-1,0), (1,1,0,1)\}$.

$\dim R(T) = 3$.

a) $N(T) = \{(0,0,0)\}$.

b) $\eta(T) = 0$

c) $R(T) = \{(1,2,1,0), (-1,-1,-1,0), (1,1,0,1)\}$

(eg) $T : \mathbb{R}^4 \to \mathbb{R}^3$

$$T(x, y, z, t) = (x - y + z + t,\ 2x - 2y + 3z + 4t,\ 3x - 3y + 4z + 5t)$$

$\rightarrow N(T)$

$\rightarrow R(T)$

$\rightarrow \eta(T)$

$\rightarrow \rho(T)$

$$T(1,0,0,0) = (1,2,3)$$
$$T(0,1,0,0) = (-1,-2,-3)$$
$$T(0,0,1,0) = (1,3,4)$$
$$T(0,0,0,1) = (1,4,5)$$

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 1 & 3 & 4 \\ 1 & 4 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\ell(T) = 2$$

$$B = \{ (1,2,3), (0,1,1) \}$$

$$R(T) = \text{span } B$$

$$R(T) = \{ (c_1, 2c_1 + c_2, 3c_1 + c_2) \}$$

$T(x, y, z, t) = (0, 0, 0)$

$(x - y + z + t,\ 2x - 2y + 3z + 4t,\ 3x - 3y + 4z + 5t) = (0, 0, 0)$

$x - y + z + t = 0$

$2x - 2y + 3z + 4t = 0$

$3x - 3y + 4z + 5t = 0$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & -2 & 3 & 4 \\ 3 & -3 & 4 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$= n - r$

$= 4 - 2 = 2$

$$\boxed{\eta(T) = 2}$$

$$x + y + z + t = 0$$

$$z + 2t = 0$$

x	y	z	t
	1		0
	0		1

$$B = \{ (1, 1, 0, 0), (1, 0, -2, 1) \}$$

$$N(T) = \text{span } B$$

$$= \text{span}\{ (1,1,0,0)\ (1, 0, -2, 1) \}$$

$$= \{ c_1 (1,1,0,0) + c_2 (1, 0, -2, 1) \mid \quad \}$$

$$= \{ (c_1 + c_2 , c_1 , -2c_2 , c_2 \}$$

1 • Asked by Anamika

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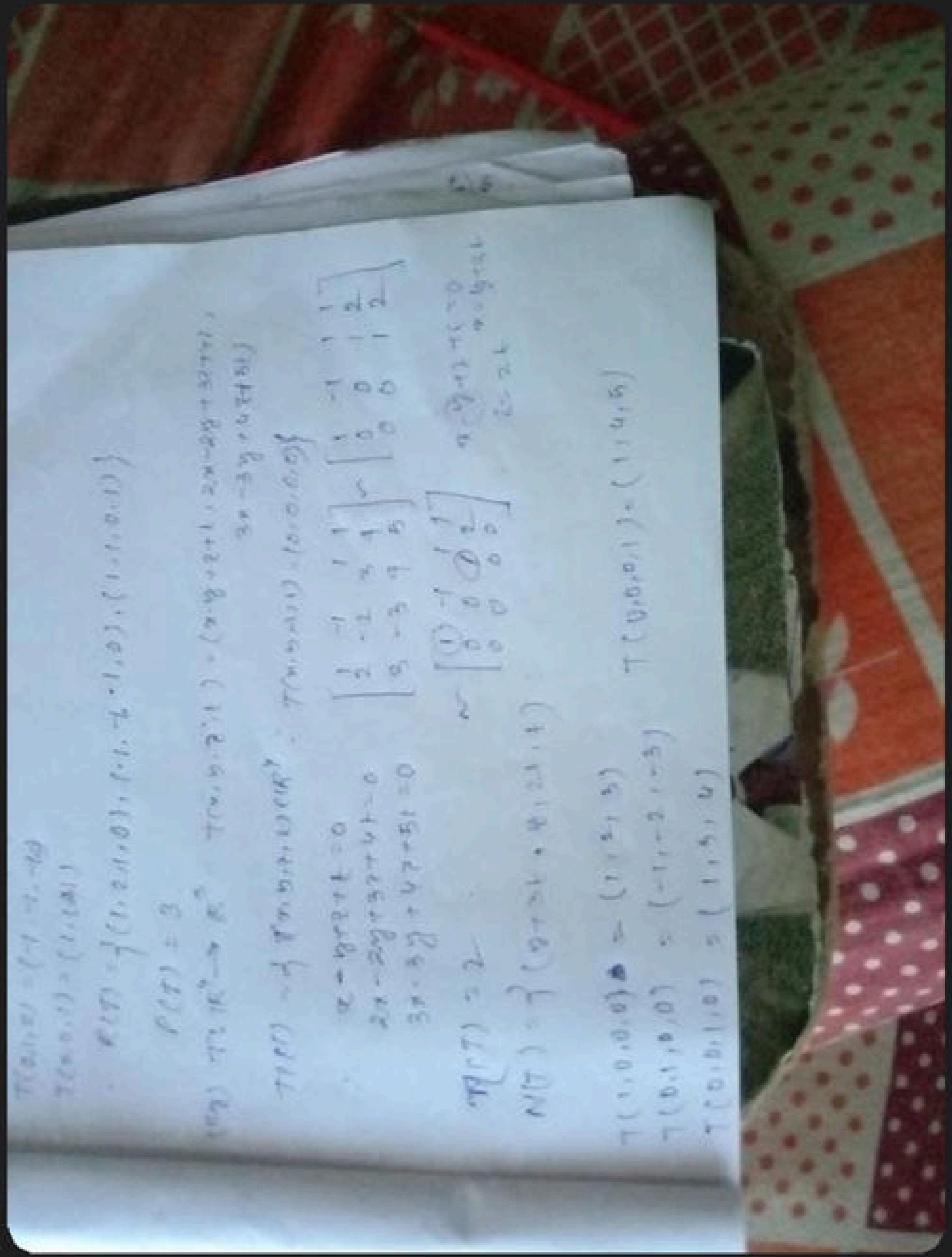

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1 • Asked by Bhanu

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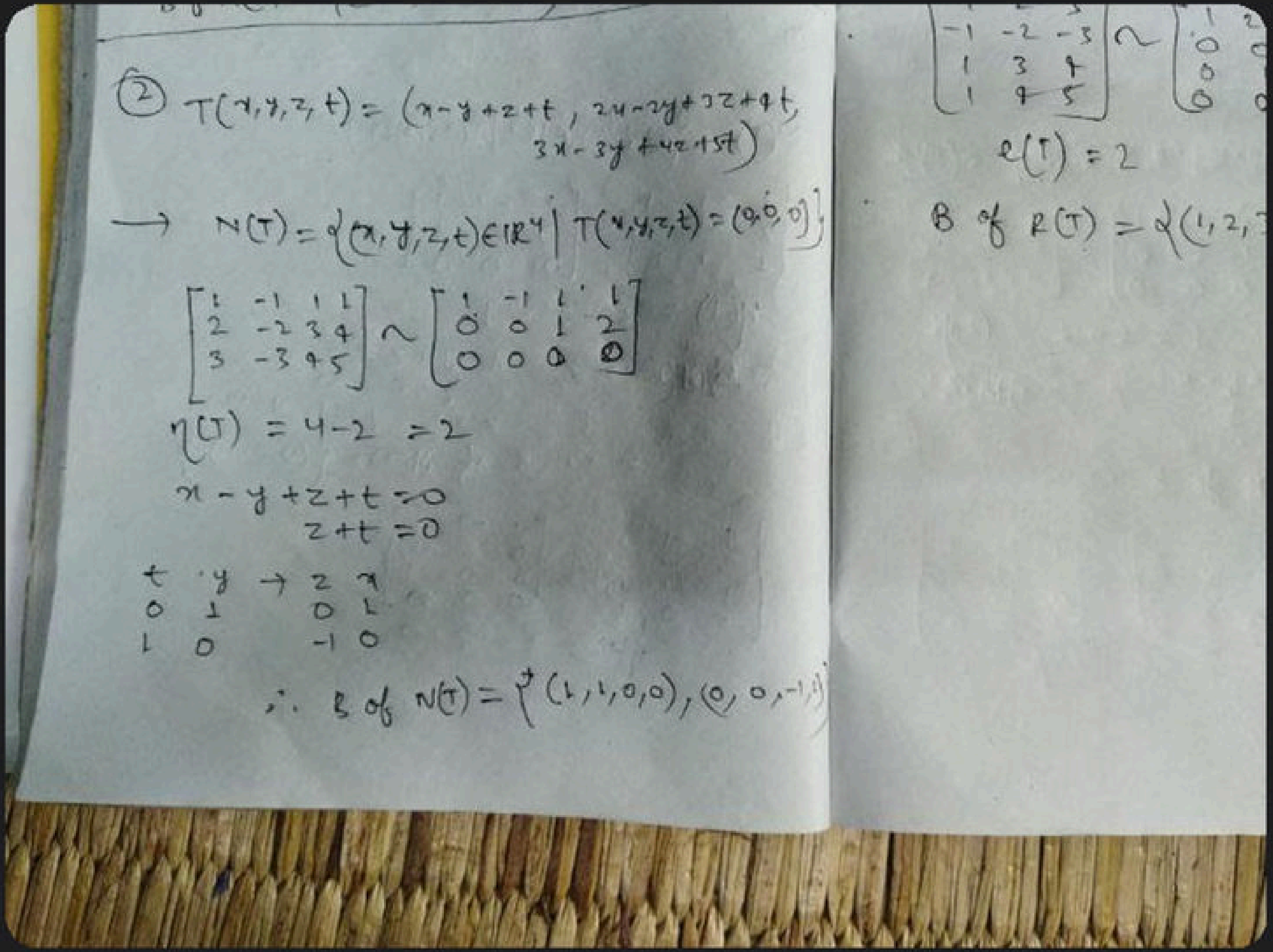

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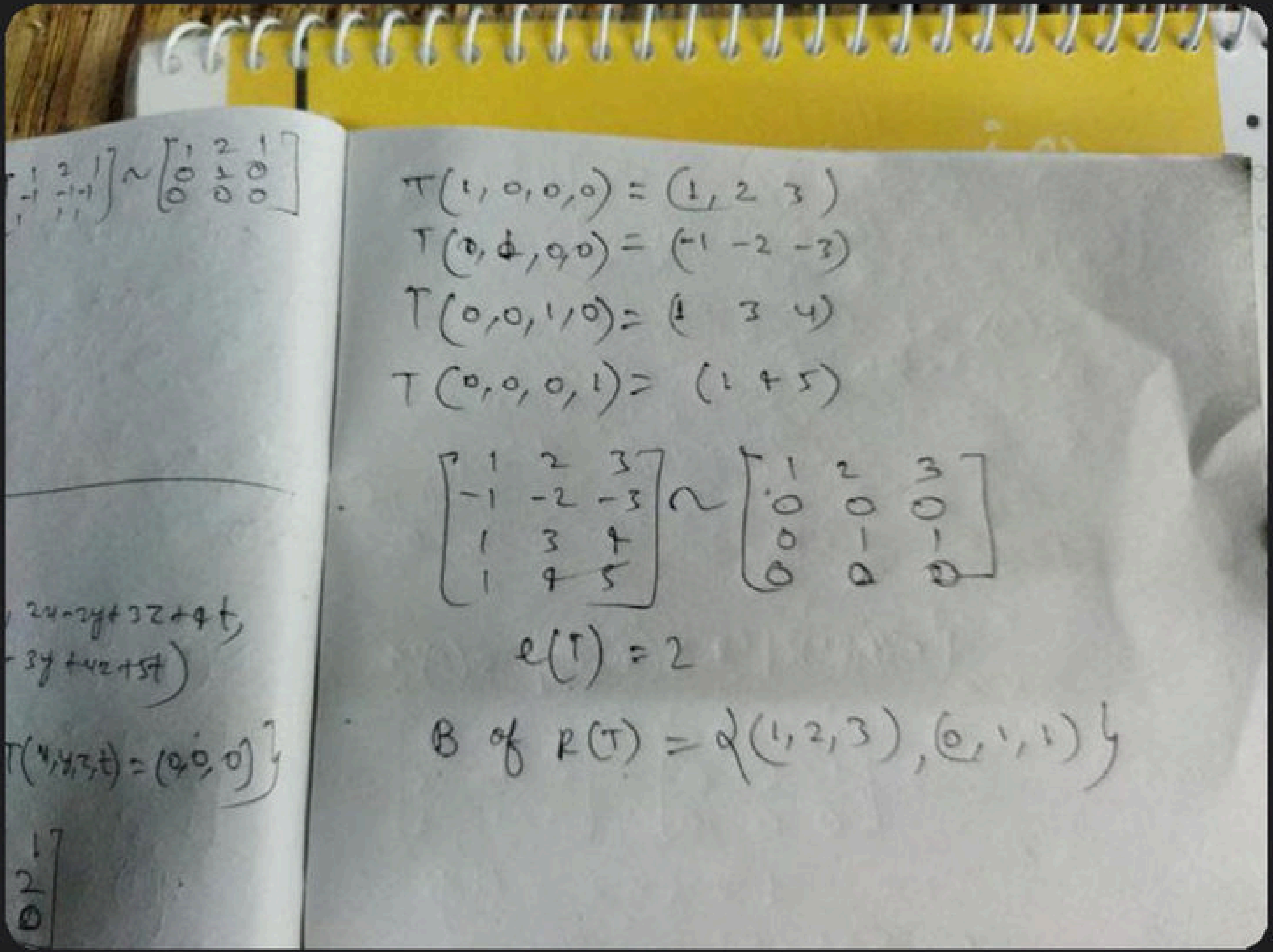

$T(1,0,0,0) = (1, 2, 3)$

$T(0,1,0,0) = (-1, -2, -3)$

$T(0,0,1,0) = (1, 3, 4)$

$T(0,0,0,1) = (1, 4, 5)$

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 1 & 3 & 4 \\ 1 & 4 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(T) = 2$

B of $R(T) = \{(1,2,3), (0,1,1)\}$

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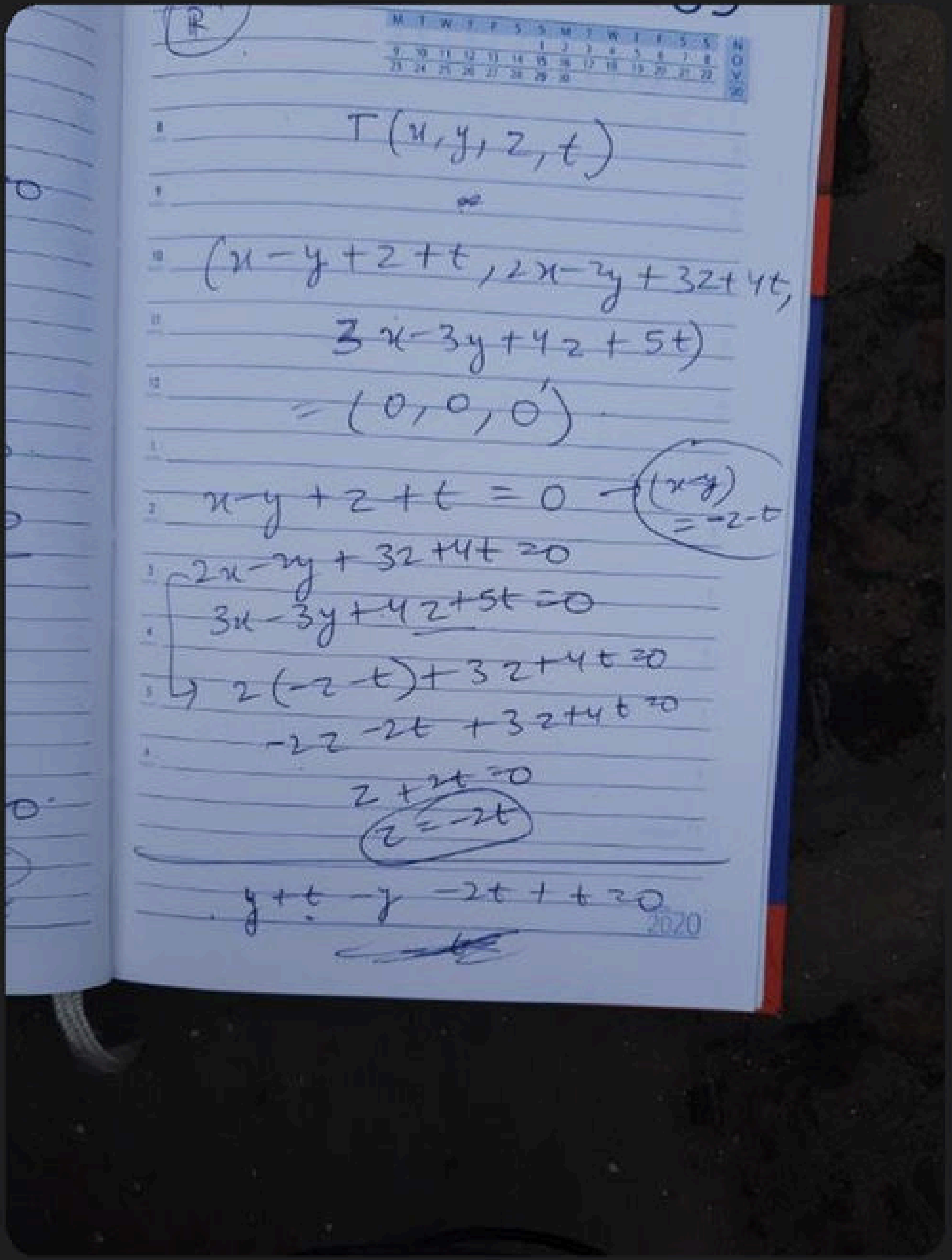
1 • Asked by Swapnajit

Please help me with this doubt



$T(x, y, z, t)$

$=$

$(x-y+z+t,\ 2x-2y+3z+4t,$

$3x-3y+4z+5t)$

$=(0,0,0)$

$x-y+z+t=0 \rightarrow (x-y) = -z-t$

$2x-2y+3z+4t=0$

$3x-3y+4z+5t=0$

$\rightarrow 2(-z-t)+3z+4t=0$

$-2z-2t+3z+4t=0$

$z+2t=0$

$z=-2t$

$y+t-y-2t+t=0$

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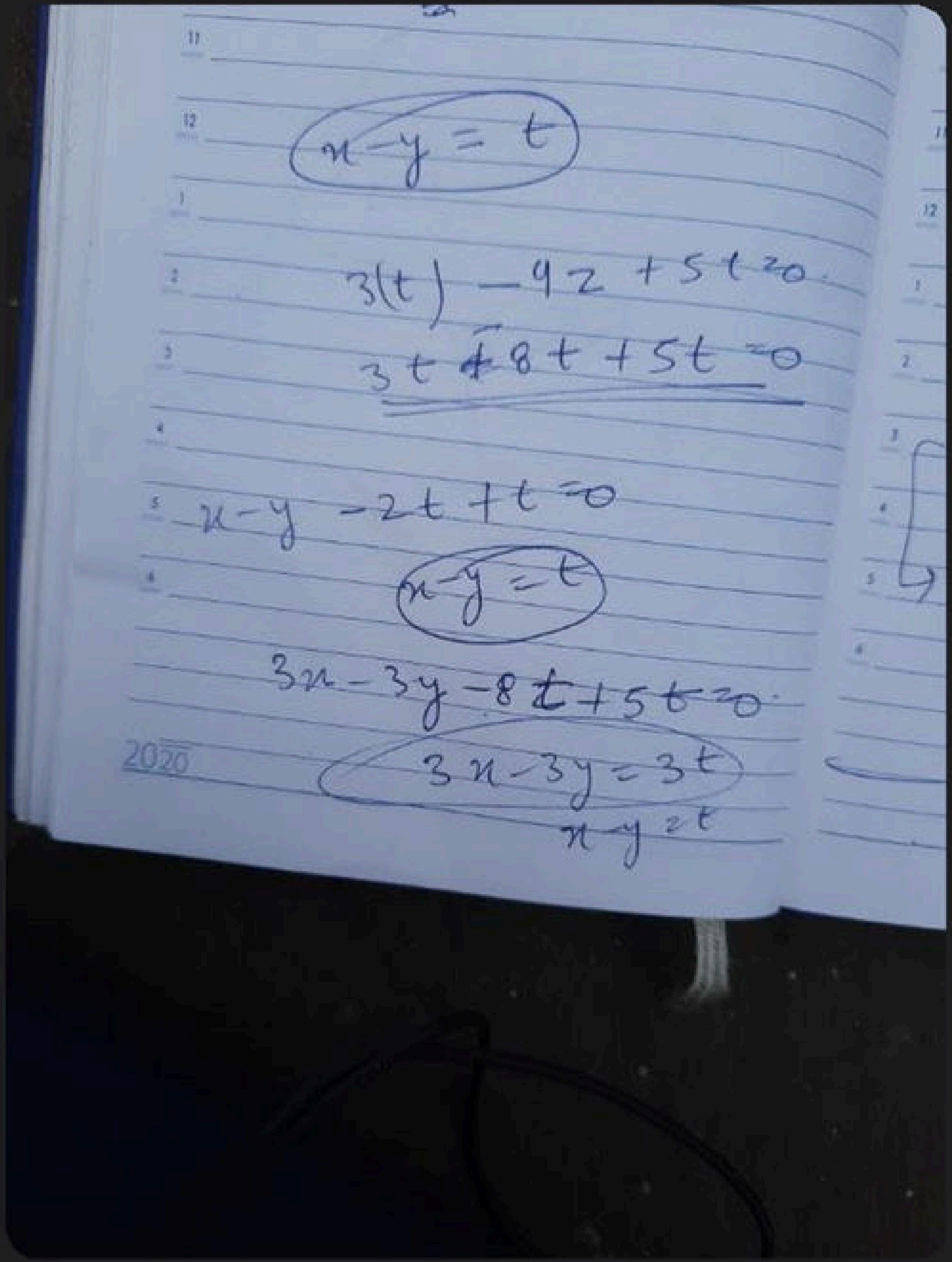

$x-y=t$

$3(t)-4z+5t=0$

$3t-\cancel{4}8t+5t=0$

$x-y-2t+t=0$

$x-y=t$

$3x-3y-8t+5t=0$

$3x-3y=3t$

$x-y=t$

SINGULAR AND NON-SINGULAR MAP

Let V and W be a V.S. over the same field F.

Let $T : V \to W$ be a linear map.

T is called as a singular map if $\exists$ a non zero $x \in V$ s.t. $T(x) = 0$.

T is a non-singular map if $\nexists$ any non. zero $x \in V$ s.t. $T(x) = 0$

If $\eta(T) > 0 \implies T$ is a singular map.

If $\eta(T) = 0 \implies T$ is a non-singular map.

T is a LT.

$0 \in V$.

T is singular map.

$$0 \in V \xrightarrow{T} 0 \in W$$

$$\exists\, x (\neq 0) \in V \xrightarrow{T} 0$$

$\Rightarrow N(T) = \{0, x$

$0 \in V \xrightarrow{T} 0 \in W$ — T is non-singular.

$N(T) = \{0\}$

(eg) $T : V \to W$

$T(x) = 0 \quad \forall\ x \in V$

T is singular.

$N(T) = V$

$\dim N(T) = \eta(T) = \underline{\dim V > 0}$

(eg) $T : V \to V \quad T(x) = x$

$N(T) = 0 \qquad$ T is non-sing.

$n(T) = 0$

(9) $T : \mathbb{R}^2 \to \mathbb{R}^2$

$T(x, y) = (x + y, y)$

$T(x, y) = (0, 0)$

$(x + y, y) = (0, 0)$

$y = 0$

$x + y = 0 \Rightarrow x = 0$

$N(T) = \{ (0, 0) \}$

T is non-singular

(eg) $T : \mathbb{R}^3 \to \mathbb{R}^2$

$$T(x, y, z) = (x+y+z, y) = (0, 0)$$

$$x+y+z=0$$
$$y=0$$

$$x+z=0$$
$$y=0$$

$$N(T) = \{ (-z, 0, z) \mid z \in \mathbb{R} \}$$

$$n(T) = \dim N(T) = 1 > 0$$

T is singular

(eg)

$T : P_3[x] \to P_4[x]$ — Non-singular.

$$T(p(x)) = \int_0^x p(t)\,dt$$

$$N(T) = \{ p(x) \in P[x] \mid T(p(x)) = 0 \}$$

$$\downarrow$$

$$\int_0^x p(t)\,dt = 0$$

$$\frac{d}{dt}\int_0^x p(t)\,dt = \frac{d}{dt}(0)$$

$$p(x) = 0$$

$$N(T) = \{0\}$$

$$\downarrow$$

$$\eta(T) = 0$$

(eg) $T : P_3(x) \to P_4(x)$ non-singular

$$T(p(x)) = p'(x) + \int_0^x p(t)\,dt$$

$$N(T) = \{ p(x) \in P_3(x) \mid T(p(x)) = 0 \}$$

$$p'(x) + \int_0^x p(t)\,dt = 0$$

$$N(T) = \{ 0 \}$$

$$p''(x) + p(x) = 0 \quad \text{by Leibni}$$

$$(D^2+1)\,p(x) = 0 \qquad m^2+1=0$$

$$\frac{d}{dx}\int_0^x p(t)\,dt = \int_0^x \frac{\partial}{\partial x} p(t)\,dt + p(x)\cdot\frac{d}{dx}(x) -$$

$$p(0)\cdot\frac{d}{dx}(0)$$

$$= 0 + p(x) - 0$$

$$= \underline{\underline{p(x)}}\,,$$