Examples of Vector Space

Comprehensive Course on Linear Algebra

Alka Singh • Lesson 3 • Sept 7, 2021

Let V be the set of all functions from S to field F is a vector space under ~~the~~ binary operation $(f+g)(x) = f(x) + g(x)$ and $(c.f)(x) = c.f(x)$ where S is a non-empty set.

$$V = \{ f \mid f : S \to F \} \qquad S \neq \phi$$

1 → $f, g \in V \implies f, g : S \to F$

by alge. $f+g : S \to F \implies f+g \in V$ (closure)

2 → $(f+g)+h = f+(g+h)$ (algebra)

3 → $0 : S \to F$ s.t $0(x) = 0$

Func.

scalar $\in F$

$$(f+0)(x) = f(x) + 0(x)$$

$$= f(x)$$

$$= (f)(x)$$

$\to \forall f \in V \quad \exists\, g \in V$ st $\quad f + g = 0 = g + f$

$f : S \to F$

$$(f+g) = 0$$

$$(f+g)(x) = 0(x)$$

$$f(x) + g(x) = 0$$

$$g(x) = -f(x) \quad \forall x$$

$\boxed{g = -f}$ is the inverse of f.

5 $\to$ $\forall\, f, g \in V.$

commutative

$f, g : S \to F$

$$(f+g)(x) = \underbrace{f(x)}_{\in F} + \underbrace{g(x)}_{\in F}$$

$$= g(x) + f(x)$$

$$= (g+f)(x)$$

$\xrightarrow{\leftarrow}$ $\forall f \in V \quad \forall \alpha \in F$

$\because \; f : S \to F$

$\alpha f : S \to F$

$$(\alpha f)(x) = \underbrace{\overset{\in F}{\alpha} \cdot \overset{\in F}{f(x)}}_{\in F} \quad \forall x \in S$$

$\alpha f \in V$

$\xrightarrow{\exists}$ $\forall\ \alpha, \beta \in F$, $\forall\ f \in V$. $\Rightarrow f : S \to F$

<u>Claim</u>: $(\alpha + \beta) f = \alpha f + \beta f$

By alg. $(\alpha + \beta) f : \underline{S \to F}$

$$\big((\alpha + \beta) f \big)(x) = (\alpha + \beta) f(x)$$
$$= \alpha f(x) + \beta f(x)$$
$$= (\alpha f + \beta f)(x)$$

8 →

9 →

1.0 →

HW

$(\alpha\beta)f = \alpha(\beta f)$

$\alpha \cdot (f+g) = \alpha f + \alpha g$

$1 \cdot \in F.$

$(1 \cdot f)(x) = 1 \cdot f(x)$

$= f(x)$

Let V be the set of all polynomials from $F \to F$ is a vector space over the field F

$$V = \left\{ a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n + \cdots \;\middle|\; a_i \in F \ \forall i \right\}$$

V be the set of all polynomials of fixed degree n.

$V = \{ a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n \mid a_i \in F \ \forall i = 1, 2 - n$

n – fixed $\}$

$\rightarrow$ $-x^2 + x = p(x)$ $\quad n = 2$

$q(x) = x^2 + x$

$p(x) + q(x) = 2x \notin V$

$\longrightarrow$

Let V be the set of all polynomials of atmost degree n.

Then is V a v.s. over the field F.

$$V = \{ a_0 + a_1 x + a_2 x^2 + \dots + a_k x^k \mid k \le n, \; a_i \in F \; \forall i = 1, 2 \dots k \}$$

Yes

Let V be the set of all polynomials of atleast degree n.

Is V a vector space over the field F.?

This is not a vector space over the field F.

$$V = \{ a_0 + a_1 x + \cdots + a_k x^k \mid k \geq n, \; a_i \in F \; \forall i \}$$

$n = 2$

$p(x) \in V \quad q(x) \in V$

$p(x) = 1 + x^2 \quad q(x) = -2x - x^2$

$p(x) + q(x) = 1 - 2x \notin V.$

let V be the set of all matrices of order $m \times n$ over the field F.

or

$S = \{ (i,j) \mid 1 \le i \le m ; 1 \le j \le n \}$

V be a set of all func from $S \to F$.

V is a V.S.

$\alpha \in F$

$M = (m_{ij})_{m \times n}$ $\quad m_{ij} \in F$

$\alpha M = (\alpha m_{ij})_{m \times n}$ $\quad \alpha m_{ij} \in F$