Doubt Clearing Session

Comprehensive Course on Linear Algebra

Alka Singh • Lesson 24 • Sept 28, 2021

MATRIX REPRESENTATION

$T : V \rightarrow V$ be a LT where V is a F.D.V.S. Let $B = \{ v_1, v_2, \dots, v_n \}$ be a basis of V. Then matrix rep. of T wrt basis B is given as, $[T]_B$.

Method:

$$T(v_1) = a_{11} v_1 + a_{12} v_2 + \dots + a_{1n} v_n$$

$$T(v_2) = a_{21} v_1 + a_{22} v_2 + \dots + a_{2n} v_n$$

$$\vdots$$

$$T(v_n) = a_{n1} v_1 + a_{n2} v_2 + \dots + a_{nn} v_n$$

$$[T]_B = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}^T$$

(eg) $T: \mathbb{R}^2 \to \mathbb{R}^2$

$T(x, y) = (x, y)$

$$[T]_B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

$B = \{ (1, 0) \quad (-1, 2) \}$

$\Rightarrow \quad T(1, 0) = (1, 0) = c_1 (1, 0) + c_2 (-1, 2)$

$= 1 \cdot (1, 0) + 0 \cdot (-1, 2)$

$\Rightarrow \quad T(-1, 2) = (-1, 2) = 0 \cdot (1, 0) + 1 \cdot (-1, 2)$

$$[T]_B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(eg)

$$T : V \to V$$

$$T(x) = 0 \quad \forall \ x \in V$$

$$B = \{v_1, v_2, \ldots, v_n\}$$

$$T(v_1) = 0 = 0 \cdot v_1 + 0 \cdot v_2 + \cdots + 0 v_n$$

$$\vdots$$

$$T(v_n) = 0 = 0 \cdot v_1 + 0 \cdot v_2 + \cdots + 0 v_n$$

$$[T]_B = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{n \times n}$$

(eg) $T : \mathbb{R}^2 \to \mathbb{R}^2$

$$T(x, y) = (2x - 5y, 3x + y)$$

$$B = \{ (0, 1) \ (-1, 2) \}$$

$\Rightarrow$ $T(0,1) = (-5, 1) = c_1 (0,1) + c_2 (-1, 2)$

$\Rightarrow$ $T(-1,2) = (-12, -1) = c_1 (0,1) + c_2 (-1, 2)$

$$(x, y) = c_1 (0, 1) + c_2 (-1, 2)$$

$$\boxed{(x, y) = (2x + y)(0, 1) + (-x)(-1, 2)}$$

$$(x,y) = (2x+y)(0,1) + (-x)(-1,2)$$

$$T(0,1) = (-5,1) = (-9)(0,1) + (.5)(-1,2)$$

$$T(-1,2) = (-12,-1) = (-25)(0,1) + (12)(-1,2)$$

$$[T]_{13} = \begin{bmatrix} -9 & 5 \\ -25 & 12 \end{bmatrix}^T = \begin{bmatrix} -9 & -25 \\ 5 & 12 \end{bmatrix}$$

$T : \mathbb{R}^2 \to \mathbb{R}^2$

$T(x, y) = (2x - 5y, 3x + y)$

$[T]_{S.B.} = \begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix}$

$\boxed{B = \{(1,0), (0,1)\}}$

$T(1,0) = (2,3) = 2(1,0) + 3(0,1)$

$T(0,1) = (-5,1) = -5(1,0) + 1 \cdot (0,1)$

$[T]_B = \begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix}_{2 \times 2}$

$$T: V \to V$$

$$[T]_{B_1} \sim [T]_{B_2} \sim [T]_{B_3} \sim \cdots \sim [T]_{B_n} \sim \cdots$$

e.v. same

trace same

det. same

$$T : \mathbb{R}^n \longrightarrow \mathbb{R}^n$$

$$T(x_1, x_2, \ldots, x_n) = \Big(\underline{a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n}, \; \underline{a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n}, \; \vdots \; \underline{a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n} \Big)$$

S.B.

$$[T]_{S.B.} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$T: \mathbb{R}^3 \to \mathbb{R}^3$

$T(x, y, z) = (\underline{2x + 3y + z}, \underline{y - 2z}, \underline{x + 3y})$

$SB. = \{ (1, 0, 0), (0, 1, 0) \quad (0, 0, 1) \}$

$$[T]_{SB} = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 0 \end{bmatrix}_{3 \times 3}$$

(eg) Let $D : P_2[x] \to P_2[x]$ such that

$D(f(x)) = \frac{df}{dx}$ is Linear Trans.

find mat. rep of D wrt basis $= \{ \underline{1}, \underline{x}, \underline{(1-x)^2} \}$

$D(1) = 0 = 0\cdot 1 + 0\cdot x + 0\cdot(1-x)^2$

$D(x) = 1 = 1\cdot 1 + 0\cdot x + 0\cdot (1-x)^2$

$D(1-x)^2 = -2(1-x) = -2\cdot 1 + 2\cdot x + 0\cdot (1-x)^2$

$$[D]_B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -2 & 2 & 0 \end{bmatrix}^T$$

$$[D]_B = \begin{bmatrix} 0 & 1 & -2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

(eg) $D : V \to V$ s.t. (V be the set of all func.)

$$D(f(x)) = \frac{df}{dx}$$

$$B = \{\sin t, \cos t, e^t\}$$

$[D]_B$

$$D(\sin t) = \cos t = 0 \cdot \sin t + 1 \cdot \cos t + 0 \cdot e^t$$

$$D(\cos t) = -\sin t = -1 \cdot \sin t + 0 \cdot \cos t + 0 \cdot e^t$$

$$D(e^t) = e^t = 0 \cdot \sin t + 0 \cdot \cos t + 1 \cdot e^t$$

$$[D]_B = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(eg) $T : \mathbb{R}^2 \to \mathbb{R}^2$, $A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$

$T(x) = Ax$

$B = \{ (1,2) \ (1,1) \}$

$[T]_B$

$T(1,2) = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} =$

$T(1,1) = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$$(x,y)^T = (y-x)(1,2)^T + (2x-y)(1,1)^T$$

$$T\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix} = 1\cdot\begin{pmatrix}1\\2\end{pmatrix} + 2\cdot\begin{pmatrix}1\\1\end{pmatrix}$$

$$T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix} = 1\cdot\begin{pmatrix}1\\2\end{pmatrix} + 1\cdot\begin{pmatrix}1\\1\end{pmatrix}$$

$$\boxed{[T]_B = \begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}} = \begin{bmatrix}1 & 2\\1 & 1\end{bmatrix}^T$$

$$(x, y) = c_1(1, 2) + c_2(1, 1)$$

$$\left[\begin{array}{cc|c} 1 & 1 & x \\ 2 & 1 & y \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & x \\ 0 & -1 & y-2x \end{array}\right]$$

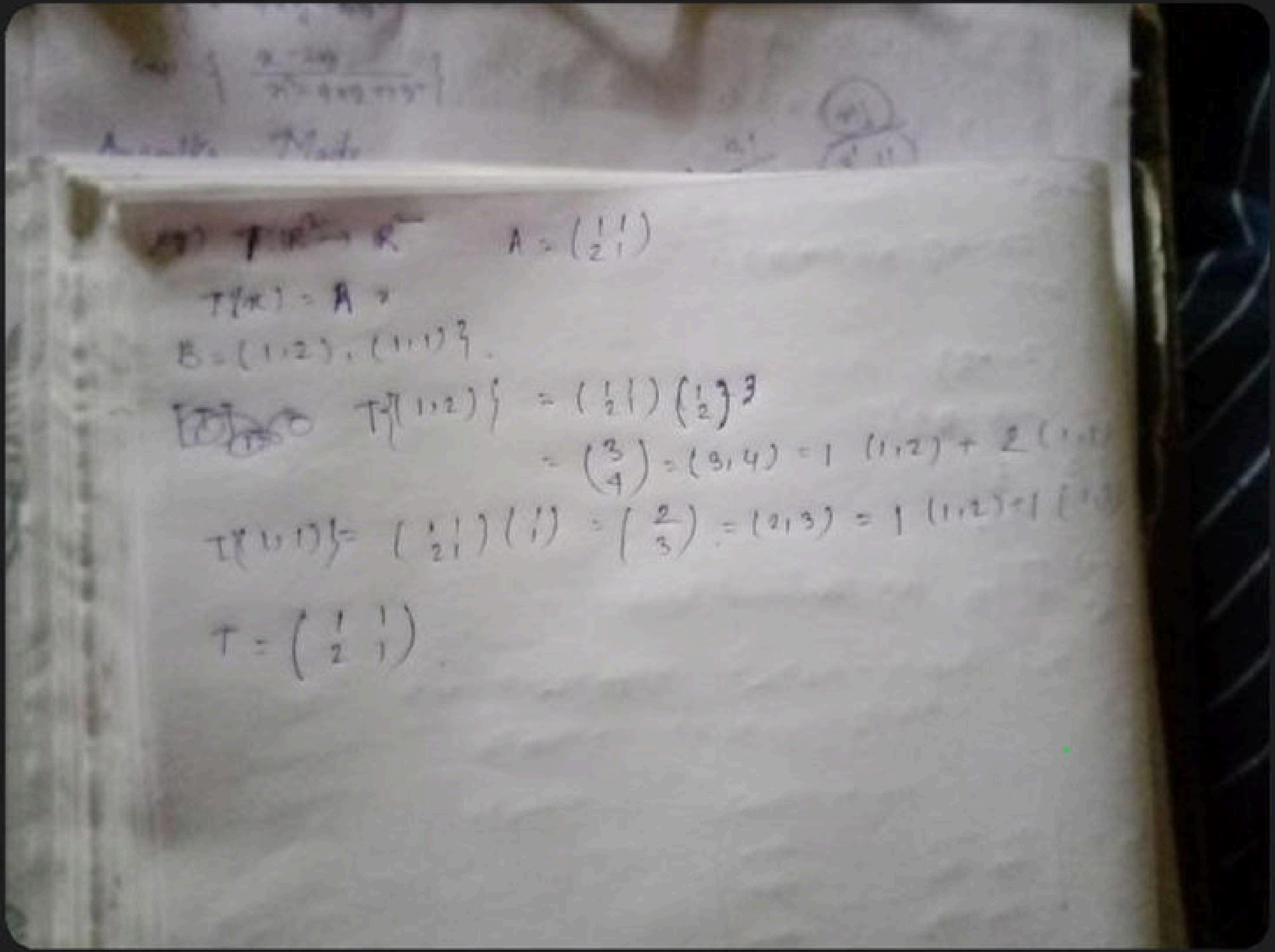
$$c_2 = 2x - y$$

$$c_1 + c_2 = x$$

$$c_1 = x - c_2 = x - 2x + y = y - x = c_1$$

1 • Asked by Anamika

Please help me with this doubt



$T: \mathbb{R}^2 \to \mathbb{R}^2 \quad A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$

$T(x) = Ax$

$B = \{(1,2), (1,1)\}$

$T\{(1,2)\} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 4 \end{pmatrix} = (3,4) = 1\,(1,2) + 2\,(1,1)$

$T\{(1,1)\} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} = (2,3) = 1\,(1,2) + 1\,(1,1)$

$T = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$

(eg) $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ HW

$T(x, y, z) = (2x + 3y - z,\ 4x - y - 2z,\ x)$

$B = \{ (1, 1, 0)\ \ (1, 2, 3)\ \ (1, 3, 5) \}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & x \\ 1 & 2 & 3 & y \\ 0 & 3 & 5 & z \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & x \\ 0 & 1 & 2 & y-x \\ 0 & 3 & 5 & z \end{array}\right] \sim$$

$c_3 = 3y - 3x - z$

$c_2 = (y - x) - 6y + 6x + 2z$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & x \\ 0 & 1 & 2 & y-x \\ 0 & 0 & -1 & z - 3y + 3x \end{array}\right]$$

(5 – 5:45) pm → Quiz

(6 – 7) pm → IC plus class COY