Matrix Representation - II

Comprehensive Course on Linear Algebra

Alka Singh • Lesson 25 • Sept 29, 2021

(1) $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$

$T: \mathbb{R}^{m} \longrightarrow \mathbb{R}^{n}$

$[T]_{n \times m}$

$T(x, y, z) = (2x + 3y - z,\ 4x - y - 2z)$

$B_1 = \{ \underset{v_1}{(1, 1, 0)}, \underset{v_2}{(1, 2, 3)}, \underset{v_3}{(1, 3, 5)} \}$

$B_2 = \{ \underset{u_1}{(1, 2)}, \underset{u_2}{(2, 3)} \}$

$[T]_{B_1}^{B_2} = A$

$(x, y) = c_1 (1, 2) + c_2 (2, 3)$

$\boxed{c_2 = 2x - y}$

$\left[\begin{array}{cc|c} 1 & 2 & x \\ 2 & 3 & y \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & x \\ 0 & -1 & y - 2x \end{array}\right]$

$c_1 = x - 2c_2$

$c_1 = x - 4x + 2y$

Sum of all elements in A =

$\boxed{c_1 = 2y - 3x}$

$$T(x,y,z) = (2x+3y-z,\ 4x-y-2z)$$

$$(x,y) = (-3x+2y)\overset{u_1}{(1,2)} + (2x-y)\overset{u_2}{(2,3)} \quad \text{—— (1)}$$

$$T(v_1) = T(1,1,0) = (5,3) = (-9)u_1 + (7)u_2$$

$$T(v_2) = T(1,2,3) = (5,-4) = (-23)u_1 + (14)u_2$$

$$T(v_3) = T(1,3,5) = (6,-9) = (-36)u_1 + (21)u_2$$

$$[T]_{B_1}^{B_2} = \begin{bmatrix} -9 & 7 \\ -23 & 14 \\ -36 & 21 \end{bmatrix}^T = \begin{bmatrix} -9 & -23 & -36 \\ 7 & 14 & 21 \end{bmatrix}$$

$$\text{Ans} = -9-23-36+7+14+21$$

$$= -2-2-22 = -26$$

(2) Let $A = \begin{pmatrix} 2 & 5 & -3 \\ 1 & -4 & 7 \end{pmatrix}$

$T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ s.t. $T(v) = Av$

$B_1 = \{ (1,1,0), (0,1,1), (0,0,1) \}$

$B_2 = \{ (1,1), (-1,1) \}$

(1) $I_i = \sum_{j=1}^{3} a_{ij}$ for $i = 1,2$

$T: \mathbb{R}^3 \to \mathbb{R}^2 \quad T(v) = Av \quad A = \begin{pmatrix} 2 & 5 & -3 \\ 1 & -4 & 7 \end{pmatrix}$

$B_1 = \{ (1,1,0), (0,1,1), (0,0,1) \} \qquad B_2 = \{ (1,1), (-1,1) \}$

$$T(v_1) = T(1,1,0) = \begin{pmatrix} 2 & 5 & -3 \\ 1 & -4 & 7 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + -5 \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$T(v_2) = T(0,1,1) = \begin{pmatrix} 2 & 5 & -3 \\ 1 & -4 & 7 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 5/2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 1/2 \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$T(v_3) = T(0,0,1) = \begin{pmatrix} 2 & 5 & -3 \\ 1 & -4 & 7 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 7 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 5 \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \left(\frac{x+y}{2}\right)(1,1) + \left(\frac{y-x}{2}\right)\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$[T] = \begin{bmatrix} 2 & -5 \\ 5/2 & 1/2 \\ 2 & 5 \end{bmatrix}^T = \begin{bmatrix} 2 & 5/2 & 2 \\ -5 & 1/2 & 5 \end{bmatrix}_{2\times 3}$$

$$I_i = \sum_{j=1}^{3} a_{ij} \qquad \underline{i = 1, 2}$$

$$I_1 = \sum_{j=1}^{3} a_{1j} = a_{11} + a_{12} + a_{13} = 2 + 5/2 + 2$$

$$= 4 + 2.5$$

$$= 6.5 = 13/2$$

$$I_2 = \sum_{j=1}^{3} a_{2j} = a_{21} + a_{22} + a_{23}$$

$$= -5 + 1/2 + 5 = 1/2 = 0.5$$

$$(x,y) = c_1(1,1) + c_2(-1,1)$$

$$\left[\begin{array}{cc|c} 1 & -1 & x \\ 1 & 1 & y \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & x \\ 0 & 2 & y-x \end{array}\right]$$

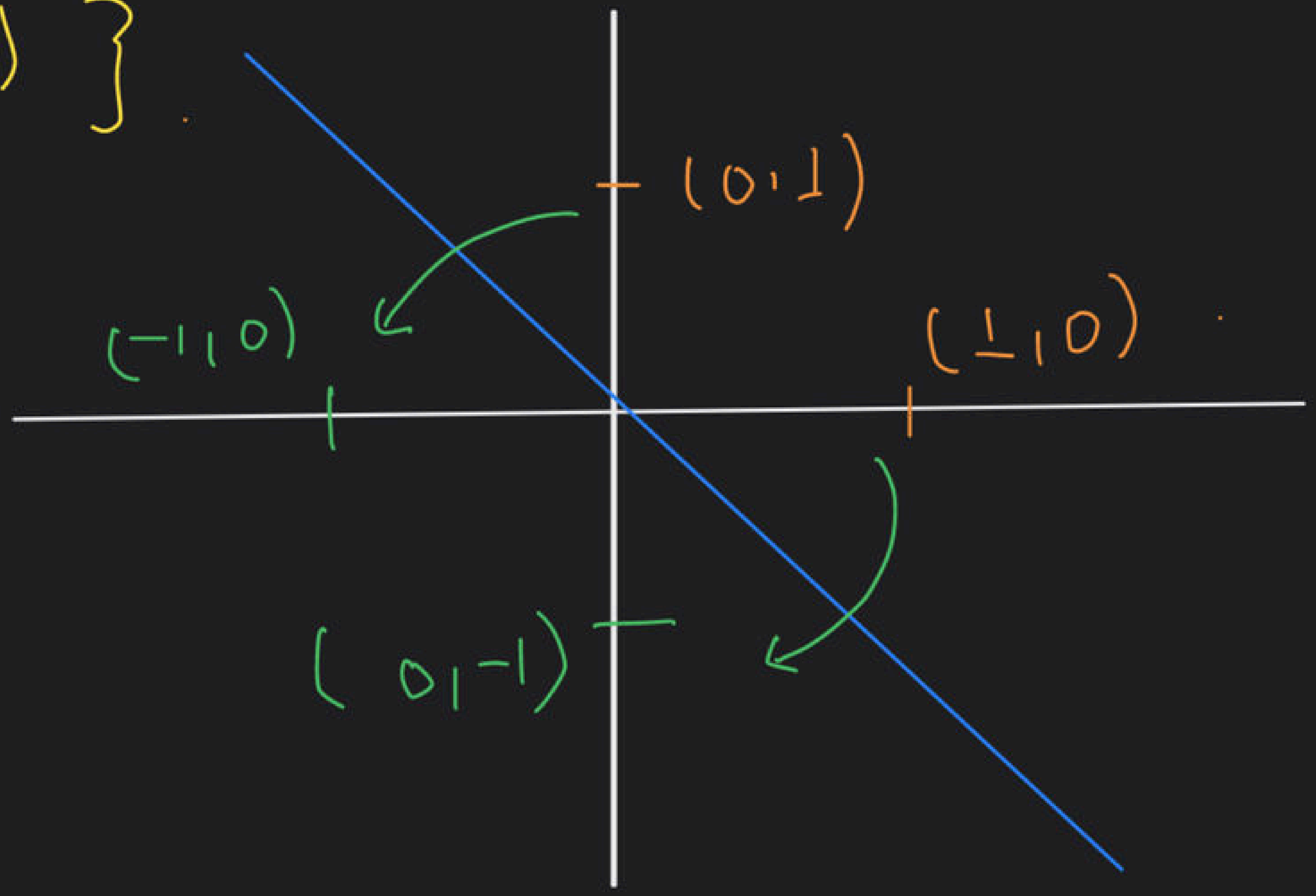
$$\boxed{c_2 = \frac{-x+y}{2}}$$

$$c_1 = x + c_2 = x + \frac{-x+y}{2}$$

$$\boxed{c_1 = \frac{x+y}{2}}$$

(eg) Let T denotes reflection in $\mathbb{R}^2$ about the line $y = -x$. Find matrix of T wrt usual basis and also wrt

$\{ (1,1), (0,1) \}$.



$$T(1,0) = (\boxed{0,}-1)$$

$$T(0,1) = (-1,0)$$

$$[T]_{S,B} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$(x,y) = x(1,0) + y(0,1)$$

$$T(x,y) = xT(1,0) + yT(0,1)$$

$$T(x,y) = x(0,-1) + y(-1,0)$$

$$\boxed{T(x,y) = (-y,-x)}$$

$$T(x,y) = (-y,-x)$$

$$\boxed{B = \{ (1,1), (0,1) \}}$$

$$(x,y) = c_1 (1,1) + c_2 (0,1)$$

$$(x,y) = x(1,1) + (y-x)(0,1)$$

$$T(1,1) = (-1,-1) = -1\cdot(1,1) + 0\cdot(0,1)$$

$$T(0,1) = (-1,0) = -1\cdot(1,1) + 1\cdot(0,1)$$

$$[T]_B = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\text{trace } T = -1+1 = 0 \qquad \det T = -1$$

Reflection about $y = -x$ line

$$T : \mathbb{R}^2 \to \mathbb{R}^2$$

$$T(x, y) = (-y, -x)$$

Ref. about $y = x$ line

$$T : \mathbb{R}^2 \to \mathbb{R}^2$$

$$T(x, y) = (y, x)$$

Ref. about line $y = x$ $\quad T : \mathbb{R}^2 \to \mathbb{R}^2$

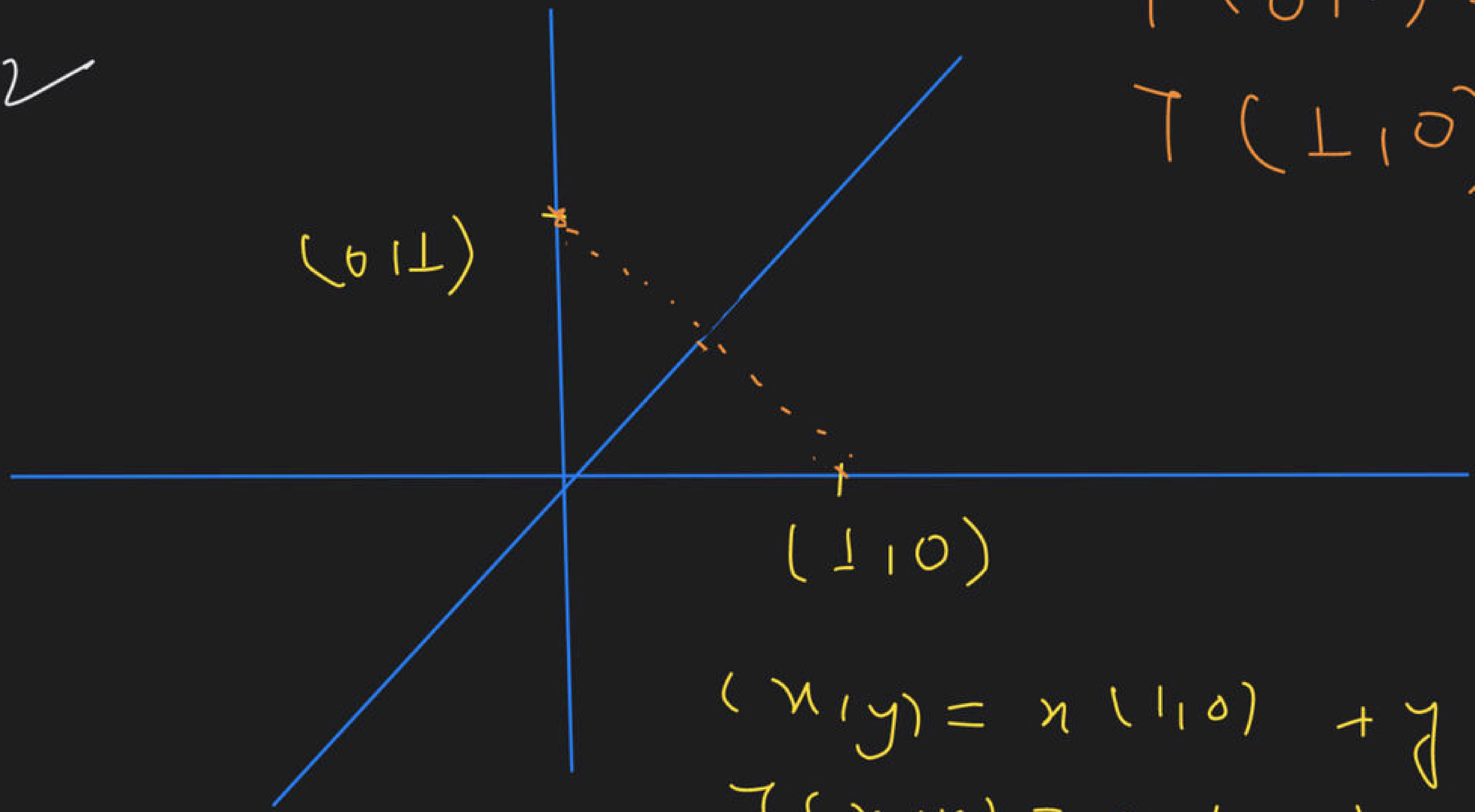
$\det T = 1$

$\operatorname{trace} T = 2$

$T(0,1) = (1,0)$

$T(1,0) = (0,1)$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$(x,y) = x(1,0) + y(0,1)$

$T(x,y) = x\cdot(0,1) + y(1,0)$

$\longleftarrow \boxed{T(x,y) = (y,x)}$

(eg) Let T be the rotation in $\mathbb{R}^2$ counterclockwise by $\pi/2$. Find M.R. of T wrt

(a) usual basis

(b) $B = \{ (1,1) \ (2,1) \}$

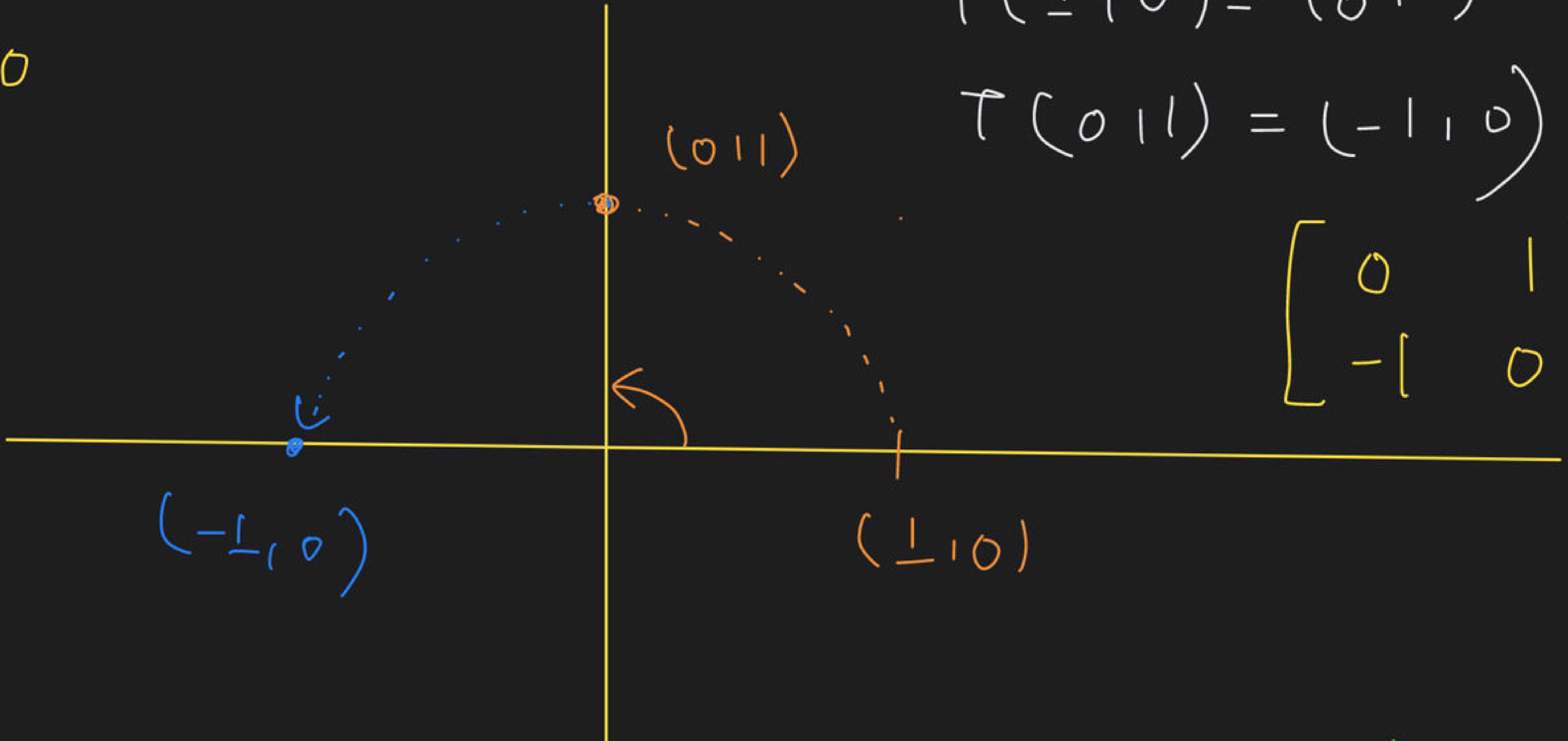
$\text{trace } T = 0$

$\det T = 1$

$T(1,0) = (0,1)$

$T(0,1) = (-1,0)$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$



$(x,y) = x(1,0) + y(0,1)$

$T(x,y) = x(0,1) + y(-1,0)$

$$\boxed{T(x,y) = (-y,x)}$$

$$T(x,y) = (-y, x)$$

$$T(1,1) = (-1, 1) = 3(1,1) + (-2)(2,1)$$

$$T(2,1) = (-1, 2) = 5(1,1) + (-3)(2,1)$$

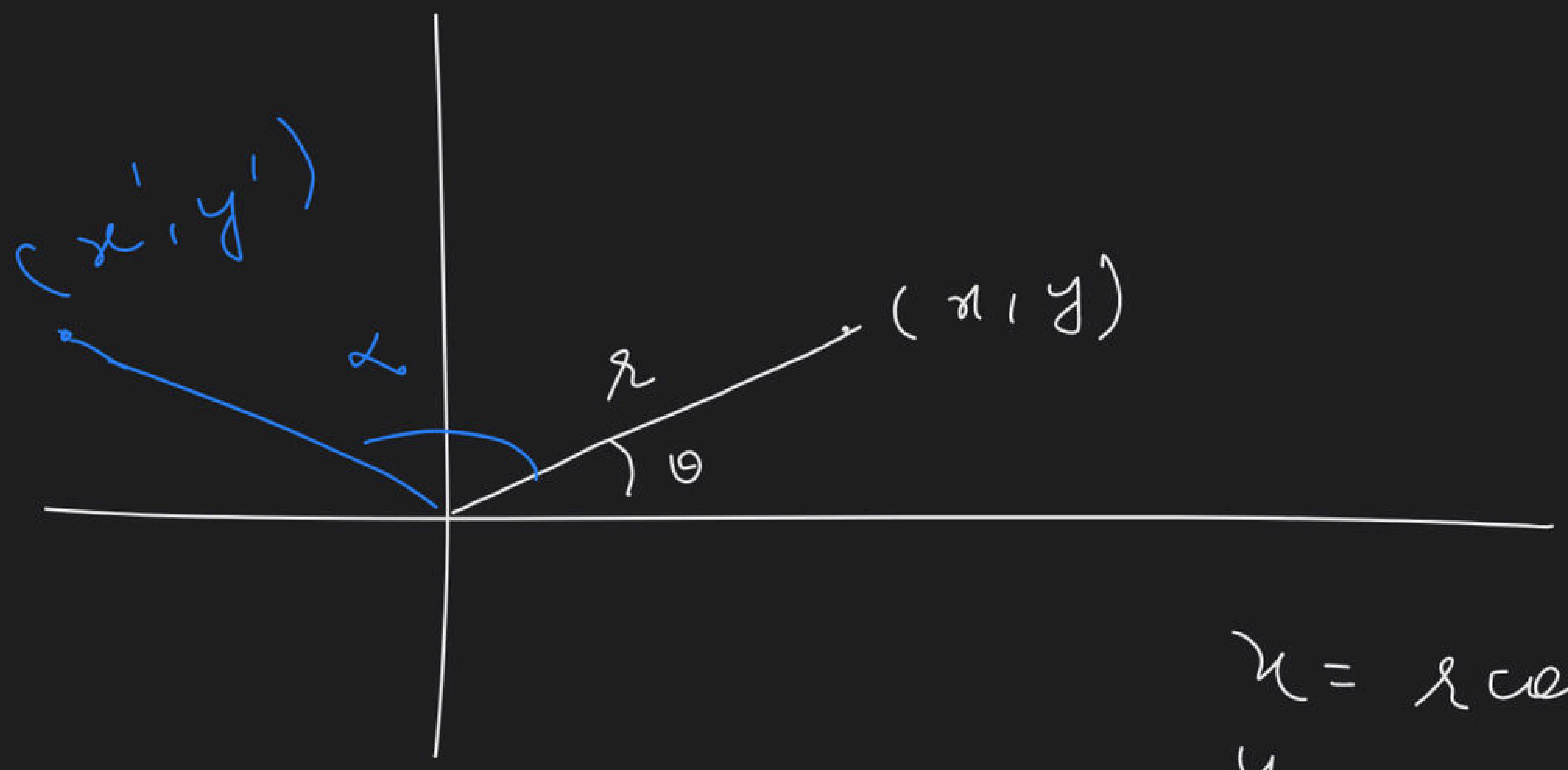
$$\left[\begin{array}{cc|c} 1 & 2 & x \\ 1 & 1 & y \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & x \\ 0 & -1 & y-x \end{array}\right] \qquad \begin{bmatrix} 3 & 5 \\ -2 & -3 \end{bmatrix}$$

$$c_2 = x - y.$$

$$-9 + 10 = 1$$

$$c_1 = x - 2c_2$$

$$c_1 = 2y - x$$



$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$x' = r\cos(\theta+\alpha)$$
$$y' = r\sin(\theta+\alpha)$$

$$x' = \underbrace{r\cos\theta}\cos\alpha - \underbrace{r\sin\theta}\sin\alpha$$

$$x' = x\cos\alpha - y\sin\alpha$$

$$y' = r\cos\theta\sin\alpha + r\sin\theta\cos\alpha$$

$$y' = x\sin\alpha + y\cos\alpha$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$