EIGEN VALUE / EIGEN VECTOR

$T: V \to V$. If for a non-zero vector $v \in V$, $\exists$ a scalar $\lambda \in F$ s.t.

$$\boxed{T(v) = \lambda v} \implies \lambda \text{ is an}$$

eigen value of T and v is eigenvector of T corresponding to e. value λ.

$T: V \to V$

$T(v) = \lambda v$

$[T]_B = A$

$\det(A - \lambda I) = 0$

λ ??

$(A - \lambda I)x = 0$

$x = $??

$T(v) = \lambda v$

$(T - \lambda I)v = 0$

$T : V \to V$

$\det T = \det [T]_B$

$\operatorname{trace} T = \operatorname{trace} [T]_B$

e·values of T = e·values of $[T]_B$

$e(T) = e([T]_B)$

(Q9) $T : \mathbb{R}^2 \to \mathbb{R}^2$ s.t. $T(x, y) = (x, 0)$

Soln.

$T(v) = \lambda v \qquad v \neq 0 \qquad \lambda \in F$

$T(x, y) = \lambda (x, y)$

$(x, 0) = (\lambda x, \lambda y)$

$x \neq 0$

$\lambda x = x \qquad \lambda y = 0$

$(\lambda - 1)x = 0$ and $\lambda y = 0$

$\lambda = 1 \quad x \neq 0 \longrightarrow y = 0$

$x = 0 \longleftarrow \lambda = 0 \quad y \neq 0$

$\lambda = 1 \qquad v = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\lambda = 0 \quad v = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T : \mathbb{R}^2 \to \mathbb{R}^2$ $\quad T(x, y) = (x, 0)$

$[T]_B = A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$\lambda = 1$

$\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$y = 0$

$x = 1$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

e.values $= 0, 1$

$\lambda = 0$

$(A - \lambda I) v = 0$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(eg) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$T(x, y) = (x+y, x+y)$

$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\lambda^2 - 2\lambda + 0 = 0$

$\lambda(\lambda - 2) = 0$

$\lambda = 0, 2$

$\lambda = 0$

$(A - \lambda I)X = 0$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_1 + x_2 = 0$

$\downarrow$

Free. va.

$x_2 = 1$

$x_1 = -1$

$\lambda = 0, \quad x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\lambda = 2. \quad (A - 2I)x = 0.$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\lambda = 2 \quad x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$-x_1 + x_2 = 0$

$x_2 = 1$

$x_1 = 1$

$\det(A - \lambda I)$ = characteristic polynomial

$\det(A - \lambda I) = 0$ — char.. equation

$\lambda \to$ eigen values / eigen roots / latent roots

charac. roots

$A_{2\times 2}$

$$\boxed{\lambda^2 - (\text{trace } A)\lambda + \det A = 0}$$

$A_{3\times 3}$

$$\boxed{\lambda^3 - (\text{trace } A)\lambda^2 + (\text{sum of cofactors of diagonal elements})\lambda - \det A = 0}$$

$$T(x,y) = (x+y,\ x+y)$$

$$Tv = \lambda v$$

$$T(x,y) = \lambda(x,y)$$

$$(x+y,\ x+y) = (\lambda x,\ \lambda y)$$

$x+y = \lambda x$ and $x+y = \lambda y$

$(1-\lambda)x + y = 0$ and $x + (1-\lambda)y = 0$

$T: \mathbb{R}^2 \to \mathbb{R}^2$

$T(x, y) = (-y, x)$

$T(v) = \lambda v$

$\boxed{\lambda \in F}$

$v (\neq 0) \in V$

$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\lambda = \pm i \notin F = \mathbb{R}$

$\nexists$ e.values.

dNE

(eg) $T : \mathbb{C}^2 \to \mathbb{C}^2$

$T(x, y) = (-y, x)$

$\lambda \in F$

$\mathbb{C}^2(\mathbb{R})$ ← V.S.

$\mathbb{C}^2(\mathbb{C})$ ← V.S.

$F = \mathbb{R} \longrightarrow$ E. VA. DNE.

If $F = \mathbb{C} \to \lambda = \pm i$

Matrix	Eigen values
diagonal matrix	diagonal entries
scalar matrix	$\begin{bmatrix} c & & 0 \\ & c & \\ 0 & & c \end{bmatrix}$, c
upper Δ. matrix	diag. entries
lower Δ matrix	"
$A_{n\times n}$; sum of all elements in each row is x	x is an e.value of A
$A_{n\times n}$; sum of all elements in each col. is x	"
If $\det A = 0$	0 is an e.va.

$A_{n \times n}$, $\lambda_1, \lambda_2, \ldots, \lambda_n$

$$\boxed{\sum_{i=1}^{n} \lambda_i = \text{trace}(A)}$$

$$\boxed{\prod_{i=1}^{n} \lambda_i = \det A}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = 3$

λ 1, 2, 3

$$\begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix} \to \lambda \quad 1, 2, 3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 5 & 6 & 3 \end{bmatrix} \to \lambda \quad 1, 2, 3$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}_{3 \times 3}$$

$R_1 = 6$

$R_2 = 6$

$R_3 = 6$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix} \qquad \pm\sqrt{1\cdot 4}, \ \pm\sqrt{2\cdot 3}$$

$$2, -2, \sqrt{6}, -\sqrt{6}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix} \qquad \pm\sqrt{1\cdot 3}, \ 2$$

$$\sqrt{3}, -\sqrt{3}, 2$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$I \sqrt{1, 1, 1}$

$1, 1, -1$

$T : \mathbb{R}^2 \to \mathbb{R}^2$

$T(x, y) = (x, x+y)$

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad \text{e.values} = 1, 1$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$(A - \lambda I)x = 0$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x = 0$

$T : \mathbb{R}^3 \to \mathbb{R}^3$

$T(x, y, z) = (x + y - z,\ x + 2y,\ y - 3z + x)$

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ 1 & 1 & -3 \end{bmatrix}$$

trace $A = 0$

det $A = -2$

$\lambda^3 - (0)\lambda^2 + (-6 - 2 + 1)\lambda - (-2) = 0$

$\lambda^3 - 7\lambda + 2 = 0$

$$\lambda^3 - 7\lambda + 2 = 0 \Rightarrow$$

$$\begin{bmatrix} 1-\lambda & 1 & -1 \\ 1 & 2-\lambda & 0 \\ 1 & 1 & -3-\lambda \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{vmatrix} 1-\lambda & 1 & -1 \\ 1 & 2-\lambda & 0 \\ 0 & \lambda-2 & -1-\lambda \end{vmatrix}$$

$1-\lambda \neq 0$

$$R_m \longrightarrow R_m \pm C \cdot R_n \qquad m \neq n$$

$$C \neq 0$$