Date

Q4 Soln. Claim: $X_{(n-13)}$ converges to θ as $n \to \infty$.

Proof:

Let $X_1, \cdots, X_n$ be identically distributed obs. from uniform distribution $(0, \theta)$ and $X_{(1)} \leq \cdots \leq X_{(n)}$ be the order statistics for these observations.

As $n \to \infty$, $\{X_{(i)}\}_{i=1}^{\infty}$ will form a sequence which is monotonically increasing and also bounded as $X_{(i)} \in (0, \theta] \ \forall \ i \in \mathbb{N}$.

Therefore $X_{(n)}$ will converge.

Now, we have to choose 'n' values from the interval $(0, \theta]$ randomly. The probability that a specific chosen value was picked from the interval $(\theta - \varepsilon, \theta]$ is ε/θ where $\varepsilon \in \mathbb{R}$, $0 < \varepsilon < \theta$.

Let the no. of points falling in the interval $(\theta - \varepsilon, \theta]$ be N_n. The expected value of $N_n = E(N_n) = \dfrac{n\varepsilon}{\theta}$.

As $n \to \infty$, $P(E(N_n) < 14) \to 0$.

And $P(E(N_n) < 14) \to 0 \Rightarrow P(X_{(n-13)} \in (\theta - \varepsilon, \theta]) \to 1$.

Therefore as $n \to \infty$, $|X_{(n-13)} - \theta| < \varepsilon \ \forall \ 0 < \varepsilon$.

$\Rightarrow X_{(n-13)}$ converges to θ as $n \to \infty$.