

$$E[x] = \sum x \cdot P(x=x)$$

$$E\left[\frac{x}{n}\right] = \frac{Q \cdot k}{n+1}$$

$$\text{Var}\left[\frac{x}{n}\right] = \frac{Q^2}{(n+1)^2(n+2)}$$

$$(k=n-13)$$

$$E(x_{n-13}) \rightarrow Q$$

$$\text{Var}[x_{n-13}] \rightarrow 0$$

as  $n \rightarrow \infty$

$$Pr(X \geq a) \leq \frac{E[X]}{a}$$

$$Pr(X - E[X] \geq a) \leq \frac{Var(X)}{a^2}$$

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$$\left[ \frac{E[X]}{a} \right]$$

$$= \frac{1}{a^2} (X - E[X])^2$$

$$= \frac{1}{a^2} (X - E[X])^2 \geq \frac{1}{a^2} Var(X)$$

$$\frac{Var(X)}{a^2} \rightarrow 0$$

$\leftarrow$

$$a \rightarrow 0$$

$$X_{n-1}$$

$\downarrow$

$$X_n$$

$\downarrow$