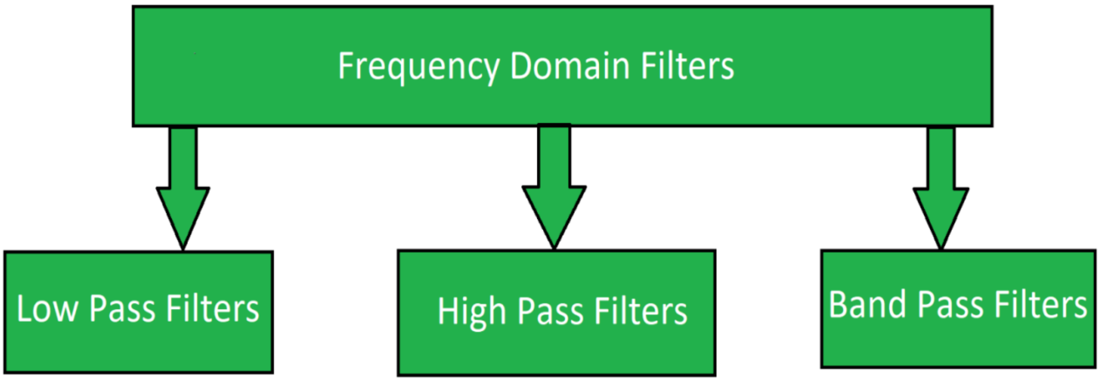
IMAGE PROCESSING

Mr.Dnyandev Khadapkar @ Gec

Image Smoothening filters Using Frequency Domain Filters in 2D

Frequency Domain Filters are used for smoothing and sharpening of image by removal of high or low frequency components. Sometimes it is possible of removal of very high and very low frequency. Frequency domain filters are different from spatial domain filters as it basically focuses on the frequency of the images.
It is basically done for two basic operations i.e., Smoothing and Sharpening.

These are of 3 types:



Classification of Frequency Domain Filters

1. **Low pass filter:**

   Low pass filter removes the high frequency components that means it keeps low frequency components. It is used for smoothing the image. It is used to smoothen the image by attenuating high frequency components and preserving low frequency components.

   Mechanism of low pass filtering in frequency domain is given by:

2. G(u, v) = H(u, v) . F(u, v)
3. where F(u, v) is the Fourier Transform of original image
4. and H(u, v) is the Fourier Transform of filtering mask

2. High pass filter:
High pass filter removes the low frequency components that means it keeps high frequency components. It is used for sharpening the image. It is used to sharpen the image by attenuating low frequency components and preserving high frequency components.
Mechanism of high pass filtering in frequency domain is given by:

5. H(u, v) = 1 - H'(u, v)
6. where H(u, v) is the Fourier Transform of high pass filtering
7. and H'(u, v) is the Fourier Transform of low pass filtering

3. Band pass filter:
Band pass filter removes the very low frequency and very high frequency components that means it keeps the moderate range band of frequencies. Band pass filtering is used to enhance edges while reducing the noise at the same time.

There are a number of filtering techniques that can be used in frequency domain.

A. Smoothing lowpass filters

We can achieve smoothing in frequency domain through high-frequency attenuation (lowpass filtering).

The 3 types of lowpass filters covers the range from very sharp (ideal) to very smooth(Gaussian) filtering.

1. Ideal Lowpass Filters (ILPF)

The lowpass filter which passes without attenuation of all frequencies within a radius **Do** from the origin and which cuts off all the frequencies outside this radius is called an ideal lowpass filter. The filter is specified by the following function:

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

$$D(u,v) = \left[(u - M/2)^2 + (v - N/2)^2\right]^{1/2}$$

where,

D0 = positive constant

D(u, v) = distance between a point (u,v) in the frequency domain and the center of the frequency rectangle.

M, N = padded sizes given as follows

M >= 2P -1

N >= 2Q — 1

P, N = array dimensions

Note: M and N are the padded sizes of the arrays.

In the ideal pass filter all the frequencies on or inside the radius Do are passed without attenuation but all frequencies frequencies outside the circle are completely filtered. The filter is completely defined by a radial cross section as it is radially symmetric about the origin.

2. Butterworth Lowpass Filters (BLPF):This filter is designed so as to have a flat frequency response in the passband. The frequency response is flat in the passband and rolls-off towards zero in the stopband. The rate of roll-off is based on the order of the filter.

The butterworth function is as:

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

$$D(u,v) = \left[(u - M/2)^2 + (v - N/2)^2\right]^{1/2}$$

where,

Do = cutoff frequency distance

D(u, v) = distance between a point (u,v) in the frequency domain and the center of the frequency rectangle.

M, N = padded sizes given as follows

M >= 2P -1

N >= 2Q — 1

P, N = array dimensions

Note: M and N are the padded sizes of the arrays.

BLPF transfer function does not have a sharp discontinuity that gives a clear cutoff between passed and the filtered frequencies. Butterworth filter with order 1 has no ringing in spatial domain which generally is imperceptible in filters of order 2 but can become significant in filters of higher order.

A Butterworth filter with order 20 has similar characteristics to Ideal pass filters.

A BLPF filter with order 2 is good compromise between effective lowpass filtering and acceptable ringing.

3. Gaussian Lowpass Filters

A Gaussian filter is a 2D convolution operator which is used to blur images and helps to eliminate noises in the image.

The main effect of this filter is to blur the image similarly to mean filters and the extend to smoothing is based on the standard deviation of the Gaussian.

GLPFs in two dimensions is given by:

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

where,

D(u, v) = distance between a point (u,v) in the frequency domain and the center of the frequency rectangle.