Portfolio Optimization

(1) Import Python Libraries

In [1]:
```
import pathlib
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy.optimize as sci_opt

from pprint import pprint
from sklearn.preprocessing import StandardScaler

from pandas_datareader import data as web
from datetime import datetime
from alpha_vantage.timeseries import TimeSeries
import time
```

In [27]:
```
pd.set_option('display.max_colwidth', -1)
pd.set_option('expand_frame_repr', False)
```

```
<ipython-input-27-eb736239aa8c>:1: FutureWarning: Passing a negative integer
is deprecated in version 1.0 and will not be supported in future version. In
stead, use None to not limit the column width.
  pd.set_option('display.max_colwidth', -1)
```

(2) Loading the Data Set

In [45]:
```
assets = ['META', 'AMZN', 'AAPL', 'NFLX', 'GOOGL', 'MSFT']
number_of_assets = len(assets)
```

In [54]:
```
api_key = 'HWI0SV3ZF6AMLE1D'
ts = TimeSeries(key=api_key, output_format='pandas')
```

In [49]:

```
df = pd.DataFrame(columns=['date', 'symbol', 'volume', 'close', 'open', 'hig

for stock in assets:
    df_stock, _ = ts.get_daily(symbol=stock, outputsize='full')
    df_stock.rename(columns={'4. close': 'close', '1. open': 'open', '2. hig
    df_stock['symbol'] = stock
    df_stock['date'] = df_stock.index
    df = df.append(df_stock[['date', 'symbol', 'volume', 'close', 'open', 'h

    time.sleep(12)

csv_filename = 'stock_data.csv'
df.to_csv(csv_filename, index=False)
print(f"Data saved to {csv_filename}")
```

```
Data saved to stock_data.csv
```

In [3]:

```
path = r"D:\IITG\portfolio_finance\portfolio_optimization\stock_data.csv"
price_data_frame: pd.DataFrame = pd.read_csv(path)
price_data_frame.head()
```

Out[3]:

	date	symbol	volume	close	open	high	low
0	2023-08-21	META	20181475.0	289.90	283.450	290.50	281.85
1	2023-08-18	META	35347925.0	283.25	279.030	285.69	274.38
2	2023-08-17	META	23950089.0	285.09	293.050	296.05	284.95
3	2023-08-16	META	18547741.0	294.29	300.195	301.08	294.28
4	2023-08-15	META	11623613.0	301.95	306.140	307.23	300.03

(3) Cleaning the Data

In [4]:

```
stockStartDate = '2015-01-01'
today = datetime.today().strftime('%Y-%m-%d')
```

In [5]:

```
price_data_frame = price_data_frame[['date', 'symbol', 'close']]
price_data_frame = price_data_frame[(price_data_frame['date'] >= stockStartD
```

In [6]:
```
price_data_frame = price_data_frame.pivot(
    index='date',
    columns='symbol',
    values='close'
)

price_data_frame.head()
```

Out[6]:

symbol	AAPL	AMZN	GOOG	META	MSFT	NFLX
date						
2015-01-02	109.33	308.52	524.81	78.450	46.760	348.940
2015-01-05	106.25	302.19	513.87	77.190	46.325	331.180
2015-01-06	106.26	295.29	501.96	76.150	45.650	325.510
2015-01-07	107.75	298.42	501.10	76.150	46.230	327.200
2015-01-08	111.89	300.46	502.68	78.175	47.590	334.455

(4) Calculating Sharpe Ratio

The `Sharpe Ratio` , is used is a measure for calculating risk-adjusted return and has been the industry standard for such calculations. The `Sharpe Ratio` allows us to quantify the relationship the average return earned in excess of the risk-free rate per unit of volatility or total risk.

Mathematically, we define the `Sharpe Ratio` as the following:

$$\text{Sharpe Ratio} = \frac{(R_p - R_f)}{\sigma_p}$$

Where:

$$\begin{aligned} R_p &= \text{Return of Portfolio} \\ R_f &= \text{Risk-Free Rate} \\ \sigma_p &= \text{Standard Deviation of Portfolio's Excess Return} \end{aligned}$$

To calculate the `expected returns` , we use the following formula:

$$R_p = (w_1 r_1) + (w_2 r_2) + \cdots + (w_n r_n)$$

Where:

$$\begin{aligned} r_i &= \text{Return of Security i} \\ w_i &= \text{Weight of Security i} \end{aligned}$$

To calculate the `standard deviation of the portfolio` , we use the following formula:

$$\sigma_p = \sqrt{(w_i^2 \sigma_i^2) + (w_j^2 \sigma_j^2) + (2 w_j w_i p_{i,j} \sigma_i \sigma_j)}$$

Where:

σ_i = Standard Deviation of Returns for Security i
w_i = Weight of Security i
ρ_{ij} = Correlation Coefficent between the returns of asset i and asset j

In [48]:
```
# Calculate the Log of returns.
log_return = np.log(1 + price_data_frame.pct_change())
log_return.head()
```

Out[48]:

symbol	AAPL	AMZN	GOOG	META	MSFT	NFLX
date						
2015-01-02	NaN	NaN	NaN	NaN	NaN	NaN
2015-01-05	-0.028576	-0.020731	-0.021066	-0.016192	-0.009346	-0.052238
2015-01-06	0.000094	-0.023098	-0.023450	-0.013565	-0.014678	-0.017269
2015-01-07	0.013925	0.010544	-0.001715	0.000000	0.012625	0.005178
2015-01-08	0.037703	0.006813	0.003148	0.026245	0.028994	0.021931

In [49]:
```
# Generate Random Weights.
random_weights = np.array(np.random.random(number_of_assets))
random_weights
```

Out[49]:
```
array([0.19438222, 0.08157005, 0.81494142, 0.65581664, 0.97832522,
       0.54393883])
```

In [50]:
```
# Generate the Rebalance Weights, these should equal 1.
rebalance_weights = random_weights / np.sum(random_weights)
rebalance_weights
```

Out[50]:
```
array([0.05946276, 0.0249528 , 0.24929575, 0.20061847, 0.29927589,
       0.16639434])
```

In [51]:
```
# Calculate the Expected Returns, annualize it by multiplying it by `252`.
exp_ret = np.sum((log_return.mean() * rebalance_weights) * 252)
exp_ret
```

Out[51]:
```
0.060717761915546134
```

In [52]:
```
# Calculate the Expected Volatility, annualize it by multiplying it by `252`
exp_vol = np.sqrt(
np.dot(
    rebalance_weights.T,
    np.dot(
        log_return.cov() * 252,
        rebalance_weights
    )
)
)
exp_vol
```

Out[52]:
```
0.39190876828430865
```

In [53]:
```
# Calculate the Sharpe Ratio.
sharpe_ratio = exp_ret / exp_vol
sharpe_ratio
```

Out[53]:
```
0.1549283068642615
```

In [54]:
```
# Put the weights into a data frame to see them better.
weights_df = pd.DataFrame(data={
'random_weights': random_weights,
'rebalance_weights': rebalance_weights
})
print('')
print('='*50)
print('PORTFOLIO WEIGHTS:')
print('-'*50)
print(weights_df)
print('-'*50)
```

```

==================================================
PORTFOLIO WEIGHTS:
--------------------------------------------------
   random_weights  rebalance_weights
0  0.194382        0.059463
1  0.081570        0.024953
2  0.814941        0.249296
3  0.655817        0.200618
4  0.978325        0.299276
5  0.543939        0.166394
--------------------------------------------------
```

In [55]:
```
# Do the same with the other metrics.
metrics_df = pd.DataFrame(data={
    'Expected Portfolio Returns': exp_ret,
    'Expected Portfolio Volatility': exp_vol,
    'Portfolio Sharpe Ratio': sharpe_ratio
}, index=[0])

print('')
print('='*90)
print('PORTFOLIO METRICS:')
print('-'*90)
print(metrics_df)
print('-'*90)
```

```
==============================================================================
==============
PORTFOLIO METRICS:
------------------------------------------------------------------------------
--------------
   Expected Portfolio Returns  Expected Portfolio Volatility  Portfolio Shar
pe Ratio
0  0.060718                    0.391909                       0.154928
------------------------------------------------------------------------------
--------------
```

(5) Portfolio Optimization : Running Monte Carlo Simulation

Monte Carlo simulations are used to model the probability of different outcomes in a process that cannot easily be predicted due to the intervention of random variables. It is a technique used to understand the impact of risk and uncertainty in prediction and forecasting models.

In [56]:
```
# We will run 5000 iterations.
num_of_portfolios = 5000
```

In [58]:
```
# Prep an array to store the weights as they are generated, 5000 iterations
all_weights = np.zeros((num_of_portfolios, number_of_assets))

# Prep an array to store the returns as they are generated, 5000 possible re
ret_arr = np.zeros(num_of_portfolios)

# Prep an array to store the volatilities as they are generated, 5000 possib
vol_arr = np.zeros(num_of_portfolios)

# Prep an array to store the sharpe ratios as they are generated, 5000 possi
sharpe_arr = np.zeros(num_of_portfolios)
```

In [59]:
```
# Start the simulations.
for ind in range(num_of_portfolios):

    # First, calculate the weights.
    weights = np.array(np.random.random(number_of_assets))
    weights = weights / np.sum(weights)

    # Add the weights, to the `weights_arrays`.
    all_weights[ind, :] = weights

    # Calculate the expected log returns, and add them to the `returns_array
    ret_arr[ind] = np.sum((log_return.mean() * weights) * 252)

    # Calculate the volatility, and add them to the `volatility_array`.
    vol_arr[ind] = np.sqrt(
        np.dot(weights.T, np.dot(log_return.cov() * 252, weights))
    )

    # Calculate the Sharpe Ratio and Add it to the `sharpe_ratio_array`.
    sharpe_arr[ind] = ret_arr[ind]/vol_arr[ind]
```

In [65]:
```
# Let's create our "Master Data Frame", with the weights, the returns, the v
simulations_data = [ret_arr, vol_arr, sharpe_arr, all_weights]

# Create a DataFrame from it, then Transpose it so it looks like our origina
simulations_df = pd.DataFrame(data=simulations_data).T

# Give the columns the Proper Names.
simulations_df.columns = ['Returns', 'Volatility', 'Sharpe Ratio', 'Portfoli

# Make sure the data types are correct, we don't want our floats to be strin
simulations_df = simulations_df.infer_objects()

simulations_df.head()
```

Out[65]:

	Returns	Volatility	Sharpe Ratio	Portfolio Weights
0	0.037474	0.403915	0.092778	[0.15760031320746729, 0.2521816043266357, 0.13442656261036467, 0.1613812251140267, 0.2193196901918524, 0.07509060454965322]
1	0.101986	0.364177	0.280045	[0.161912130351683, 0.01444301121439593, 0.0187123767963907, 0.30223935976837063, 0.206711820594815, 0.29598130127434474]
2	0.039241	0.379845	0.103308	[0.24284465697753363, 0.07558232995370391, 0.16903906861362336, 0.1186067412841262, 0.17259667376572496, 0.2213305294052879]
3	0.051213	0.389795	0.131384	[0.05425825934348524, 0.19139141595587986, 0.0986916236576 6704, 0.27476310180423524, 0.16581580043975436, 0.2150797987989784]
4	0.010418	0.453962	0.022950	[0.0911606195984837, 0.24414633884271253, 0.1360603290871 6834, 0.07754506319337028, 0.15102577822124463, 0.3000618710570206]

Grabbing the Important Metrics

`max_sharpe_ratio` : This is Sharpe Ratio where the excess return of the portfolio is greatest given the amount of risk we are taking on. In other words, these portfolios are "best" because they provide the largest risk-adjusted returns.

`min_volatility` : These are the weights where the overall volatility of the portfolio is at it's lowest. In other words, if you want to take on the least amount of risk, these are the weights you would want to have.

In [70]:
```python
max_sharpe_ratio = simulations_df.loc[simulations_df['Sharpe Ratio'].idxmax(

print('')
print('='*80)
print('MAX SHARPE RATIO:')
print('-'*80)
print(max_sharpe_ratio)
print('-'*80)
```

```
================================================================================
====
MAX SHARPE RATIO:
--------------------------------------------------------------------------------
----
Returns              0.165343
Volatility           0.291222
Sharpe Ratio         0.567755
Portfolio Weights    [0.25356877302148945, 0.010627637935533044, 0.001178549
5421920082, 0.15387190616798102, 0.577131546194449, 0.003621587138355487]
Name: 2723, dtype: object
--------------------------------------------------------------------------------
----
```

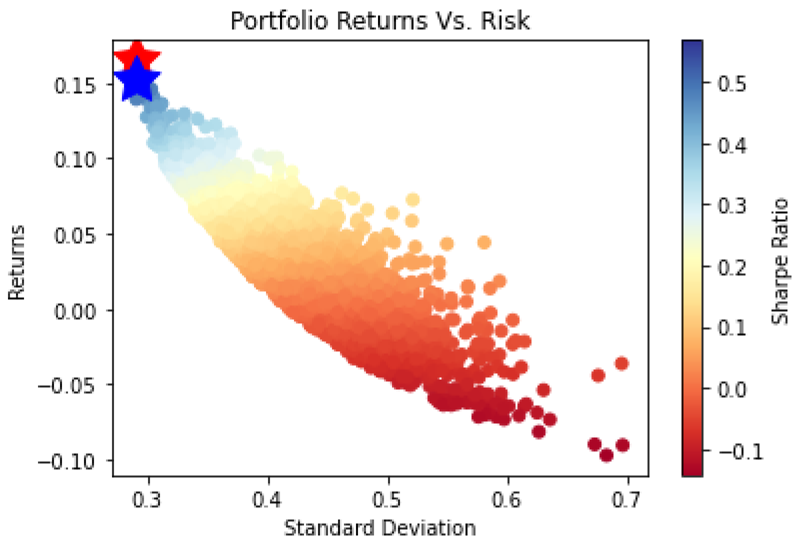
In [71]:
```
min_volatility = simulations_df.loc[simulations_df['Volatility'].idxmin()]

print('')
print('='*80)
print('MIN VOLATILITY:')
print('-'*80)
print(min_volatility)
print('-'*80)
```

```
================================================================================
MIN VOLATILITY:
--------------------------------------------------------------------------------
Returns              0.15261
Volatility           0.290849
Sharpe Ratio         0.524704
Portfolio Weights    [0.19961213707555278, 0.013596448286011475, 0.005093923130234636, 0.30964496520514867, 0.42889604911671464, 0.043156477186337854]
Name: 1310, dtype: object
--------------------------------------------------------------------------------
```

Plotting: Risk v/s Returns

In [73]:

```
%matplotlib inline

plt.scatter(
    y=simulations_df['Returns'],
    x=simulations_df['Volatility'],
    c=simulations_df['Sharpe Ratio'],
    cmap='RdYlBu'
)

plt.title('Portfolio Returns Vs. Risk')
plt.colorbar(label='Sharpe Ratio')
plt.xlabel('Standard Deviation')
plt.ylabel('Returns')

# Plot the Max Sharpe Ratio, using a `Red Star`.
plt.scatter(
    max_sharpe_ratio[1],
    max_sharpe_ratio[0],
    marker=(5, 1, 0),
    color='r',
    s=600
)

# Plot the Min Volatility, using a `Blue Star`.
plt.scatter(
    min_volatility[1],
    min_volatility[0],
    marker=(5, 1, 0),
    color='b',
    s=600
)

plt.show()
```



(6) Portfolio Optimization : Using Scipy Optimization Module

The draw back to using the Monte Carlo Simulation is that it's not the most efficient way to find the optimal values. Instead we can use mathematical techniques to easily arrive at the optimal values. We run the optimization using the `scipy.optimization` module.

The first function is the `get_metrics()` function which will return the metrics for a given set of weights provided. In other words, think of it as a "lookup function" where we can return the results of a certain weight. The second function is the `grab_negative_sharpe()` function which is used as a minimization function. The minimization function is used to help find the values which results in the lowest sharpe ratio. In Scipy's optimize function, there's no `maximize`, so as an objective function you need to pass something that should be minimized.

The third function is `check_sum` function which is used as a constraint. What is will do is make sure that the weights that are passed through meet the constraint that we must have a portfolio allocation equal 100%, not more and not less.

In [74]:
```python
def get_metrics(weights: list) -> np.array:

    weights = np.array(weights)

    ret = np.sum(log_return.mean() * weights) * 252
    vol = np.sqrt(
        np.dot(weights.T, np.dot(log_return.cov() * 252, weights))
    )
    sr = ret / vol

    return np.array([ret, vol, sr])
```

In [101]:
```python
def grab_negative_sharpe(weights: list) -> np.array:
    return -(1 + get_metrics(weights)[2])

def grab_volatility(weights: list) -> np.array:
    return get_metrics(weights)[1]

def check_sum(weights: list) -> float:
    return np.sum(weights) - 1
```

Let's run the optimization using the `scipy.optimization` module. This module has a function called `minimize` which we can use to help our optimal values. we can run the optimization by passing through the arguments defined and defining the `method` as `SLSQP` which is short for `Sequential Least Squares Programming`.

In [102]:

```
# Define the boundaries for each symbol. Remember I can only invest up to 10
bounds = tuple((0, 1) for symbol in range(number_of_assets))

# Define the constraints, here I'm saying that the sum of each weight must n
constraints = ({'type': 'eq', 'fun': check_sum})

# We need to create an initial guess to start with, and usually the best ini
init_guess = number_of_assets * [1 / number_of_assets]
```

Maximizing Sharpe Ratio

In [103]:

```
# Perform the operation to minimize the risk.

optimized_sharpe = sci_opt.minimize(
    grab_negative_sharpe, # minimize this.
    init_guess, # Start with these values.
    method='SLSQP',
    bounds=bounds, # don't exceed these bounds.
    constraints=constraints # make sure you don't exceed the 100% constraint
)
```

In [104]:

```
print('')
print('='*90)
print('OPTIMIZED SHARPE RATIO:')
print('-'*90)
print(optimized_sharpe)
print('-'*90)
```

```
============================================================================
==============
OPTIMIZED SHARPE RATIO:
----------------------------------------------------------------------------
--------------
     fun: -1.7962099152537103
     jac: array([ 0.41213581,  0.98618677,  1.21474755,  0.09539299, -0.
,
        0.53260311])
 message: 'Optimization terminated successfully'
    nfev: 28
     nit: 4
    njev: 4
  status: 0
 success: True
       x: array([1.59990393e-16, 7.77383541e-16, 4.84604742e-16, 0.00000000e
+00,
       1.00000000e+00, 5.23484780e-16])
----------------------------------------------------------------------------
--------------
```

Grabbing the Important Metrics

In [105]:
```
optimized_metrics = get_metrics(weights=optimized_sharpe.x)
```

In [106]:
```
print('')
print('='*80)
print('OPTIMIZED WEIGHTS:')
print('-'*80)
print(optimized_sharpe.x)
print('-'*80)
```

```
================================================================================
====
OPTIMIZED WEIGHTS:
--------------------------------------------------------------------------------
----
[1.59990393e-16 7.77383541e-16 4.84604742e-16 0.00000000e+00
 1.00000000e+00 5.23484780e-16]
--------------------------------------------------------------------------------
----
```

In [107]:
```
print('')
print('='*80)
print('OPTIMIZED METRICS:')
print('-'*80)
print(optimized_metrics)
print('-'*80)
```

```
================================================================================
====
OPTIMIZED METRICS:
--------------------------------------------------------------------------------
----
[0.22382411 0.28111194 0.79620992]
--------------------------------------------------------------------------------
----
```

Minimizing Risk (Volatility)

In [96]:
```
# Perform the operation to minimize the risk.

optimized_volatility = sci_opt.minimize(
    grab_volatility, # minimize this.
    init_guess, # Start with these values.
    method='SLSQP',
    bounds=bounds, # don't exceed these bounds.
    constraints=constraints # make sure you don't exceed the 100% constraint
)
```

In [97]:
```
print('')
print('='*80)
print('OPTIMIZED VOLATILITY RATIO:')
print('-'*80)
print(optimized_volatility)
print('-'*80)
```

```
================================================================================
====
OPTIMIZED VOLATILITY RATIO:
--------------------------------------------------------------------------------
----
     fun: 0.2738308506943697
     jac: array([0.27380429, 0.27398584, 0.27392484, 0.27383472, 0.27383466,
       0.27362216])
 message: 'Optimization terminated successfully'
    nfev: 56
     nit: 8
    njev: 8
  status: 0
 success: True
       x: array([0.06093243, 0.00994952, 0.01096501, 0.13513252, 0.76205518,
       0.02096535])
--------------------------------------------------------------------------------
----
```

Grabbing the Important Metrics

In [98]:
```
optimized_metrics = get_metrics(weights=optimized_volatility.x)
```

In [99]:
```
print('')
print('='*80)
print('OPTIMIZED WEIGHTS:')
print('-'*80)
print(optimized_volatility.x)
print('-'*80)
```

```
================================================================================
====
OPTIMIZED WEIGHTS:
--------------------------------------------------------------------------------
----
[0.06093243 0.00994952 0.01096501 0.13513252 0.76205518 0.02096535]
--------------------------------------------------------------------------------
----
```

In [100]:

```
print('')
print('='*80)
print('OPTIMIZED METRICS:')
print('-'*80)
print(optimized_metrics)
print('-'*80)
```

```
============================================================================
====
OPTIMIZED METRICS:
----------------------------------------------------------------------------
----
[0.19205796 0.27383085 0.70137444]
----------------------------------------------------------------------------
----
```

In [ ]: