Calculation for Jacobi Set in 3D

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Abstract

Algebraic simplifications

1 Introduction

We are going to see when a face becomes critical w.r.t f on the level set of g and h.

2 Condition

Suppose a,b and c be the vertices of a face abc. When abc becomes critical,we have

$$f^a + \lambda_1 g^a + \lambda_2 h^a = f^b + \lambda_1 g^b + \lambda_2 h^b = f^c + \lambda_1 g^c + \lambda_2 h^c \tag{1}$$

2.1 Elimination of λ_1 and λ_2

$$\lambda_1(g^a - g^b) + \lambda_2(h^a - h^b) + (f^a - f^b) = 0 \tag{2}$$
$$\lambda_1(g^b - g^c) + \lambda_2(h^b - h^c) + (f^b - f^c) = 0 \tag{3}$$

Can be re written as

$$A_1\lambda_1 + B_1\lambda_2 + C_1 = 0 \tag{4}$$
$$A_2\lambda_1 + B_2\lambda_2 + C_2 = 0 \tag{5}$$

where

$$A_1 = (g^a - g^b) \tag{6}$$
$$B_1 = (h^a - h^b) \tag{7}$$
$$C_1 = (f^a - f^b) \tag{8}$$
$$A_2 = (g^b - g^c) \tag{9}$$
$$B_2 = (h^b - h^c) \tag{10}$$
$$C_2 = (f^b - f^c) \tag{11}$$

Solving for λ_1 and λ_2 we get,

$$\frac{\lambda_1}{B_1C_2 - B_2C_1} = \frac{\lambda_2}{A_2C_1 - A_1C_2} = \frac{1}{A_1B_2 - A_2B_1} \tag{12}$$

2.2 Condition for Lower-Link

A vertex u is in lower link of face abc, then $h_\lambda(u) \leq h_\lambda(a)$. This inequality, with values of λ_1 and λ_2 plugged in transforms to

$$f^u + \frac{B_1C_2 - B_2C_1}{A_1B_2 - A_2B_1}g^u + \frac{A_2C_1 - A_1C_2}{A_1B_2 - A_2B_1}h^u \leq f^a + \frac{B_1C_2 - B_2C_1}{A_1B_2 - A_2B_1}g^a + \frac{A_2C_1 - A_1C_2}{A_1B_2 - A_2B_1}h^a \tag{13}$$

$$(f^a - f^u)(A_1B_2 - A_2B_1) + (g^a - g^u)(B_1C_2 - B_2C_1) + (h^a - h^u)(A_2C_1 - A_1C_2) \geq 0 \;, \quad (A_1B_2 - A_2B_1) > 0 \tag{14}$$

or

$$(f^a - f^u)(A_1B_2 - A_2B_1) + (g^a - g^u)(B_1C_2 - B_2C_1) + (h^a - h^u)(A_2C_1 - A_1C_2) \leq 0 \;, \quad (A_1B_2 - A_2B_1) < 0 \tag{15}$$

2.3 Determining Comparison Order

Eq 14 can be written as,

$$X = f^aP + f^bQ + f^cR + f^uS \tag{16}$$

with

$$\begin{aligned} P &= \left[(g^a - g^b)(h^b - h^c) - (g^b - g^c)(h^a - h^b) - (g^a - g^u)(h^b - h^c) + (g^b - g^c)(h^a - h^u)\right] && (17)\\ &= \left[(g^u - g^b)(h^b - h^c) + (g^b - g^c)(h^b - h^c)\right] && (18)\\ &= g^b(h^c - h^u) + g^c(h^u - h^b) + g^(h^b - h^c) && (19)\end{aligned}$$

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$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n}\sum_{i}^{n} X_i$$

denote their mean. Then as n approaches infinity, the random variables $\sqrt{n}(S_n - \mu)$ converge in distribution to a normal $\mathcal{N}(0, \sigma^2)$.

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References

[Gre93] George D. Greenwade. The Comprehensive Tex Archive Network (CTAN). *TUGBoat*, 14(3):342–351, 1993.