ISEN 616: Design and Analysis of Industrial Experiments

Optimization of Design of a Paper Helicopter through use of Orthogonal Arrays and Hamada Wu method.
Final Project

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Executive Summary

The goal of this project is to find the optimal design for a paper helicopter and to optimize the settings for the construction of the helicopter which will increase the flight time of the helicopter made from 8.5” x 11” paper. The initial design is made from the Orthogonal Array by the help of Plackett-Burman Design OA($12,2^{11}$). Further the significance of the factors is accounted by the ‘Hamada Wu Effect Sparsity Analysis’ method.

(1) Step – 1: The first task was to construct a design matrix in accordance to the Placket-Burman Design corresponding to the Orthogonal Array OA($12,2^{11}$). From this design further proceeding with the generation of flight times with the help of Helicopters.

(2) Step – 2: The next task was to construct paper helicopters according to the measurements related to the 12 runs in the orthogonal array. The helicopters were made form the same type of paper.

(3) Step – 3: Randomizing between the Runs was the next task where the order of the runs was randomized to get better results. The principle of Replication was also used in the sense that the helicopters were replicated for each reading (3 readings in this case)

(4) Step – 4: Following this the helicopters were launched, and the time was recorded for each run. The data was averaged and variance for the data was calculated. Main effects of each factor and 2fi was calculated. Based on these values Half-Normal plots were generated to find out the significant factors.

(5) Step – 5: The Hamada Wu Effect Sparsity stepwise regression method was applied on the data and Two equations for location and dispersion were found out. Based on these equations the levels of the significant factors were set to an optimal level. For validation, a new model with the optimized levels was made and tested.

Following the above steps, we ended up having a model that has a flight time of **2.69** , which increased from an average value of **2.22** . This value is for the validation model which was obtained after the stepwise regression method.

The outcome of this project was the practical knowledge about methods to optimize a certain design of the product. The major outcome of this project is the overall confidence obtained about the implementation of DOE in any given test case. (Optimizing the flight time in this case)

Abstract

This paper is focused towards optimizing the design of a paper helicopter so that the flight time is maximized on a set of constraints for the dimensions of the helicopter. We employ Orthogonal Arrays for achieving the design matrix and develop a model for testing. Using randomization and replication we then perform stepwise regression for the analysis of the significant factors and hence perform validation.

Keywords: Orthogonal Arrays (OA); design of experiments (DoE);Hamada Wu method; factorial design; significant factors

Introduction

For any product design the experimentation is an integral part. This experimentation is always targeted towards achieving a particular target. This aim is affected by many factors in real life situations, which are multi-dimensional, Design of Experiments comes in play in these kinds of situations where it only considers the factors that significantly affect the output of the project. The further design will be based on these parameters and will contribute towards achieving the desired target.
In this project we construct a design matrix using Plackett Burman method and construct 12 helicopters according to the runs and the dimension of every part in each run. The parameters that affect the dimensions and their levels are shown below in the table:

Factors	symbol	Dimensions	
		- level	+ level
Wing length	l	3 inches	4.5 inches
Wing width	w	1.8 inches	2.4 inches
Body length	L	3 inches	4.5 inches
Body width	W	1.25 inches	2 inches
Middle body length	d	1 inch	1.5 inches
Fold at tip	F	no	yes

table (1)

Considering combinations of these Factor dimensions we must construct various helicopters according to a design as shown below:

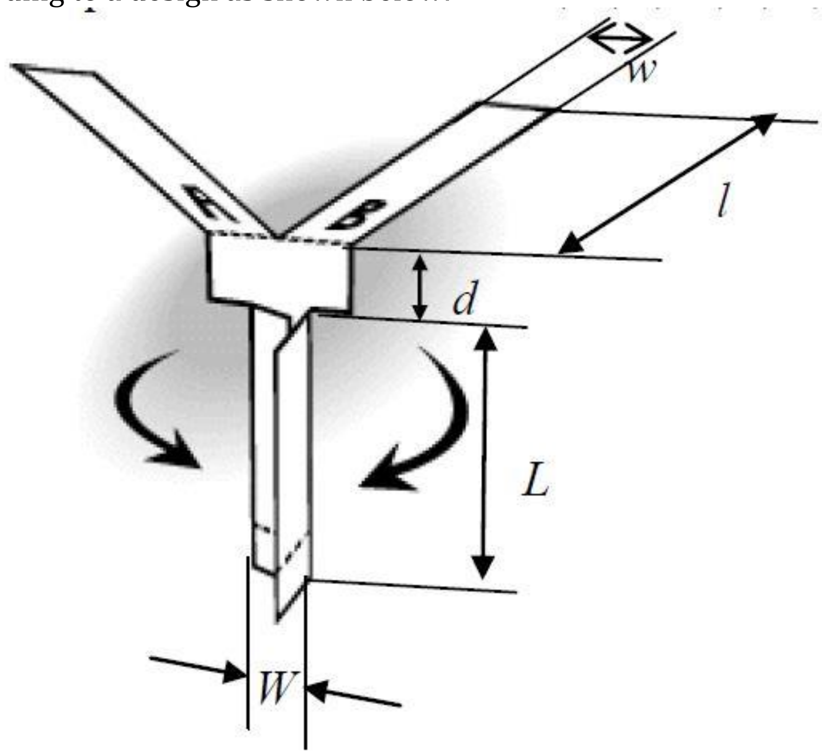


Figure (1)

According to the Design Table we must construct 12 helicopters and record flight time for each of them. Perform Randomization and replications on the runs and find out the significant factors.

Methodology and Analysis

We design an optimal model for helicopter for maximum flight time, the steps for which are mentioned in the summary and are elaborated in the methodology.

Design Table:
We must construct a table by the Plackett Burman method by Orthogonal Array of OA $(12,2^{11})$
Orthogonal Arrays:
An orthogonal array array $OA(N,s1^{m1} \ldots s\, \gamma^{m\gamma},t)$ of strength t is an N ×m matrix,
$m = m1 + \ldots + m\gamma$, in which mi columns have si(≥ 2) symbols or levels such that, for any t columns, all possible combinations of symbols appear equally often in the matrix.
There are two reasons behind using the orthogonal arrays which are,

1) Run size economy: Instead of using 2 factor or 3 factor design, we use a mixed factor design which in turn reduces the number of runs and hence improve the economy of runs.
2) Flexibility: The number of runs is limited in the previous case, whereas in this case the design is more flexible in the sense of the number of runs that are available to use.

For constructing the OA, we consider constructing it row wise; in the sense that the first row will be represented and then the entire table will be constructed with respect to that row. Hence the OA $(12,2^{11})$ will have the following format.

***N* = 12.**

Run	1	2	3	4	5	6	7	8	9	10	11
1	+	+	−	+	+	+	−	−	−	+	−
2	−	+	+	−	+	+	+	−	−	−	+
3	+	−	+	+	−	+	+	+	−	−	−
4	−	+	−	+	+	−	+	+	+	−	−
5	−	−	+	−	+	+	−	+	+	+	−
6	−	−	−	+	−	+	+	−	+	+	+
7	+	−	−	−	+	−	+	+	−	+	+
8	+	+	−	−	−	+	−	+	+	−	+
9	+	+	+	−	−	−	+	−	+	+	−
10	−	+	+	+	−	−	−	+	−	+	+
11	+	−	+	+	+	−	−	−	+	−	+
12	−	−	−	−	−	−	−	−	−	−	−

table (2)

In this experiment we have only 6 factors to consider hence rest of the factors will be kept as numbers only. (7-11). The generated table with mean flight time and logarithmic value of standard deviation are shown below.

Helicopter Construction

After this step we build helicopters corresponding to the dimensions in each case and then replicate the procedure for recording the flight times for 3 times.

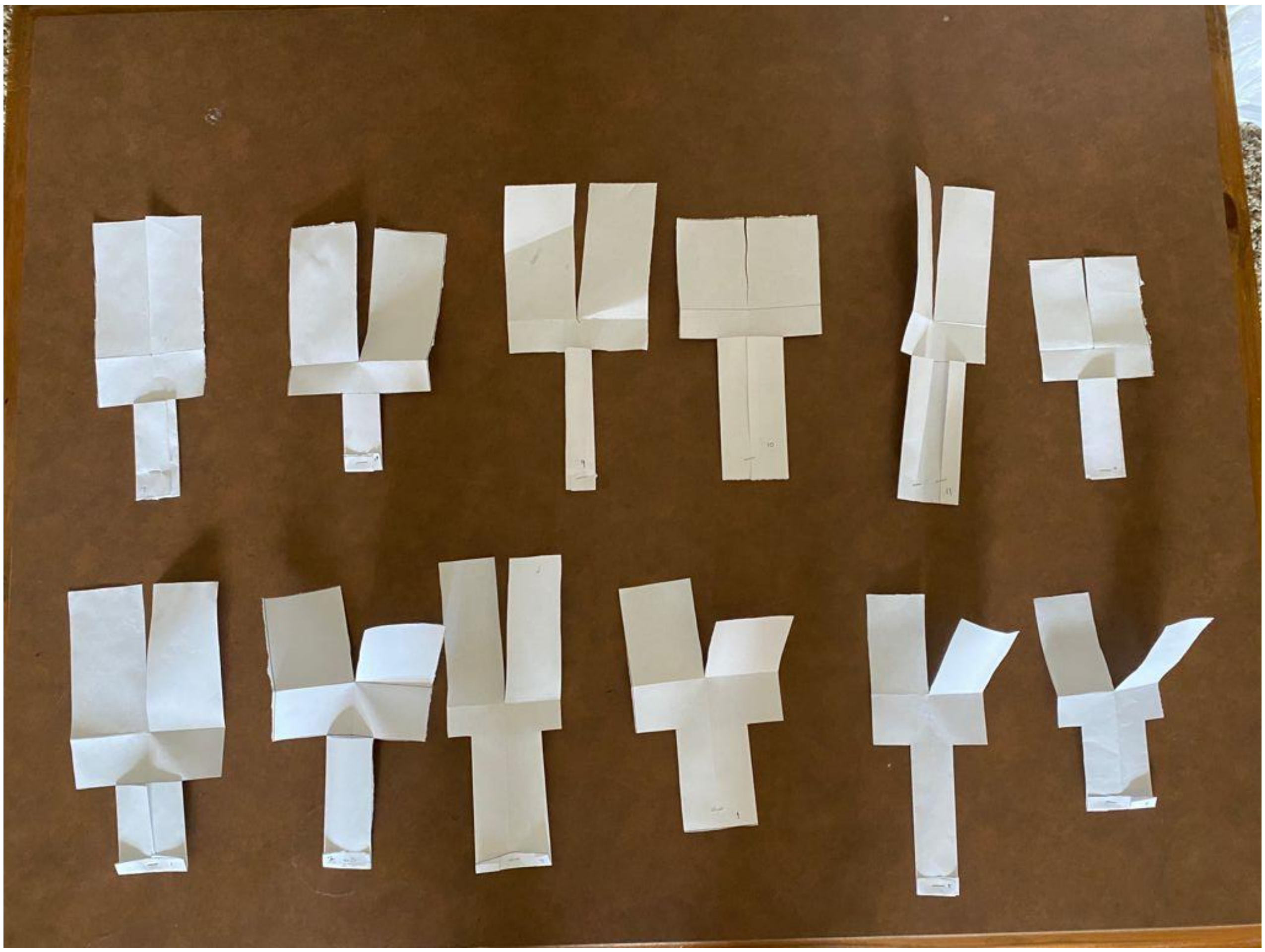


Figure (2)

After the construction and recording of data on helicopters we can use that data for calculating the mean flight time and the logarithmic value of variance. The traditional method will be used to find out the significant main factors and then we will do the analysis of the procedure by Complex aliasing but simplified by Effect sparsity principal by Hamada Wu method.

In this experiment we have only 6 factors to consider hence rest of the factors will be kept as numbers only. (7-11). The generated table with mean flight time and logarithmic value of standard deviation are shown below.

Run	A	B	C	D	E	F	7	8	9	10	11	1	2	3	Mean flight time	ln(Standard Deviation)
1	1	1	-1	1	1	1	-1	-1	-1	1	-1	2.1	2.77	2.7	2.523333333	-1.997800113
2	-1	1	1	-1	1	1	1	-1	-1	-1	1	2.1	2.1	2.3	2.166666667	-4.317488114
3	1	-1	1	1	-1	1	1	1	-1	-1	-1	2.23	2.5	3.07	2.6	-1.693363147
4	-1	1	-1	1	1	-1	1	1	1	-1	-1	2.75	2.1	2.18	2.343333333	-2.074387667
5	-1	-1	1	-1	1	1	-1	1	1	1	-1	1.91	2.6	1.78	2.096666667	-1.638695094
6	-1	-1	-1	1	-1	1	1	-1	1	1	1	2.24	2.04	2.22	2.166666667	-4.411798793
7	1	-1	-1	-1	1	-1	1	1	-1	1	1	1.97	2.49	2.67	2.376666667	-2.023943765
8	1	1	-1	-1	-1	1	-1	1	1	-1	1	2.88	3.1	3	2.993333333	-4.411798793
9	1	1	1	-1	-1	-1	1	-1	1	1	-1	3.13	2.75	2.9	2.926666667	-3.306796706
10	-1	1	1	1	-1	-1	-1	1	-1	1	1	2.34	1.87	2.07	2.093333333	-2.888972737
11	1	-1	1	1	1	-1	-1	-1	1	-1	1	2.13	3.5	2.6	2.743333333	-0.724362688
12	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	2.6	2.01	2.4	2.336666667	-2.407575307

table (3)

Replication:

For Replication we launched the same helicopter for 3 times and got the readings of the three runs. Hence the same method was replicated on 3 different models and the observations were recorded.

Randomization:

The runs showed in the table (3) are re-organized for computational purposes and the initial order of the runs is as shown below. Hence the runs were randomized, and a new table was constructed as shown in table (3).

Randomized Order	Re-assigned order
3	1
2	2
6	3
4	4
10	5
7	6
9	7
8	8
12	9
1	10
5	11
11	12

Traditional analysis on Main factors:

The tables for location and dispersion equations containing Main effects are as following:

RUN	MEY
A	0.493333
B	0.121111
C	-0.01889
D	-0.07111
E	-0.14444
F	-0.04556
7	-0.03444
8	-0.06
9	0.195556
10	-0.16667
11	-0.04778

RUN	MED
A	0.596809
B	-1.01625
C	0.459604
D	0.719269
E	1.057271
F	-0.84082
7	-0.62643
8	0.405777
9	-0.20645
10	-0.10651
11	-0.94329

table (4)

Based on these main effects, we can get a general idea of the significant factors:

Plotting Half normal plots for Location and Dispersion:

```
x = halfnorm(HALFNORMALY$MEY, nlab= 11, labs = as.character(HALFNORMALY$RUN),
ylab = "Location",)
```

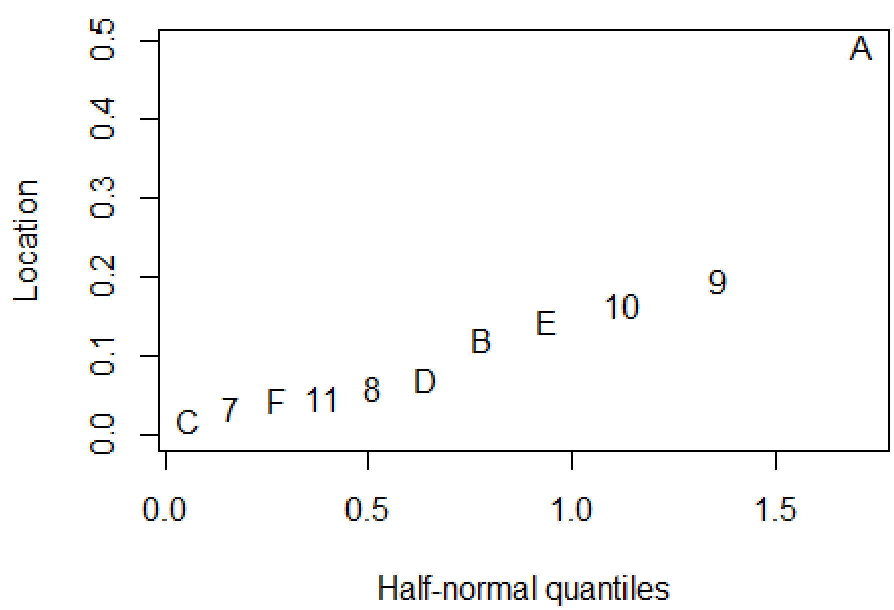
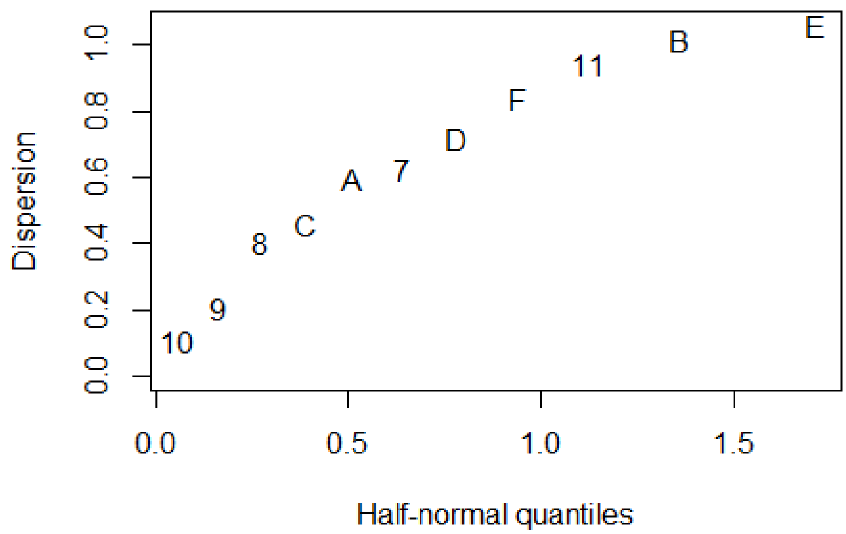


Figure (3)

<u>From this Half-normal plot, we understand that the Factor A is the most significant factor.</u>

```
y = halfnorm(HALFNORMALD$MED, nlab= 11, labs = as.character(HALFNORMALD$RUN),
ylab = "Dispersion",)
```



<u>From this Half-normal plot we understand that the Factor E is the most significant factor, followed by factor B.</u>

Figure (4)

From location half normal plot we formulate the location equation as:

$Y= 2.447 + 0.2467\ X_A$

Similarly, we get a dispersion equation as:

$Z= -2.658 - 0.5081\ X_B + 0.5286\ X_E$

<u>Performing two step method on these equations we set the levels of factors B as (+) and factor E as (-) to decrease the dispersion.</u>

<u>Similarly, we set the level of A as (+) to increase the flight time.</u>
<u>The resulting levels of the factors will result in the dimensions of the validation helicopter.</u>

Analysis Strategy:

Initially to analyze the orthogonal array methods like complex aliasing partial aliasing were used. But these methods were computationally very demanding and expensive, hence we will use a different method to analyze our data in this project.

Hamada Wu Effect Sparsity:

In this method we will turn the complex aliasing into our advantage. We will apply the effect sparsity principle in our complex aliasing procedure and hence will limit the model to 2 factor interactions only.

The steps used in Hamad Wu strategy are:
Step 1) For each factor X, consider X and all 2fi's XY involving X. Use stepwise regression to identify significant effects. Repeat this for each X and keep the best model.
Step 2) Use stepwise regression to identify significant effects among effects identified in 1 and the main effects.
Step 3) Using effect heredity, consider (i) effects identified in 2 and (ii) 2fi's with at least one parent factor appearing in the main effects in (i). Use stepwise regression to identify significant effects.
Step 4) Iterate between 2 and 3 until model stops changing.

Hamada Wu analysis for Location parameters:
The table containing all the 2 factor interactions and the main effects is given below:

RUN	A	B	C	D	E	F	7	8	9	10	11	AB	AC	AD	AE	AF	BC	BD	BE	BF	CD	CE	CF	DE	DF	EF	Y
1	1	1	-1	1	1	1	-1	-1	-1	1	-1	1	-1	1	1	1	-1	1	1	1	-1	-1	-1	1	1	1	2.523333
2	-1	1	1	-1	1	1	1	-1	-1	-1	1	-1	-1	1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	1	2.166667
3	1	-1	1	1	-1	1	1	1	-1	-1	-1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	2.6
4	-1	1	-1	1	1	-1	1	1	1	-1	-1	-1	1	-1	-1	1	-1	1	1	-1	-1	-1	1	1	-1	-1	2.343333
5	-1	-1	1	-1	1	1	-1	1	1	1	-1	1	-1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	2.096667
6	-1	-1	-1	1	-1	1	1	-1	1	1	1	1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1	1	-1	2.166667
7	1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	1	-1	1	-1	2.376667
8	1	1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	1	-1	1	-1	-1	2.993333
9	1	1	1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1	-1	-1	-1	-1	-1	1	1	1	2.926667
10	-1	1	1	1	-1	-1	-1	1	-1	1	1	-1	-1	-1	1	1	1	1	-1	-1	1	-1	-1	-1	-1	1	2.093333
11	1	-1	1	1	1	-1	-1	-1	1	-1	1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	1	-1	-1	2.743333
12	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2.336667

Table (5)

Step	Regression Input	Output
1	A,AB,AC,AD,AE,AF	A
2	B,BA,BS,BD,BE,BF	BD
3	C,CA,CB,CD,CE,CF	CF
4	D,DA,DB,DC,DE,DF	DE,BD
5	E,EA,EB,EC,ED,EF	DE
6	F,FA,FB,FC,FD,FE	CF

From the initial 6 steps we can see that the significant 2fi are- BD, CF, DE, CF. Hence for the second step we need to include all the main effects and the 2fi that are significant in the first step.

Iteration 1)

Iteration	Regression input	Output
1	A,B,C,D,E,F,BD,CF,DE	A,DE,BD

From the first iteration we see that the significant effects are- A, DE AND BD. Applying Effect Heredity principal we can see that the 2 fi- CE and BD are not significant as none of their parent terms are significant. Hence removing them and performing the second iteration as following:

Iteration 2)

Iteration	Regression input	Output
2	A,B,C,D,E,F	A

From the results we can see that only Factor A is significant which refers to "l" in the helicopter which is the "wing length"

Hamada Wu analysis for Dispersion parameters:

The table containing all the 2 factor interactions and the main effects is given below:

RUN	A	B	C	D	E	F	7	8	9	10	11	AB	AC	AD	AE	AF	BC	BD	BE	BF	CD	CE	CF	DE	DF	EF	S
1	1	1	-1	1	1	1	-1	-1	-1	1	-1	1	-1	1	1	1	-1	1	1	1	-1	-1	-1	1	1	1	-1.9978
2	-1	1	1	-1	1	1	1	-1	-1	-1	1	-1	-1	1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	1	-4.31749
3	1	-1	1	1	-1	1	1	1	-1	-1	-1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	-1.69336
4	-1	1	-1	1	1	-1	1	1	1	-1	-1	-1	1	-1	-1	1	-1	1	1	-1	-1	-1	1	1	-1	-1	-2.07439
5	-1	-1	1	-1	1	1	-1	1	1	1	-1	1	-1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	-1.6387
6	-1	-1	-1	1	-1	1	1	-1	1	1	1	1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1	1	-1	-4.4118
7	1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	1	-1	1	-1	-2.02394
8	1	1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	1	-1	1	-1	-1	-4.4118
9	1	1	1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	-1	1	-1	-1	-1	-1	-1	-1	1	1	1	-3.3068
10	-1	1	1	1	-1	-1	-1	1	-1	1	1	-1	-1	-1	1	1	1	1	-1	-1	1	-1	-1	-1	-1	1	-2.88897
11	1	-1	1	1	1	-1	-1	-1	1	-1	1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	1	-1	-1	-0.72436
12	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-2.40758

Table (6)

We must repeat the same procedure for Dispersion equation.

Step	Regression Input	Output
1	A,AB,AC,AD,AE,AF	AD
2	B,BA,BS,BD,BE,BF	BC
3	C,CA,CB,CD,CE,CF	BC
4	D,DA,DB,DC,DE,DF	AD
5	E,EA,EB,EC,ED,EF	E
6	F,FA,FB,FC,FD,FE	F

From the first step we understand that the significant 2fi are as following-AD and BC Hence we perform the first iteration including these interaction terms and the main effects.

Iteration	Regression input	Output
1	A,B,C,D,E,F,AD,BC	BC,F,AD,B

Iteration 1)
Hence from the first iteration we can conclude that the significant terms are : BC, F, AD, B from Effect Heredity principal we remove factor interaction AD as none of its parent factors are coming as significant.
Now we have to perform second iteration by removing AD.

Iteration	Regression input	Output
2	A,B,C,D,E,F,BC	BC,F,B

Iteration 2)

After performing Second iteration we can see that the significant effects are BC, F and B which will not further change.

From the analysis we get two equations for location and dispersion:

Location equation:

$Y = 2.44722 + 0.2467\ X_A$

Dispersion equation:

$Z = -2.658 - 0.508\ X_B - 0.420\ X_F - 0.568\ X_{BC}$

For maximizing the flight time, we need to change the values for the coefficient of the significant factors in Dispersion and Location equation.

In dispersion equation to minimize the dispersion we need to set the factor B and F to higher level (+) to minimize the sign of the coefficient. Hence in the interaction term to make the coefficient positive we need to set the value of factor C as higher value (+), as we know that the factor B is already at higher level.

Applying the same policy in the location equation we get a higher level of Factor A.

From both the equations we get an overall design where the levels of factors are at following levels:

B : (+)

F : (+)

C : (+)

A : (+)

Hence for optimizing the design to get an increase flight time the value of the factors should be set to the following levels.

Wing width (w): 2.4 inches

Body length (L): 4.5 inches

Fold at tip (F): Yes

Wing length(l): 4.5 inches.

Validation and Recommendations

We now need to reconstruct the model with these settings and record the flight time again to see the results of the analysis.

From trial and error, we set the values of the other parameters as following to achieve the maximum flight time.

Factors	symbol	Optimal Dimensions
Wing length	l	4.5 inches
Wing width	w	2.4 inches
Body length	L	4.5 inches
Body width	W	1.25 inches
Middle body length	d	1 inch
Fold at tip	F	Yes

Table (7)

Hence after making a model corresponding to these dimensions, we find that the flight time has increased from an average of **2.22 seconds** to **2.69 seconds.**

The recommendations are the values in the table for each parameter of the helicopter which should be implemented to increase the flight time.

The model with the above models is shown below:

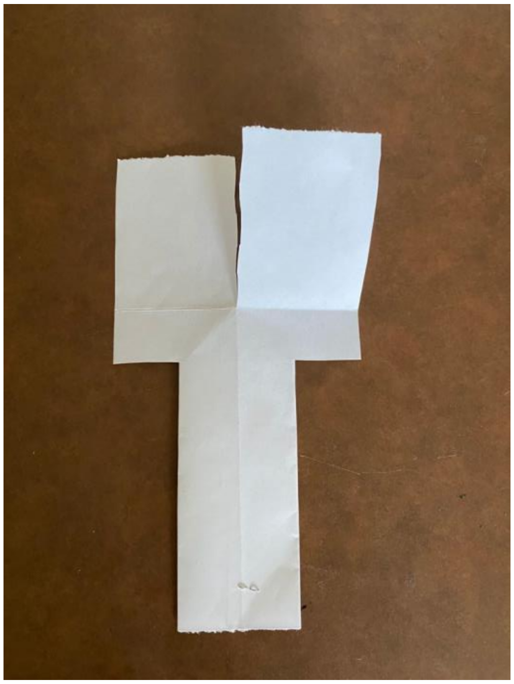


Figure (5)

Comparison and conclusion.

The Design matrix was constructed with the help of Plackett-Burman method and the Orthogonal arrays for 12 runs. The helicopters were designed on that basis and then were used for recording the data.

Initially with the traditional Half normal plotting method, we found that the significant factor for location is Wing Length (l), and the significant factors for dispersion were Middle body length (d) and wing width (w).

With the second method we did the analysis by Effect sparsity principal used in Complex Aliasing by Hamada Wu method we found out that the significant factor for location is Wing length (l), and the significant factors for Dispersion were- Wing width (w), Fold at tip (F) and the interaction between wing width and Body length (L).

Hence, to compare between traditional method and Hamada Wu method, traditional method gives significant factors as – l, F, w. Whereas the Hamada Wu is giving significant factors as w, F, and L.

The comparison of the levels set by traditional method and Hamada Wu method is give below with the factors highlighted (Only the factors which are significant and have their levels set)

Factors	symbol	Traditional Method	Hamda Wu Method
Wing length	l	+	+
Wing width	w	+	+
Body length	L		+
Body width	W		
Middle body length	d	-	
Fold at tip	F		+

We set the levels of these factors to maximize the flight time and the dimensions for the validation model by Hamada Wu method were set as shown in the table in Validation.

There is a **21.17 % increase in the flight time.** We have successfully used the concepts of Randomization, Replication, use of Orthogonal Arrays, Stepwise regression, Half Normal plots, Hamada Wu analysis that we learned in the course and have implemented it to achieve good results.

APPENDIX 1- R-code and output for Location Equation.

```
library(ISLR)
library(leaps)
library(olsrr)
## Warning: package ‘olsrr’ was built under R version 4.0.5
## Registered S3 methods overwritten by ‘car’:
##   method                          from
##   influence.merMod                lme4
##   cooks.distance.influence.merMod lme4
##   dfbeta.influence.merMod         lme4
##   dfbetas.influence.merMod        lme4
##
## Attaching package: ‘olsrr’
## The following object is masked from ‘package:faraway’:
##
##     hsb
## The following object is masked from ‘package:datasets’:
##
##     rivers
regfitLOCATION=lm(Y~A+B+C+D+E+F,data=HAMADAY)
ols_step_both_p(regfitLOCATION)
##
##                                Stepwise Selection Summary
## ---------------------------------------------------------------------------
----------
##                      Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
## ---------------------------------------------------------------------------
----------
##    1       A        addition       0.677       0.645    -0.0570    -2.4196
0.1866
## ---------------------------------------------------------------------------
----------
summary(regfitLOCATION)
##
## Call:
## lm(formula = Y ~ A + B + C + D + E + F, data = HAMADAY)
##
## Residuals:
##         1         2         3         4         5         6         7
8
## -0.110000 -0.025556 -0.037778  0.157778  0.025556  0.003333 -0.252222
0.144444
##         9        10        11        12
##  0.051111 -0.217778  0.204444  0.056667
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)  2.447222   0.060452  40.482 1.73e-07 ***
## A            0.246667   0.060452   4.080  0.00954 **
## B            0.060556   0.060452   1.002  0.36247
## C           -0.009444   0.060452  -0.156  0.88196
## D           -0.035556   0.060452  -0.588  0.58198
## E           -0.072222   0.060452  -1.195  0.28578
## F           -0.022778   0.060452  -0.377  0.72179
## ---
## Signif. Codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2094 on 5 degrees of freedom
## Multiple R-squared:  0.7967, Adjusted R-squared:  0.5527
## F-statistic: 3.265 on 6 and 5 DF,  p-value: 0.1075
regfitA=lm(Y~A+AB+AC+AD+AE+AF,data=HAMADAY)
ols_step_both_p(regfitA)
##
##                              Stepwise Selection Summary
## ----------------------------------------------------------------------------
---------
##                      Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## ----------------------------------------------------------------------------
---------
##    1       A        addition       0.677       0.645    3.8370    -2.4196
0.1866
## ----------------------------------------------------------------------------
---------
summary(regfitA)
##
## Call:
## lm(formula = Y ~ A + AB + AC + AD + AE + AF, data = HAMADAY)
##
## Residuals:
##        1        2        3        4        5        6        7        8
## -0.08278  0.09500 -0.17833 -0.01389 -0.09500 -0.09389 -0.11278  0.16722
##        9       10       11       12
##  0.02500  0.02833  0.18167  0.07944
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.44722    0.04952  49.418 6.41e-08 ***
## A            0.24667    0.04952   4.981  0.00417 **
## AB           0.06000    0.04952   1.212  0.27979
## AC           0.07222    0.04952   1.458  0.20453
## AD          -0.03611    0.04952  -0.729  0.49858
## AE          -0.07389    0.04952  -1.492  0.19588
## AF           0.03444    0.04952   0.696  0.51770
## ---
## Signif. Codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 0.1715 on 5 degrees of freedom
## Multiple R-squared:  0.8636, Adjusted R-squared:  0.6998
## F-statistic: 5.275 on 6 and 5 DF,  p-value: 0.04415
regfitB=lm(Y~B+AB+BC+BD+BE+BF,data=HAMADAY)
ols_step_both_p(regfitB)
##
##                              Stepwise Selection Summary
## -----------------------------------------------------------------------------
--------
##                      Added/                  Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)      AIC
RMSE
## -----------------------------------------------------------------------------
--------
##    1       BD       addition       0.258       0.184    1.5900    7.5640
0.2829
## -----------------------------------------------------------------------------
--------
summary(regfitB)
##
## Call:
## lm(formula = Y ~ B + AB + BC + BD + BE + BF, data = HAMADAY)
##
## Residuals:
##         1         2         3         4         5         6         7
8
##  0.020000 -0.315556  0.185556  0.112222 -0.315556 -0.162222  0.137778
0.003333
##         9        10        11        12
##  0.294444 -0.114444 -0.005556  0.160000
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.44722    0.08340  29.344 8.62e-07 ***
## B            0.06056    0.08340   0.726    0.500
## AB           0.06000    0.08340   0.719    0.504
## BC          -0.10278    0.08340  -1.232    0.273
## BD          -0.15222    0.08340  -1.825    0.128
## BE          -0.09111    0.08340  -1.092    0.324
## BF           0.07611    0.08340   0.913    0.403
## ---
## Signif. Codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ‘ 1
##
## Residual standard error: 0.2889 on 5 degrees of freedom
## Multiple R-squared:  0.6131, Adjusted R-squared:  0.1487
## F-statistic:  1.32 on 6 and 5 DF,  p-value: 0.389
regfitC=lm(Y~C+AC+BC+CD+CE+CF,data=HAMADAY)
ols_step_both_p(regfitC)
```

```
##
##                             Stepwise Selection Summary
## ----------------------------------------------------------------------------
---------
##                         Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## ----------------------------------------------------------------------------
---------
##    1       CF       addition       0.180       0.098    -0.7820    8.7593
0.2974
## ----------------------------------------------------------------------------
---------
summary(regfitC)
##
## Call:
## lm(formula = Y ~ C + AC + BC + CD + CE + CF, data = HAMADAY)
##
## Residuals:
##         1         2         3         4         5         6         7
8
## -0.044444  0.137778  0.007778 -0.114444 -0.137778 -0.280000  0.115556
0.332222
##         9        10        11        12
##  0.438889 -0.403333 -0.043333 -0.008889
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.447222   0.101035  24.222 2.24e-06 ***
## C           -0.009444   0.101035  -0.093    0.929
## AC           0.072222   0.101035   0.715    0.507
## BC          -0.102778   0.101035  -1.017    0.356
## CD           0.076667   0.101035   0.759    0.482
## CE          -0.030000   0.101035  -0.297    0.778
## CF          -0.127222   0.101035  -1.259    0.264
## ---
## Signif. Codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ‘ 1
##
## Residual standard error: 0.35 on 5 degrees of freedom
## Multiple R-squared:  0.4321, Adjusted R-squared:  -0.2494
## F-statistic: 0.634 on 6 and 5 DF,  p-value: 0.7041
regfitD=lm(Y~D+AD+BD+CD+DE+DF,data=HAMADAY)
ols_step_both_p(regfitD)
##
##                             Stepwise Selection Summary
## ----------------------------------------------------------------------------
---------
##                         Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
```

```
## ----------------------------------------------------------------------------
---------
##    1       DE       addition       0.433       0.376    6.4260     4.3376
0.2473
##    2       BD       addition       0.691       0.622    1.8690    -0.9365
0.1925
## ----------------------------------------------------------------------------
---------
summary(regfitD)
##
## Call:
## lm(formula = Y ~ D + AD + BD + CD + DE + DF, data = HAMADAY)
##
## Residuals:
##        1        2        3        4        5        6        7        8
##  0.13833 -0.11722  0.15167 -0.03167  0.11722 -0.20056  0.08944  0.08944
##        9       10       11       12
##  0.09389 -0.04056 -0.01722 -0.27278
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.44722    0.05944  41.169 1.59e-07 ***
## D           -0.03556    0.05944  -0.598   0.5758
## AD          -0.03611    0.05944  -0.607   0.5701
## BD          -0.15222    0.05944  -2.561   0.0506 .
## CD           0.07667    0.05944   1.290   0.2536
## DE           0.19722    0.05944   3.318   0.0211 *
## DF           0.04111    0.05944   0.692   0.5200
## ---
## Signif. Codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2059 on 5 degrees of freedom
## Multiple R-squared:  0.8034, Adjusted R-squared:  0.5675
## F-statistic: 3.406 on 6 and 5 DF,  p-value: 0.09994
regfitE=lm(Y~E+AE+BE+CE+DE+EF,data=HAMADAY)
ols_step_both_p(regfitE)
##
##                              Stepwise Selection Summary
## ----------------------------------------------------------------------------
--------
##                      Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)      AIC
RMSE
## ----------------------------------------------------------------------------
--------
##    1       DE       addition       0.433       0.376    3.0820    4.3376
0.2473
## ----------------------------------------------------------------------------
--------
summary(regfitE)
```

```
##
## Call:
## lm(formula = Y ~ E + AE + BE + CE + DE + EF, data = HAMADAY)
##
## Residuals:
##        1        2        3        4        5        6        7        8
##  0.17611  0.12611  0.17500 -0.33167 -0.12611 -0.05056  0.06167  0.05167
##        9       10       11       12
##  0.10500 -0.18611  0.09389 -0.09500
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.44722    0.06782  36.084 3.08e-07 ***
## E           -0.07222    0.06782  -1.065   0.3356
## AE          -0.07389    0.06782  -1.089   0.3256
## BE          -0.09111    0.06782  -1.343   0.2369
## CE          -0.03000    0.06782  -0.442   0.6767
## DE           0.19722    0.06782   2.908   0.0335 *
## EF          -0.09000    0.06782  -1.327   0.2419
## ---
## Signif. Codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2349 on 5 degrees of freedom
## Multiple R-squared:  0.7441, Adjusted R-squared:  0.437
## F-statistic: 2.423 on 6 and 5 DF,  p-value: 0.175
regfitF=lm(Y~F+AF+BF+CF+DF+EF,data=HAMADAY)
ols_step_both_p(regfitF)
##
##                             Stepwise Selection Summary
## ----------------------------------------------------------------------------
---------
##                      Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## ----------------------------------------------------------------------------
---------
##    1       CF       addition       0.180       0.098    -1.4670    8.7593
0.2974
## ----------------------------------------------------------------------------
---------
summary(regfitF)
##
## Call:
## lm(formula = Y ~ F + AF + BF + CF + DF + EF, data = HAMADAY)
##
## Residuals:
##         1         2         3         4         5         6         7
8
## -0.090000 -0.041111  0.213333 -0.006667  0.041111 -0.405556 -0.138889
0.282222
```

```
##          9         10         11         12
##  0.488889 -0.331111  0.055556 -0.067778
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.44722    0.10621  23.042 2.86e-06 ***
## F           -0.02278    0.10621  -0.214    0.839
## AF           0.03444    0.10621   0.324    0.759
## BF           0.07611    0.10621   0.717    0.506
## CF          -0.12722    0.10621  -1.198    0.285
## DF           0.04111    0.10621   0.387    0.715
## EF          -0.09000    0.10621  -0.847    0.435
## ---
## Signif. Codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3679 on 5 degrees of freedom
## Multiple R-squared:  0.3725, Adjusted R-squared:  -0.3806
## F-statistic: 0.4946 on 6 and 5 DF,  p-value: 0.7918
```

From the initial 6 steps we can see that the significant 2fi are- BD, CF, DE, CF. Hence for the second step we need to include all the main effects and the 2fi that are significant in the first step.

Iteration 1)

```
regfitall1=lm(Y~A+B+C+D+E+F+BD+CF+DE+CF,data=HAMADAY)
ols_step_both_p(regfitall1)
##
##                               Stepwise Selection Summary
## ----------------------------------------------------------------------------
-----------
##                       Added/                      Adj.
## Step    Variable     Removed     R-Square     R-Square     C(p)        AIC
RMSE
## ----------------------------------------------------------------------------
-----------
##    1        A        addition       0.677       0.645     16.7720     -
2.4196    0.1866
##    2        DE       addition       0.843       0.808      6.0750     -
9.0425    0.1374
##    3        BD       addition       0.933       0.908      1.1320    -
17.3111    0.0950
## ----------------------------------------------------------------------------
-----------
summary(regfitall1)
##
## Call:
## lm(formula = Y ~ A + B + C + D + E + F + BD + CF + DE + CF, data =
```

```
HAMADAY)
##
## Residuals:
##          1          2          3          4          5          6          
7 
## -7.143e-02 -6.413e-02  3.469e-17  6.413e-02  6.413e-02 -7.302e-03 -4.510e-
17 
##          8          9         10         11         12 
##  7.873e-02 -1.460e-02  7.302e-03  7.302e-03 -6.413e-02 
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  2.447222   0.034231  71.491 0.000196 ***
## A            0.180952   0.040914   4.423 0.047510 *  
## B            0.020913   0.039878   0.524 0.652318    
## C            0.011111   0.041925   0.265 0.815805    
## D           -0.028571   0.037721  -0.757 0.527863    
## E           -0.053135   0.039878  -1.332 0.314252    
## F           -0.002222   0.041925  -0.053 0.962546    
## BD          -0.078214   0.047538  -1.645 0.241645    
## CF           0.020952   0.047538   0.441 0.702458    
## DE           0.139881   0.047538   2.943 0.098693 .  
## ---
## Signif. Codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1186 on 2 degrees of freedom
## Multiple R-squared:  0.9739, Adjusted R-squared:  0.8566 
## F-statistic:   8.3 on 9 and 2 DF,  p-value: 0.1121
```

From the first iteration we see that the significant effects are- A, DE AND BD. Applying Effect Heredity principal we can see that the 2 fi- CE and BD are not significant as none of their parent terms are significant. Hence removing them and performing the second iteration as following:

Iteration 2)

```
regfitall2=lm(Y~A+B+C+D+E+F,data=HAMADAY)
ols_step_both_p(regfitall2)
##
##                               Stepwise Selection Summary                               
## -----------------------------------------------------------------------------
----------
##                        Added/                  Adj.                                   
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC        
RMSE
## -----------------------------------------------------------------------------
----------
##    1       A        addition       0.677       0.645    -0.0570    -2.4196
```

```
0.1866
## -----------------------------------------------------------------------------
----------
summary(regfitall2)
##
## Call:
## lm(formula = Y ~ A + B + C + D + E + F, data = HAMADAY)
##
## Residuals:
##         1         2         3         4         5         6         7
8
## -0.110000 -0.025556 -0.037778  0.157778  0.025556  0.003333 -0.252222
0.144444
##         9        10        11        12
##  0.051111 -0.217778  0.204444  0.056667
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.447222   0.060452  40.482 1.73e-07 ***
## A            0.246667   0.060452   4.080  0.00954 **
## B            0.060556   0.060452   1.002  0.36247
## C           -0.009444   0.060452  -0.156  0.88196
## D           -0.035556   0.060452  -0.588  0.58198
## E           -0.072222   0.060452  -1.195  0.28578
## F           -0.022778   0.060452  -0.377  0.72179
## ---
## Signif. Codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2094 on 5 degrees of freedom
## Multiple R-squared:  0.7967, Adjusted R-squared:  0.5527
## F-statistic: 3.265 on 6 and 5 DF,  p-value: 0.1075
```

From the results we can see that only Factor A is significant which refers to "l" in the helicopter which is the "wing length"

Appendix 2- R-code for Dispersion Equation.

```
regfitDISPERSION=lm(S~A+B+C+D+E+F,data=HAMADAD)
ols_step_both_p(regfitDISPERSION)
##
##                              Stepwise Selection Summary
## ----------------------------------------------------------------------------
---------
##                      Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## ----------------------------------------------------------------------------
---------
##    1       E        addition       0.205       0.126    6.4100    40.9929
1.1391
## ----------------------------------------------------------------------------
---------
summary(regfitDISPERSION)
##
## Call:
## lm(formula = S ~ A + B + C + D + E + F, data = HAMADAD)
##
## Residuals:
##         1         2         3         4         5         6         7
8
##  0.631944 -0.831271  0.517797  0.311347  0.831271 -1.144226 -0.532000 -
0.005515
##         9        10        11        12
## -0.200935  0.094429 -0.411292  0.738449
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -2.6581     0.2739  -9.703 0.000197 ***
## A             0.2984     0.2739   1.089 0.325708
## B            -0.5081     0.2739  -1.855 0.122773
## C             0.2298     0.2739   0.839 0.439785
## D             0.3596     0.2739   1.313 0.246263
## E             0.5286     0.2739   1.930 0.111523
## F            -0.4204     0.2739  -1.535 0.185448
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9489 on 5 degrees of freedom
## Multiple R-squared:  0.7243, Adjusted R-squared:  0.3934
## F-statistic: 2.189 on 6 and 5 DF,  p-value: 0.2038
regfit1=lm(S~A+AB+AC+AD+AE+AF,data=HAMADAD)
ols_step_both_p(regfit1)
##
##                               Stepwise Selection Summary
```

```
## -----------------------------------------------------------------------------
----------
##                         Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
## -----------------------------------------------------------------------------
----------
##    1       AD       addition       0.205       0.126    -0.6680    40.9980
1.1394
## -----------------------------------------------------------------------------
----------
summary(regfit1)
##
## Call:
## lm(formula = S ~ A + AB + AC + AD + AE + AF, data = HAMADAD)
##
## Residuals:
##        1        2        3        4        5        6        7        8
##  0.09827 -1.71039 -0.28468  0.98751  1.71039 -0.94772  0.54473 -0.76132
##        9       10       11       12
##  0.05848  0.11832  0.34451 -0.15810
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -2.6581     0.3841  -6.920 0.000966 ***
## A             0.2984     0.3841   0.777 0.472367
## AB           -0.3710     0.3841  -0.966 0.378474
## AC            0.2217     0.3841   0.577 0.588829
## AD            0.5282     0.3841   1.375 0.227509
## AE            0.2490     0.3841   0.648 0.545400
## AF            0.0791     0.3841   0.206 0.844969
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.331 on 5 degrees of freedom
## Multiple R-squared:  0.4579, Adjusted R-squared:  -0.1926
## F-statistic: 0.7039 on 6 and 5 DF,  p-value: 0.6626
regfit2=lm(S~B+AB+BC+BD+BE+BF,data=HAMADAD)
ols_step_both_p(regfit2)
##
##                              Stepwise Selection Summary
## -----------------------------------------------------------------------------
---------
##                         Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## -----------------------------------------------------------------------------
---------
##    1       BC       addition       0.237       0.161     5.6400    40.5038
1.1161
```

```
## --------------------------------------------------------------------------
---------
summary(regfit2)
##
## Call:
## lm(formula = S ~ B + AB + BC + BD + BE + BF, data = HAMADAD)
##
## Residuals:
##       1       2       3       4       5       6       7       8       9
10
##  0.6333 -0.3200  0.1737 -0.1635 -0.3200 -0.6667 -0.3331 -1.1263  1.1365 -
0.1600
##      11      12
##  0.8029  0.3433
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.65808    0.27588  -9.635 0.000204 ***
## B           -0.50813    0.27588  -1.842 0.124852
## AB          -0.37100    0.27588  -1.345 0.236475
## BC          -0.56801    0.27588  -2.059 0.094563 .
## BD           0.48619    0.27588   1.762 0.138310
## BE          -0.15899    0.27588  -0.576 0.589387
## BF           0.01092    0.27588   0.040 0.969957
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9557 on 5 degrees of freedom
## Multiple R-squared:  0.7203, Adjusted R-squared:  0.3848
## F-statistic: 2.147 on 6 and 5 DF,  p-value: 0.2097
regfit3=lm(S~C+AC+BC+CD+CE+CF,data=HAMADAD)
ols_step_both_p(regfit3)
##
##                              Stepwise Selection Summary
## --------------------------------------------------------------------------
----------
##                      Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
## --------------------------------------------------------------------------
----------
##    1       BC       addition       0.237       0.161    -0.0140    40.5038
1.1161
## --------------------------------------------------------------------------
----------
summary(regfit3)
##
## Call:
## lm(formula = S ~ C + AC + BC + CD + CE + CF, data = HAMADAD)
##
```

```
## Residuals:
##          1          2          3          4          5          6
7
##  0.8151522 -0.7713825 -0.9805887 -0.3025172  0.7713825 -0.2518118
0.7278654
##          8          9         10         11         12
## -1.5439291 -0.2608231  0.0009115  1.2405003  0.5552406
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -2.6581     0.3605  -7.372 0.000722 ***
## C             0.2298     0.3605   0.637 0.551922
## AC            0.2217     0.3605   0.615 0.565523
## BC           -0.5680     0.3605  -1.575 0.175979
## CD            0.2997     0.3605   0.831 0.443646
## CE           -0.3272     0.3605  -0.908 0.405745
## CF            0.2988     0.3605   0.829 0.444941
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.249 on 5 degrees of freedom
## Multiple R-squared:  0.5224, Adjusted R-squared:  -0.05082
## F-statistic: 0.9113 on 6 and 5 DF,  p-value: 0.5518
regfit4=lm(S~D+AD+BD+CD+DE+DF,data=HAMADAD)
ols_step_both_p(regfit4)
##
##                              Stepwise Selection Summary
## ----------------------------------------------------------------------------
---------
##                      Added/                  Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)       AIC
RMSE
## ----------------------------------------------------------------------------
---------
##    1       AD       addition       0.205       0.126    1.0650    40.9980
1.1394
## ----------------------------------------------------------------------------
---------
summary(regfit4)
##
## Call:
## lm(formula = S ~ D + AD + BD + CD + DE + DF, data = HAMADAD)
##
## Residuals:
##       1       2       3       4       5       6       7       8       9
10
## -0.6028 -0.8532  0.4164  0.4127  0.8532 -0.6461  0.8891 -0.8325  0.8362 -
0.6594
##      11      12
##  1.0792 -0.8928
```

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.65808    0.34545  -7.695 0.000591 ***
## D            0.35963    0.34545   1.041 0.345548
## AD           0.52820    0.34545   1.529 0.186807
## BD           0.48619    0.34545   1.407 0.218332
## CD           0.29975    0.34545   0.868 0.425239
## DE           0.17096    0.34545   0.495 0.641653
## DF           0.01787    0.34545   0.052 0.960750
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.197 on 5 degrees of freedom
## Multiple R-squared:  0.5615, Adjusted R-squared:  0.03534
## F-statistic: 1.067 on 6 and 5 DF,  p-value: 0.4817
regfit5=lm(S~E+AE+BE+CE+DE+EF,data=HAMADAD)
ols_step_both_p(regfit5)
##
##                              Stepwise Selection Summary
## ----------------------------------------------------------------------------
----------
##                      Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
## ----------------------------------------------------------------------------
----------
##    1       E        addition       0.205       0.126    -1.6200    40.9929
1.1391
## ----------------------------------------------------------------------------
----------
summary(regfit5)
##
## Call:
## lm(formula = S ~ E + AE + BE + CE + DE + EF, data = HAMADAD)
##
## Residuals:
##       1       2       3       4       5       6       7       8       9
10
## -0.3551 -1.1804  1.6436 -0.1366  1.1804 -0.9184 -0.5602 -1.0803 -0.4268 -
0.1650
##      11      12
##  1.0519  0.9468
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -2.6581     0.4117  -6.456  0.00133 **
## E             0.5286     0.4117   1.284  0.25540
## AE            0.2490     0.4117   0.605  0.57169
## BE           -0.1590     0.4117  -0.386  0.71525
```

```
## CE           -0.3272     0.4117  -0.795  0.46278
## DE            0.1710     0.4117   0.415  0.69517
## EF           -0.1015     0.4117  -0.246  0.81511
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.426 on 5 degrees of freedom
## Multiple R-squared:  0.3772, Adjusted R-squared:  -0.3701
## F-statistic: 0.5048 on 6 and 5 DF,  p-value: 0.7853
regfit6=lm(S~F+AF+BF+CF+DF+EF,data=HAMADAD)
ols_step_both_p(regfit6)
##
##                              Stepwise Selection Summary
## -----------------------------------------------------------------------------
----------
##                      Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
## -----------------------------------------------------------------------------
----------
##    1       F        addition       0.130       0.043    -2.5070    42.0818
1.1920
## -----------------------------------------------------------------------------
----------
summary(regfit6)
##
## Call:
## lm(formula = S ~ F + AF + BF + CF + DF + EF, data = HAMADAD)
##
## Residuals:
##       1       2       3       4       5       6       7       8       9
10
##  1.3731 -1.3503  0.8988 -0.2873  1.3503 -1.0638 -0.1363 -1.2081 -0.5967 -
0.3013
##      11      12
##  1.7967 -0.4752
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.65808    0.46427  -5.725  0.00227 **
## F           -0.42041    0.46427  -0.906  0.40671
## AF           0.07910    0.46427   0.170  0.87140
## BF           0.01092    0.46427   0.024  0.98214
## CF           0.29884    0.46427   0.644  0.54815
## DF           0.01787    0.46427   0.038  0.97079
## EF          -0.10147    0.46427  -0.219  0.83563
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.608 on 5 degrees of freedom
```

```
## Multiple R-squared:  0.208,  Adjusted R-squared:  -0.7424
## F-statistic: 0.2189 on 6 and 5 DF,  p-value: 0.9542
```

<u>From the first step we understand that the significant 2fi are as following-AD and BC Hence we perform the first iteration including these interaction terms and the main effects.</u>

<u>Iteration 1)</u>

```
regfitAll1=lm(S~A+B+C+D+E+F+AD+BC,data=HAMADAD)
ols_step_both_p(regfitAll1)
##
##                              Stepwise Selection Summary
## ---------------------------------------------------------------------------
----------
##                       Added/                   Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
## ---------------------------------------------------------------------------
----------
##    1       BC       addition       0.237       0.161    14.8840    40.5038
1.1161
##    2       F        addition       0.544       0.443     7.6640    36.3159
0.9091
##    3       AD       addition       0.757       0.666     3.2790    30.7591
0.7038
##    4       B        addition       0.840       0.748     2.8100    27.7863
0.6116
## ---------------------------------------------------------------------------
----------
summary(regfitAll1)
##
## Call:
## lm(formula = S ~ A + B + C + D + E + F + AD + BC, data = HAMADAD)
##
## Residuals:
##          1          2          3          4          5          6
7
##  4.522e-01 -4.522e-01  2.384e-01 -6.769e-02  4.522e-01 -3.061e-01  3.061e-
01
##          8          9         10         11         12
## -3.845e-01  7.844e-02  3.738e-01 -6.907e-01  4.163e-17
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -2.6581     0.2130 -12.480  0.00111 **
## A             0.1338     0.2341   0.571  0.60768
## B            -0.3934     0.2341  -1.680  0.19153
```

```
## C              0.1150     0.2341   0.491  0.65690
## D              0.1950     0.2341   0.833  0.46595
## E              0.2493     0.2459   1.014  0.38545
## F             -0.6998     0.2459  -2.845  0.06536 .
## AD             0.3443     0.2917   1.181  0.32286
## BC            -0.4938     0.2917  -1.693  0.18901
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7378 on 3 degrees of freedom
## Multiple R-squared:    0.9,  Adjusted R-squared:  0.6333
## F-statistic: 3.375 on 8 and 3 DF,  p-value: 0.1728
```

Hence from the first iteration we can conclude that the significant terms are : BC, F, AD, B from Effect Heredity principal we remove factor interaction AD as none of its parent factors are coming as significant.

Now we have to perform second iteration by removing AD

Iteration 2)

```
regfitAll2=lm(S~A+B+C+D+E+F+BC,data=HAMADAD)
ols_step_both_p(regfitAll2)
##
##                              Stepwise Selection Summary
## ---------------------------------------------------------------------------
----------
##                      Added/                        Adj.
## Step    Variable    Removed     R-Square    R-Square     C(p)        AIC
RMSE
## ---------------------------------------------------------------------------
----------
##    1       BC       addition       0.237       0.161    12.8340    40.5038
1.1161
##    2       F        addition       0.544       0.443     6.4400    36.3159
0.9091
##    3       B        addition       0.734       0.635     3.2580    31.8510
0.7366
## ---------------------------------------------------------------------------
----------
summary(regfitAll2)
##
## Call:
## lm(formula = S ~ A + B + C + D + E + F + BC, data = HAMADAD)
##
## Residuals:
##        1        2        3        4        5        6        7        8
##  0.81950 -0.26861  0.33024 -0.25132  0.26861 -0.58156  0.03067 -0.56818
```

```
##        9       10       11       12
## -0.01338  0.28198 -0.59885  0.55089
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -2.6581     0.2232 -11.907 0.000285 ***
## A             0.1108     0.2445   0.453 0.673830
## B            -0.5081     0.2232  -2.276 0.085137 .
## C             0.2298     0.2232   1.029 0.361442
## D             0.1721     0.2445   0.704 0.520423
## E             0.3411     0.2445   1.395 0.235539
## F            -0.6080     0.2445  -2.486 0.067758 .
## BC           -0.5627     0.2995  -1.879 0.133476
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7733 on 4 degrees of freedom
## Multiple R-squared:  0.8535, Adjusted R-squared:  0.5972
## F-statistic:  3.33 on 7 and 4 DF,  p-value: 0.131
```

After performing Second iteration we can see that the significant effects are BC, F and B which will not further change.