CSE340: Theory of Computation (Final Exam – Part 2)

21st November, 2021

Subjective Questions   Total Points: 55

Question 1. (10 points) Design the context-free grammar (CFG) for the following language,

$$L = \{w \in \{a, b\}^* \mid w \text{ has more } a\text{'s than } b\text{'s}\}$$

Question 2. (10 points) Design a PDA for the language $L = \{0^p 1^q 0^r 1^s \mid p = q \text{ and } r \neq s\}$

Question 3. (10 points) An *weight function* in a graph $G = (V, E)$ is a function $w : E \to \mathbb{N}$ that assigns a non-negative integer to every edge in G. A graph with a weight function is called a weighted graph. The *weight* of a walk in G is the sum of the weights of the edges in the walk.

$$\mathsf{TSP} = \{\langle G, w, B\rangle \quad | \quad G \text{ is a weighted directed graph with weight function } w \text{ and } G \text{ has a walk of}$$
$$\text{weight at most } B, \text{ that visits every vertex in } G \text{ and returns to the starting vertex}\}$$

Prove that TSP is NP-complete.

(Hint: You can give a reduction from the $\mathsf{HamPath}$ problem.)

Question 4. Consider the following two languages

$$\begin{aligned} L_1 &= \{\langle M_1, M_2\rangle \mid M_1, M_2 \text{ are two TMs and } L(M_1) \subseteq L(M_2)\} \\ L_2 &= \{\langle D_1, D_2\rangle \mid D_1, D_2 \text{ are two DFAs and } L(D_1) \subseteq L(D_2)\} \end{aligned}$$

(a) (7 points) Show that L_1 is undecidable.

(b) (6 points) Show that L_2 is decidable.

Question 5. In this question, for two languages L_1 and L_2, if $L_1 \leq_m L_2$ then we denote it by $L_1 \longrightarrow L_2$. Consider the relation between the languages given by the following diagram.
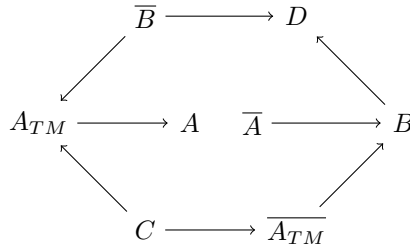
Consider the following 4 class of languages:

- Dec: decidable languages
- TR′: undecidable but Turing recognizable languages
- coTR′: undecidable but co-Turing recognizable languages
- NTR: neither Turing recognizable nor co-Turing recognizable languages

Now for each of the following languages, mention which class does it belong to by giving proper justification.

(a) (3 points) A

(b) (3 points) B

(c) (3 points) C

(d) (3 points) D