Name: Rollno:

CS340: Theory of Computation (Homework Assignment 4)

Due Date: 8th November, 2021, 11:59 PM

Total Number of Pages: 1 Total Points 50

Question 1. Which of the following languages are decidable/undecidable? Prove your answer by either giving an algorithm or a proof of undecidability.

(a) (5 points) $L_1 = \{\langle M, N\rangle \mid M, N$ are two TMs and M takes fewer steps than N on input $\epsilon\}$

(b) (5 points) $L_2 = \{\langle M\rangle \mid M$ takes at most 2^{340} steps on some input$\}$

(c) (5 points) $L_3 = \{\langle M\rangle \mid$ there are infinitely many TMs equivalent to $M\}$

(d) (5 points) $L_4 = \{\langle M, N\rangle \mid L(M) \cap L(N)$ is infinite$\}$

Question 2. (10 points) Let L be Turing recognizable (TR) and let $\bar{L}$ be non-TR. Consider the language:

$$L' = \{0w \mid w \in L\} \cup \{1w \mid w \notin L\}$$

Can you say for certain whether L' or its complement are decidable, TR or non-TR? Justify your answer.

Question 3. Prove the undecidability of each of the following languages.

(a) (10 points) $INFINITE_{TM} = \{\langle M\rangle \mid M$ is a TM and $L(M)$ is an infinite language$\}$

(b) (10 points) $ALL_{TM} = \{\langle M\rangle \mid M$ is a TM and $L(M) = \Sigma^*\}$