Name: Rollno:

# CSE340: Theory of Computation (Homework Assignment 1)

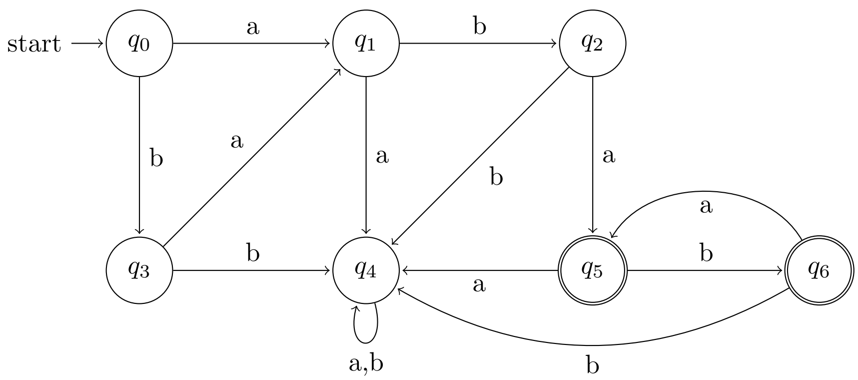
September 27, 2021

Total Number of Pages: 5 Total Points 50

**Question 1**. (18 points) Give DFAs for the following languages.

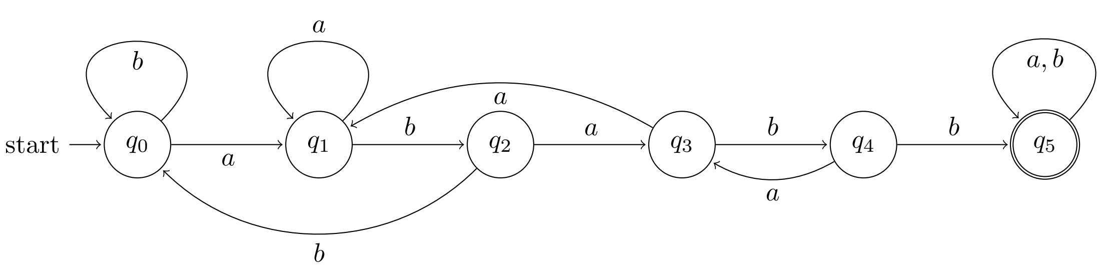
(a) $A = \{x \in \{a, b\}^* \mid x \text{ alternates between } a \text{ and } b \text{ and has at least 2 } a\text{'s}\}$

**Solution:**



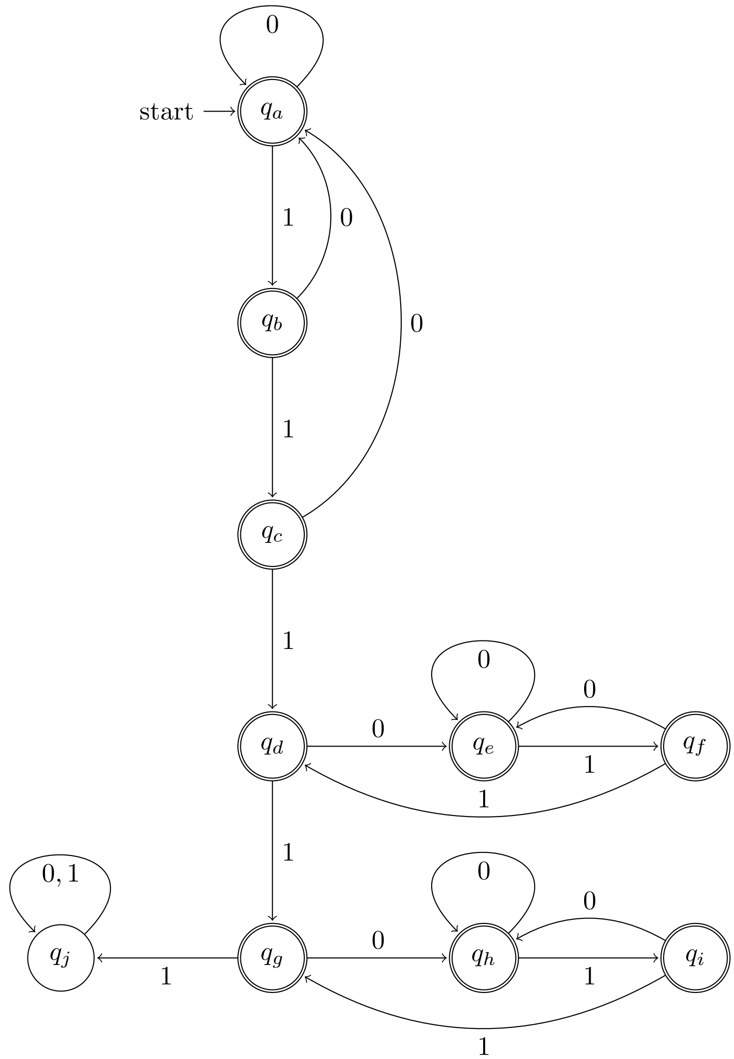

(b) $B = \{x \in \{a, b\}^* \mid x \text{ has } ababb \text{ as a substring}\}$

**Solution:**




(c) $C = \{x \in \{0, 1\}^* \mid x \text{ has at most 2 occurrences of 3 consecutive 1's with possible overlapping}\}$
(For example the string 1111 is in the language $C$ but the string 11111 is not in the language $C$.)

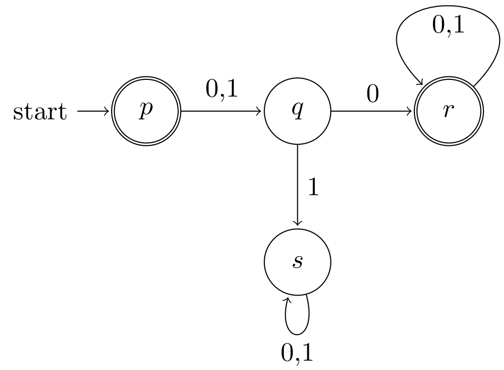
**Solution:**




**Question 2**. (12 points) Give DFAs accepting the same language as the following regular expressions using the minimum number of states. Give reason why you cannot have a DFA with lesser number of states.

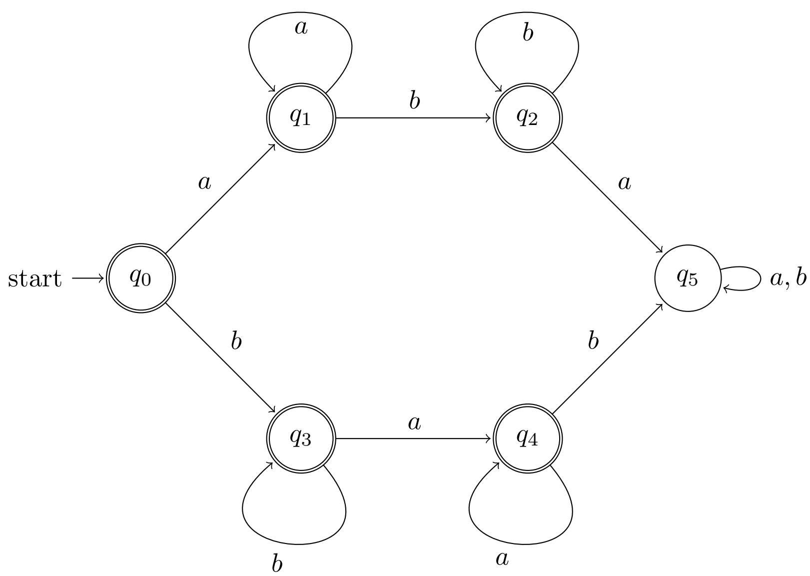
(a) $\epsilon + (0+1)0(0+1)^*$

**Solution:**




(b) $(a^*b^* + b^*a^*)$

**Solution:**




**Question 3**. (10 points) For languages $A$ and $B$ over $\Sigma$, define

$$f(A, B) = \{w \in \Sigma^* \mid w = a_1b_1a_2b_2 \dots a_kb_k, \text{ where } a_1, a_2, \dots, a_k \in A \text{ and } b_1, b_2, \dots, b_k \in B, \text{ each } a_i, b_i \in \Sigma^*\}.$$

Show that if $A$ and $B$ are regular then $f(A, B)$ is also regular.

**Solution:** $f(A, B)$ is $(AB)^*$. By closure properties of regular languages, $(AB)^*$ is regular. Hence, $f(A, B)$ is regular.

P.S:-Those who have correctly constructed the NFA for $(AB)^*$ has also been given full marks. (unless some terms in the constructed NFA are not properly defined, in which case some marks are cut).

Another WRONG solution which is frequently given is as follows:-

**Solution:** Let $D_1 = (Q_1, \Sigma, \delta_1, q_{01}, F_1)$ and $D_2 = (Q_2, \Sigma, \delta_2, q_{02}, F_2)$ be two DFAs such that $A = L(D_1)$ and $B = L(D_2)$. We define an NFA $N = (Q, \Sigma, \delta q_s, F)$ that accepts $f(A, B)$ as follows:

- $Q = Q_1 \times Q_2$ is the set of states.
- The transition function $\delta$ of $N$ is defined as

$$\delta((x, y), a) \quad = \quad \{(\delta_1(x, a), y), (x, \delta_2(y, a))\}$$

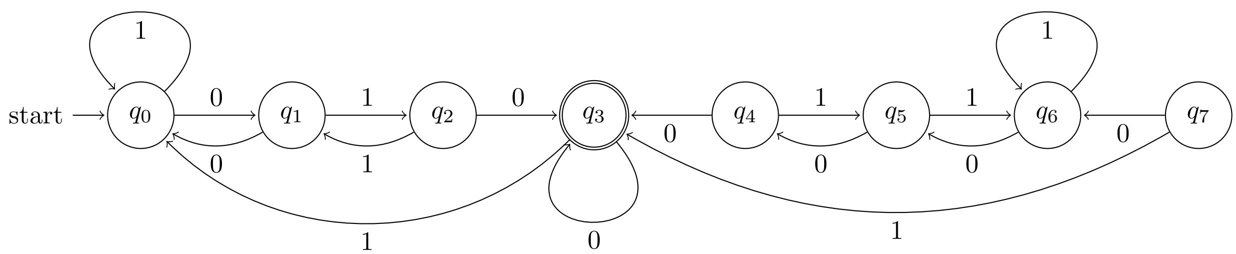
- $(q_{01}, q_{02})$ is the start state.
- $F = F_1 \times F_2$ is the set of accept states.

The above solution will work only if $f(A, B) = \{w \in \Sigma^* \mid w = a_1 b_1 a_2 b_2 \ldots a_k b_k,$ where $a_1 a_2 \ldots a_k \in A$ and $b_1 b_2 \ldots b_k \in B,$ each $a_i, b_i \in \Sigma^*\}$. Notice the difference of commas between $a_1$, $a_2$ etc. Hence, anyone who has given this solution is awarded 0 marks.

Some people have given a mixture of the correct solution and wrong solution. They have been awarded partial marks.

Some people have considered $k$ to be an arbitrary constant instead of a variable. Since that is not clear from the question, full marks have been awarded in such cases (unless there are other problems).

**Question 4**. (10 points) Find the minimum-state finite automton corresponding to the following DFA. Show in details all the steps of minimization.




**Solution:** Remove the unreachable states $q_4 . q_5, q_6, q_7$.

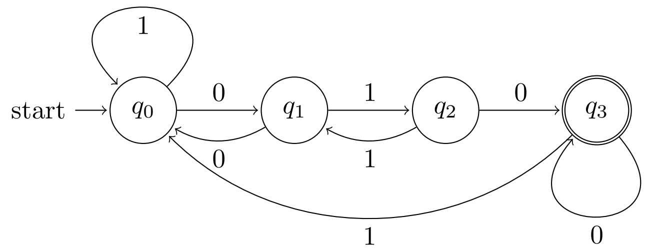
Group the states into two sets of accept and reject states:- $\{q_3\}$ and $\{q_0, q_1, q_2\}$

Repeatedly check if the states within the same set are equivalent or not by considering whether the transitions on 0 and 1 lead these states to the same sets or not.

In this way, we see $q_0$ and $q_1$ on 0 and 1 remains in $\{q_0, q_1, q_2\}$ but $q_2$ on 1 remains in $\{q_0, q_1, q_2\}$ and on 0 moves to $q_3$. Thus, $q_2$ is made into a seperate set from $\{q_0, q_1\}$.

Similarly, we can see in the next round that $q_0$ and $q_1$ are also not equivalent. Hence, we now have four set of states $\{q_0\}, \{q_1\}, \{q_2\}, \{q_3\}$. Thus no further minimization is possible.

The minimized DFA is as follows:-




P.S:- Anyone who has followed any other minimization process has been given full marks(unless there is some other problem). Some students have not removed the unreachable states. They have been awarded partial marks.

**In Q1 and Q2, if there are minor mistakes, then partial marks have been given but if there are mistakes like wrong transitions, wrong or no accept states etc. then no marks have been awarded. This is followed for everyone. Similarly, in Q3, no marks have been awarded for the WRONG solution (discussed above). No further requests for regrading or partial marks in this regard will be entertained as it will require all papers to be reevaluated. Only ask for regrading if you find some counting mistake in marks, or some answer is not checked/ignored etc.**