Name: Rollno:

CSE340: Theory of Computation (Mid Semester Exam)

September 15, 2021

Total Number of Pages: 8 Total Points 80

Instructions

1. Read these instructions carefully.
2. Write you name and roll number on all the pages of the answer book.
3. Cheating or resorting to unfair means will be severely penalized.
4. Do not exchange question books or change the seat after obtaining question paper.
5. Using pens (blue/black ink) and not pencils. Do not use red pens for answering.
6. Every question should be done on a new page. Parts of the same question can be done on the same page. Failure to do this will result in a penalty of 5 marks.

Helpful hints

1. It is advisable to solve a problem first before writing down the solution.
2. The questions are *not* arranged according to the increasing order of difficulty. Do a quick first round where you answer the easy ones and leave the difficult ones for the subsequent rounds.

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1 Objective Questions

Question 1. (1 point) Consider the languages $L_1 = \{\}$ and $L_2 = \{\epsilon\}$. Which of the following is equivalent to $\{\}$?

A. L_1L_2

B. $L_1^*L_2$

C. $L_1L_2^* \cup L_1^*$

D. $L_1^*L_2^* \cup L_1$

Question 2. (1 point) What is the minimum number of states required to construct a DFA to recognize $L = \{0^{5n} | n > 0\}$?

A. 4

B. 5

C. 6

D. 7

Question 3. (2 points) Which of the following regular expression are equivalent?

$$
\begin{aligned}
R_1: &\quad a^*(ab^*a^* + b^*) + b^*(ba^*b^* + a^*) \\
R_2: &\quad (a^* + b^*)(a^* + b^*)(b^* + a^*) \\
R_3: &\quad a^*b^*a^* + b^*a^*b^* \\
R_4: &\quad a^*b^* + bb^*a^* + aa^*b^*a^*
\end{aligned}
$$

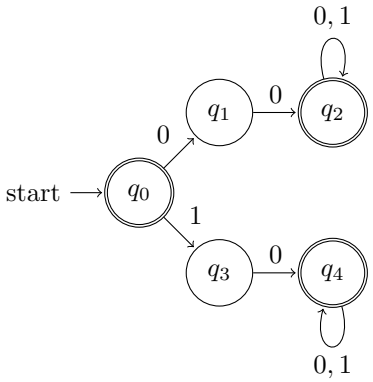
A. Only R_1 and R_3

B. Only R_3 and R_4

C. Only R_2, R_3 and R_4

D. Only R_1, R_2 and R_3

Question 4. (2 points) What is the regular expression corresponding to the following NFA?



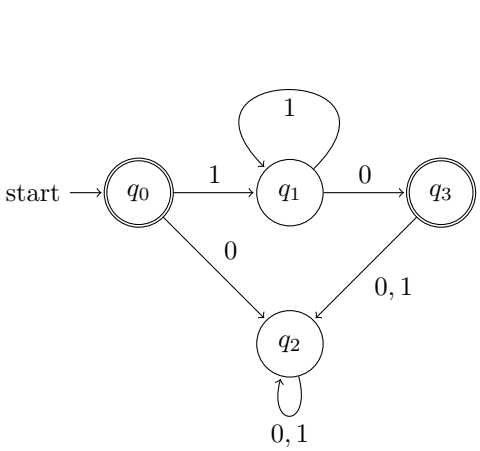
A. $(\epsilon + (0+1)0)(0+1)^*$

B. $(\epsilon + 0 + 1)0(0+1)^*$

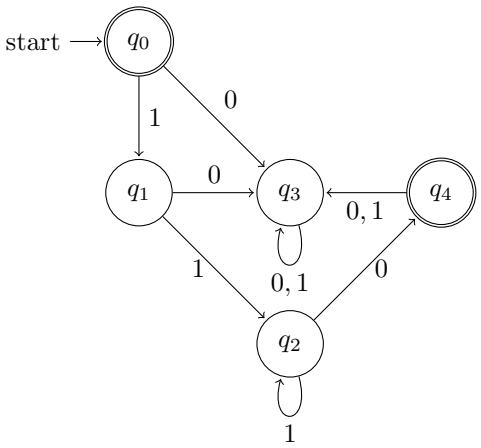
C. $(\epsilon + 00 + 10)(0+1)^*$

D. $\epsilon + (0+1)0(0+1)^*$

Question 5. (5 points) Consider the following two DFAs A_1 and A_2



DFA A_1



DFA A_2

Which one of the following is true?

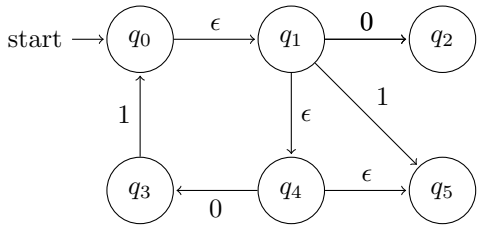
A. $L(A_1) \subsetneq L(A_2)$

B. $L(A_2) \subsetneq L(A_1)$

C. $L(A_1) = L(A_2)$

D. None of the above

Question 6. (1 point) Consider the following NFA.



Which states belong to the ϵ-closure of the state q_1 in the above NFA?

A. q_0

B. q_1

C. q_4

D. q_5

Question 7. (1 point) Context-free languages are closed under which of the following operations?

A. Set difference

B. Complementation

C. Concatenation

D. Union

Question 8. (1 point) Which of the following production rules are not allowed in a grammar in Chomsky Normal Form (S is the start symbol)?

A. $A \longrightarrow BCD$

B. $S \longrightarrow SA$

C. $A \longrightarrow AB$

D. $A \longrightarrow aB$

Question 9. (2 points) A CFG G is given below with S as the start symbol.

$$\begin{aligned} S &\longrightarrow aS \mid A \\ A &\longrightarrow aAb \mid bAa \mid \epsilon \end{aligned}$$

Which of the following strings is generated by the grammar?

A. aabbaba

B. aababab

C. abababb

D. aabbaab

Question 10. (2 points) Consider the language

$$L = \{w \in \{0,1\}^* \mid w \text{ begins with 11 and has an even number of 1's}\}.$$

How many states will the minimal DFA for L have?

A. 6

B. 5

C. 3

D. 4

Question 11. (1 point) Identify the language generated by the following grammar -

$$\begin{aligned} S &\longrightarrow AB \\ A &\longrightarrow aAb \mid \epsilon \\ B &\longrightarrow bB \mid b \end{aligned}$$

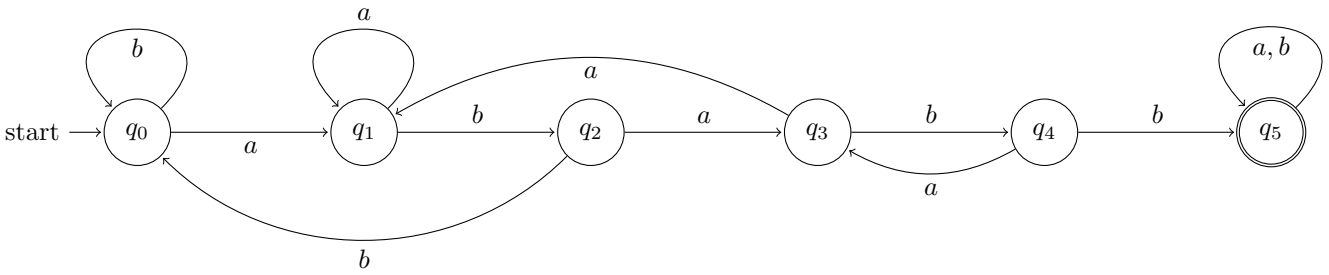
A. $\{a^m b^n \mid n \geq m, m > 0\}$

B. $\{a^m b^n \mid n \geq m, m \geq 0\}$

C. $\{a^m b^n \mid n > m, m > 0\}$

D. $\{a^m b^n \mid n > m, m \geq 0\}$

Question 12. (2 points) What is the language accepted by the following DFA?



A. The set of string containing *ababb* as substring

B. The set of strings beginning with *ababb*

C. The set of strings ending with *ababb*

D. The set of strings with *bababb* as substring

Question 13. (2 points) Consider the following two grammars: G_1:

$$S \longrightarrow SbS \mid a$$

G_2:

$$\begin{aligned} S &\longrightarrow aB \mid ab \\ A &\longrightarrow AB \mid a \\ B &\longrightarrow ABb \mid b \end{aligned}$$

Which of the following option is correct?

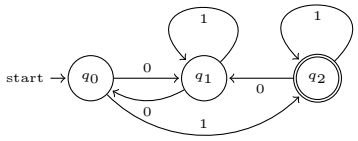
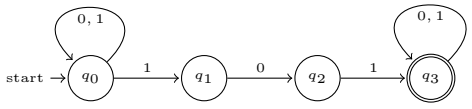
A. Only G_1 is ambiguous.

B. Only G_2 is ambiguous.

C. Both G_1 and G_2 are ambiguous.

D. Neither G_1 nor G_2 are ambiguous.

Question 14. (5 points) Match the NFAs , DFAs and REs in column A with their appropriate languages in column B.

Column A	**Column B**
(1) 	(a) $\{w \in \{0,1\}^* \mid w$ ends with 1 and has even no. of 0's$\}$
(2) $00^*1(0+1)^*$	(b) $\{w \in \{0,1\}^* \mid w$ has 101 as a substring$\}$
(3) $0^*(\epsilon + 10^*(\epsilon + 10^*))$	(c) $\{w \in \{0,1\}^* \mid w$ has at most two 1's$\}$
(4) 	(d) $\{w \in \{0,1\}^* \mid w$ begins with 0 and has at least one 1$\}$

Choose the correct matching from following choices.

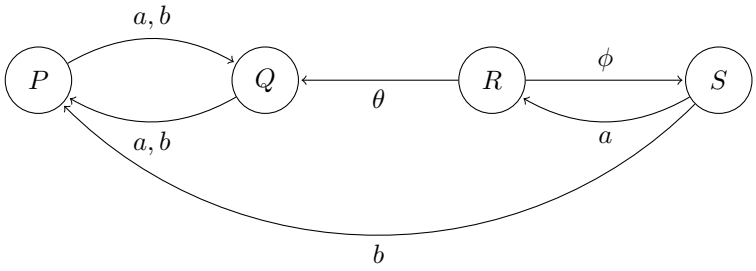
A. 1-b, 2-d, 3-a, 4-c

B. 1-b, 2-c, 3-a, 4-d

C. 1-a, 2-d, 3-c, 4-b

D. 1-d, 2-c, 3-a, 4-b

Question 15. (5 points) Consider the following finite state machine A



Choose the correct modifications so that A becomes a DFA and

$$L(A) = \{w \in \{a,b\}^* \mid w \text{ has odd length with at least one } b\}.$$

A. Replace θ by $'b'$ and ϕ by $'a'$ and make P starting state and S accept state

B. Replace θ by $'b'$ and ϕ by $'a'$ and make S starting state and P accept state

C. Replace θ by $'a'$ and ϕ by $'b'$ and make P starting state and Q accept state

D. Replace θ by $'a'$ and ϕ by $'b'$ and make Q starting state and P accept state

Question 16. (1 point) If $L = \{a, ab\}$, which of the following are in L^*?

A. aabaaab

B. abababaab

C. abababbab

D. bababaaba

Question 17. (2 points) Consider the following grammar.

$$\begin{aligned} S &\longrightarrow AB \\ A &\longrightarrow a \mid BaB \\ B &\longrightarrow bbA \end{aligned}$$

Which of the following statements is/are true?

A. The length of every string produced by this grammar is even.

B. No string produced by this grammar has three consecutive as.

C. The grammar is in Chomsky Normal Form.

D. The string *abbaab* is generated by this grammar.

Question 18. (2 points) Let R be a regular language and L be a context-free language. Which of the following is necessarily true?

A. $L \cap R$ is regular

B. $\overline{R}$ is regular

C. If $L \cup R$ is regular then L is regular

D. $L \cup R$ is context-free

Question 19. (2 points) Which of the following languages are regular?

A. $\{w \in \{0,1\}^* \mid w$ **has an even number of 0's**$\}$

B. $\{w \in \{0,1\}^* \mid w$ has equal number of 0's and 1's$\}$

C. $\{w \in \{a,b,c\}^* \mid w$ **has no** c **to the left of an** $a\}$

D. $\{w \in \{a,b,c\}^* \mid w$ **has 10** c**'s after every** $a\}$

Question 20. (5 points) Let L be a language over Σ. Define

$$L' = \{w \in \Sigma^* \mid wx \in L \text{ for some } x \in \Sigma^* \text{ and } |w| = |x|\}.$$

Which one of the following statements are correct?

A. If L is regular then L' is regular

B. If L is regular then L' is non-regular

C. There exists a regular language L such that L' is not regular

D. There exists a non-regular language L such that L' is regular

2 Subjective Questions

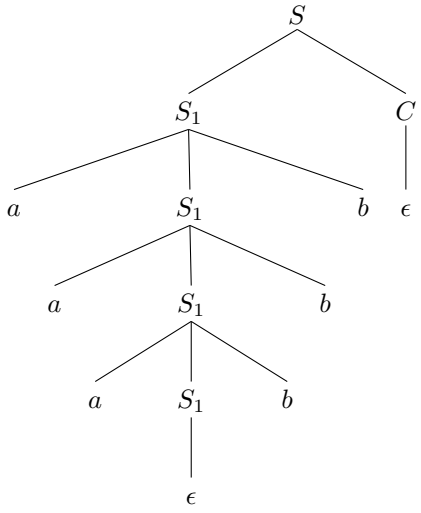
Question 21. Consider the CFG G given by the following production rules

$$\begin{aligned} S &\longrightarrow S_1C \mid AS_2 \\ S_1 &\longrightarrow aS_1b \mid \epsilon \\ S_2 &\longrightarrow bS_2c \mid \epsilon \\ A &\longrightarrow aA \mid \epsilon \\ C &\longrightarrow cC \mid \epsilon \end{aligned}$$

(a) (2 points) The language of G,
$L(G) = \{w \in \{a,b,c\}^* \mid$ $w = a^i b^j c^k,\ i = j$ **or** $j = k$ $\}$

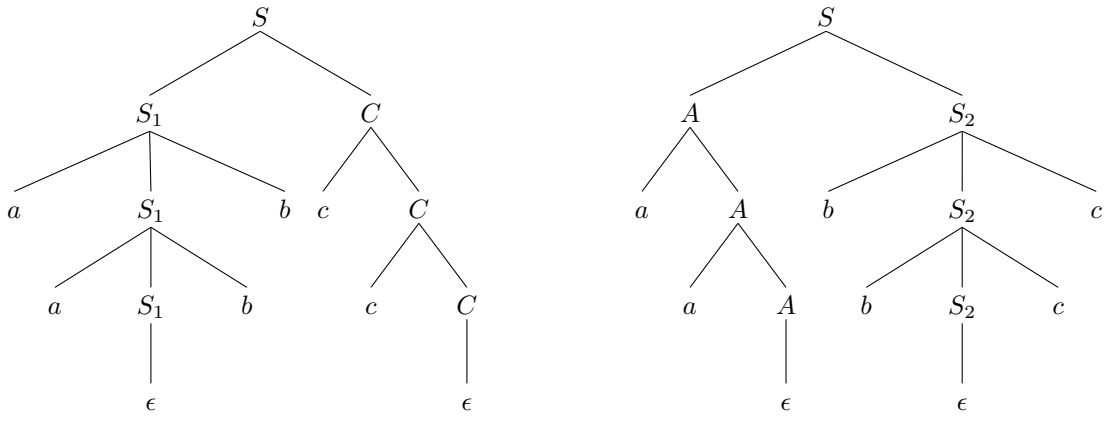
(b) (4 points) Give a string of length 6 in $L(G)$ that has a unique parse tree with respect to G. Draw the parse tree.

Solution: a^3b^3 (b^3c^3, abc^4 and a^4bc are also correct answers)



(c) (4 points) Give a string of length 6 in $L(G)$ that has two parse trees with respect to G. Draw the two parse trees.

Solution: $a^2b^2c^2$



Question 22. (10 points) Give a CFG for the language

$$L = \{a^i b^j c^k \mid j \leq i + k \leq 2j, i, j, k \geq 0\}.$$

For each variable used in your CFG, describe the language generated by the variable.

Solution:

$$\begin{aligned} S &\longrightarrow T_1T_2 \mid aT_1bT_2c \\ T_1 &\longrightarrow aT_1b \mid aaT_1b \mid \epsilon \\ T_2 &\longrightarrow bT_2c \mid bT_2cc \mid \epsilon \end{aligned}$$

- Language generated by T_1 is $\{a^i b^{j_1} \mid j_1 \leq i \leq 2j_1\}$
- Language generated by T_2 is $\{b^{j_2} c^k \mid j_2 \leq k \leq 2j_2\}$
- Language generated by S is $\{a^i b^j c^k \mid j \leq i + k \leq 2j\}$

Question 23. (5 points) Prove that no infinite subset of the language $L = \{0^n 1^n \mid n \geq 0\}$ is regular.

Solution: Given $p > 0$, set $w = 0^q 1^q$ where q is some number greater than p and $0^q 1^q$ is in the language. Since it is an infinite subset, therefore existence of such a string is guaranteed. Rest of the proof is similar to the proof that L is not regular using pumping lemma.

Question 24. (10 points) Let $A \subseteq \{0,1\}^*$ and let

$$A' = \{xy \mid x1y \in A\}.$$

That is, A' contains all strings obtained from a string in A by deleting exactly one 1. Show that if A is regular, then A' is also regular (give the construction only).

Solution: Let $D = (Q, \{0,1\}, \delta, q_0, F)$ be a DFA for A. We construct an NFA $N = (Q', \{0,1\}, \delta', q_0', F')$ for A', where $Q' = Q \times (\alpha, \beta)$, $q_0' = (q_0, \alpha)$, $F' = \{(f, \beta) \mid f \in F\}$ and δ' is defined as follows:

$$\begin{aligned} \delta'((q,\alpha), a) &= \{(\delta(q,a), \alpha)\} \\ \delta'((q,\beta), a) &= \{(\delta(q,a), \beta)\} \\ \delta'((q,\alpha), \epsilon) &= \{(\delta(q,1), \beta)\} \end{aligned}$$

Essentially, states of the form (q, α) corresponds to the automata not having skipped a 1 yet and states of the form (q, β) corresponds to the fact that the automata has skipped a 1.