Suppose you are standing in a well-illuminated room with a large window, and you take a picture of the scene outside. The window undesirably acts as a semi-reflecting surface, and hence the picture will contain a reflection of the scene inside the room, besides the scene outside. While solutions exist for separating the two components from a single picture, here you will look at a simpler-to-solve version of this problem where you would take two pictures. The first picture g_1 is taken by adjusting your camera lens so that the scene outside (f_1) is in focus (we will assume that the scene outside has negligible depth variation when compared to the distance from the camera, and so it makes sense to say that the entire scene outside is in focus), and the reflection off the window surface (f_2) will now be defocussed or blurred. This can be written as $g_1 = f_1 + h_2 * f_2$ where h_2 stands for the blur kernel that acted on f_2 . The second picture $g2$ is taken by focusing the camera onto the surface of the window, with the scene outside being defocussed. This can be written as $g_2 = h_1 * f_1 + f_2$ where h_1 is the blur kernel acting on f_1 . Given g_1 and g_2 , and assuming h_1 and h_2 are known, your task is to derive a formula to determine f_1 and f_2 . Note that we are making the simplifying assumption that there was no relative motion between the camera and the scene outside while the two pictures were being acquired, and that there were no changes whatsoever to the scene outside or inside. Even with all these assumptions, you will notice something inherently problematic about the formula you will derive. What is it? $[8 + 7 = 15 points]$

Solution :

From the question we have,

$$g_1 = f_1 + h_2 \star f_2 \quad \text{and}$$
$$g_2 = h_1 \star f_1 + f_2$$

Taking Discrete Fourier Transform, we obtain :

$$\Longrightarrow \qquad G_1 = F_1 + H_2 F_2 \quad \text{and}$$
$$\Longrightarrow \qquad G_2 = H_1 F_1 + F_2$$

Now, Solving the linear equations in F_1 and F_2,

$$F_1 = \frac{G_1 - H_2 G_2}{1 - H_1 H_2}$$
$$F_2 = \frac{G_2 - H_1 G_1}{1 - H_1 H_2}$$

For generality, adding some noise n_1 and n_2 we would get,

$$F_1 = \frac{G_1 - H_2 G_2}{1 - H_1 H_2} - \frac{N_1}{1 - H_1 H_2} + \frac{H_2 N_2}{1 - H_1 H_2}$$
$$F_2 = \frac{G_2 - H_1 G_1}{1 - H_1 H_2} - \frac{N_2}{1 - H_1 H_2} + \frac{H_1 N_1}{1 - H_1 H_2}$$

Hence f_1 and f_2 can be computed as,

$$f_1 = F^{-1}(F_1)$$
$$f_2 = F^{-1}(F_2)$$

The problem with this solution is the denominator part $(1 - H_1 H_2)$. H_1 and H_2 are blur kernels and do not amplify the images. As a result, for lower frequencies both $H_1 \to 1$ and $H_2 \to 1$. This makes the denominator $(1 - H_1 H_2) \to 0$, making the system unstable at lower frequencies. The DC component isn't affected by blur and hence $H_1(0, 0) = H_2(0, 0) = 1$, which leads to infinity for f_1 and f_2.