

CS-663 Assignment 4 Q1

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Given a matrix A of size $m \times n$, write a MATLAB routine called MySVD which takes this matrix as input and outputs the left and right singular vectors (i.e. column vectors of U and V under usual notation) and the singular values (i.e. diagonal entries of S) of A . You are not allowed to use the `svd` or `svds` functions of MATLAB directly. You should use only the eigenvalue decomposition routines `eig` or `eigs` for this task. Cross-check your answer by verifying that $A = USV^T$ based on your computation. [15 points]

In order to use eigen value decomposition (EVD) to calculate the singular value decomposition (SVD) components(U, Σ, V^T), for a given matrix A , we need to first calculate either of AA^T or $A^T A$, calculate their eigen vectors and eigen values.

Here, we calculate first the eigen vectors(V) and eigen values(Λ) of $A^T A$, sort in descending order and calculate the other unitary vector matrix U from the relation $AV = \Sigma U$.

mySVD.py

```
1  import numpy as np
2
3  def SVD(matrix):
4      """Calculates SVD using eigen value decomposition method and
5          returns the U,S,V_T
6          :param matrix: any random matrix
7          :output U,S,V_T : the singular value components
8          """
9      # calculating A'A
10     temp1 = np.matmul(matrix.T,matrix)
11     # calculating eigen values and vectors of A'A
12     lambda_,V = np.linalg.eig(temp1)
13
14     # sorting the singular values and in descending order
15     sorted_lambda = np.sort(lambda_)::-1
16
17     idx = np.argsort(lambda_)::-1
18     V_sort = np.zeros_like(V)
19     _,c = V_sort.shape
20
21     for i in range(c):
22         V_sort[:,i] = V[:,idx[i]]
23
24     U_sort = np.matmul(matrix,V_sort)
25     _,c = U_sort.shape
26     for i in range(c):
27         U_sort[:,i] = U_sort[:,i]/np.sqrt(np.sum(np.square(U_sort[:,i])))
28
29     sigma = np.sqrt(sorted_lambda)
30     return U_sort,sigma,V_sort.T
31
32
33 if __name__=="__main__":
34     m,n = 4,4
```

```

35     matrix = np.random.randint(1,10,(m,n))
36     print("Input matrix of size {}x{}:\n {}".format(m,n,matrix))
37     print("*****")
38     U,S,V_T = SVD(matrix)
39     print("U matrix\n",U)
40     print("Singular Values\n",S)
41     print("V'(V transpose) Matrix\n",V_T)
42     print("*****")
43     print("calculated matrix: \n",np.matmul(np.matmul(U,np.diag(S)),V_T))

```

Sample Program Output

Input matrix of size 4x4:

```
[[6 8 6 2]
```

```
[5 9 2 2]
```

```
[9 1 7 9]
```

```
[6 4 9 3]]
```

U matrix

```
[[ 0.49902061 -0.40367479 -0.17508303 -0.74657286]
```

```
[ 0.39465136 -0.60441194  0.50622627  0.4718808 ]
```

```
[ 0.57958458  0.68105716  0.4395442  -0.08392709]
```

```
[ 0.50922535  0.08884829 -0.72102829  0.46142575]]
```

Singular Values

```
[22.48631246  9.09254006  4.43526137  0.14115208]
```

V'(V transpose) Matrix

```
[[ 0.58875788  0.45185323  0.55249373  0.37939885]
```

```
[ 0.13401049 -0.8394415  0.21293963  0.48170102]
```

```
[ 0.25034854  0.16026185 -0.77796787  0.55354109]
```

```
[-0.7567899  0.255896  0.21017466  0.56357156]]
```

calculated matrix:

```
[[6. 8. 6. 2.]
```

```
[5. 9. 2. 2.]
```

```
[9. 1. 7. 9.]
```

```
[6. 4. 9. 3.]]
```