Soham Patil
RDC
Assignment 1

1. a) Rigid Body Transformation
→ Any transformation on a set of points that preserves distance between all the points after the transformation can be termed as a Rigid Body Transform.

→ Mathematically it should satisfy the below:
Given a set of points that are transformed using function $g(p)$ and $p = \{a_1, a_2, \ldots\}$ (points) following should be satisfied

i) Length is preserved

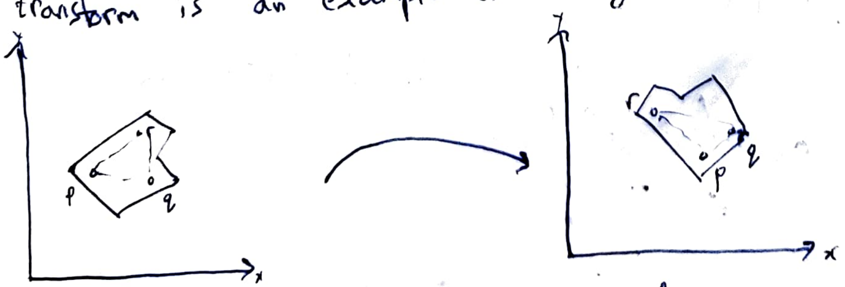
$$\|a_1 - a_2\| = \|g(a_1) - g(a_2)\|$$

ii) Cross Product is preserved for $v_1(a_1 \to a_2)$ and $v_2(a_2 \to a_3)$

$$g^*(v_1 \times v_2) = g^*(v_1) \times g^*(v_2)$$ (Avoids Reflection)

Note: $g^* \neq g$ · g^* is the transformation for the vectors

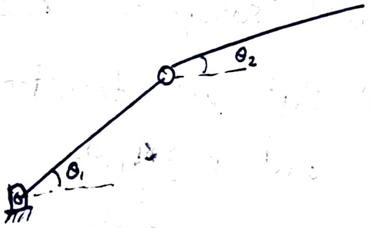
→ Example
Any (rotational) or (translational) or (combination of rotational and translation) transform is an example of Rigid Body Transform.



∴ $\|\vec{pq}\|$, $\|\vec{qr}\|$ & $\|\vec{pr}\|$ are preserved
& Direction of $(\vec{pq} \times \vec{qr})$ is preserved as well

b) Configuration Space

- The configuration of a robot is the specification for all the points in it. (Also called Joint Space).
- The configuration space is the metric space of all such configurations.
- Example - 2R Manipulator



$\theta_1 \in S_1 \quad [0, 2\pi]$

$\theta_2 \in S_1 \quad [0, 2\pi]$

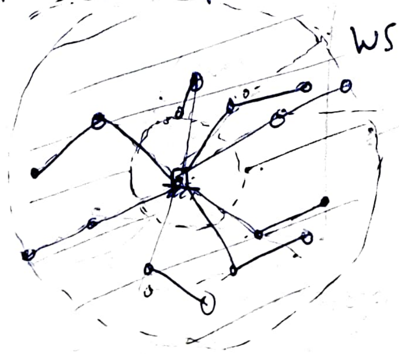
An example of a configuration for the above robot would be $[\theta_1 = 2.5\text{ rad}, \theta_2 = 1.3\text{ rad}]$

A space of all such configurations would be the ordered pairs between all such possibilities

$\therefore \boxed{\text{C-Space} = S^1 \times S^1}$

c) Workspace

- This is the reachable configuration of a robot's end effector.
- For Planar Robots, $W \subseteq \mathbb{R}^2 \times S^1$
  For Spatial Robots, $W \subseteq \mathbb{R}^3 \times T^3$
- Example - 2R Manipulator



WS = Shaded Portion (Annulus)

d) Task Space

→ This is the space in which the task ~~are~~ expressed

→ Can be considered as the C-Space of the end effector.

→ Example - 2R Manipulator

$\boxed{TS = \mathbb{R}^2}$

e) Degree of Freedom

→ Minimum variables required to fully characterise the configurations of your robot

→ It is also the dimension of the C-Space

→ DoF = (Total Individual Freedoms) - (Total Individual Constraints)

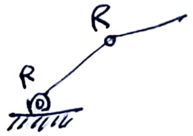
→ As such, a Gruebler's formula also exists derived from the above to calculate DoFs on the basis of links and joints

$$DoF = \underbrace{K(n-1}_{\text{Links}} \underbrace{- j)}_{\text{Joints}} + \underbrace{\sum f_j}_{\text{Individual Freedoms}}$$

K = 3 (planar)

K = 6 (spatial)

→ Example - 2R Manipulator



In a planar system, each joint would have 3 freedoms (x, y, θ)

Due to revolute joint, x & y are constrained.

$\therefore$ DoF = 3+3 - 2 - 2

$= \boxed{2}$

2. a) → ~~E~~ Explicit Parametrization -

- When n-dimensional space is represented by n parameters. $y = f(x)$ form
- This is usually straightforward to work with however it is plagued by problems of singularity.
- Can be avoided by limiting task space.

Ex - Euler Angle representation

→ Implicit Parametrization

- When n-dimensional space is represented by m parameters, $m > n$, $(m-n)$ constraints.

$f(x, y)$ form

- Harder to ~~x~~ formulate and work with
- Solves the problems of singularity.

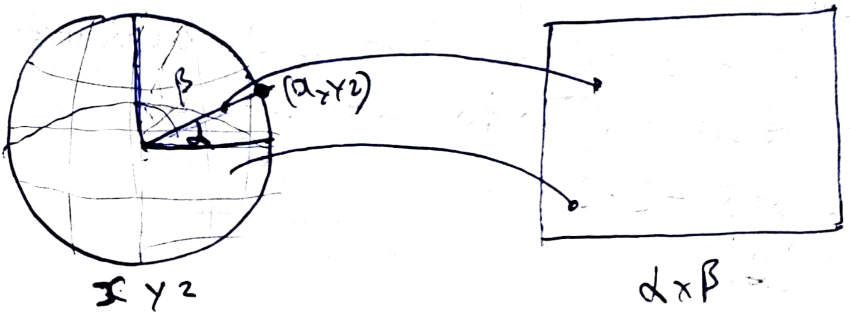
Ex - Quaternion.

b) → Coordinate Singularity

- Is a type of discontinuity or redundancy in a co-ordinate frame.
- Solved by using another frame / implicit parametrization.

Ex - Representing Spherical Coordinates on Earth with Latitude(α) and Logitude (β)

$\alpha \times \beta$ representation is a plane, and is topologically different from sphere



$\therefore$ Poles have infinite representations

as $(0, 0, 1)$ is satisfied by
$\begin{array}{cc} x & y \quad z \end{array}$

$\left(\theta, \frac{\pi}{2}\right)$ for all values of θ.
$\begin{array}{cc} \alpha & \beta \end{array}$

Thus $\alpha = \arcsin(z)$
$\beta = \operatorname{arctan2}(x, y)$ which is undefined

for $x = y = 0$

→ Switching at edges of plane $\alpha\beta$ also causes issues in velocity calculation.

3. a) $\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix} \begin{array}{l} \in S^1 \\ \in S^1 \\ \in S^1 \\ \in S^1 \\ \in S^1 \\ \in S^1 \end{array}$

Note: Assuming the robot is not allowed to come out of plane of blackboard. and duster is used by also rotating in plane of blackboard

$\therefore$ Joints Space $= S^1 \times S^1 \times S^1 \times S^1 \times S^1 \times S^1$
$= \mathbb{T}^6$

b) Task Space $= \mathbb{R}^2 \times S^1$ ~~$\times S^1 \times S^1$~~ [Blackboard]

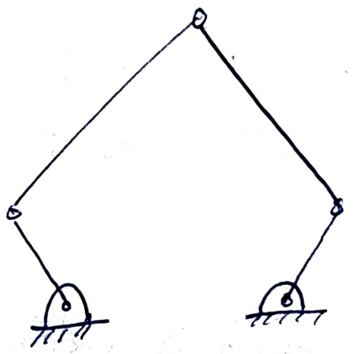
Represent with ~~$[x, y, z, \theta_1, \theta_2, \theta_3]^T$~~ $[x, y, \theta]$

c) Workspace
$W \subseteq \mathbb{R}^3 \times S^1 \times S^1 \times S^1$

Represented with
$[x, y, z, \theta_1, \theta_2, \theta_3]$

The topology will be quite complicated to visualise but will have some similarity to a 3D ~~Annulus (Spherical)~~ Sphere - 3D sphere with parts cut out, if we consider constraint real body. otherwise, Workspace is a Sphere (Solid).

4. a)



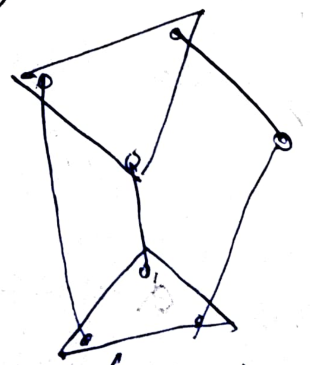
∴ $N=5$ ∴ $k=3$

∴ $J=5$

∴ $DoF = 3(5-1-5) + 5$

$= \boxed{2}$

b)



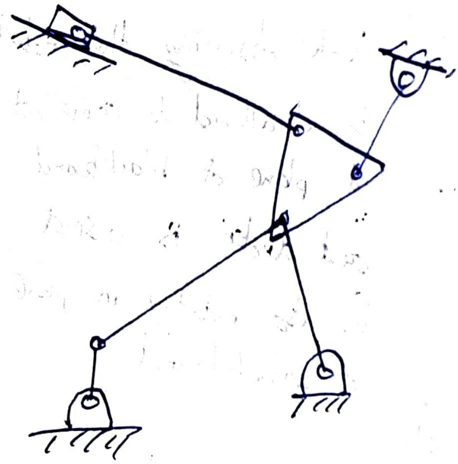
∴ $N=6$ ∴ $k=3$

∴ $J=7$

∴ $DoF = 3(6-1-7) + 7$

$= \boxed{1}$

c)



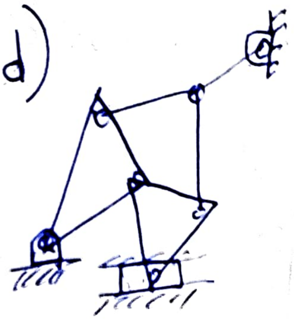
∴ $N=8$ ∴ $k=3$

∴ $J=10$

∴ $DoF = 3(8-1-10) + 10$

$= \boxed{1}$

d)



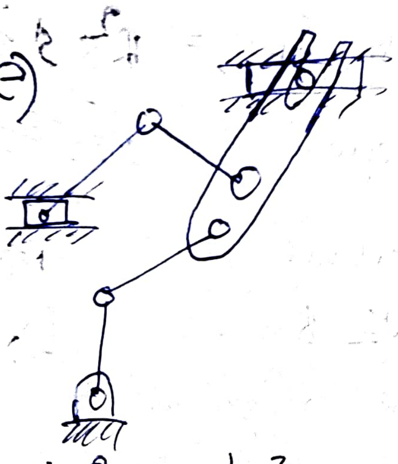
∴ $N=7$ ∴ $k=3$

∴ $J=9$

∴ $DoF = 3(7-1-9) + 9$

$= \boxed{0}$

e)



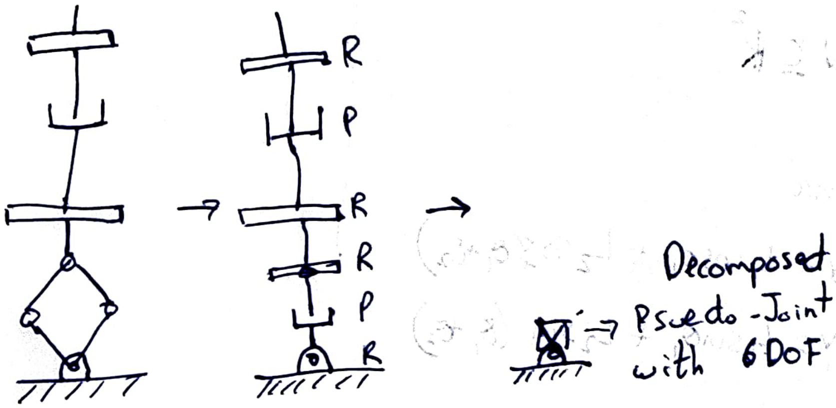
∴ $N=8$ ∴ $k=3$

∴ $J=9$

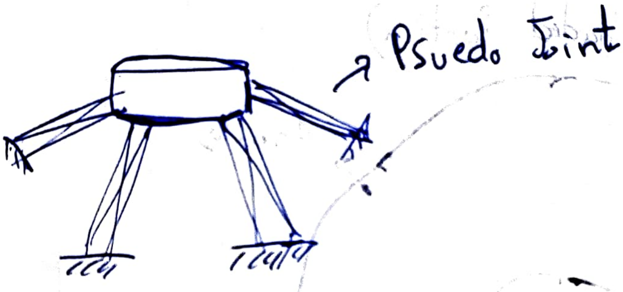
∴ $DoF = 3(8-1-9) + 10$

$= \boxed{4}$

5. a) Lets consider each leg as a seperate unit and decompose it into a psuedo-joint.



$\therefore$ For Entire Mechanism



$\therefore$ $N=2$, $J=4$, $K=6$

$\therefore$ $D.F = 6(2-1-4) + 4\times 6$

$= \boxed{6}$

b) Generalised Formula, lets take J joints/Legs

$\therefore$ $D.F = 6(2-1-J) + 6\times J$

$= 6(2-1-J+J)$

$= \boxed{6}$ (Independent from number of Joints/Legs)

6. a) C-Space

$S^1 \times S^1$

$= T^2 \longrightarrow$ Torus

Note: $\theta^1, \theta^2 \in S^1$

$[0 \leq \theta_i < 2\pi]$

Constraints

$x = 2\cos\theta_1 + \cos(\theta_1 + \theta_2)$

$y = 2\sin\theta + \sin(\theta_1 + \theta_2)$

b) $W \subseteq \mathbb{R}^2$

Since

$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$

$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$

$\therefore L_1 = 2$ units

$L_2 = 1$ unit

$\therefore$ Workspace (Shaded Portion)