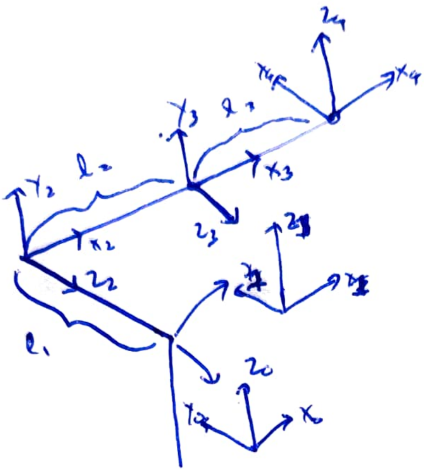
Assignment-3

1)

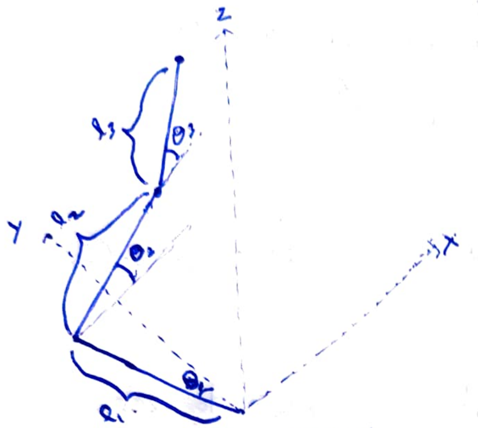
1>



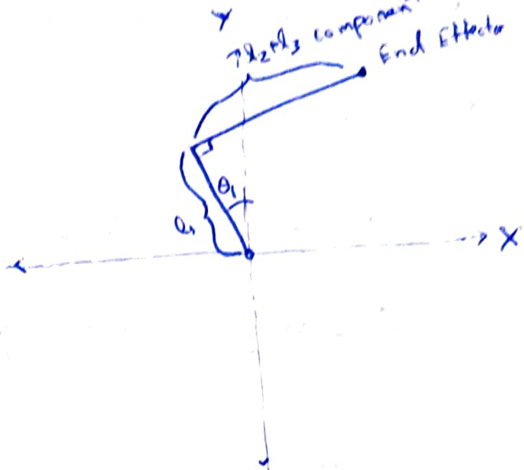
J	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$\pi/2$	0	$-l_1$	θ_2
3	0	l_2	0	θ_3
4	$-\pi/2$	l_3	0	0

2)

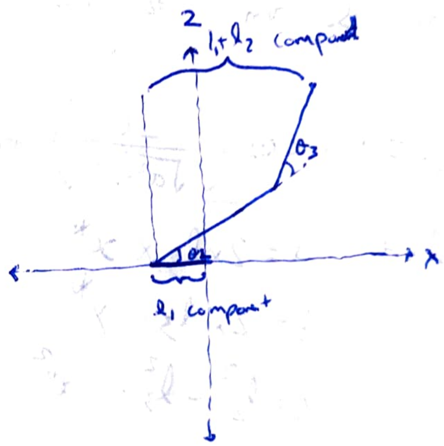
2.1)



ISOMETRIC VIEW



TOP VIEW



SIDE VIEW

$\therefore x = (l_2 c\theta_2 + l_3 c\theta_{23}) c\theta_1 - l_1 s\theta_1$ — ①

$\therefore y = (l_2 c\theta_2 + l_3 c\theta_{23}) s\theta_1 + l_1 c\theta_1$ — ②

$\therefore z = l_2 s\theta_2 + l_3 s\theta_{23}$ — ③

Lets find θ_1

$\therefore$ From ① and ②

$$\frac{x + l_1 s\theta_1}{c\theta_1} = \frac{y - l_1 c\theta_1}{s\theta_1}$$

$\therefore x s\theta_1 + l_1 s^2\theta_1 = y c\theta_1 - l_1 c^2\theta_1$

$\therefore y c\theta_1 - x s\theta_1 - l_1 = 0$

This is of the form $a c\theta + b s\theta + c = 0$ where $a = y$, $b = -x$, $c = -l_1$, whose solution we have derived in class

$$\therefore \theta_1 = \pm \arccos\left(\frac{-c}{\sqrt{a^2+b^2}}\right) + \arctan(b, a) \quad \text{(2 solutions)}$$

$$\therefore \boxed{\theta_1 = \pm \arccos\left(\frac{l_1}{\sqrt{x^2+y^2}}\right) + \arctan(-x, y)} \quad — ④$$

Since we know θ_1 through ④, we can use ①, ②, ③ to create a standard 2R Manipulator Formulation (whose Solution we already knows) using x^* any y^*

$$x^* = l_2 c\theta_2 + l_3 c\theta_{23} \quad - ⑤$$
$$y^* = l_2 s\theta_2 + l_3 s\theta_{23} \quad - ⑥$$

} 2 sets of equations will form, one for each solution of ④

where $y^* = z$ ....from ①

$$x^* = \frac{x + l_1 s\theta_1}{c\theta_1} \quad \text{or} \quad \frac{y - l_1 c\theta_1}{s\theta_1}$$ ....from ② or ③

Chose above Depending upon if denominator is close to zero

The solution to ⑤ and ⑥ is already known as

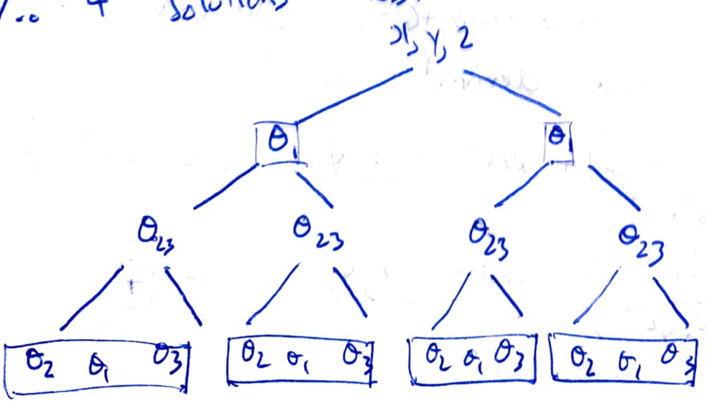
$$\boxed{\theta_{23} = \pm \arccos\left(\frac{-c}{\sqrt{a^2 + b^2}}\right) + \arctan(b, a) \quad - ⑦}$$ (Two Solutions)

where $a = 2 \times l_3 \times x^*$
$b = 2 \times l_3 \times y^*$
$c = l_2^2 - l_3^2 - x^{*2} - y^{*2}$

$$\therefore \boxed{\theta_2 = \arctan(y^* - l_3 s\theta_{23}, x^* - l_3 c\theta_{23})}$$ (One for each solution of ⑦)

$$\therefore \boxed{\theta_3 = \theta_{23} - \theta_2}$$

2.3) ∴ 4 solutions Possible



→ 4 Solutions