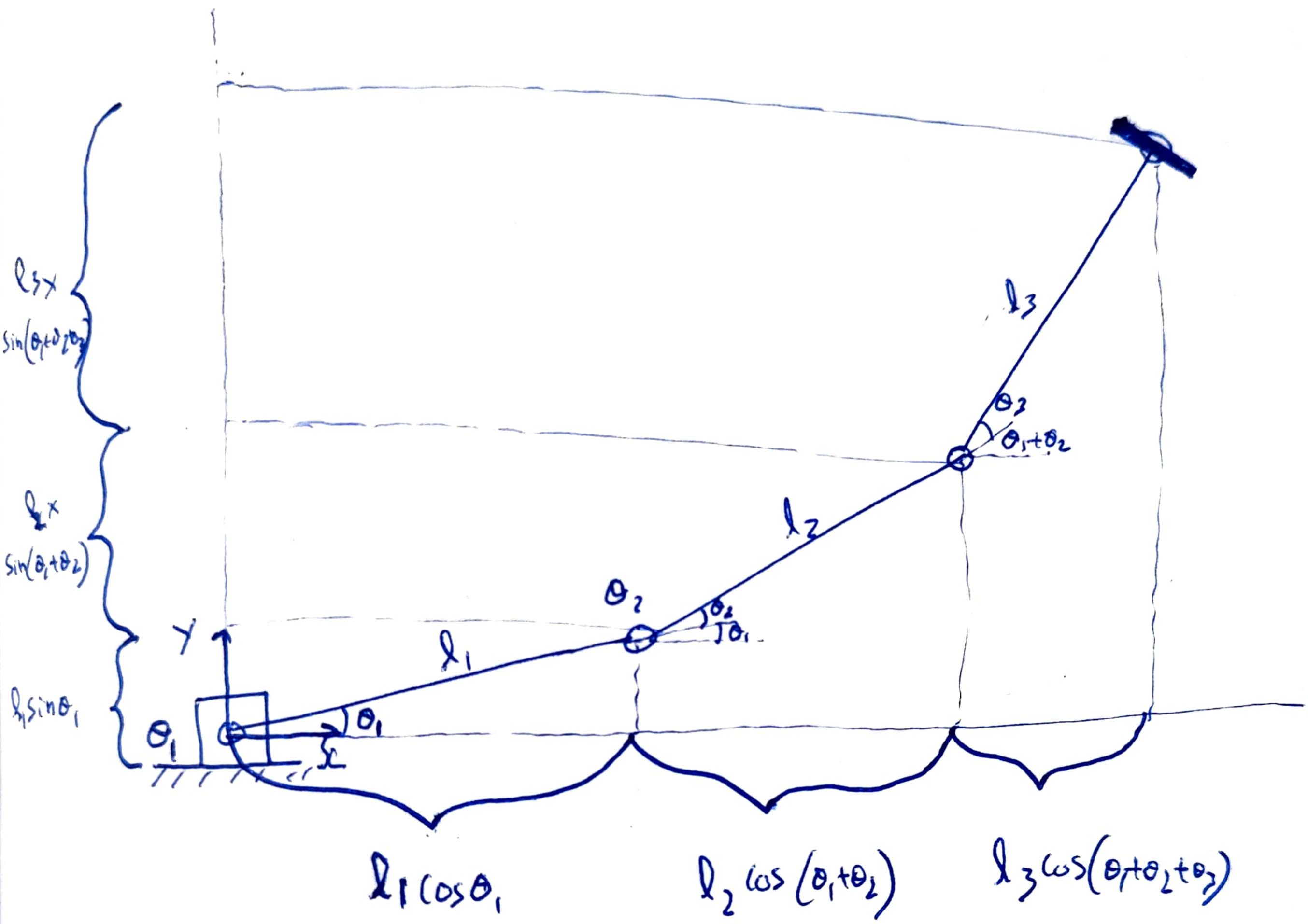
2)

2.1)



$$\therefore P_e = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 c\theta_1 + l_2 c\theta_{12} + l_3 c\theta_{123} \\ l_1 s\theta_1 + l_2 s\theta_{12} + l_3 s\theta_{123} \end{bmatrix}$$

$$\boxed{phi = \theta_1 + \theta_2 + \theta_3}$$

3)

3.1)

We know $R = I + (\sin\theta[\hat{n}]) + ((1-\cos\theta)[\hat{n}]^2)$ where $n = [n_1, n_2, n_3]^T$, (the axis of rotation) and θ is the magnitude of rotation.

$\therefore$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \sin\theta \begin{bmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{bmatrix} + (1-\cos\theta) \begin{bmatrix} -n_3^2 - n_2^2 & n_1 n_2 & n_1 n_3 \\ n_2 n_1 & -n_3^2 - n_1^2 & n_2 n_3 \\ n_3 n_1 & n_3 n_2 & -n_1^2 - n_2^2 \end{bmatrix}$$

$$R = \begin{bmatrix} c\theta + n_1^2(1-c\theta) & n_1 n_2(1-c\theta) - n_3 s\theta & n_1 n_3(1-c\theta) + n_2 s\theta \\ n_1 n_2(1-c\theta) + n_3 s\theta & c\theta + n_2^2(1-c\theta) & n_2 n_3(1-c\theta) - n_1 s\theta \\ n_1 n_3(1-c\theta) - n_2 s\theta & n_2 n_3(1-c\theta) + n_1 s\theta & c\theta + n_3^2(1-c\theta) \end{bmatrix}$$

Lets find n and θ from above given a general

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$\Rightarrow$ Case 1: $R = I$

$\therefore$ $\theta = 0$ and there is no defined axis

$\rightarrow$ Case 2:

Lets take trace (R)

$\therefore r_{11} + r_{22} + r_{33} = 3c\theta + (1-c\theta)(n_1^2 + n_2^2 + n_3^2)$ ← unit vector norm

$\therefore Tr(R) = 1 + 2c\theta$

$\therefore$ for any θ satisfying $1+2c\theta = Tr(R)$, such that

$\theta \neq \pm n\pi$, $R = e^{[\hat{n}]\theta}$

$\therefore$ if $Tr(R) = -1$

$\therefore$ $\theta = \pm\pi, \pm 3\pi, \ldots$

$\therefore$ $R = e^{[\hat{n}]\pi} = I + 2[\hat{n}]^2$

$$R = \begin{bmatrix} -2n_2^2 - 2n_3^2 + 1 & 2n_1 n_2 & 2n_1 n_3 \\ 2n_1 n_2 & -2n_1^2 - 2n_3^2 + 1 & 2n_2 n_3 \\ 2n_1 n_3 & 2n_2 n_3 & -2n_1^2 - 2n_2^2 + 1 \end{bmatrix}$$

$\therefore$ Multiple, same solutions exist, any method can be used

$$\hat{n} = \frac{1}{\sqrt{2(1+r_{33})}} \begin{bmatrix} r_{13} \\ r_{23} \\ 1+r_{33} \end{bmatrix}$$

or

$$\hat{n} = \frac{1}{\sqrt{2(1+r_{22})}} \begin{bmatrix} r_{12} \\ 1+r_{22} \\ r_{32} \end{bmatrix}$$

or

$$\hat{n} = \frac{1}{\sqrt{2(1+r_{11})}} \begin{bmatrix} 1+r_{11} \\ r_{21} \\ r_{31} \end{bmatrix}$$

Case 3:

$\therefore Tr(R) = 1 + 2c\theta$

$\therefore \theta = \cos^{-1}\left(\frac{1}{2}(Tr(R) - 1)\right) \in [0, \pi)$

We can find $\hat{n}$ by

$r_{32} - r_{23} = 2n_1 s\theta$

$r_{13} - r_{31} = 2n_2 s\theta$

$r_{21} - r_{12} = 2n_3 s\theta$

As long as $\theta \neq \pm n\pi$

$\therefore n_1 = \frac{1}{2s\theta}(r_{32} - r_{23})$

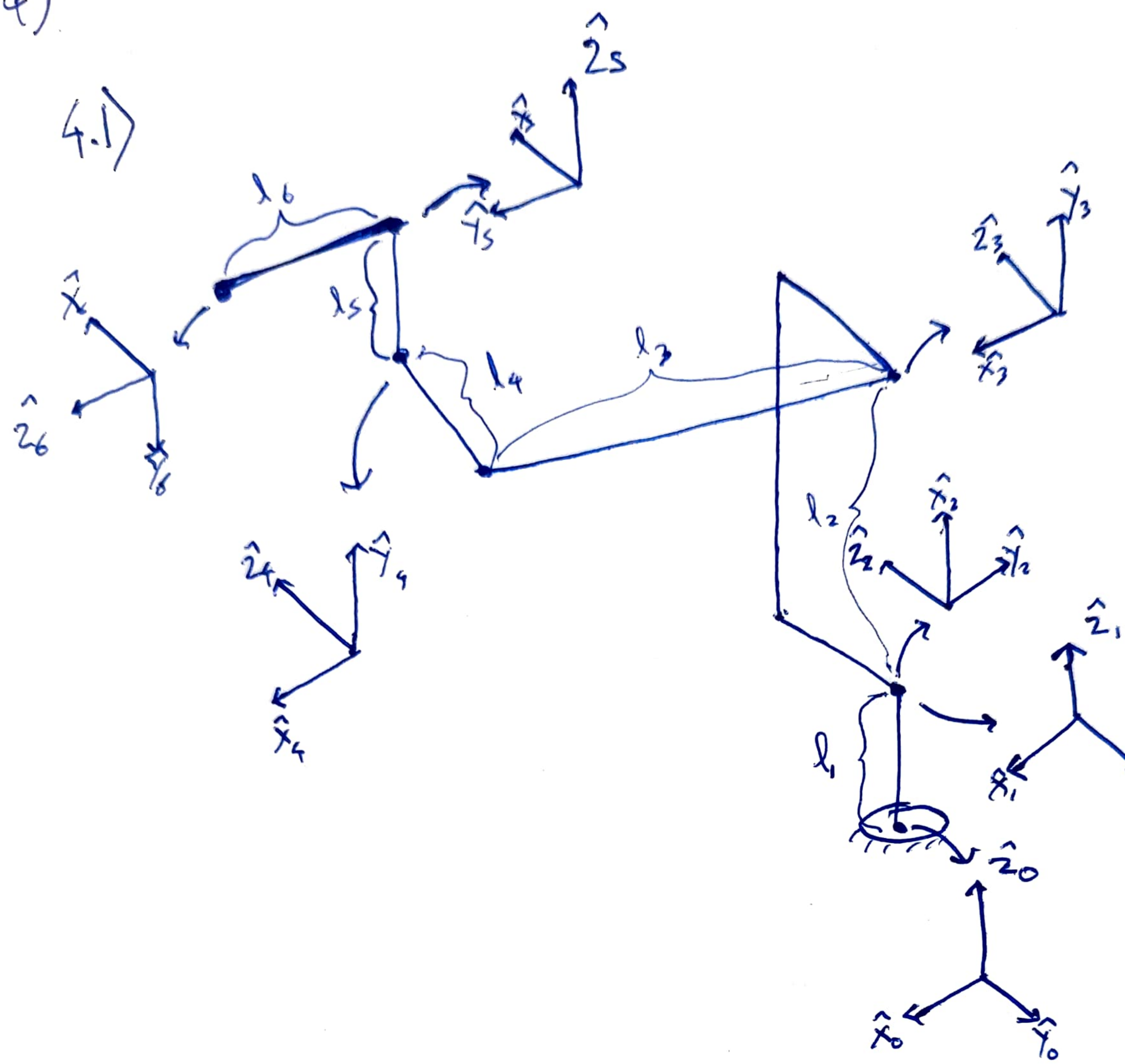
$\therefore n_2 = \frac{1}{2s\theta}(r_{13} - r_{31})$

$\therefore n_3 = \frac{1}{2s\theta}(r_{21} - r_{12})$

$\therefore \hat{n} = \frac{1}{2s\theta}(R - R^T)$

4)

4.1)



DH Table

Joint No.	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	l_1	θ_1
2	$\pi/2$	0	0	$\theta_2+\pi/2$
3	0	l_2	0	$\theta_3-\pi/2$
4	0	l_3	l_4	θ_4
5	$-\pi/2$	0	l_5	$\theta_5-\pi/2$
6	$\pi/2$	0	l_6	θ_6.