

An abstract graphic on the left side of the slide featuring swirling, marbled patterns in various shades of blue (from light sky blue to deep navy) against a white background. The patterns resemble fluid motion or perhaps a stylized representation of a flow field.

# Modeling Lid-Driven Cavity Flow

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# Abstract: Simulating Incompressible Viscous Flows

## Benchmark Problem

Lid-driven cavity flow: a classical CFD problem for incompressible viscous flows.

## Dual Approach

Simulated using OpenFOAM (icoFoam) and a custom Python solver.

## Key Analysis

Compared numerical behaviour under varying Reynolds numbers ( $Re = 100, 500, 5000$ ) and model parameters.

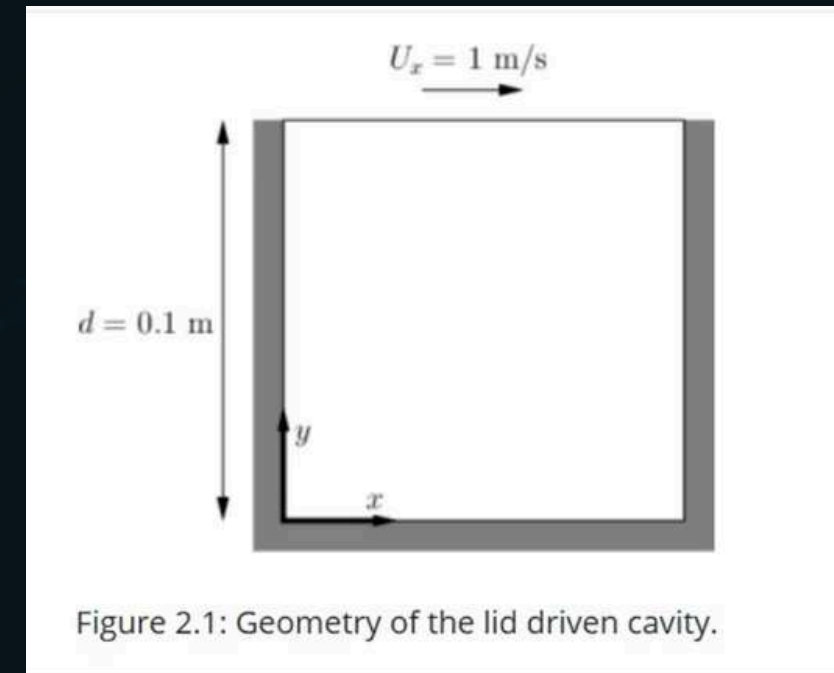
## Validation & Trade-offs

Validated reliability of numerical methods, highlighting accuracy, stability, and computational cost trade-offs.

# Introduction: The Lid-Driven Cavity

## Background & Motivation

- Closed cavity flows are important benchmark problems in CFD.
- The lid-driven cavity is widely studied due to its simple geometry and rich flow physics.
- It consists of an incompressible viscous fluid inside a square cavity.
- The top wall moves at constant velocity while other walls remain stationary.
- The flow displays complex behavior such as vortex formation and flow separation.
- Lack of analytical solutions makes numerical simulation essential.
- It is commonly used to validate CFD solvers and study Reynolds number effects.
- This work compares OpenFOAM and Python solvers for accuracy at different Reynolds numbers.



## Problem Statement & Objectives

Simulate and analyze 2D incompressible lid-driven cavity flow using OpenFOAM and a Python solver.

- Setup OpenFOAM simulations.
- Develop Python-based solver.
- Analyze parameter sensitivity.
- Compare velocity and vorticity profiles.

# Literature Review: Foundations of CFD

Ghia et al. (1982)

1

Established benchmark for 2D lid-driven cavity flow using finite-difference multigrid solver, providing reference data up to  $Re = 10,000$ .

2

Botella & Peyret (1998)

Refined benchmarks with spectral Chebyshev collocation, emphasizing grid refinement and higher-order accuracy.

Erturk & Gökçöl (2006)

3

Developed fourth-order compact finite-difference scheme for efficient high Reynolds number solutions.

4

Xiang & Shi (2022)

Applied Lattice Boltzmann Method (LBM) for accurate and parallelizable simulations, motivating LBM inclusion.



# Methodology: Three Computational Approaches

## Finite Volume Method (FVM)

OpenFOAM: Ensures flux conservation.



## Finite Difference Method (FDM)

Python: Algorithmic simplicity.

## Lattice Boltzmann Method (LBM)

Python: Mesoscopic kinetic approach, parallelizable.



# Governing Equations & Boundary Conditions

## Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

## Momentum Equations

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

Where  $Re = \frac{UL}{\nu}$  is the Reynolds number.

## Boundary Conditions

$$\left. \begin{array}{l} u = U, \\ v = 0 \\ u = 0, \\ v = 0 \end{array} \right\} \begin{array}{l} \text{at } y = L \\ \text{at } y = 0, \\ x = 0, \\ x = L \text{ (stationary walls)} \end{array}$$

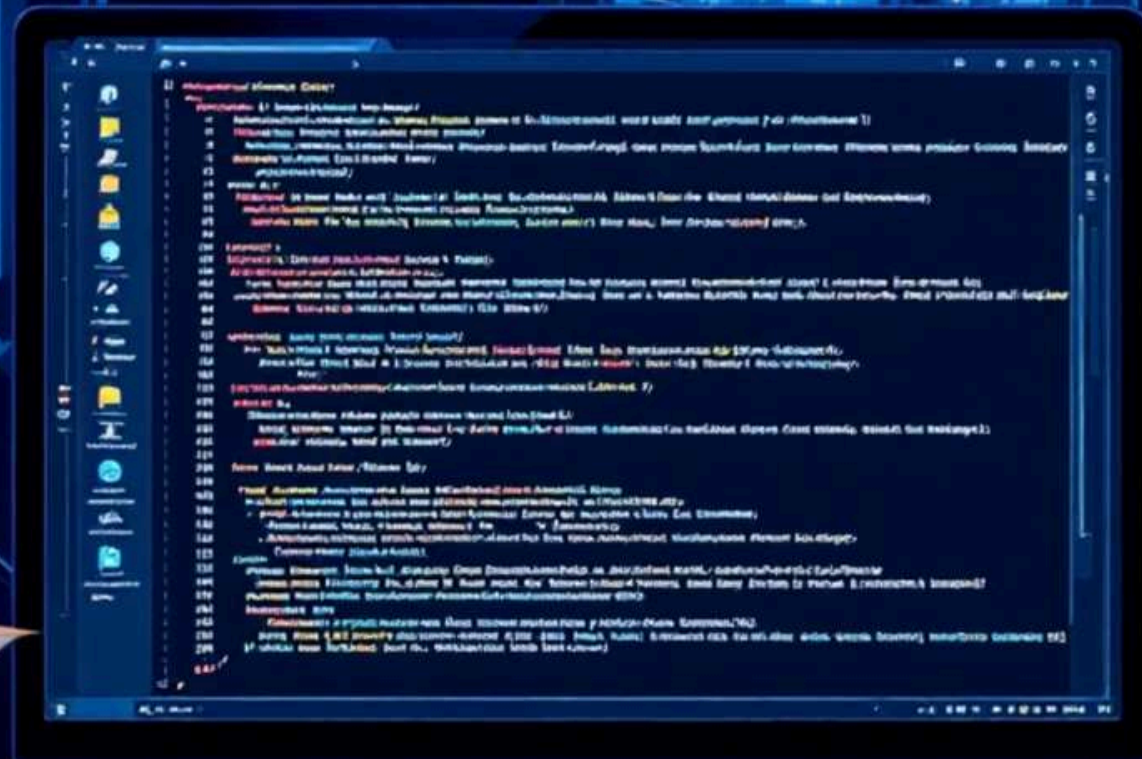
The moving lid drives fluid motion, creating primary and secondary vortices dependent on the Reynolds number.

# OpenFOAM

## Implementation Details

### OpenFOAM Setup

- Used icoFoam (laminar) and pimpleFoam (transient) solvers
- Domain: Square cavity of unit length ( $L = 1$ )
- Mesh: Structured grid with  $128^2$ , and  $256^2$  cells (grid independence verified)
- Schemes: Second-order upwind for convection; central differencing for diffusion
- Solver Control:  $\Delta t = 0.001$ , convergence tolerance =  $10^{-6}$
- Boundary Conditions: Moving lid ( $U, 0$ ) at the top, no-slip on other walls, zero-gradient for pressure



# Python Solvers: FDM & LBM Implementation

1

## Python FDM Solver

- Streamfunction-vorticity formulation
- Second-order central differences, explicit Euler time stepping
- Gauss-Seidel for streamfunction
- Validated against OpenFOAM and Ghia et al.
- Spatial derivatives: second-order central differences
- Time stepping:  $\Delta t = 0.001$
- Grid:  $128 \times 128$  structured grid
- Solver for  $\psi$  : Gauss-Seidel iterative method
- Boundary Conditions:
  - $\psi = 0$  on stationary walls
  - $\psi = \text{constant}$  at the moving lid
  - $\omega = -2\psi/\Delta y^2$  on the lid;  $\omega = 0$  elsewhere

FDM SOLVER

2

## Python LBM Solver

- D2Q9 model with BGK collision operator
- Bounce-back rule for no-slip walls, tangential velocity for moving lid
- Achieved stable results for  $Re = 100$  and  $500$

Evolution Equation:

$$f_i(\mathbf{x} + e_i\Delta t, t + \Delta t) - f_i(\mathbf{x}, t) = -\frac{1}{\tau} [f_i(\mathbf{x}, t) - f_i^{eq}(\mathbf{x}, t)]$$

Boundary Conditions:

- Bounce-back: for no-slip walls
- Moving lid: Zou-He velocity boundary condition
- Relaxation time:  $\tau = 3\nu + 0.5$  (BGK collision operator)

LBM SOLVER

# Results: Visualizing Flow Patterns

Contour plots of velocity, vorticity, streamlines for visual comparison.

## OpenFOAM Simulation Results :

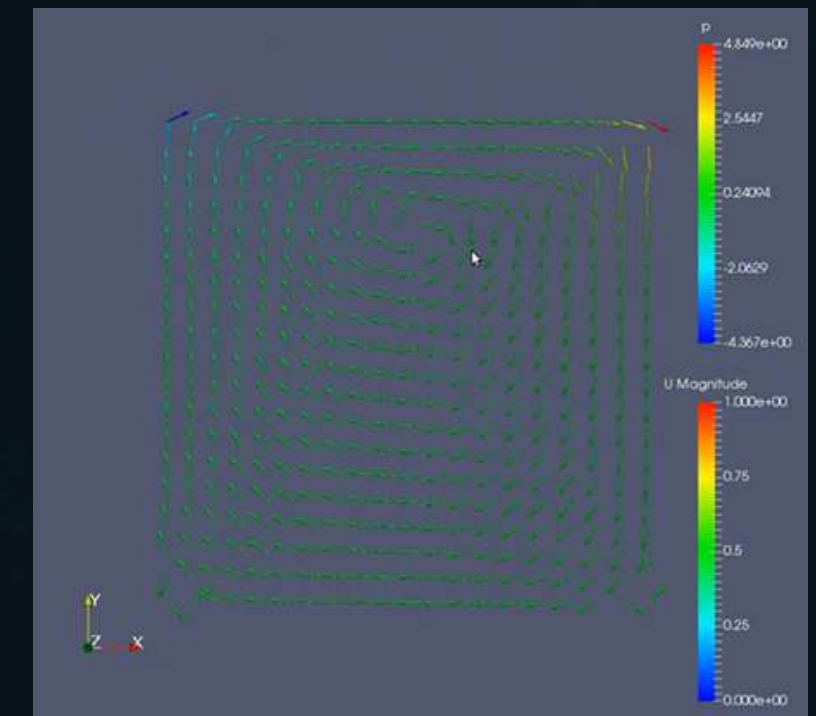
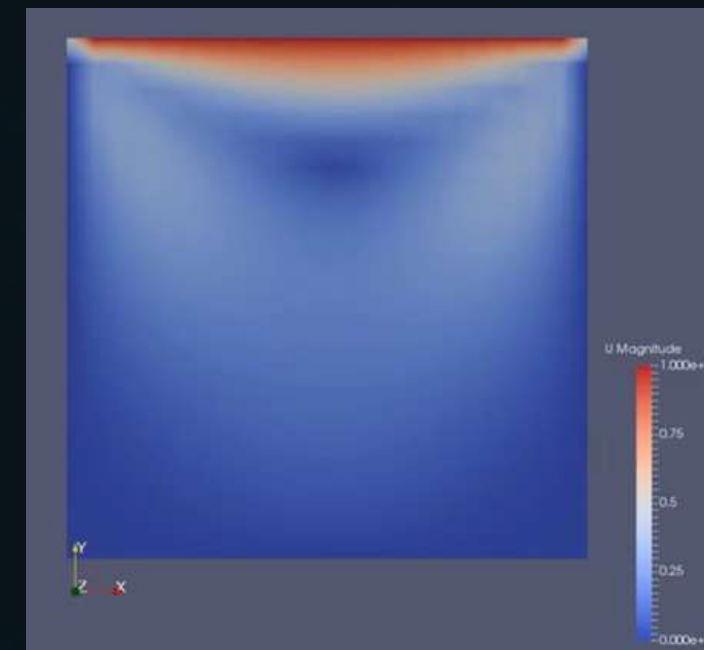
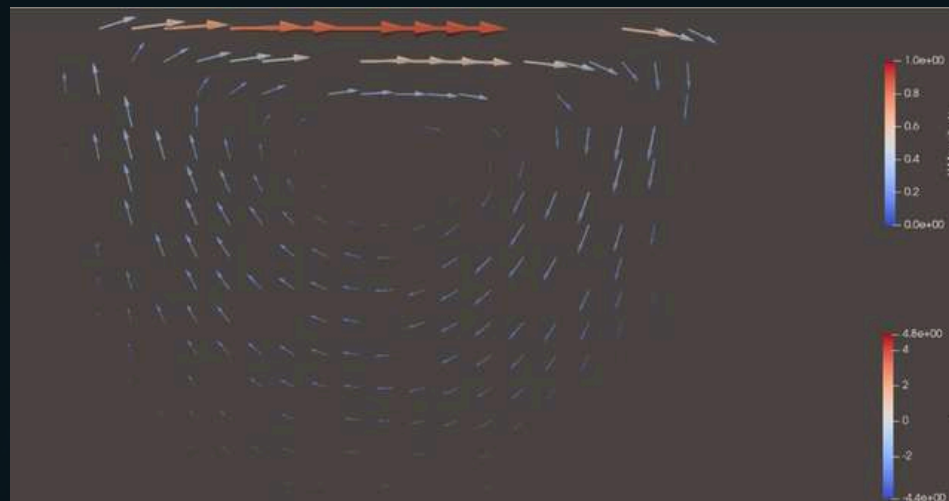
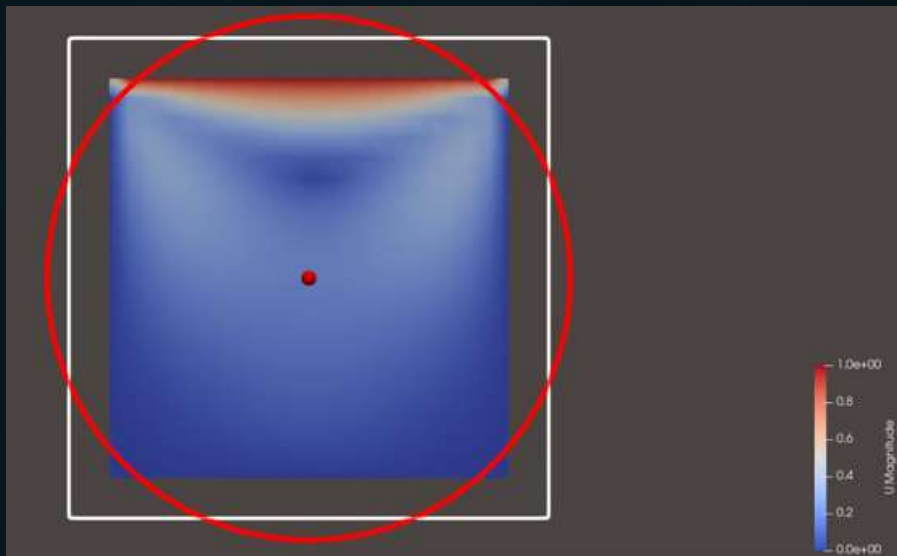


Figure 1. Velocity Magnitude (Re = 5000)  
High-speed flow near the lid with a strong central vortex and thin shear layer. Corner vortices start to form due to strong inertia.

Figure 2. Streamlines (Re = 5000)  
Flow is highly rotational with intense recirculation. Primary vortex dominates and small corner vortices appear.

Figure 3. Velocity Magnitude (Re = 1000)  
At Re = 1000, the flow is smoother and less aggressive compared to Re = 5000. The primary vortex is well-formed but less concentrated. The velocity gradients near the top lid are thicker and more diffused.

Figure 4. Streamlines (Re = 1000)  
Streamlines show a stable, symmetric flow pattern with one dominant vortex. Corner vortices are smaller and weaker because inertial forces are lower at this Reynolds number.

## Finite Difference Method (FDM) Results :

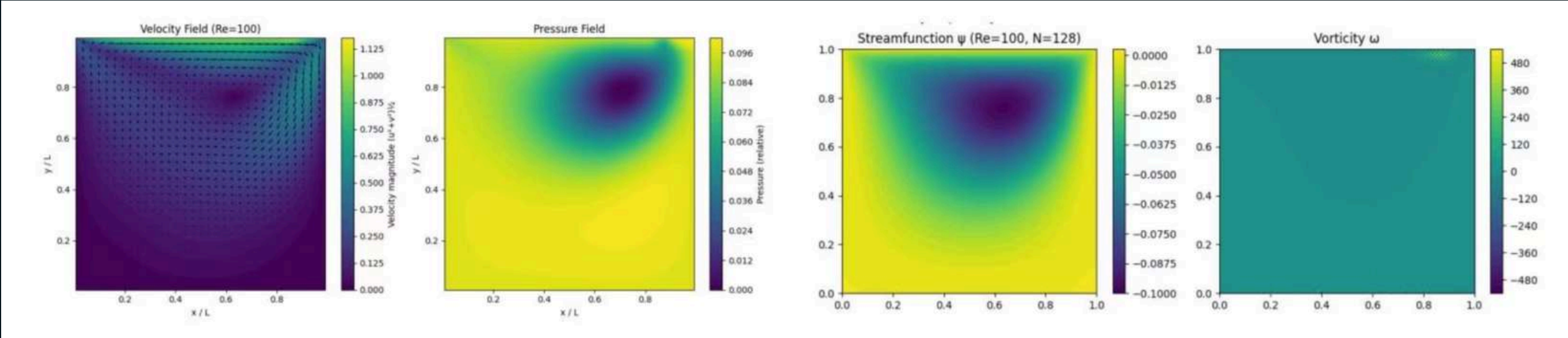


Figure 5. The velocity and stream function fields clearly show the primary circulating vortex formed by the lid-driven motion, with the highest speeds near the moving top wall. The pressure field reflects a high-pressure zone on the upper-left and a low-pressure zone on the upper-right, consistent with fluid acceleration. The vorticity plot indicates rotational behavior concentrated near the boundaries, though the wide color scale reduces visible contrast across the domain.

## Lattice Boltzmann Method (LBM) Results :

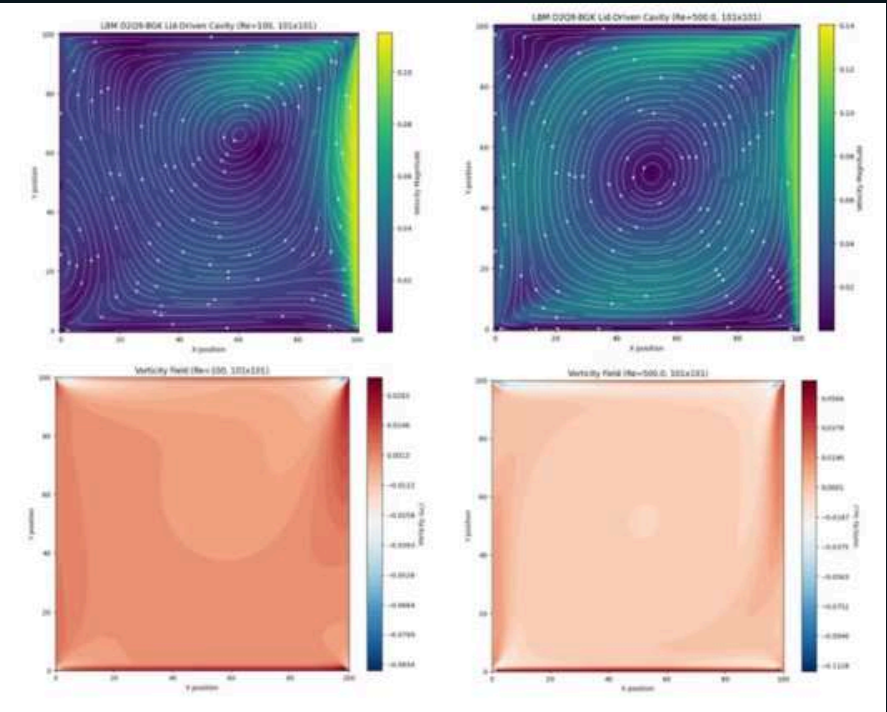


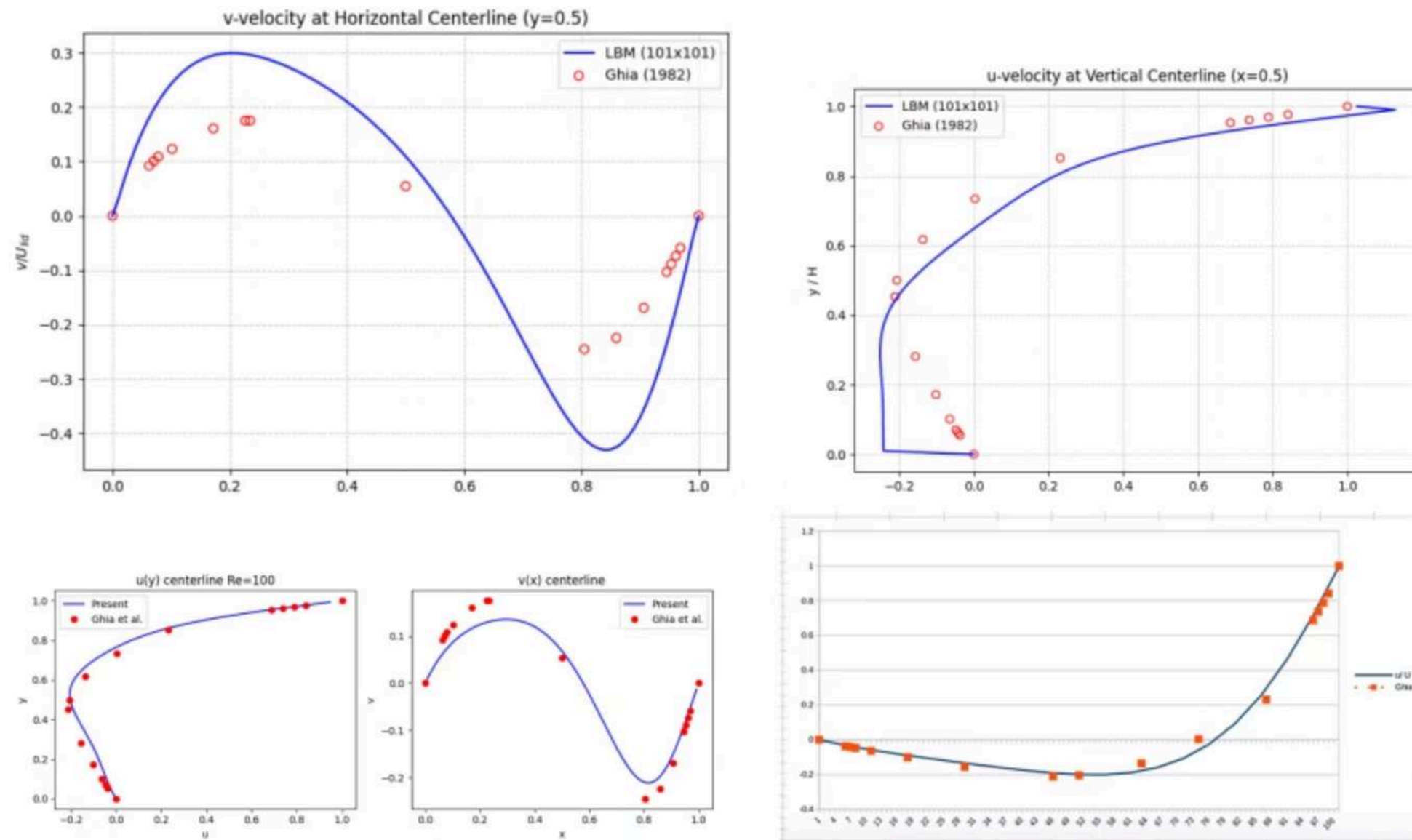
Figure 6. These four LBM results compare the cavity flow at  $Re = 100$  and  $Re = 500$  :

At  $Re = 100$ , the velocity and vorticity fields are smooth, showing a stable central vortex with weak rotation.

At  $Re = 500$ , the flow becomes stronger and more energetic, with a sharper shear layer, stronger central vortex, and higher vorticity near the walls and corners.

# Validation: Benchmarking Accuracy

Quantitative comparison showed strong agreement with benchmark data, with deviations below 5% for velocity profiles at  $Re = 100$ .



**Figure 5.4:** Comparison with Ghia et al. (1982): (a)  $u$  centerline velocity, (b)  $v$  centerline velocity, (c) streamlines, (d) vorticity contours.

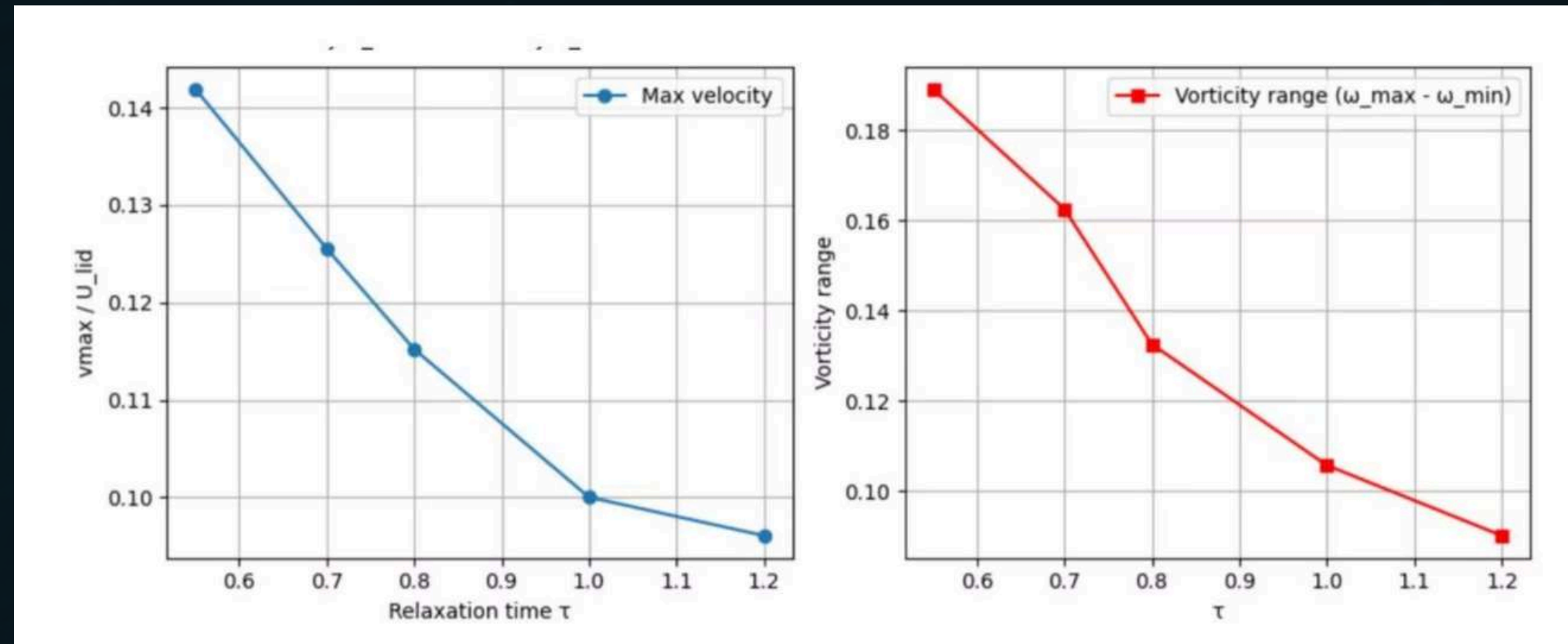
OpenFOAM provided the closest match due to its advanced discretization and pressure-velocity coupling.

# Parameter Sensitivity Analysis

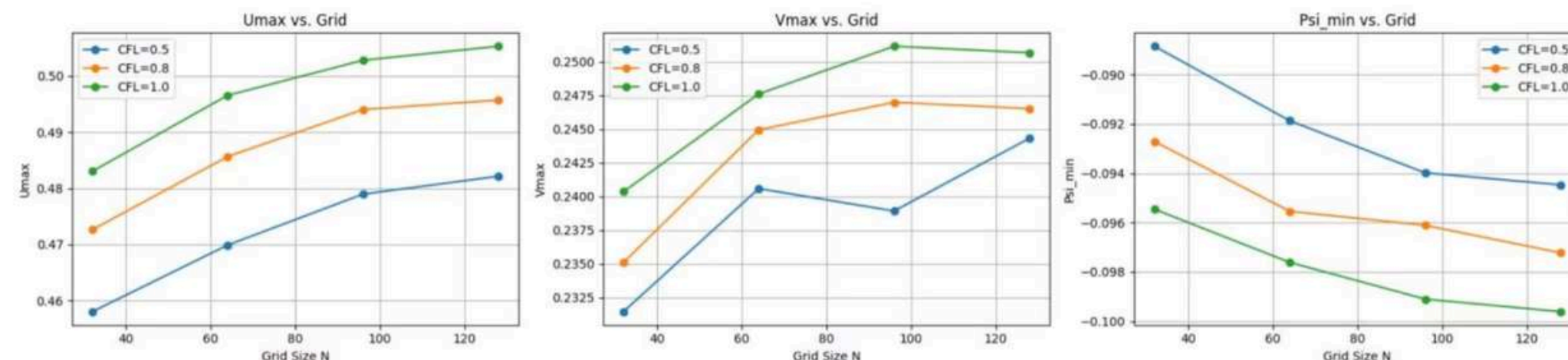
## Parameter Sensitivity

LBM sensitivity to relaxation time  $\tau$ : higher  $\tau$  increased numerical diffusion, weakening vortex intensity.

FDM sensitivity to mesh density and CFL number: Higher CFL values produce sharper gradients and stronger vortices, whereas lower CFL values are more stable but slightly diffusive. finer grids and higher CFL values improved accuracy, with  $128^2$  resolution achieving grid independence.



Sensitivity Analysis: Metrics vs. Grid Size for Different CFL



# Conclusion: Reliable Simulation of Cavity Flows

## Accurate Flow Physics

All three solvers (OpenFOAM, FDM, LBM) accurately captured essential flow physics, including vortex formation and eddies.

## Validated Reliability

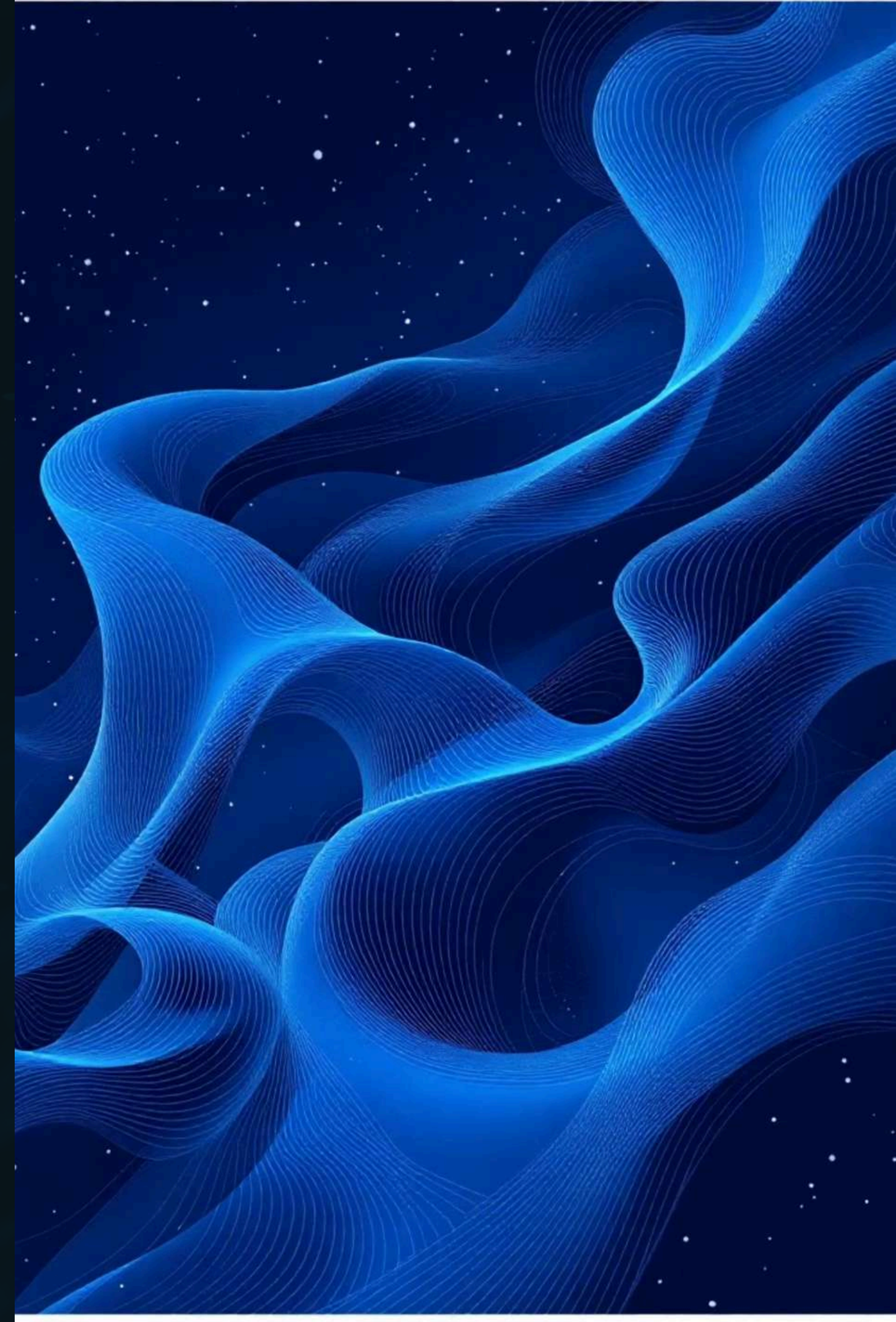
Validation against Ghia et al. (1982) confirmed reliability, with errors consistently below 5% for velocity profiles.

## Optimal Parameters

Grid size of  $128^2$  and  $\Delta t = 0.001$  provided an optimal balance of cost and accuracy. (Evaluated this during the Simulation Process)

## Future Work

Extend to 3D geometries, turbulent regimes, and parallelized Python solvers for enhanced efficiency.



# Challenges & Lessons Learned

Throughout the project, we encountered several key challenges that refined our understanding of CFD and solver implementation.

## Computational Cost

Simulating higher Reynolds numbers (e.g.,  $Re=5000$ ) and finer meshes significantly increased computational demands, particularly for the Python FDM and LBM solvers.

## Convergence Stability

Ensuring stability and achieving convergence, especially with explicit time-stepping schemes in FDM, required careful selection of time step ( $\Delta t$ ) and relaxation parameters.

## LBM Boundary Conditions

Correctly implementing the Zou-He velocity boundary condition for the moving lid in the LBM solver was critical and required precise adjustments for accuracy.

## Validation Data Comparison

Meticulously comparing velocity profiles and streamfunction contours against established benchmark data (Ghia et al. 1982) was time-consuming but essential for validating solver accuracy.

# Team Contributions

The successful execution of this project was a collaborative effort, with each team member contributing their expertise to advance our understanding of lid-driven cavity flows.

1

## Prince

Led the implementation and analysis of the Lid-Driven Cavity flow using OpenFOAM, focusing on setting up complex geometries and boundary conditions for higher Reynolds numbers.

2

## Soham

Contributed significantly to both the OpenFOAM simulations and developed the Python-based Finite Difference Method (FDM) solver, ensuring accurate discretization and stability for incompressible flows.

3

## Garvit

Developed the Lattice Boltzmann Method (LBM) solver in Python, including the implementation of specialized boundary conditions, and conducted the comprehensive parameter sensitivity analysis to optimize solver performance.