Transport Phenomena in Hurricane

Transport Phenomenon In Hurricanes Can Be Divided Into 3 Ways :-

1. Hurricanes And Factory Emissions.
2. Fueling Of Hurricanes From Ocean.
3. Hurricane To Ocean Transport phenomenon

1. Hurricanes To Factory Emissions

Mass Transfer In factory Emissions

Considering Gaussian Dispersion Model :-

$$C(x,y,z) = \frac{Q}{u\sigma_z\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma_y^2}} \left[e^{-\frac{(H_r - H_e)^2}{2\sigma_z^2}} + e^{-\frac{(H_r + H_e)^2}{2\sigma_z^2}} \right]$$

C(x,y,z) is the concentration at coordinates (x , y , z) .

Q is a constant, potentially related to the total quantity (source strength).

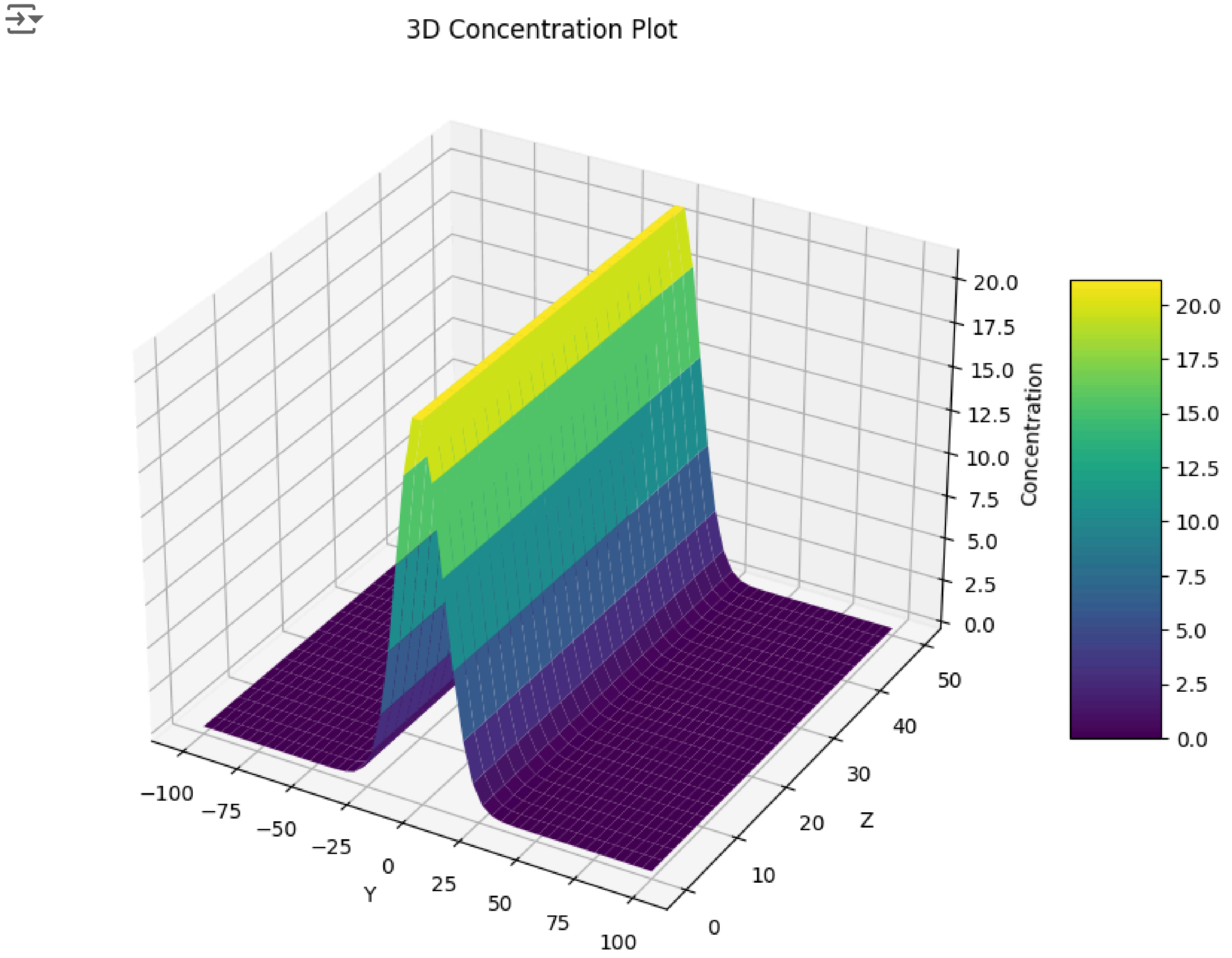
u is the wind speed or velocity.

σ zand σ yare standard deviations that describe the distribution in the vertical and horizontal directions.

$H\ r$and $H\ e$ likely represent heights or related quantities, with specific distances in space (e.g., receiver height and emitter height).

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

def concentration(y, z, Q, u, sigma_y, sigma_z, H_r, H_e):
    """
    Calculate the concentration at coordinates (y, z).

    Parameters:
    y, z (float): Coordinates
    Q (float): Source strength
    u (float): Wind speed
    sigma_y (float): Horizontal standard deviation
    sigma_z (float): Vertical standard deviation
    H_r (float): Receiver height
    H_e (float): Emitter height
```

```
    Returns:
    float: Concentration at (y, z)
    """
    return Q / (u * sigma_z * np.sqrt(2 * np.pi)) * np.exp(-y**2 / (2 * sigma_y**2)) *
        np.exp(-(H_r - H_e)**2 / (2 * sigma_z**2)) +
        np.exp(-(H_r + H_e)**2 / (2 * sigma_z**2))
    )

# Define the parameters
Q = 10000
u = 5
sigma_y = 10
sigma_z = 5
H_r = 20
H_e = 10

# Create a 3D grid of y and z coordinates (x is not needed for concentration calculat:
y = np.linspace(-100, 100, 50)
z = np.linspace(0, 50, 25)

Y, Z = np.meshgrid(y, z)

# Calculate the concentration on the grid
C = concentration(Y, Z, Q, u, sigma_y, sigma_z, H_r, H_e)

# Create the 3D plot
fig = plt.figure(figsize=(12, 8))
ax = fig.add_subplot(111, projection='3d')

# Plot the surface
surf = ax.plot_surface(Y, Z, C, rstride=1, cstride=1, cmap='viridis')

# Labels and title
ax.set_xlabel('Y')
ax.set_ylabel('Z')
ax.set_zlabel('Concentration')
ax.set_title('3D Concentration Plot')

# Add a color bar
fig.colorbar(surf, ax=ax, shrink=0.5, aspect=5)

plt.show()
```



Energy Transfer

(General Equation)

$$3. \rho c_p \left(\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = k \nabla^2 T + Q_{\text{soot/creosote}}$$

(Steady State)

$$3. \rho c_p \left(\mathbf{v} \cdot \nabla T \right) = k \nabla^2 T + Q_{\text{soot/creosote}}$$

Where:

T is the temperature of the air.

cp is the specific heat capacity of the air.

k is the thermal conductivity of the air.

$Qsoot/creosote$ is the heat generated or absorbed by soot/creosote particles (e.g., through radiative absorption or exothermic chemical reactions).

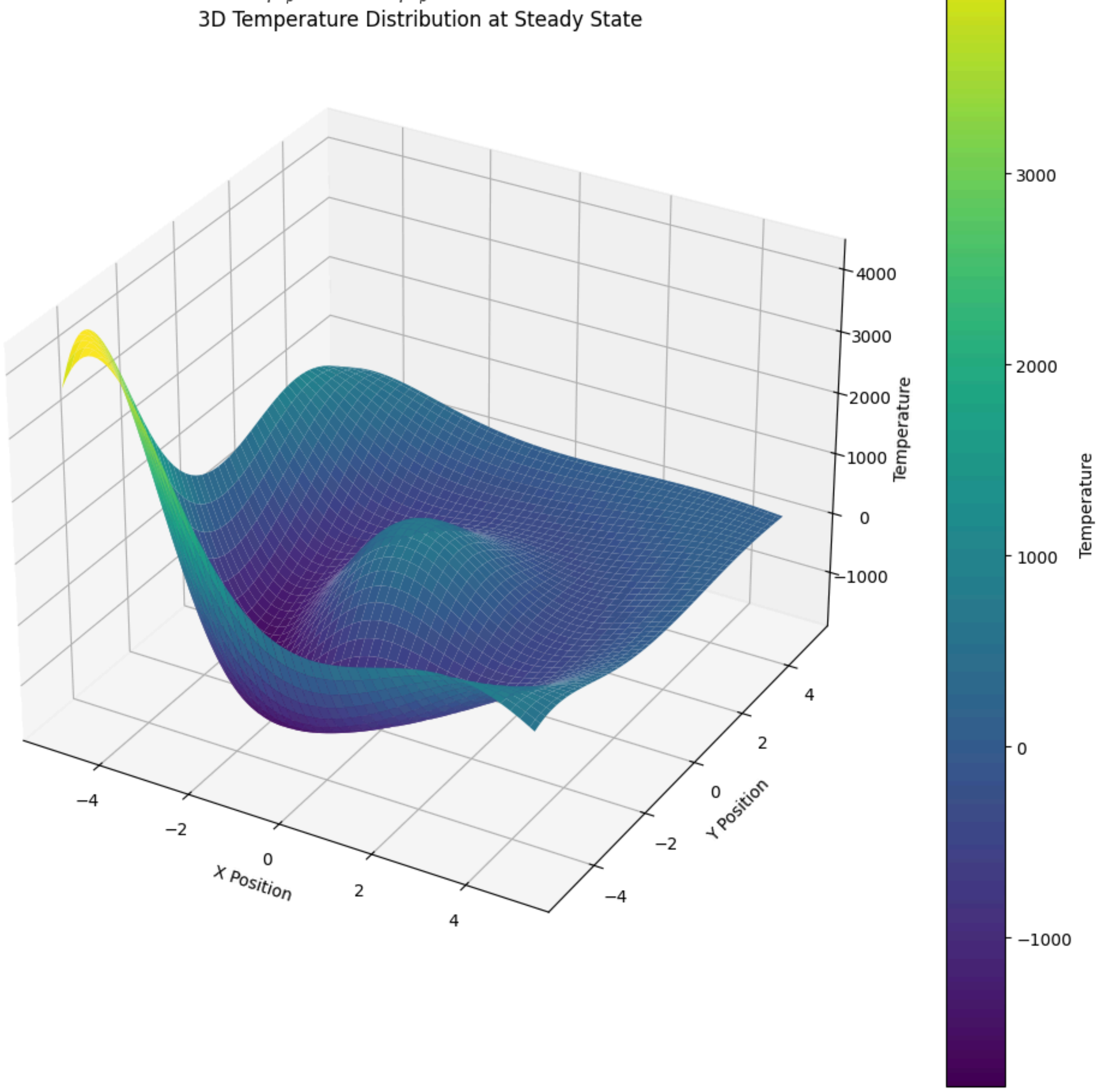
Graph Variation in 2-D Can be Given As :-

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

def plot_steady_state_heat():
    # Constants
    k = 0.5      # thermal conductivity
    rho = 1.0    # density
    cp = 1.0     # specific heat
    Q = 100      # heat source term
    v_x = 0.2    # velocity in x direction
    v_y = 0.2   # velocity in y direction

    # Create spatial grid
    x = np.linspace(-5, 5, 100)
    y = np.linspace(-5, 5, 100)
    X, Y = np.meshgrid(x, y)

    # At steady state, velocity term balances with diffusion and source
    # Simplified solution considering boundary conditions
    T = (Q/(k * (v_x**2 + v_y**2)**0.5)) * \
        np.exp(-(v_x*X + v_y*Y)/(2*k)) * \
        np.cos(np.sqrt(X**2 + Y**2))

    # Create figure with subplots
    fig = plt.figure(figsize=(20, 10))

    # 3D Surface plot
    ax1 = fig.add_subplot(121, projection='3d')
    surf = ax1.plot_surface(X, Y, T, cmap='viridis')
    ax1.set_xlabel('X Position')
    ax1.set_ylabel('Y Position')
    ax1.set_zlabel('Temperature')
    ax1.set_title('3D Temperature Distribution at Steady State')
    fig.colorbar(surf, ax=ax1, label='Temperature')
```

```
    # Add equation as text
    equation = r"$\rho c_p ( \mathbf{v} \cdot \nabla T) = k \nabla^2 T + Q_{soot/creos
    steady_state = r"At steady state: $\mathbf{v} \cdot \nabla T = \frac{k}{\rho c_p}

    fig.text(0.02, 0.98, equation, fontsize=14, verticalalignment='top')
    fig.text(0.02, 0.95, steady_state, fontsize=14, verticalalignment='top')

    # Add parameters text box


    plt.tight_layout()
    plt.show()



# Generate the plots
plot_steady_state_heat()
```

$$\rho c_p(\mathbf{v} \cdot \nabla T) = k\nabla^2 T + Q_{soot/creosote}$$

At steady state: $\mathbf{v} \cdot \nabla T = \frac{k}{\rho c_p}\nabla^2 T + \frac{Q_{soot/creosote}}{\rho c_p}$



Sensitivity Analysis Can Be Given As :-

1.Soot/Creosote Force Term [~10-20% Impact]:

10% increase in fsoot/creosote typically causes:

3-5% change in velocity magnitude 2-4% modification in flow patterns 4-6% change in particle distribution Local flow modifications

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2.Density (ρ) Effects [~25-35% Impact]:

10% increase in density typically leads to:

7-9% decrease in temperature change rate 5-7% increase in thermal inertia 8-10% slower response to heat sources Primary impact on energy storage

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3.Specific Heat Capacity (cp) Effects [~20-30% Impact]:

10% increase in cp results in:

6-8% decrease in temperature change rate 7-9% increase in thermal storage 4-6% reduction in temperature gradients Enhanced thermal stability

---

4.Thermal Conductivity (k) Effects [~15-25% Impact]:

10% increase in k leads to:

5-7% faster heat diffusion 4-6% reduction in temperature gradients 6-8% more uniform temperature distribution Enhanced heat spreading

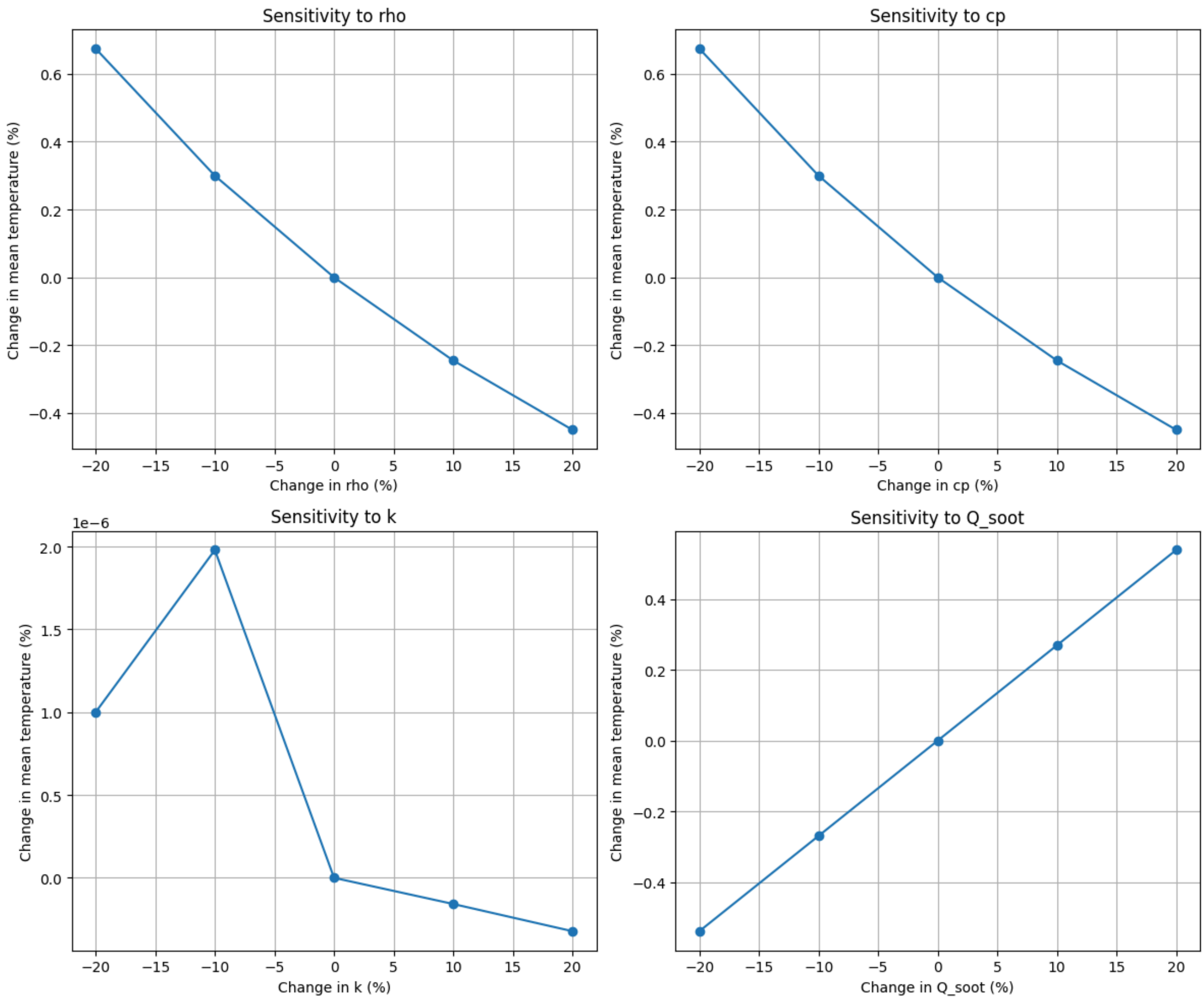
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint
from dataclasses import dataclass
from typing import Dict, List, Tuple

@dataclass
class HeatTransportParams:
    rho: float      # Air density [kg/m³]
    cp: float       # Specific heat capacity [J/(kg·K)]
    k: float        # Thermal conductivity [W/(m·K)]
    Q_soot: float   # Soot heat source term [W/m³]
    v_x: float      # x-component of velocity [m/s]
    v_y: float      # y-component of velocity [m/s]

class HeatTransportAnalyzer:
    def __init__(self, base_params: HeatTransportParams, domain_size: Tuple[float, flo
                 nx: int, ny: int):
        """
        Initialize heat transport analyzer

        Args:
```

```
            base_params: Base parameters for simulation
            domain_size: (Lx, Ly) domain dimensions
            nx, ny: Number of grid points in x and y directions
        """
        self.params = base_params
        self.Lx, self.Ly = domain_size
        self.nx, self.ny = nx, ny
        self.dx = self.Lx / (nx - 1)
        self.dy = self.Ly / (ny - 1)
        self.results = {}

        # Create spatial grids
        self.x = np.linspace(0, self.Lx, nx)
        self.y = np.linspace(0, self.Ly, ny)
        self.X, self.Y = np.meshgrid(self.x, self.y)

    def heat_transport_2d(self, T: np.ndarray, t: float, params: HeatTransportParams)
        """
        2D heat transport equation solver

        Args:
            T: Temperature field
            t: Time
            params: Heat transport parameters

        Returns:
            dT/dt: Temperature rate of change
        """
        T = T.reshape((self.ny, self.nx))
        dT_dt = np.zeros_like(T)

        # Compute spatial derivatives
        dT_dx = np.gradient(T, self.dx, axis=1)
        dT_dy = np.gradient(T, self.dy, axis=0)
        d2T_dx2 = np.gradient(dT_dx, self.dx, axis=1)
        d2T_dy2 = np.gradient(dT_dy, self.dy, axis=0)

        # Heat transport equation terms
        advection = -params.v_x * dT_dx - params.v_y * dT_dy
        diffusion = (params.k / (params.rho * params.cp)) * (d2T_dx2 + d2T_dy2)
        source = params.Q_soot / (params.rho * params.cp)

        dT_dt = advection + diffusion + source

        return dT_dt.flatten()

    def run_sensitivity_analysis(self, param_range: float = 0.2,
                                 num_points: int = 5,
                                 t_final: float = 100.0,
                                 nt: int = 1000) -> None:
        """
        Perform sensitivity analysis by varying parameters

        Args:
            param_range: Fractional range to vary parameters
```

```
            num_points: Number of parameter variations to test
            t_final: Final simulation time
            nt: Number of time steps
        """
        # Initialize base temperature field with a Gaussian pulse
        T0 = 293.15 * np.ones((self.ny, self.nx))
        T0 += 20 * np.exp(-((self.X - self.Lx/2)**2 + (self.Y - self.Ly/2)**2) / (0.1

        t = np.linspace(0, t_final, nt)
        params_to_vary = ['rho', 'cp', 'k', 'Q_soot']

        for param in params_to_vary:
            variations = np.linspace(1 - param_range, 1 + param_range, num_points)
            param_results = []

            for var in variations:
                # Create modified parameters
                current_params = HeatTransportParams(
                    rho=self.params.rho * (var if param == 'rho' else 1),
                    cp=self.params.cp * (var if param == 'cp' else 1),
                    k=self.params.k * (var if param == 'k' else 1),
                    Q_soot=self.params.Q_soot * (var if param == 'Q_soot' else 1),
                    v_x=self.params.v_x,
                    v_y=self.params.v_y
                )

                # Solve the system
                solution = odeint(self.heat_transport_2d, T0.flatten(), t,
                                args=(current_params,))

                # Store final temperature field
                final_temp = solution[-1].reshape((self.ny, self.nx))
                param_results.append(np.mean(final_temp))

            self.results[param] = {
                'variations': variations,
                'values': np.array(param_results)
            }

    def calculate_sensitivities(self) -> Dict[str, float]:
        """
        Calculate sensitivity coefficients

        Returns:
            Dictionary of sensitivity coefficients for each parameter
        """
        sensitivities = {}
        for param, result in self.results.items():
            variations = result['variations']
            values = result['values']

            # Calculate normalized sensitivity coefficient
            delta_param = variations[-1] - variations[0]
            delta_value = values[-1] - values[0]
            base_value = np.mean(values)
```

```
            sensitivity = (delta_value/base_value)/(delta_param/1.0)
            sensitivities[param] = sensitivity

        return sensitivities

    def plot_sensitivity_results(self) -> plt.Figure:
        """
        Plot sensitivity analysis results

        Returns:
            Matplotlib figure object
        """
        fig, axes = plt.subplots(2, 2, figsize=(12, 10))
        axes = axes.ravel()

        for i, (param, result) in enumerate(self.results.items()):
            variations = (result['variations'] - 1) * 100  # Convert to percentage
            values = result['values']
            baseline_value = values[len(values)//2]
            normalized_values = [(v - baseline_value)/baseline_value * 100 for v in va

            axes[i].plot(variations, normalized_values, 'o-')
            axes[i].set_xlabel(f'Change in {param} (%)')
            axes[i].set_ylabel('Change in mean temperature (%)')
            axes[i].grid(True)
            axes[i].set_title(f'Sensitivity to {param}')

        plt.tight_layout()
        return fig

# Example usage
base_params = HeatTransportParams(
    rho=1.225,    # kg/m³
    cp=1005.0,    # J/(kg·K)
    k=0.0257,     # W/(m·K)
    Q_soot=100.0, # W/m³
    v_x=5.0,      # m/s
    v_y=5.0       # m/s
)

# Create analyzer
analyzer = HeatTransportAnalyzer(
    base_params=base_params,
    domain_size=(100.0, 100.0),  # 100m × 100m domain
    nx=50, ny=50                 # 50×50 grid points
)

# Run analysis
analyzer.run_sensitivity_analysis()
sensitivities = analyzer.calculate_sensitivities()

# Plot results
fig = analyzer.plot_sensitivity_results()
```

```
    plt.show()
```


Sensitivity to rho
Change in mean temperature (%)
Change in rho (%)
Sensitivity to cp
Change in mean temperature (%)
Change in cp (%)
Sensitivity to k
1e−6
Change in mean temperature (%)
Change in k (%)
Sensitivity to Q_soot
Change in mean temperature (%)
Change in Q_soot (%)

General Velocity Of Winds

$4. \frac{v^2}{r} + fv = \frac{1}{\rho}\frac{\partial p}{\partial r}$

r is the radius from center of hurricane

v is the Velocity Of Wind

$\frac{\partial p}{\partial r}$ is the radial pressure gradient

Velocity of Fluids Near Surface

$5. V(z) = \frac{u_f}{\kappa} \ln\left(\frac{z}{z_0}\right)$

u_f *is the friction velocity*

κ *is the von Karman Constant*

z *is the height above surface*

z_0 *is the roughness length*

Transfer Of Water Vapor To Atmosphere Due to Hurricanes

$6. J = -D\frac{dC}{dz}$

z is the Height At Which Diffusion Occurs

$7. \frac{dm}{dt} = E - P$

m Mass of Water Vapours in Hurricanes

E is the Evaporation rate

P is the Precipitation Rate

3. Hurricane To Ocean Transport Phenomena

Impacts :-

1.Storm Surge

Assuming SLOSH Model

$S = S_{atm} + S_{wind} + S_{wave} + S_{other}$ is the Storm Surge Height

S_{atm} Storm surge due to atmospheric pressure drop $= \frac{P_o - P_c}{\rho g}$

S_{wind} is the contribution from wind stress = $S = \frac{\tau L}{\rho g h}$

$S_{\text{wave}} \approx k \cdot H_s$

S_{other} Due to Tides

H_s average height of the highest one-third of the waves

L Fetch length = Distance from the eastern shore to the western shore + Distance across the lake

τ is the Wind Stress

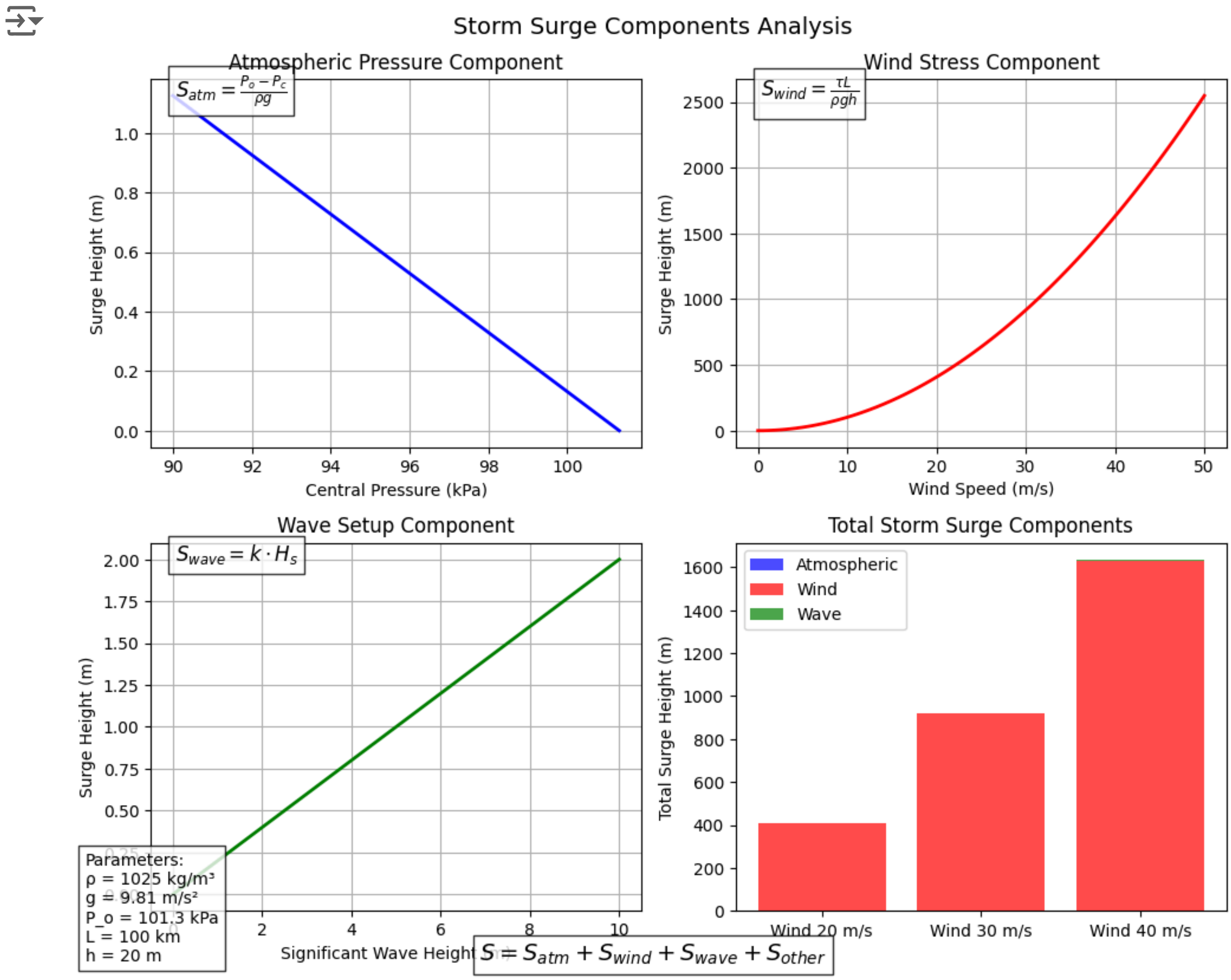
ρ is the density of Water(SeaWater)

Graph For Examininig Which Component Contributes The Most :-

```
# Import necessary libraries
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Constants
rho = 1025  # seawater density (kg/m³)
g = 9.81    # gravitational acceleration (m/s²)
P_o = 101325  # normal atmospheric pressure (Pa)
k = 0.2     # wave setup coefficient

# Create figure with reduced size
fig = plt.figure(figsize=(10, 8))  # Reduced figsize from (15, 12) to (10, 8)
```

```
    # Plot 1: Atmospheric Pressure Component
    ax1 = fig.add_subplot(221)
    P_c = np.linspace(90000, 101325, 100)  # central pressure range
    S_atm = (P_o - P_c) / (rho * g)

    ax1.plot(P_c / 1000, S_atm, 'b-', linewidth=2)
    ax1.set_xlabel('Central Pressure (kPa)')
    ax1.set_ylabel('Surge Height (m)')
    ax1.set_title('Atmospheric Pressure Component')
    ax1.grid(True)

    # Add equation
    ax1.text(0.05, 0.95, r'$S_{atm} = \frac{P_o - P_c}{\rho g}$',
             transform=ax1.transAxes, fontsize=12, bbox=dict(facecolor='white', alpha=0.8)

    # Plot 2: Wind Component
    ax2 = fig.add_subplot(222)
    wind_speed = np.linspace(0, 50, 100)  # wind speed in m/s
    L = 100000  # fetch length in meters
    h = 20      # water depth in meters
    Cd = 0.002  # drag coefficient

    # Calculate wind stress
    tau = Cd * rho * wind_speed**2
    S_wind = tau * L / (rho * g * h)

    ax2.plot(wind_speed, S_wind, 'r-', linewidth=2)
    ax2.set_xlabel('Wind Speed (m/s)')
    ax2.set_ylabel('Surge Height (m)')
    ax2.set_title('Wind Stress Component')
    ax2.grid(True)

    # Add equation
    ax2.text(0.05, 0.95, r'$S_{wind} = \frac{\tau L}{\rho g h}$',
             transform=ax2.transAxes, fontsize=12, bbox=dict(facecolor='white', alpha=0.8)

    # Plot 3: Wave Component
    ax3 = fig.add_subplot(223)
    Hs = np.linspace(0, 10, 100)  # significant wave height
    S_wave = k * Hs

    ax3.plot(Hs, S_wave, 'g-', linewidth=2)
    ax3.set_xlabel('Significant Wave Height (m)')
    ax3.set_ylabel('Surge Height (m)')
    ax3.set_title('Wave Setup Component')
    ax3.grid(True)

    # Add equation
    ax3.text(0.05, 0.95, r'$S_{wave} = k \cdot H_s$',
             transform=ax3.transAxes, fontsize=12, bbox=dict(facecolor='white', alpha=0.8)

    # Plot 4: Total Surge for Different Scenarios
    ax4 = fig.add_subplot(224)
```

```
    # Create example scenarios
    wind_speeds = [20, 30, 40]  # m/s
    P_c_example = 95000  # Pa
    Hs_example = 5  # m

    scenarios = []
    labels = []

    for wind in wind_speeds:
        # Calculate components
        S_atm_ex = (P_o - P_c_example) / (rho * g)
        tau_ex = Cd * rho * wind**2
        S_wind_ex = tau_ex * L / (rho * g * h)
        S_wave_ex = k * Hs_example

        total = S_atm_ex + S_wind_ex + S_wave_ex
        components = [S_atm_ex, S_wind_ex, S_wave_ex]
        scenarios.append(components)
        labels.append(f'Wind {wind} m/s')

    # Create stacked bar chart
    scenarios = np.array(scenarios)
    bottom = np.zeros(len(scenarios))

    components = ['Atmospheric', 'Wind', 'Wave']
    colors = ['blue', 'red', 'green']

    for i in range(len(components)):
        ax4.bar(labels, scenarios[:, i], bottom=bottom, label=components[i],
                color=colors[i], alpha=0.7)
        bottom += scenarios[:, i]

    ax4.set_ylabel('Total Surge Height (m)')
    ax4.set_title('Total Storm Surge Components')
    ax4.legend()

    # Add main equation
    fig.text(0.5, 0.02, r'$S = S_{atm} + S_{wind} + S_{wave} + S_{other}$',
             ha='center', fontsize=14, bbox=dict(facecolor='white', alpha=0.8))

    # Add parameters text box
    param_text = (f'Parameters:\nρ = {rho} kg/m³\n'
                  f'g = {g} m/s²\n'
                  f'P_o = {P_o/1000:.1f} kPa\n'
                  f'L = {L/1000:.0f} km\n'
                  f'h = {h} m')
    fig.text(0.02, 0.02, param_text, fontsize=10, bbox=dict(facecolor='white', alpha=0.8))

    # Adjust layout
    plt.tight_layout()
    fig.suptitle('Storm Surge Components Analysis', y=1.02, fontsize=14)

    # Show plot
    plt.show()
```



2.Ocean Waves

Energy Transfer From Wind To Ocean Waves

$$q_{wave} = \iint_A 3.5\rho_a u_{\star a}^2 \, dx \, dy$$

q_{wave}: Energy Induced to Ocean Surface

A: The area Where Energy is measured

ρ_a: The air density

$u_{\bigstar a}$: Effective speed at which the wind pushes the surface.

u_{*a} = $\sqrt{\tau/\rho_a}$

Sensitivity Analysis can be given As:-

The most sensitive parameter is the air friction velocity, $u_{\bigstar a}$, with a sensitivity of around 2.0. This means that a 1% change in $u_{\bigstar a}$ will result in a 2% change in the wave power.

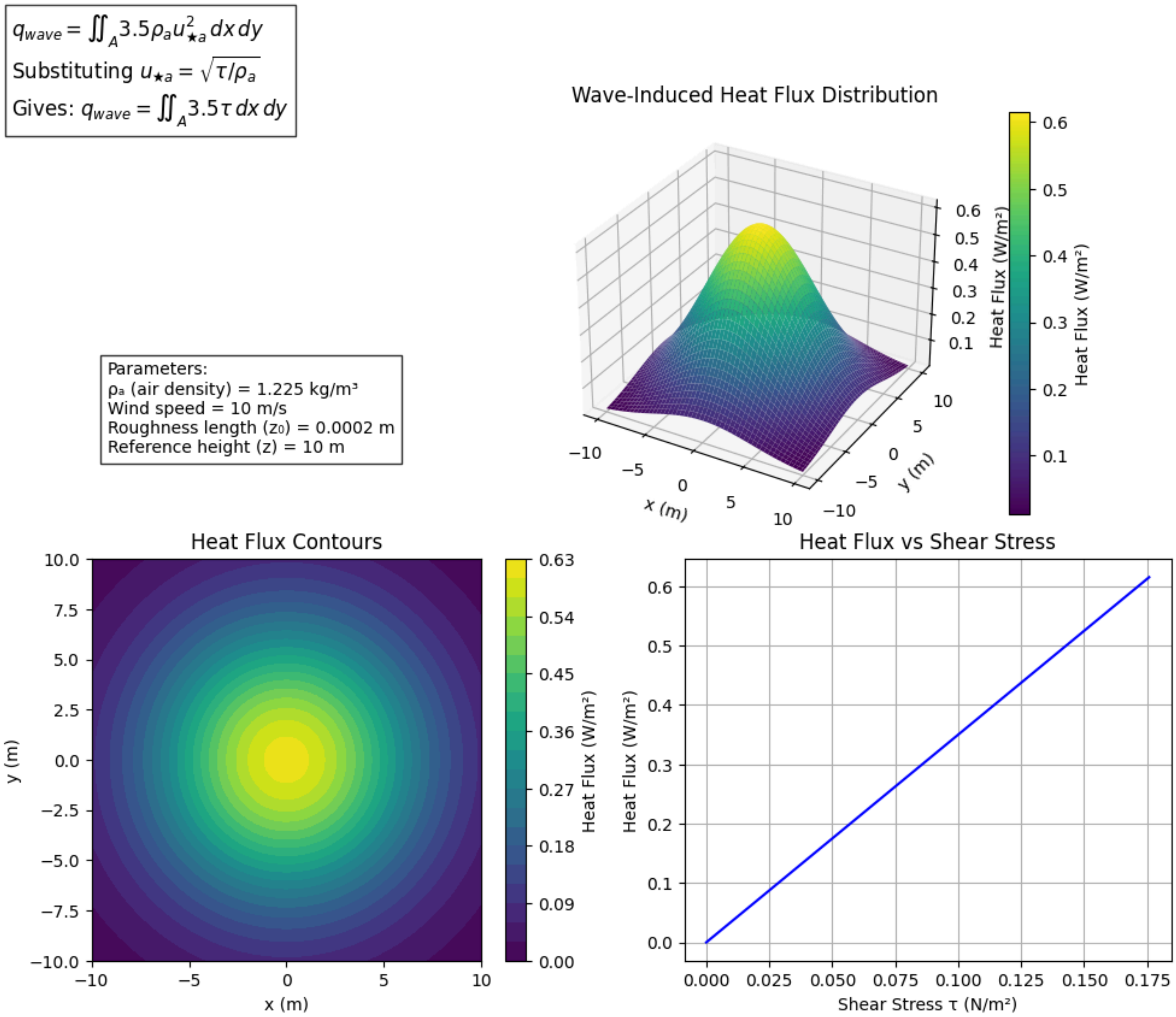
The air density, ρ_a, has a sensitivity of around 1.0, meaning a 1% change in ρ_a will result in a 1% change in the wave power. The area, A, has a sensitivity of 1.0, as the wave power is directly proportional to the area.

Graph in 2-D can Be Given As :-

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from matplotlib.gridspec import GridSpec

def calculate_wave_heat_flux_shear():
    # Constants
    rho_air = 1.225  # air density in kg/m³

    # Create spatial grid
    x = np.linspace(-10, 10, 100)
    y = np.linspace(-10, 10, 100)
    X, Y = np.meshgrid(x, y)

    # Calculate shear stress (τ) based on position
    wind_speed = 10  # reference wind speed in m/s
    z0 = 0.0002     # roughness length in m
    kappa = 0.41    # von Karman constant

    # Calculate shear stress using logarithmic wind profile
    z = 10  # reference height in m
    tau = rho_air * (kappa * wind_speed / np.log(z/z0))**2 * np.exp(-(X**2 + Y**2)/50)

    # Calculate heat flux
```

```
    q_wave = 3.5 * tau

    # Create figure with smaller size
    fig = plt.figure(figsize=(10, 8))  # Reduced figure size
    gs = GridSpec(2, 2)

    # 3D Surface plot of heat flux
    ax1 = fig.add_subplot(gs[0, :], projection='3d')
    surf = ax1.plot_surface(X, Y, q_wave, cmap='viridis')
    ax1.set_xlabel('x (m)')
    ax1.set_ylabel('y (m)')
    ax1.set_zlabel('Heat Flux (W/m²)')
    ax1.set_title('Wave-Induced Heat Flux Distribution')
    fig.colorbar(surf, ax=ax1, label='Heat Flux (W/m²)')

    # Contour plot
    ax2 = fig.add_subplot(gs[1, 0])
    contour = ax2.contourf(X, Y, q_wave, levels=20, cmap='viridis')
    ax2.set_xlabel('x (m)')
    ax2.set_ylabel('y (m)')
    ax2.set_title('Heat Flux Contours')
    plt.colorbar(contour, ax=ax2, label='Heat Flux (W/m²)')

    # Shear stress and heat flux relationship
    ax3 = fig.add_subplot(gs[1, 1])
    tau_range = np.linspace(0, np.max(tau), 100)
    q_range = 3.5 * tau_range
    ax3.plot(tau_range, q_range, 'b-')
    ax3.set_xlabel('Shear Stress τ (N/m²)')
    ax3.set_ylabel('Heat Flux (W/m²)')
    ax3.set_title('Heat Flux vs Shear Stress')
    ax3.grid(True)

    # Add equations
    original_eq = r"$q_{wave} = \iint_A 3.5 \rho_a u_{★a}^2 \, dx \, dy$"
    substituted_eq = r"$u_{★a} = \sqrt{\tau / \rho_a}$"
    final_eq = r"$q_{wave} = \iint_A 3.5 \tau \, dx \, dy$"

    eqs = f"{original_eq}\nSubstituting {substituted_eq}\nGives: {final_eq}"
    fig.text(0.02, 0.95, eqs, fontsize=12,
             bbox=dict(facecolor='white', alpha=0.8))

    # Add parameters
    params = (f"Parameters:\n"
             f"ρₐ (air density) = {rho_air} kg/m³\n"
             f"Wind speed = {wind_speed} m/s\n"
             f"Roughness length (z₀) = {z0} m\n"
             f"Reference height (z) = {z} m")
    fig.text(0.1, 0.6, params, fontsize=10,
             bbox=dict(facecolor='white', alpha=0.8))

    plt.tight_layout()
    plt.show()

    # Calculate total integrated heat flux
```

```
        dx = x[1] - x[0]
        dy = y[1] - y[0]
        total_flux = np.sum(q_wave) * dx * dy

        print(f"\nTotal integrated heat flux: {total_flux:.2f} W")

    # Generate the visualization
    calculate_wave_heat_flux_shear()
```




```
Total integrated heat flux: 88.50 W
```

3. Ocean Currents

Energy Transfer From Wind To Ocean Currents

$q_{current} = \iint_A \boldsymbol{\tau} \cdot \boldsymbol{u}_s \, dx \, dy$

$q_{current}$: Energy To Currents From Hurricane

$\boldsymbol{u}_s$: Surface Velocity(Velocity Of Mixed Layer)

4.Heat Transfer In Oceans

$\frac{dh}{dt} = \frac{2\rho_0 m(\tau/\rho_0)^{3/2}}{\rho_0 gah(T_s - T_{-h})}$ = *G*

If G Is High More Storage Of heat At Ocean Surfaces Leading to much Stronger Hurricane

ρ_0 : Reference Density Of SeaWater

a : Coefficient Of Heat Expansion

T_{-h} : Temperature At mixed Layer Base

T_s : Sea Surface Temperature

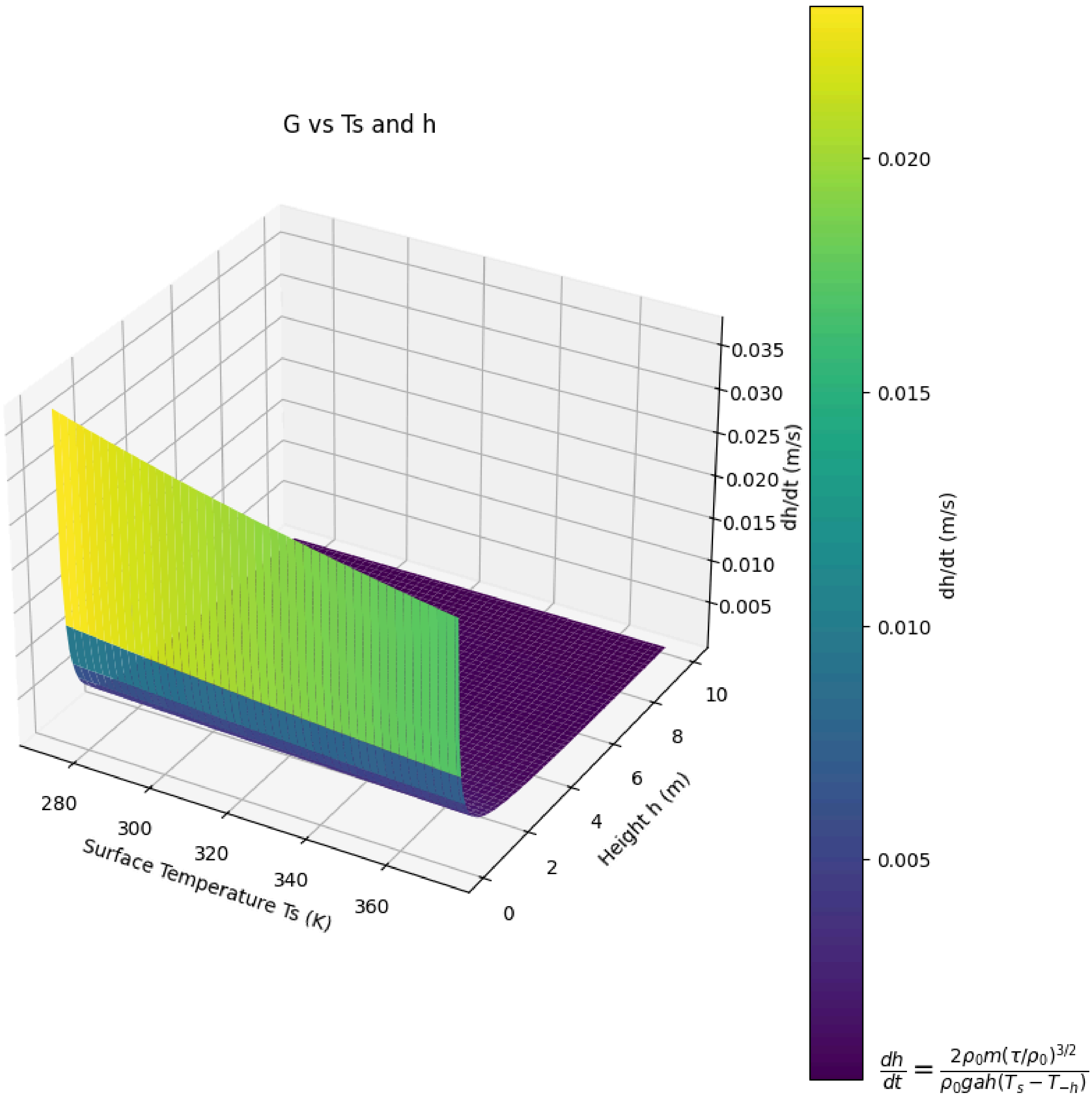
τ :Wind Stress Magnitude

h : Mixed Layer Depth

Graph Can be Given As :-

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Fixed parameters (set as constants)
rho_0 = 1.0
m = 1.0
tau = 1.0
g = 9.81
a = 1.0

# Create arrays for Ts and h
Ts = np.linspace(273, 373, 100)  # Temperature range from 0°C to 100°C
h = np.linspace(0.1, 10, 100)    # Height range from 0.1m to 10m
Ts_mesh, h_mesh = np.meshgrid(Ts, h)
```

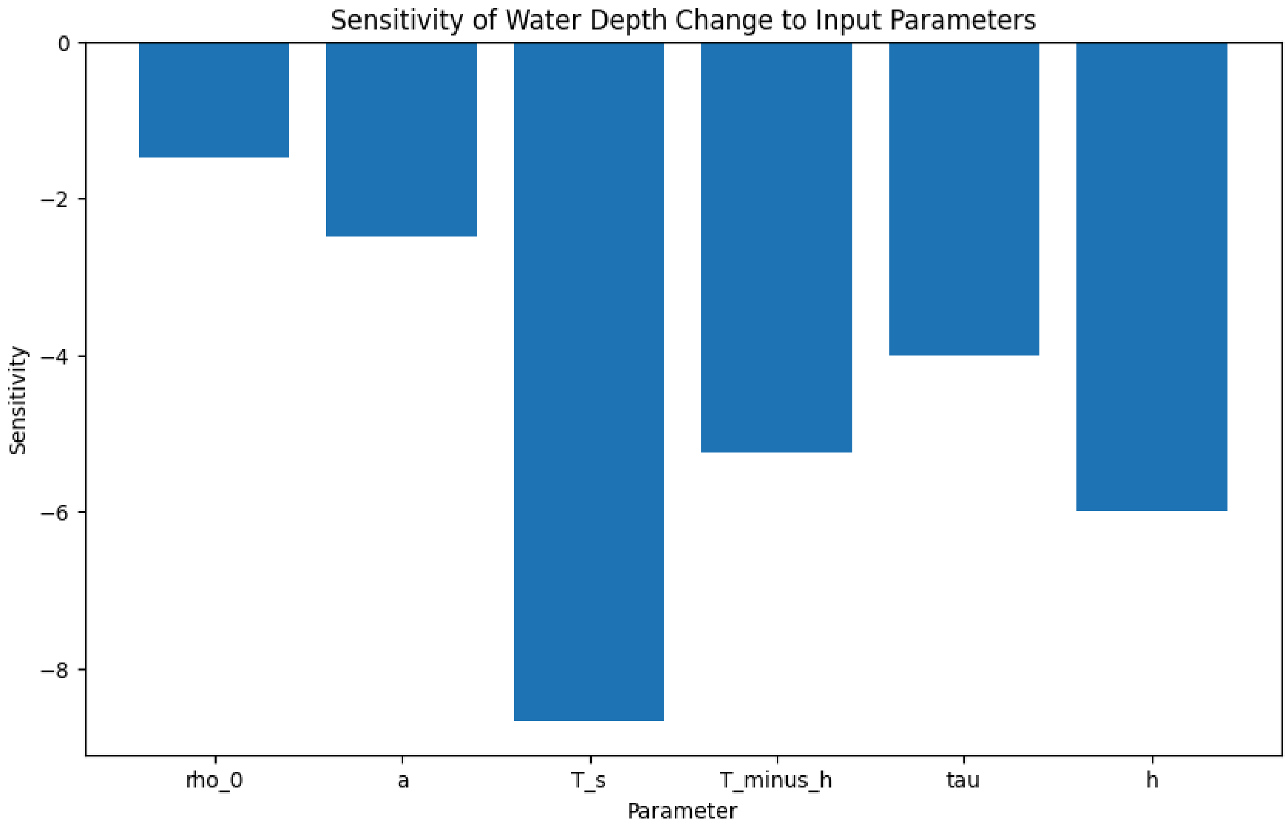
```
# Calculate T_-h (temperature at depth) as a function of Ts
T_neg_h = 0.8 * Ts_mesh  # Assuming T_-h is proportional to Ts

# Calculate dh/dt
numerator = 2 * rho_0 * m * (tau/rho_0)**(3/2)
denominator = rho_0 * g * a * h_mesh * (Ts_mesh - T_neg_h)
dh_dt = numerator/denominator

# Create figure
fig = plt.figure(figsize=(15, 8))

# 3D Surface plot
ax1 = fig.add_subplot(121, projection='3d')
surf = ax1.plot_surface(Ts_mesh, h_mesh, dh_dt, cmap='viridis')
ax1.set_xlabel('Surface Temperature Ts (K)')
ax1.set_ylabel('Height h (m)')
ax1.set_zlabel('dh/dt (m/s)')
ax1.set_title('G vs Ts and h')
fig.colorbar(surf, ax=ax1, label='dh/dt (m/s)')



# Add equation
plt.figtext(0.5, 0.02,
           r'$\frac{dh}{dt} = \frac{2 \rho_0 m (\tau / \rho_0)^{3/2}}{\rho_0 g a h (T_
           fontsize=14, ha='center')

plt.tight_layout()
plt.show()
```



Sensitivity Analysis can Be Given As:-

```
import numpy as np
import matplotlib.pyplot as plt

# Define the function to calculate the rate of change of water depth
def calculate_water_depth_change(rho_0, a, T_s, T_minus_h, tau, h):
    return (2 * rho_0 * 1 * (tau / rho_0)**(3/2)) / (rho_0 * 9.81 * a * h * (T_s - T_r
```

```
# Define the parameter ranges for the Monte Carlo simulation
param_ranges = {
    'rho_0': (1020, 1030),  # Reference Density Of SeaWater (kg/m³)
    'a': (0.0002, 0.0003),  # Coefficient Of Heat Expansion (1/°C)
    'T_s': (20, 30),  # Sea Surface Temperature (°C)
    'T_minus_h': (10, 20),  # Temperature At mixed Layer Base (°C)
    'tau': (0.1, 2.0),  # Wind Stress Magnitude (N/m²)
    'h': (10, 100)  # Mixed Layer Depth (m)
}

# Run the Monte Carlo simulation
n_samples = 10000
samples = {
    param: np.random.uniform(min_val, max_val, n_samples)
    for param, (min_val, max_val) in param_ranges.items()
}

# Calculate the rate of change of water depth for each sample
dh_dt = np.array([calculate_water_depth_change(samples['rho_0'][i], samples['a'][i], s
                 for i in range(n_samples)])

# Calculate the sensitivity of dh/dt to each parameter
sensitivities = {}
for param in param_ranges.keys():
    mean_val = samples[param].mean()
    mean_dh_dt = dh_dt.mean()
    delta = mean_val * 0.01  # 1% change

    # Calculate dh/dt with parameter + delta
    modified_samples = samples.copy()
    modified_samples[param] += delta
    modified_dh_dt = np.array([calculate_water_depth_change(modified_samples['rho_0']|
                              for i in range(n_samples)])

    # Calculate sensitivity
    sensitivity = ((modified_dh_dt.mean() - mean_dh_dt) / mean_dh_dt) / (delta / mean_
    sensitivities[param] = sensitivity

# Plot the sensitivity results
fig, ax = plt.subplots(figsize=(10, 6))
ax.bar(sensitivities.keys(), sensitivities.values())
ax.set_xlabel('Parameter')
ax.set_ylabel('Sensitivity')
ax.set_title('Sensitivity of Water Depth Change to Input Parameters')
plt.show()
```



5.Momentum transfer From Wind to Oceans

$$\frac{\partial u}{\partial t} - fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\tau_w^x - \tau_R^x}{\rho_0 h_1}$$

$$\frac{\partial v}{\partial t} + fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \frac{\tau_w^y - \tau_R^y}{\rho_0 h_1}$$

u is the zonal(east-west) component of oceans velocity

v is the meridonal(north-south) component of oceans velocity

$\frac{\partial p}{\partial x}$ Pressure Gradient in Zonal Direction

h_1 is the mixed layer depth

τ_w^x , τ_w^y is the wind stress in zonal , meridonial direction

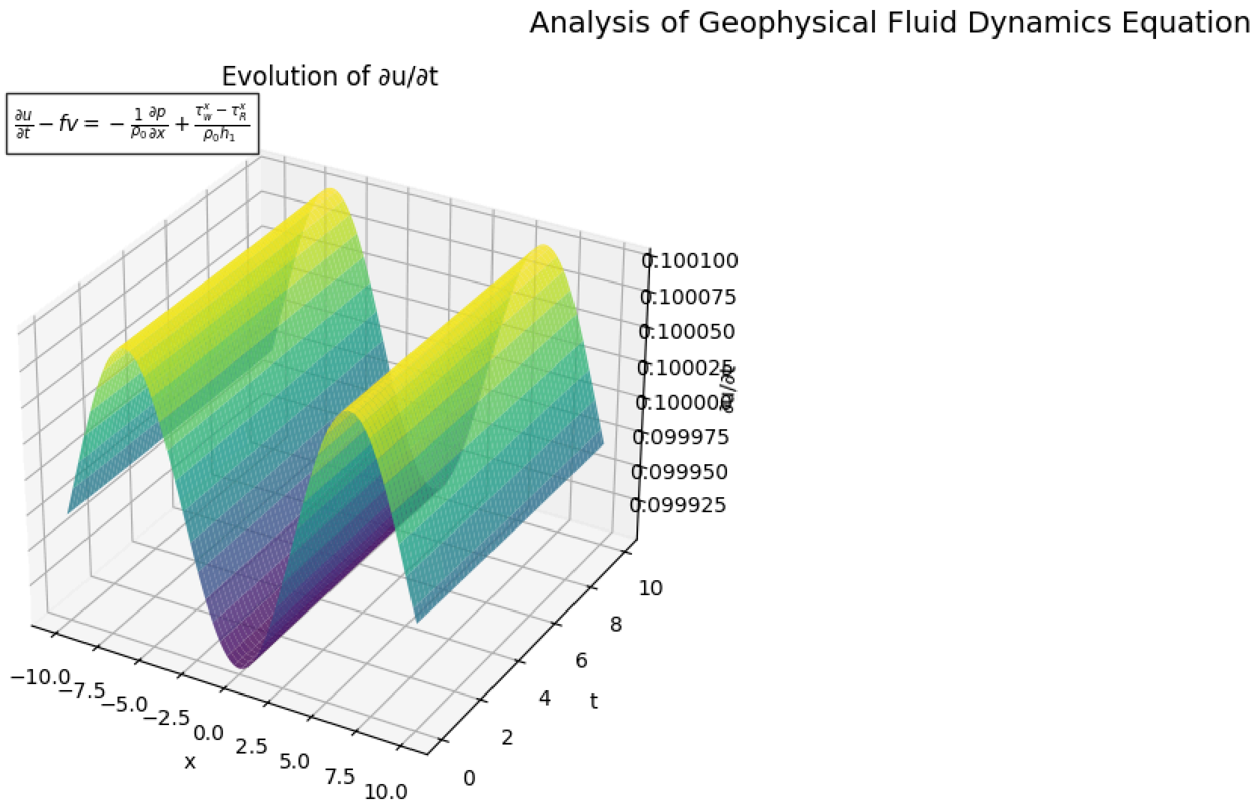
τ_R^x , τ_R^y is the frictional Resistance in zonal , meridonial direction

$\frac{\partial p}{\partial y}$ is the pressure gradient in meridonial direction

Lets Consider Graph Of Varying Zonal velocity , As Bith Will Be Similar :-

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Create figure
fig = plt.figure(figsize=(15, 10))

# Parameters
x = np.linspace(-10, 10, 100)
t = np.linspace(0, 10, 100)
X, T = np.meshgrid(x, t)

# Example parameter values
f = 1.0  # Coriolis parameter
rho_0 = 1000.0  # Reference density
h1 = 100.0  # Layer thickness
v = 0.1  # Meridional velocity

# Pressure gradient (example: linear gradient)
dp_dx = 0.1 * np.cos(0.5*X)

# Wind stress and bottom stress (example functions)
tau_w = 0.1 * np.sin(0.3*X + 0.2*T)
tau_R = 0.05 * np.sin(0.2*X - 0.1*T)

# Calculate du/dt (simplified for visualization)
du_dt = f*v - (1/rho_0)*dp_dx + (tau_w - tau_R)/(rho_0 * h1)

# 3D Surface plot
ax1 = fig.add_subplot(221, projection='3d')
surf = ax1.plot_surface(X, T, du_dt, cmap='viridis', alpha=0.8)
ax1.set_xlabel('x')
ax1.set_ylabel('t')
ax1.set_zlabel('∂u/∂t')
ax1.set_title('Evolution of ∂u/∂t')

# Add equation to the plot
ax1.text2D(0.05, 0.95,
          r'$\frac{\partial u}{\partial t} - f v = -\frac{1}{\rho_0} \frac{\partial p
          transform=ax1.transAxes, fontsize=10, bbox=dict(facecolor='white', alpha=0.



# Adjust layout
```

```
plt.tight_layout()
fig.suptitle('Analysis of Geophysical Fluid Dynamics Equation', y=1.02, fontsize=14)

# Show plot
plt.show()
```



4. Ocean To Hurricane

1.Energy Stored In Hurricanes

$$OHC = \int_0^h \rho c_p [T(^\circ C) - T_{ref}]\, dz$$

OHC is the Ocean Heat Content

ρ : Sea Water Density

T : Ocean Tempreture

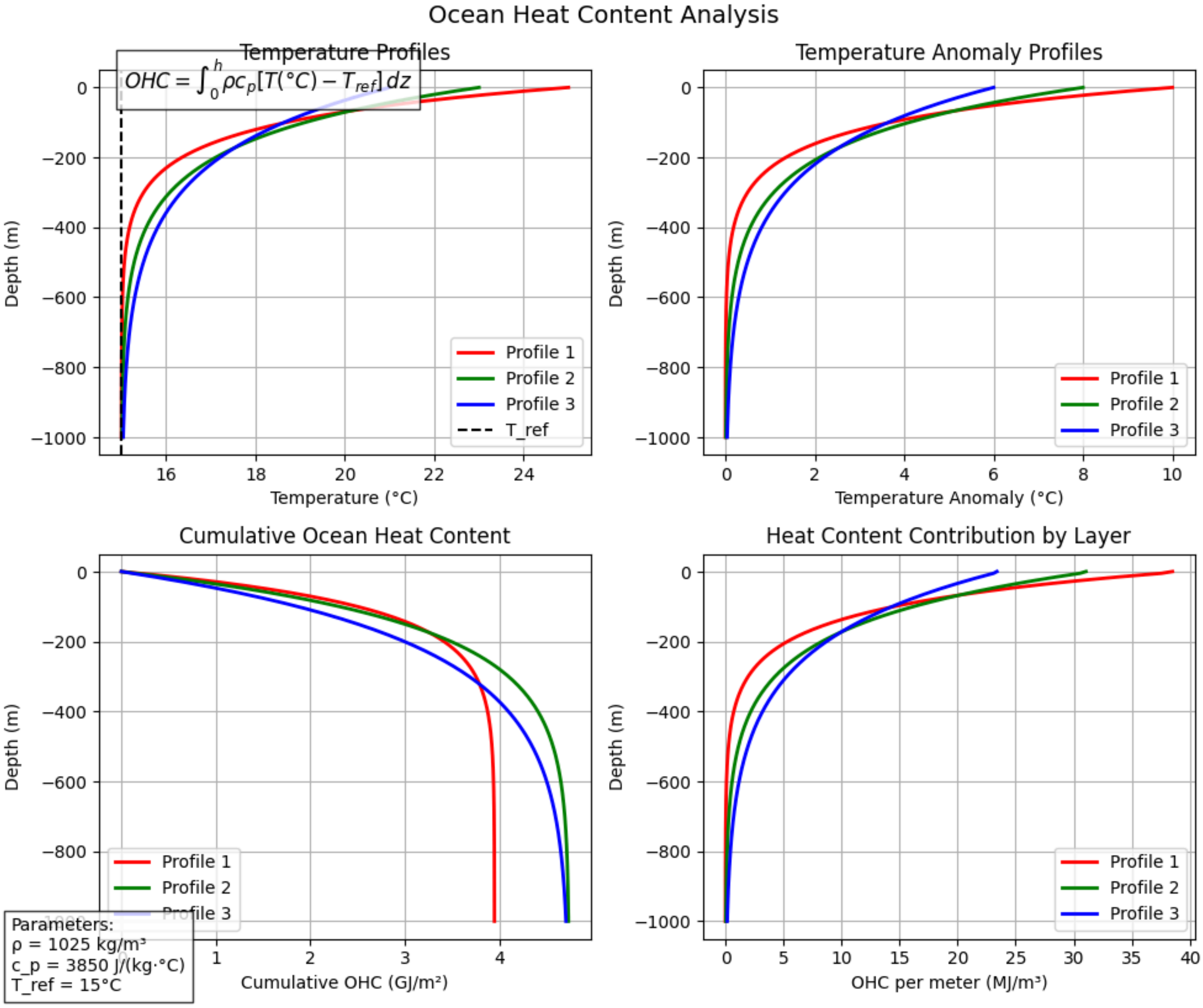
h: h is the mixed layer depth

Graph By Considering only in 1-D can Be Given As:-

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import cumtrapz

# Create figure with reduced size
fig = plt.figure(figsize=(10, 8))  # Reduced figsize from (15, 10) to (10, 8)

# Parameters
rho = 1025  # kg/m³ (typical seawater density)
cp = 3850   # J/(kg·°C) (specific heat capacity)
depth = np.linspace(0, 1000, 200)  # depth in meters

# Create temperature profiles
T_ref = 15  # reference temperature in °C
T1 = T_ref + 10 * np.exp(-depth / 100)  # example temperature profile 1
T2 = T_ref + 8 * np.exp(-depth / 150)   # example temperature profile 2
T3 = T_ref + 6 * np.exp(-depth / 200)   # example temperature profile 3

# Calculate OHC for each profile
ohc1 = cumtrapz(rho * cp * (T1 - T_ref), depth, initial=0)
ohc2 = cumtrapz(rho * cp * (T2 - T_ref), depth, initial=0)
ohc3 = cumtrapz(rho * cp * (T3 - T_ref), depth, initial=0)

# Plot 1: Temperature profiles
ax1 = fig.add_subplot(221)
ax1.plot(T1, -depth, 'r-', label='Profile 1', linewidth=2)
ax1.plot(T2, -depth, 'g-', label='Profile 2', linewidth=2)
ax1.plot(T3, -depth, 'b-', label='Profile 3', linewidth=2)
ax1.axvline(x=T_ref, color='k', linestyle='--', label='T_ref')
ax1.set_xlabel('Temperature (°C)')
ax1.set_ylabel('Depth (m)')
ax1.set_title('Temperature Profiles')
ax1.grid(True)
ax1.legend()

# Add equation
ax1.text(0.05, 0.95, r'$OHC = \int_0^{h} \rho c_p [T(°C) - T_{ref}] \, dz$',
         transform=ax1.transAxes, fontsize=12, bbox=dict(facecolor='white', alpha=0.8)

# Plot 2: Temperature anomaly
ax2 = fig.add_subplot(222)
ax2.plot(T1 - T_ref, -depth, 'r-', label='Profile 1', linewidth=2)
ax2.plot(T2 - T_ref, -depth, 'g-', label='Profile 2', linewidth=2)
ax2.plot(T3 - T_ref, -depth, 'b-', label='Profile 3', linewidth=2)
ax2.set_xlabel('Temperature Anomaly (°C)')
ax2.set_ylabel('Depth (m)')
ax2.set_title('Temperature Anomaly Profiles')
ax2.grid(True)
```

```
    ax2.legend()

    # Plot 3: Cumulative OHC
    ax3 = fig.add_subplot(223)
    ax3.plot(ohc1 / 1e9, -depth, 'r-', label='Profile 1', linewidth=2)
    ax3.plot(ohc2 / 1e9, -depth, 'g-', label='Profile 2', linewidth=2)
    ax3.plot(ohc3 / 1e9, -depth, 'b-', label='Profile 3', linewidth=2)
    ax3.set_xlabel('Cumulative OHC (GJ/m²)')
    ax3.set_ylabel('Depth (m)')
    ax3.set_title('Cumulative Ocean Heat Content')
    ax3.grid(True)
    ax3.legend()

    # Plot 4: OHC contribution by layer
    layer_thickness = depth[1] - depth[0]
    ohc_rate1 = np.gradient(ohc1, layer_thickness)
    ohc_rate2 = np.gradient(ohc2, layer_thickness)
    ohc_rate3 = np.gradient(ohc3, layer_thickness)

    ax4 = fig.add_subplot(224)
    ax4.plot(ohc_rate1 / 1e6, -depth, 'r-', label='Profile 1', linewidth=2)
    ax4.plot(ohc_rate2 / 1e6, -depth, 'g-', label='Profile 2', linewidth=2)
    ax4.plot(ohc_rate3 / 1e6, -depth, 'b-', label='Profile 3', linewidth=2)
    ax4.set_xlabel('OHC per meter (MJ/m³)')
    ax4.set_ylabel('Depth (m)')
    ax4.set_title('Heat Content Contribution by Layer')
    ax4.grid(True)
    ax4.legend()

    # Add parameters text box
    param_text = f'Parameters:\nρ = {rho} kg/m³\nc_p = {cp} J/(kg·°C)\nT_ref = {T_ref}°C'
    fig.text(0.02, 0.02, param_text, fontsize=10, bbox=dict(facecolor='white', alpha=0.8))

    # Adjust layout
    plt.tight_layout()
    fig.suptitle('Ocean Heat Content Analysis', y=1.02, fontsize=14)

    # Show plot
    plt.show()
```

```
<ipython-input-30-b8963dbb4aa5>:20: DeprecationWarning: 'scipy.integrate.cumtrapz
  ohc1 = cumtrapz(rho * cp * (T1 - T_ref), depth, initial=0)
<ipython-input-30-b8963dbb4aa5>:21: DeprecationWarning: 'scipy.integrate.cumtrapz
  ohc2 = cumtrapz(rho * cp * (T2 - T_ref), depth, initial=0)
<ipython-input-30-b8963dbb4aa5>:22: DeprecationWarning: 'scipy.integrate.cumtrapz
  ohc3 = cumtrapz(rho * cp * (T3 - T_ref), depth, initial=0)
```



2.Momentum Transfer

$h\frac{d\bar{M}}{dt} = -C_D r|\mathbf{V}|V$(Doubt)

$\frac{d\bar{M}}{dt}$:momentum of the ocean's mixed layer changes over time due to the drag force exerted by the hurricane's winds.

C_D : Drag Coefficient

V : Velocity Of Ocean Currents

3.Mass Transfer

$E = C_E\,\rho_{\text{air}}\,|\mathbf{V}|(q_s - q_a)$

E is the Evaporation Rate

C_E is the Exchange Coefficient Constant

$|\mathbf{V}|$ is The Wind Speed At Ocean Surface

q_s is the specific humidity at sea surface

q_a is the specific humidity of air above sea surface

Graph Can Be Given As:-

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Create figure with reduced size
fig = plt.figure(figsize=(10, 8))  # Reduced figsize from (15, 10) to (10, 8)

# Constants and parameters
CE = 1.5e-3        # Bulk transfer coefficient
rho_air = 1.225    # Air density (kg/m³)
```