Retrofit Flight Control Using an Adaptive Chebyshev Function Approximator

Sung-Sik Shin[1]; Byoung-Mun Min[2]; and Min-Jea Tahk[3]

Abstract: In this paper, a novel adaptive control approach, named adaptive Chebyshev retrofit control (ACRC), retrofitting an existing baseline controller with an adaptive Chebyshev function approximator is presented. The approximator is composed of a linear combination of a parameter and a basis function. Instead of using neural networks as a function approximator, the new approach utilizes a Chebyshev polynomial as a basis function for function approximation, and a parameter update law is derived via a Lyapunov-like analysis method. The benefits of the proposed method are twofold. First, the computational time is approximately 1.7 times faster than that of the method using the neural network. Second, the implementation is very efficient, because the structure of the approximator is significantly simpler in comparison with those of neural network approaches. Because the complexity of the software is the major contributing factor to software reliability, the high complexity of the implementation of a control algorithm that adopts neural networks could lead to a reduction in software reliability. Therefore, the new adaptive control method is valuable in terms of the improvement in software reliability. In particular, it is important in the field of aerospace control, which requires exceptional reliability for flight control software. Moreover, the short computational time in comparison with neural network approaches is very crucial for small unmanned aerial vehicles that have restricted on-board hardware performance. From simulation results, it is found that the performance of the proposed method in several responses is on par with that of the neural network method in the presence of varying flight conditions. Considering the computation time and simplicity of the proposed method, the authors conclude that the proposed approach is very effective, particularly relative to the neural network method. **DOI: 10.1061/(ASCE)AS.1943-5525.0000191.**

CE Database subject headings: Control systems; Neural networks; Adaptive systems; Aerospace engineering.

Author keywords: Retrofit flight control; Adaptive Chebyshev function approximator; Neural networks.

Introduction

A number of systems including aerospace systems are normally operated by a linear controller in the nominal region. This type of controller has been applied for many years, and it accomplishes the objectives in the nominal domain well. However, during operation, the system state may deviate from the nominal region for several reasons. For example, the aircraft may encounter a disturbance whose magnitude is larger than the threshold for which the nominal controller was designed and tested. In this case, the existing baseline controller's performance is radically degraded, and the system may even become unstable. To solve this problem, it is possible to replace the nominal controller with an advanced controller such as a dynamic inversion controller.

However, for an approach such as dynamic inversion, the existing baseline control architecture must be based on inversion inherently. Therefore, in this case, the existing baseline control architecture must be redesigned to apply a dynamic inversion approach. Considering that the vast majority of existing baseline controllers are not based on inversion, it would be highly desirable to retrofit such systems with an adaptive element rather than replace them with a dynamic inversion controller. In particular, within the aircraft industry, there is a legacy of experience with existing baseline controller architectures, and thus, engineers in this industry would much prefer to augment their plants with an adaptive element rather than replace them with a totally new architecture. Moreover, from the point of view of verification and validation (V&V), the sunk costs and time invested for nominal controller design would be wasted. In addition, the V&V of the new controller would need to be performed from the initial stage. The V&V method for flight control software is based on tedious computer simulations of the flight control algorithms in different flight regimes. For these reasons, it is undesirable to develop completely new control algorithms, which would require entire V&V and software certification procedures for the performance upgrade of existing aircrafts.

Therefore, retrofit control, a method to prevent performance deterioration of existing baseline controllers without controller redesign, has been previously developed. Several methods to add an adaptive element to the existing controller architecture have recently been reported in Calise et al. (2002), Campa et al. (2000), and Sharma and Calise (2005). These methods augment existing linear controllers by using a neural network approach. In the field of adaptive control,

[1]Patent Examiner, Transport Machinery Exam. Division, Korean Intellectual Property Office, Gov. Complex Daejeon Bldg. 4, 189, Cheongsa-ro, Seo-gu, Daejeon 302-701, Republic of Korea; formerly, Senior Researcher and Head of Group, Flight Control Group, Korean Air Research and Development Center, 461-1 Jeonmin-dong, Yuseong-gu, Daejeon 305-811, Republic of Korea. E-mail: ssshin@kipo.go.kr

[2]Researcher, Flight Control Group, Korean Air Research and Development Center, 461-1 Jeonmin-dong, Yuseong-gu, Daejeon 305-811, Republic of Korea. E-mail: bmmin@koreanair.com

[3]Professor, Dept. of Aerospace Engineering, Korea Advanced Institute of Science and Technology (KAIST), Daejeon 305-701, Republic of Korea (corresponding author). E-mail: mjtahk@fdcl.kaist.ac.kr

Note. This manuscript was submitted on May 6, 2011; approved on December 14, 2011; published online on December 16, 2011. Discussion period open until March 1, 2014; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Aerospace Engineering***, Vol. 26, No. 4, October 1, 2013. ©ASCE, ISSN 0893-1321/2013/4-735–749/$25.00.

J. Aerosp. Eng., 2013, 26(4): 735-749

neural networks have received much attention and shown good performance. A popular method in using neural networks for control is to combine the neural network with dynamic inversion. This approach has been investigated extensively in the field of flight control, particularly for missiles and aircraft applications, as documented in Johnson et al. (2000), Kim and Calise (1997), Kim (2003), Lewis et al. (1996), Nguyen (2008), Pashilkar et al. (2006), and Yesildirek and Lewis (1995). In these approaches, a neural network is used to cancel inversion errors of the plant dynamics. However, implementing methods that use a neural network–based adaptive element in the software of a flight control computer are very complicated, and the computational burden is very large as well. This may deteriorate the reliability of flight control software. Other approaches that deal with the issue of retrofitting an adaptive module without using a neural network for an existing baseline controller were proposed in Bošković et al. (2005, 2007), Bošković and Mehra (2002), and Wohletz et al. (2000). In these approaches, a standard adaptive controller is applied for retrofitting the software to handle uncertainties such as actuator failure, control effector damage, and state-dependent disturbances.

Meanwhile, the application of Chebyshev polynomials can be found in various fields of engineering (Dinu et al. 2006; Lee and Jeng 1998; Patra and Kot 2002; Zou and Kumar 2011). In Dinu et al. (2006), Chebyshev polynomials are used for unsteady calculations in aeroservoelasticity. Lee and Jeng (1998) proposed the function approximation method, which includes a direct transformation and indirect transformation, to obtain Chebyshev-polynomials-based (CPB) unified model neural networks for feedforward/recurrent neural networks via the approximation of Chebyshev polynomials. In Patra and Kot (2002), for the purpose of dynamic nonlinear system identification, a computationally efficient artificial neural network (ANN) that eliminates the need for a hidden layer by expanding the input pattern by Chebyshev polynomials is proposed. More recently, Zou and Kumar (2011) proposed adaptive output feedback control of spacecraft formation flying using Chebyshev neural networks. The proposed control scheme employs a linear reduced-order observer to generate pseudovelocity-related signals and Chebyshev neural networks to approximate unknown nonlinearities in the system dynamics.

In this paper, a novel approach for retrofitting an existing baseline controller with an adaptive Chebyshev function approximator is presented. The structure of the approximator is composed of a linear combination of a parameter and a basis function. The approach proposed in this paper has an advantage over neural network–based approaches in the sense that its implementation can be relatively less complicated and that its computational load can be alleviated as well. In particular, it will be very effective for small unmanned aerial vehicles (UAVs) with restricted on-board hardware performance. Hence, in this paper, the authors consider the linear and nonlinear flight dynamics of a small UAV.

From simulation results, it is shown that the performance of the proposed method in several responses is on par with that of the neural network method in the presence of varying flight conditions. Considering the computation time and simplicity of the algorithm, the authors can conclude that the proposed method is very effective, particularly in comparison with the neural network method.

Problem Statement

Consider a general nonlinear dynamic system of the form

$$\dot{\mathbf{x}}_T = \mathbf{f}(\mathbf{x}_T) + \mathbf{g}(\mathbf{x}_T)u \tag{1}$$

Assumption 1: Here, $\mathbf{g}(\mathbf{x}_T)$ is a known constant vector, because it is composed of constant control derivatives of an aircraft.

Remark 1: In flight control applications, control derivatives often were regarded as constants, with the exception of the structural damage of control surfaces.

From Assumption 1, one can take $\mathbf{g}(\mathbf{x}_T) = \mathbf{b}$ and replace u with retrofit control u_{retro}

$$\dot{\mathbf{x}}_T = \mathbf{f}(\mathbf{x}_T) + \mathbf{b}u_{\text{retro}} = \mathbf{A}\mathbf{x}_T + \mathbf{f}_U(\mathbf{x}_T) + \mathbf{b}u_{\text{retro}} \tag{2}$$

where $\mathbf{A}\mathbf{x}_T$ = known part; $\mathbf{f}_U(\mathbf{x}_T)$ = unknown part of a dynamic system $\mathbf{f}(\mathbf{x}_T)$; and $\mathbf{x}_T$ = state of the true plant, respectively.

Assumption 2: The system matrix $\mathbf{A}$ is Hurwitz.

Assumption 3: The quantity $\mathbf{A}\mathbf{x}_T$ is known to be designed, and $\mathbf{f}_U(\mathbf{x}_T)$ is unknown, but its states can be measured.

Also, u_{retro} is the retrofit control that is obtained from the summation of an existing linear controller u_{ec} and adaptive control input u_{ad} (i.e., $u_{\text{retro}} = u_{\text{ec}} + u_{\text{ad}}$), which is composed of the multiplication of the pseudo-inverse of the control input vector $\mathbf{b}$ and $\hat{\mathbf{f}}_U(\mathbf{x}_T)$ [i.e., $u_{\text{ad}} = -\mathbf{b}^{\dagger}\hat{\mathbf{f}}_U(\mathbf{x}_T)$].

Eq. (2) can be rewritten to obtain the following equation after manipulation:

$$\begin{aligned}\dot{\mathbf{x}}_T &= \mathbf{A}\mathbf{x}_T + \mathbf{f}_U(\mathbf{x}_T) + \mathbf{b}u_{\text{retro}} = \mathbf{A}\mathbf{x}_T + \mathbf{f}_U(\mathbf{x}_T) + \mathbf{b}(u_{\text{ec}} + u_{\text{ad}}) \\ &= \mathbf{A}\mathbf{x}_T + \mathbf{f}_U(\mathbf{x}_T) + \mathbf{b}\left[u_{\text{ec}} - \mathbf{b}^{\dagger}\hat{\mathbf{f}}_U(\mathbf{x}_T)\right] \\ &= \mathbf{A}\mathbf{x}_T + \mathbf{f}_U(\mathbf{x}_T) - \mathbf{b}\cdot\mathbf{b}^{\dagger}\hat{\mathbf{f}}_U(\mathbf{x}_T) + \mathbf{b}u_{\text{ec}}\end{aligned} \tag{3}$$

The authors assume that there exists a nonlinear approximator having a good approximation ability and that $\hat{\mathbf{f}}_U(\mathbf{x}_T)$ is the approximation of the unknown part $\mathbf{f}_U(\mathbf{x}_T)$. From Eq. (3), if the unknown part $\mathbf{f}_U(\mathbf{x}_T)$ can be approximately canceled by $\mathbf{b}\cdot\mathbf{b}^{\dagger}\hat{\mathbf{f}}_U(\mathbf{x}_T)$, as delineated in Eq. (4), then the dynamic system becomes a linear system with some small or weak residual uncertainty attributable to the approximation error, as given in Eq. (5).

Assumption 4: In Eq. (3), the uncertainty satisfies the matching condition

$$\tilde{\mathbf{f}}_U(\mathbf{x}_T) = \mathbf{f}_U(\mathbf{x}_T) - \mathbf{b}\cdot\mathbf{b}^{\dagger}\hat{\mathbf{f}}_U(\mathbf{x}_T) \approx \mathbf{0} \tag{4}$$

$$\dot{\mathbf{x}}_T = \mathbf{A}\mathbf{x}_T + \mathbf{b}u_{\text{ec}} \tag{5}$$

As a result, Eq. (1) becomes the existing baseline control system as given in Eq. (5) by retrofitting the unknown dynamic system with an adaptive Chebyshev function approximator.

Adaptive Chebyshev Retrofit Control

Retrofit Flight Control Concept

The proposed approach is illustrated in Fig. 1. As seen in the figure, the adaptive Chebyshev function approximator for the retrofit control is added to the existing baseline controller to compensate for uncertainties such as the change of flight conditions. Also, to compare the performance of the proposed method, a retrofit flight control architecture described in Kim (2003) that uses a neural network for a function approximation, as shown in Fig. 2, is considered.

J. Aerosp. Eng., 2013, 26(4): 735-749

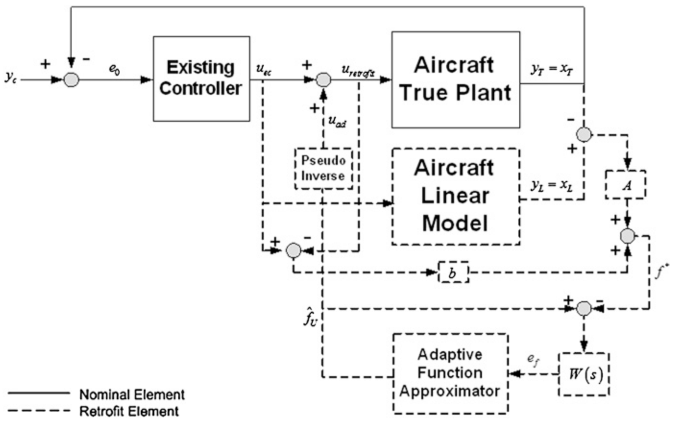

Fig. 1. Structure of the proposed ACRC

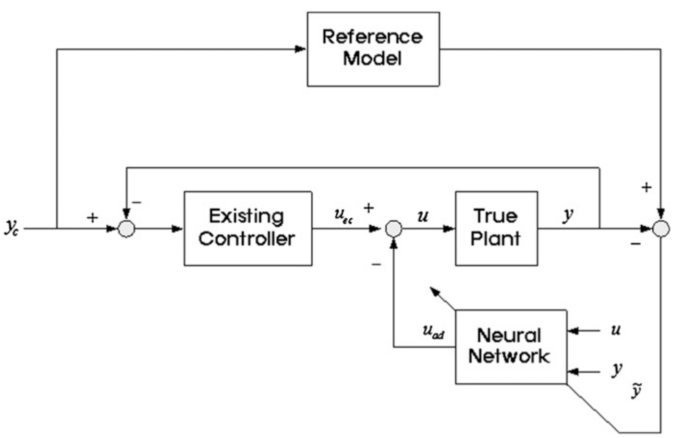

Fig. 2. Structure of the retrofit control using neural network

Chebyshev Polynomials

In mathematics, the Chebyshev polynomials, named after Pafnuty Chebyshev, are a sequence of orthogonal polynomials that are related to de Moivre's formula and are easily defined recursively, similar to Fibonacci or Lucas numbers. There are several kinds of Chebyshev polynomials. In this paper, Chebyshev polynomials of the first kind are used, and for the application, the interval for interpolation is transformed.

Recurrence Formulas and the Solution of Chebyshev Polynomials (Mason and Handscomb 2003)

The following recurrence relations have been used in the Chebyshev polynomials:

$$\phi_0(x) = 1, \quad \phi_1(x) = x$$

$$\phi_n(x) = 2x\phi_{n-1}(x) - \phi_{n-2}(x), \quad n \geq 1 \tag{6}$$

Next, the following condition is used to find the Chebyshev polynomials solution:

$$\phi_n(x) = 0 \tag{7}$$

where n = rank of the Chebyshev polynomial. Eq. (6) gives the following solution:

$$x = \cos[(2j+1)\pi/2n] \tag{8}$$

Transforming the Interval for Interpolation (Mason and Handscomb 2003; Mathews and Fink 2004)

For the application, it is necessary to take a problem stated on an interval $[a, b]$ and reformulate the problem on the interval $[c, d]$ where the solution is known. If the approximation $p_n(x)$ to $f(x)$ is to be obtained on the interval $[a, b]$, then one can change the variable so that the problem is reformulated on $[-1, 1]$

$$x = \frac{b-a}{2}t + \frac{a+b}{2} \quad \text{or} \quad t = 2\frac{x-a}{b-a} - 1 \tag{9}$$

where $a \leq x \leq b$ and $-1 \leq t \leq 1$. The required Chebyshev nodes of $\phi_{n+1}(x)$ on $[-1, 1]$ are

$$t_k = \cos\left(\frac{2n+1-2k}{2n+2}\pi\right) \quad \text{for } k = 0, 1, \ldots, n \tag{10}$$

and the interpolating nodes $\{x_k\}_{k=0}^n$ on $[a, b]$ are obtained using the change of variable

$$x_k = \frac{b-a}{2}t_k + \frac{a+b}{2} \quad \text{for } k = 0, 1, \ldots, n \tag{11}$$

Remark 2: From Rice (1964, Theorem 3-10), if $\phi(x)$ is a Chebyshev basis function, a best approximation $\mathbf{f}_U^*(\mathbf{x}_T)$ exists.

Remark 3: From Remark 2, one can deduce that $\|\mathbf{f}_U(\mathbf{x}_T) - \mathbf{f}_U^*(\mathbf{x}_T)\| < \delta$, where δ is an arbitrary constant that depends on the order of the Chebyshev polynomials, that is, the larger the order is, the smaller δ is. But if the order of the Chebyshev polynomial is beyond some value, δ does not decrease further. From the simulation, the authors find that the performance of retrofit control is about the same for orders greater than 5.

Adaptive Retrofit Control Law Based on Chebyshev Function Approximator

A function approximator with parameter linearity can be represented as

$$\hat{f}_U(\mathbf{x}) = \hat{\boldsymbol{\theta}}^T \boldsymbol{\phi}(\mathbf{x}) \tag{12}$$

where $\mathbf{x} \in D \subset \mathbb{R}^n$; $\boldsymbol{\theta} \in \mathbb{R}^N$; $\hat{f}_U : D \mapsto \mathbb{R}^1$; and D is assumed to be compact. Note that $\hat{f}_U$ is assumed to map a subset of $\mathbb{R}^n$ onto $\mathbb{R}^1$, $\hat{\boldsymbol{\theta}}$ is a parameter vector that will be adjusted, and $\boldsymbol{\phi}(\mathbf{x})$ is a basis function that it is used as a Chebyshev polynomial in this paper.

From Fig. 1, the dynamic system for an aircraft linear model is delineated as

$$\dot{\mathbf{x}}_L = \mathbf{A}\mathbf{x}_L + \mathbf{b}u_{\text{ec}} \tag{13}$$

Here, the authors assume that all state variables can be measured (i.e., $\mathbf{y}_T = \mathbf{x}_T$ and $\mathbf{y}_L = \mathbf{x}_L$).

By subtracting Eqs. (2) and (13), one has the following equation:

$$\begin{aligned}\dot{\mathbf{y}}_T - \dot{\mathbf{y}}_L &= \mathbf{A}(\mathbf{x}_T - \mathbf{x}_L) + \mathbf{f}_U(\mathbf{x}_T) + \mathbf{b}(u_{\text{retro}} - u_{\text{ec}}) \\ &= \mathbf{A}(\mathbf{x}_T - \mathbf{x}_L) + \mathbf{f}_U(\mathbf{x}_T) + \mathbf{b}u_{\text{ad}} \\ &= \mathbf{A}(\mathbf{x}_T - \mathbf{x}_L) + \mathbf{f}_U(\mathbf{x}_T) - \mathbf{b} \cdot \mathbf{b}^\dagger \hat{\mathbf{f}}_U(\mathbf{x}_T)\end{aligned} \tag{14}$$

J. Aerosp. Eng., 2013, 26(4): 735-749

To improve the robustness of parameter estimation, various filters are used, which are quite standard in the adaptive control methodology (Farrell and Polycarpou 2006).

Applying a first-order filter on both sides of Eq. (14) gives

$$\frac{\lambda}{s+\lambda}\left(\dot{\mathbf{y}}_T - \dot{\mathbf{y}}_L\right) = \frac{\lambda}{s+\lambda}\left[\mathbf{A}(\mathbf{x}_T - \mathbf{x}_L) + \mathbf{f}_U(\mathbf{x}_T) - \mathbf{b}\cdot\mathbf{b}^{\dagger}\hat{\mathbf{f}}_U(\mathbf{x}_T)\right] \tag{15}$$

Here, let $W(s) = \lambda/(s+\lambda)$. Rewriting Eq. (15) gives

$$\begin{aligned}\mathbf{e}_f &= W(s)\left[\mathbf{A}(\mathbf{x}_T - \mathbf{x}_L) + \mathbf{f}_U(\mathbf{x}_T) - \mathbf{b}\cdot\mathbf{b}^{\dagger}\hat{\mathbf{f}}_U(\mathbf{x}_T)\right]\\ &= W(s)\left[\mathbf{A}(\mathbf{x}_T - \mathbf{x}_L) + \tilde{\mathbf{f}}_U(\mathbf{x}_T)\right]\\ &= W(s)\left[\mathbf{A}(\mathbf{x}_T - \mathbf{x}_L) + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}(\mathbf{x})\right]\end{aligned} \tag{16}$$

where $\mathbf{e}_f$ = filtered error; and the parameter estimation error $\tilde{\boldsymbol{\theta}}(t) = \hat{\boldsymbol{\theta}}(t) - \boldsymbol{\theta}^*$.

The authors assume that the filter $\mathbf{W}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$ is strictly positive real (SPR), where ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$) is a minimal state-space realization of $\mathbf{W}(s)$. The state-space model is

$$\dot{\mathbf{e}}_0 = \mathbf{A}\mathbf{e}_0 + \tilde{\mathbf{f}}_U(\mathbf{x}_T) = \mathbf{A}\mathbf{e}_0 + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}(\mathbf{x}) \tag{17}$$

$$\mathbf{e}_f = C\mathbf{e}_0$$

where e_0 = error state variable of the realization. To apply the Lyapunov synthesis method, the Lyapunov candidate function is selected as

$$V\left(\mathbf{e}_0, \tilde{\boldsymbol{\theta}}\right) = \frac{\rho}{2}\mathbf{e}_0^T\mathbf{P}\mathbf{e}_0 + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}} \tag{18}$$

where $\mathbf{P} > 0$ = positive matrix; ρ = positive constant to be selected; and $\boldsymbol{\Gamma}$ = positive-definite matrix that will ultimately appear in the adaptive law for updating $\hat{\boldsymbol{\theta}}(t)$ as the learning rate or adaptive gain. By taking the time derivative of V and using the fact that $\boldsymbol{\theta}^*$ is constant (i.e., $\dot{\tilde{\boldsymbol{\theta}}} = \dot{\hat{\boldsymbol{\theta}}}$), one can obtain that

$$\dot{V} = \frac{\rho}{2}\mathbf{e}_0^T\left(\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{e}_0 + \rho\tilde{\boldsymbol{\theta}}^T\mathbf{P}\boldsymbol{\phi}(\mathbf{x})\mathbf{e}_0 + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}} \tag{19}$$

Now, using the Kalman-Yakubovich-Popov lemma in Farrell and Polycarpou (2006), because $\mathbf{W}(s)$ is SPR, there exist positive-definite matrices $\mathbf{P}$ and $\mathbf{Q}$ such that $\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q}$

$$\begin{aligned}\dot{V} &= -\frac{\rho}{2}\mathbf{e}_0^T\mathbf{Q}\mathbf{e}_0 + \rho\tilde{\boldsymbol{\theta}}^T\mathbf{P}\boldsymbol{\phi}(\mathbf{x})\mathbf{e}_0 + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}}\\ &= -\frac{\rho}{2}\mathbf{e}_0^T\mathbf{Q}\mathbf{e}_0 + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}^{-1}\left[\dot{\hat{\boldsymbol{\theta}}} + \rho\boldsymbol{\Gamma}\mathrm{P}\boldsymbol{\phi}(\mathbf{x})\mathbf{e}_0\right]\end{aligned} \tag{20}$$

To obtain desirable stability and convergence properties, the derivative of V should be at least negative semidefinite. Then, one can obtain the parameter adaptive law as the following equation:

$$\dot{\hat{\boldsymbol{\theta}}} = -\rho\boldsymbol{\Gamma}\mathrm{P}\boldsymbol{\phi}(\mathbf{x})\mathbf{e}_0 \tag{21}$$

From the simulation viewpoint, $\rho\boldsymbol{\Gamma}\mathrm{P}$ is a positive constant that can be taken into a single constant.

Simulation Results

In this section, the authors evaluate the performance of the proposed ACRC for compensating for the uncertainties of dynamic systems. Several methods such as ACRC, the neural network method, and baseline control without augmenting an adaptive element were simulated and compared in the presence of change of flight conditions.

The basic flight vehicle frame used in this paper is a blended wing body, pusher-type UAV with a twin vertical tail and boom. The physical properties of the UAV are summarized in Table 1. A full nonlinear UAV model including full envelope aerodynamics, engine dynamics, actuator dynamics, and saturation is considered for the simulation. The simulation was performed in *MATLAB*.

The nominal baseline controller of this test-bed UAV is designed through the conventional and classical control design technique. Also, the performance of the controller is verified by several flight tests. All actuators are characterized by first-order dynamics of the form

$$\frac{u}{u_c} = \frac{20}{s+20}$$

where u = actual position of the control surface; and u_c = position command for that control surface generated by the controllers. The position and rate limits for all control surfaces are, respectively, $[-30, 30°]$ and $[-60, 60°/\mathrm{s}]$.

To test the robustness of the proposed ACRC, the authors varied the flight conditions in the simulation. The nominal model is linearized at the trim condition of a steady-state level flight with an altitude of 200 m and speed of 120 km/h. In case 1, the flight conditions were changed from a nominal trim condition to an altitude of 1,000 m and speed of 100 km/h. In case 2, the flight conditions were changed from a nominal trim condition to an altitude of 3,000 m and speed of 95 km/h. The existing baseline controller was designed through the conventional and classical control design technique. The autopilot is composed of speed, altitude, and heading angle control loops. In the simulation, Vt and q signals are used for nonlinear function approximation, and adaptive gains were chosen as $\gamma_{V_t} = \gamma_q = 3.0$. The order of the Chebyshev polynomials is chosen to be fifth order. In addition, a first-order filter with parameter $\lambda = 10.0$ is used for the simulation.

Case 1

The flight conditions were changed from a nominal trim condition to an altitude of 1,000 m and speed of 100 km/h to test the performance of the retrofit control schemes.

Fig. 3 shows the results of the altitude responses. In the results, one can see the effectiveness of the proposed ACRC. In the case of the proposed ACRC using the existing baseline controller augmented by the adaptive Chebyshev approximator, it is seen

Table 1. Specifications of UAV Model in Nonlinear Simulation

Specification	Value	Unit
Weight		
Empty	50	kg
Max take-off	55	
Wing span	3.0	m
Fuselage length	2.5	m
Height	0.56	m
Engine	13.97	kW
Endurance	40	min

that the response is very desirable despite the change of the flight conditions. However, in the case of the existing baseline controller not augmented by the adaptive Chebyshev approximator, it is seen that the altitude error response is more unacceptable than that in the cases of the proposed ACRC and the neural network method. The response of the proposed ACRC is similar to that of the neural network method. The altitude error responses are compared in Fig. 4. First, the proposed ACRC represents the best performance among the evaluated methods. The altitude error of it is the smallest, and the convergence speed of the error is also

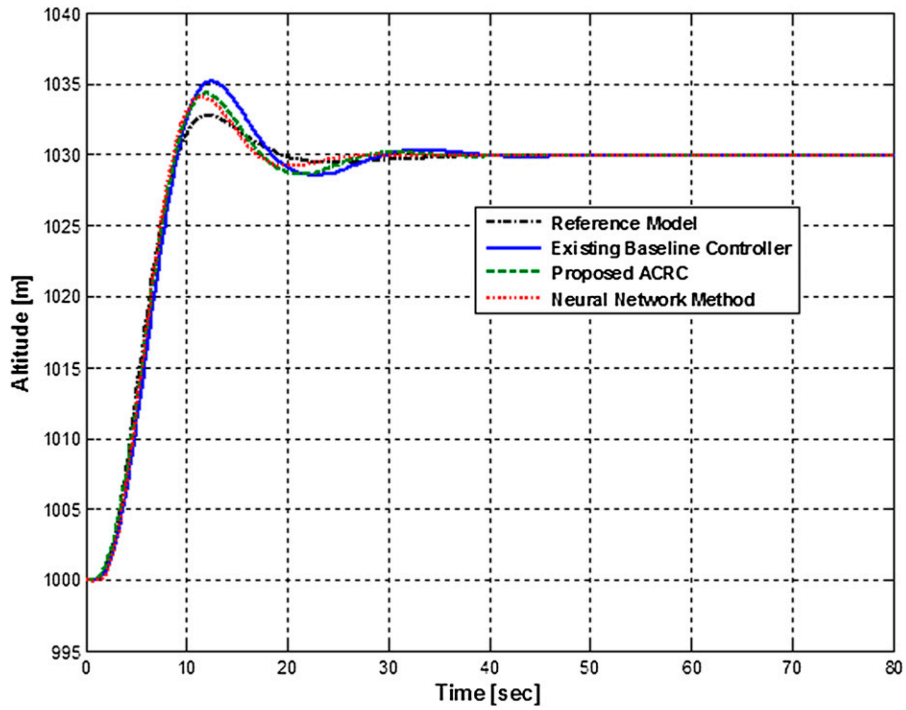

Fig. 3. Altitude response in case 1

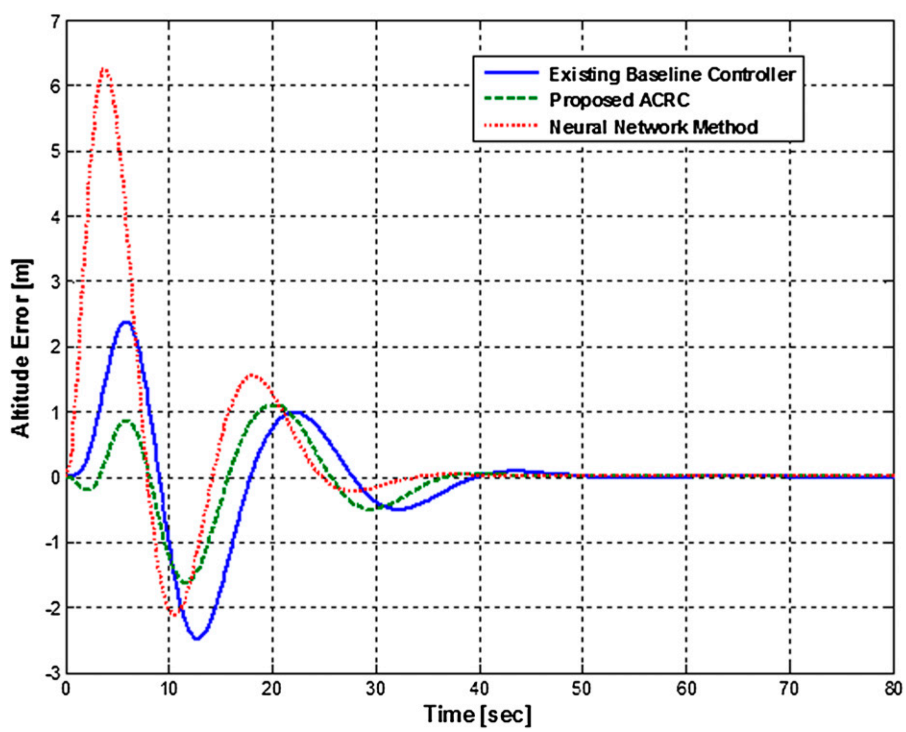

Fig. 4. Altitude error response in case 1

J. Aerosp. Eng., 2013, 26(4): 735-749

fast as well. In the case of the neural network method, its response shows a large altitude error as compared with those of the nonaugmented baseline controller and the proposed ACRC. However, after a simulation time of 10 s, the error of the neural network method is rapidly reduced. On the other hand, the baseline controller without retrofitting shows the slowest convergence among the three methods. The response of the velocity is shown in Fig. 5. In the result, one can see that the neural network method has the smallest deviation from the trim condition, and the baseline controller has the largest deviation. In terms of the angle of attack (AoA) response, shown in Fig. 6, the proposed ACRC shows the best performance among the three

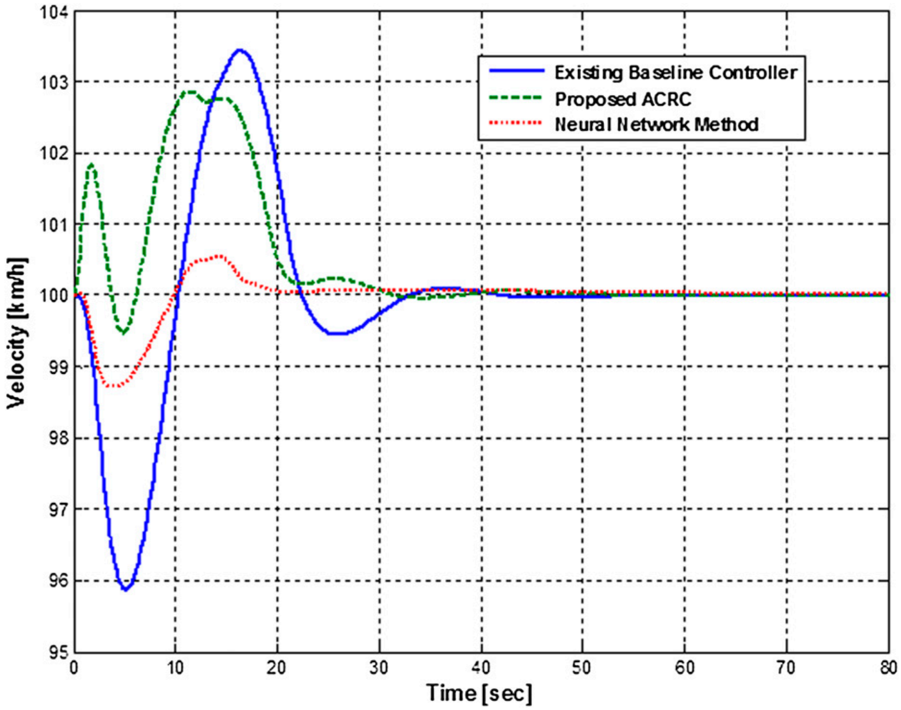

Fig. 5. Velocity response in case 1

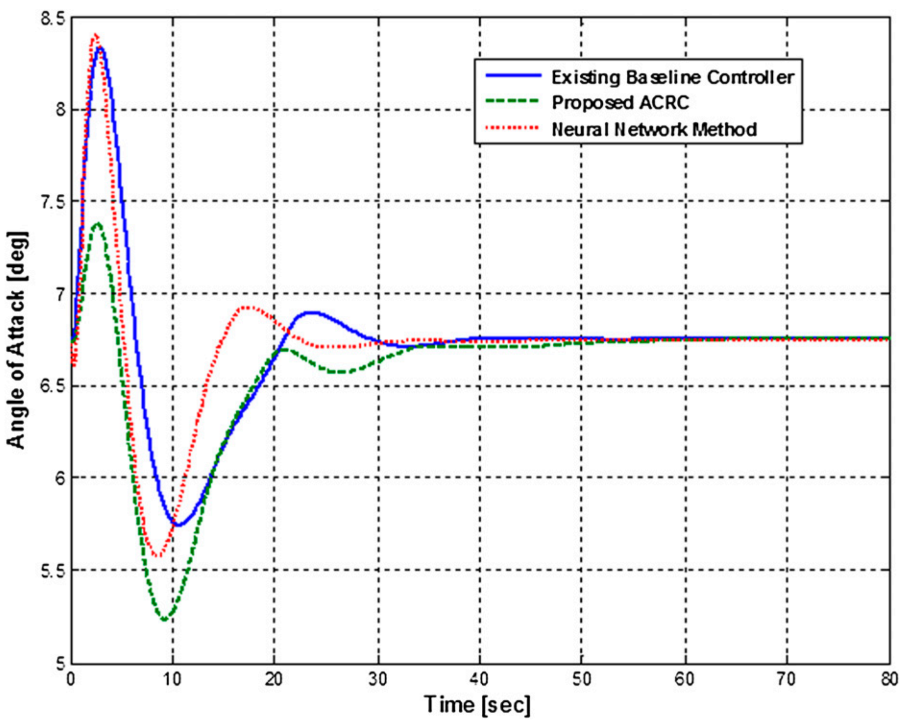

Fig. 6. Response of AoA in case 1

J. Aerosp. Eng., 2013, 26(4): 735-749

methods; that is, it has the smallest deviation from the trim condition. The pitch rate and pitch angle responses are expressed in Figs. 7 and 8, where it is seen that the performance differences among the three methods are not large. In the case of the normalized throttle, presented in Fig. 9, the performances of the three methods are relatively equal. However, in the cases of the left and right elevators, shown in Figs. 10 and 11, the neural network method uses the smallest response. Also, the adaptive signals of the proposed ACRC are displayed in Fig. 12. In the result, one can see that the elevator adaptive signal, which is related to altitude control, is actively changed, whereas the throttle adaptive signal maintains a constant value.

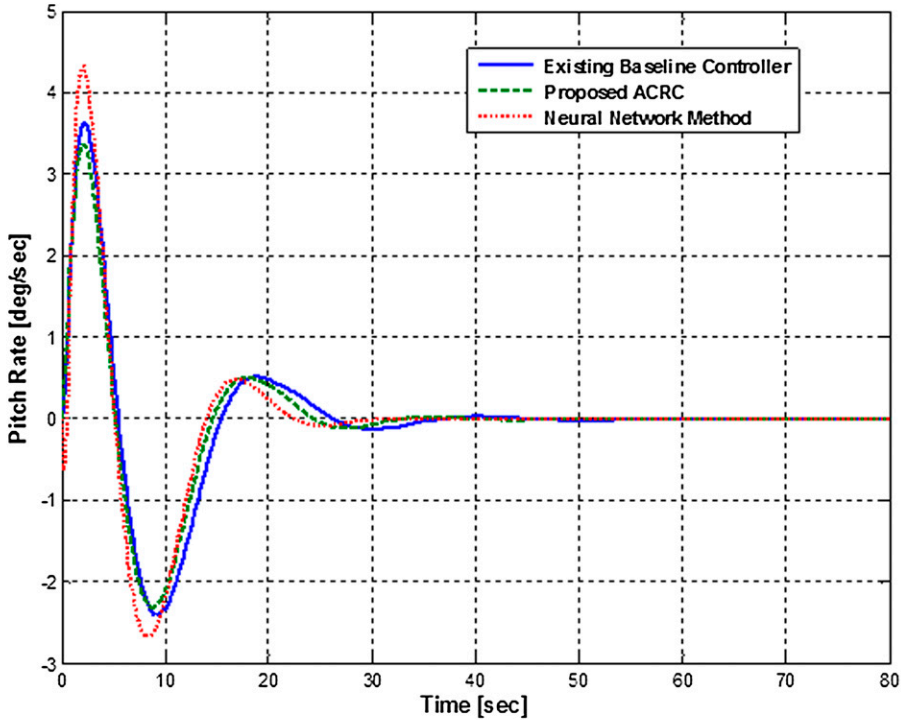

Fig. 7. Pitch rate response in case 1

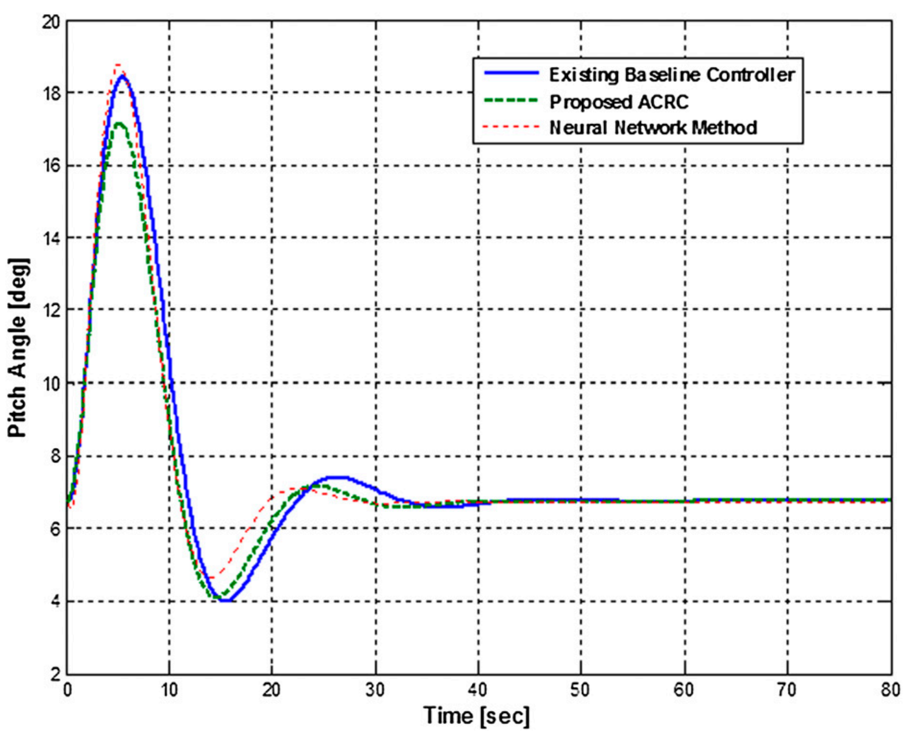

Fig. 8. Pitch angle response in case 1

J. Aerosp. Eng., 2013, 26(4): 735-749

The computational speeds of the proposed ACRC and the neural network method are compared in Table 2. For the simulation test, a laptop computer with Windows 7 and an Intel Core i5 CPU is used. The simulation conditions are that the simulation time is 80 s and that the integration solver is the Runge-Kutta fourth-order method. To compare the computational speeds of the two methods, the authors use the profiler tool of *Simulink*. The simulation models are all the same in the ACRC and neural network method except for the function approximation module. Hence, the authors measured the computation time of only the function approximation modules as given in Table 2. The results show that the proposed ACRC is approximately 1.7 times faster than the method using the neural network.

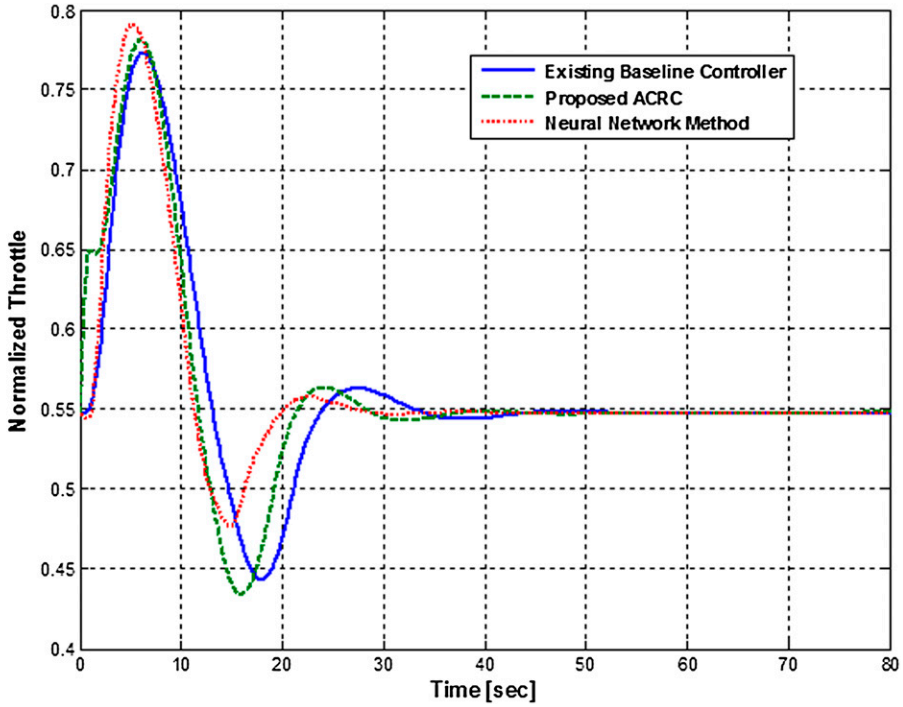

Fig. 9. Normalized throttle input response in case 1

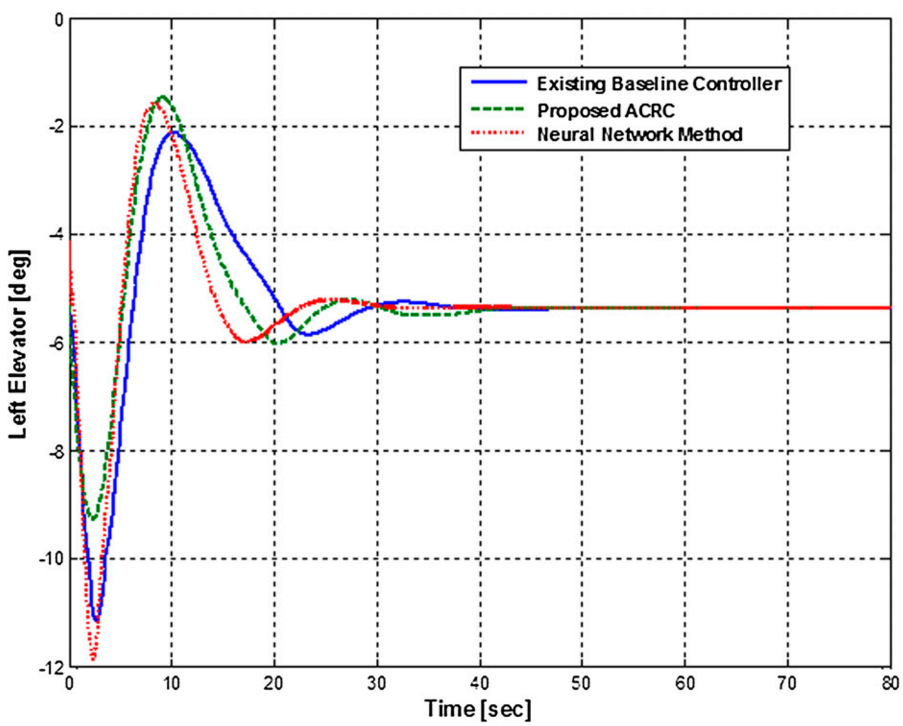

Fig. 10. Left elevator input response in case 1

Case 2

The flight conditions were changed from a nominal trim condition to an altitude of 3,000 m and a speed of 95 km/h to test the performance of retrofit control schemes. In this example, the proposed ACRC is superior to the existing baseline controller without an adaptive element. Fig. 13 shows the results of the altitude responses. From the results, one can see that the effectiveness of the proposed ACRC is clearly shown. In the case of the proposed ACRC using the existing baseline controller augmented by the adaptive Chebyshev approximator, it is seen that the response is very desirable despite the change of the flight conditions.

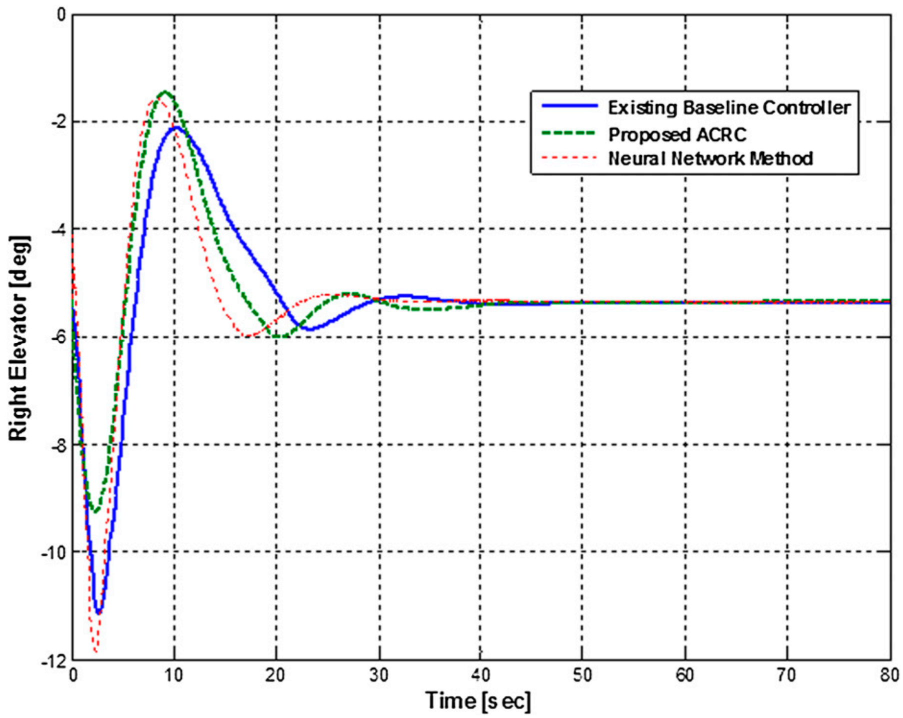

Fig. 11. Right elevator input response in case 1

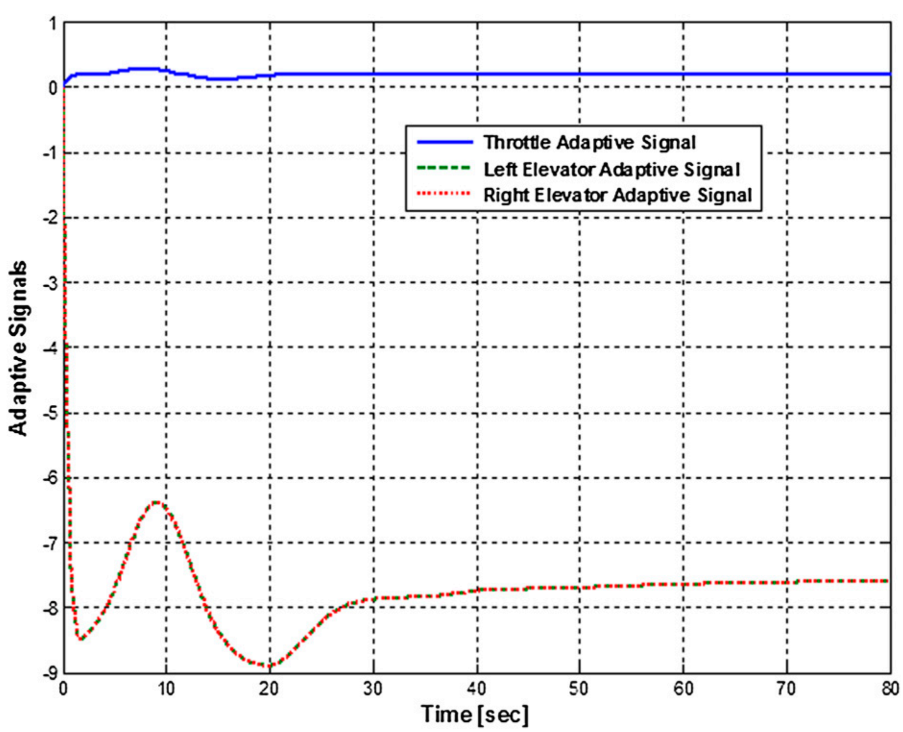

Fig. 12. Adaptive signals of the proposed ACRC in case 1

J. Aerosp. Eng., 2013, 26(4): 735-749

Table 2. Comparison of Computational Speed for the Two Methods

Item	Proposed method	Neural network method
Total recorded time	91.0 s	116.24 s
Clock precision	0.00000004 s	
Clock speed	2.53 GHz	

However, for the case of the existing nonaugmented baseline controller without the adaptive Chebyshev approximator, it is seen that the altitude error response is considerably unacceptable compared to that in the case of the proposed ACRC and the neural network method. The response of the proposed ACRC is similar to that of the neural network method. The altitude error responses are

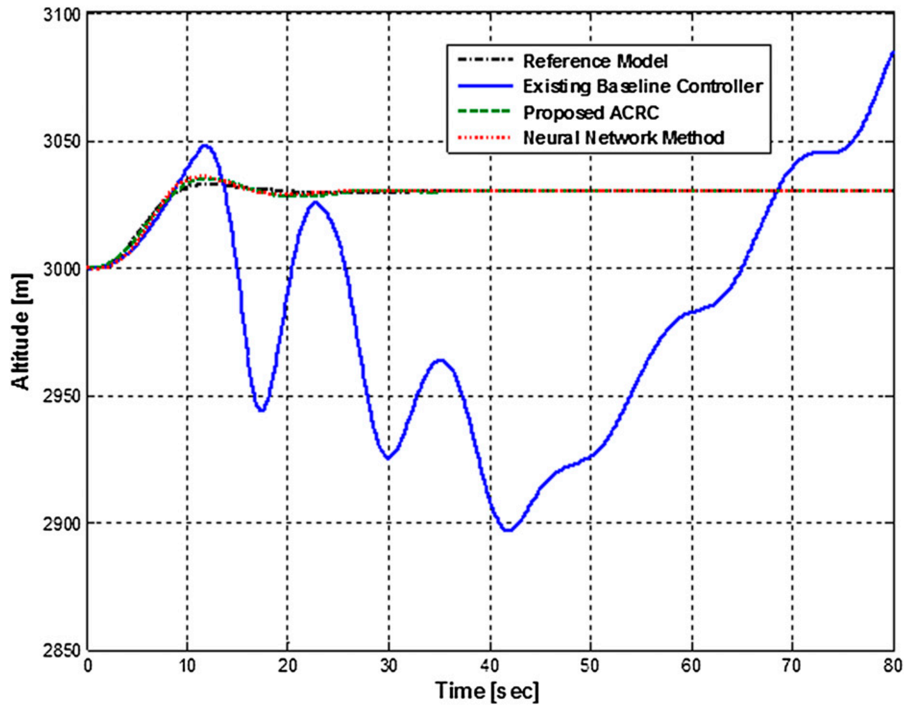

Fig. 13. Altitude response in case 2

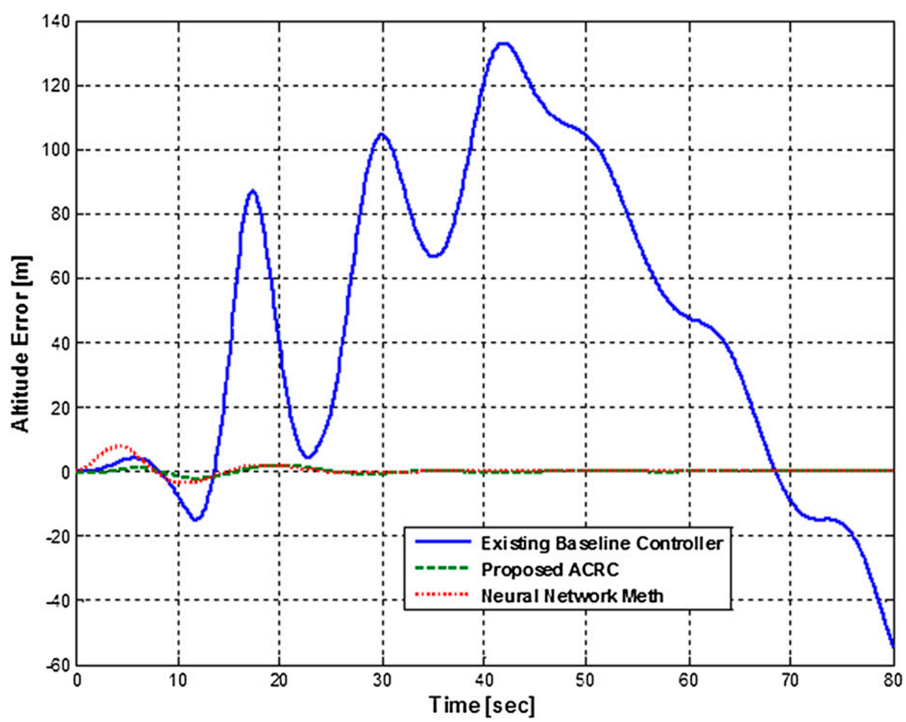

Fig. 14. Altitude error response in case 2

J. Aerosp. Eng., 2013, 26(4): 735-749

compared in Fig. 14. First, the proposed ACRC represents the best performance among the evaluated methods. The altitude error is the smallest, and the convergence speed of the error is fast as well. In the case of the neural network method, its response shows a large altitude error as compared with the nonaugmented baseline controller and the proposed ACRC. However, after a simulation time of 10 s, the error of the neural network method is rapidly reduced. On the other hand, the baseline controller without retrofitting shows very large error and does not converge to zero. The response of the velocity is shown in Fig. 15. In this result, one can see that the neural network method has the smallest deviation from the trim condition, and the baseline controller has the largest

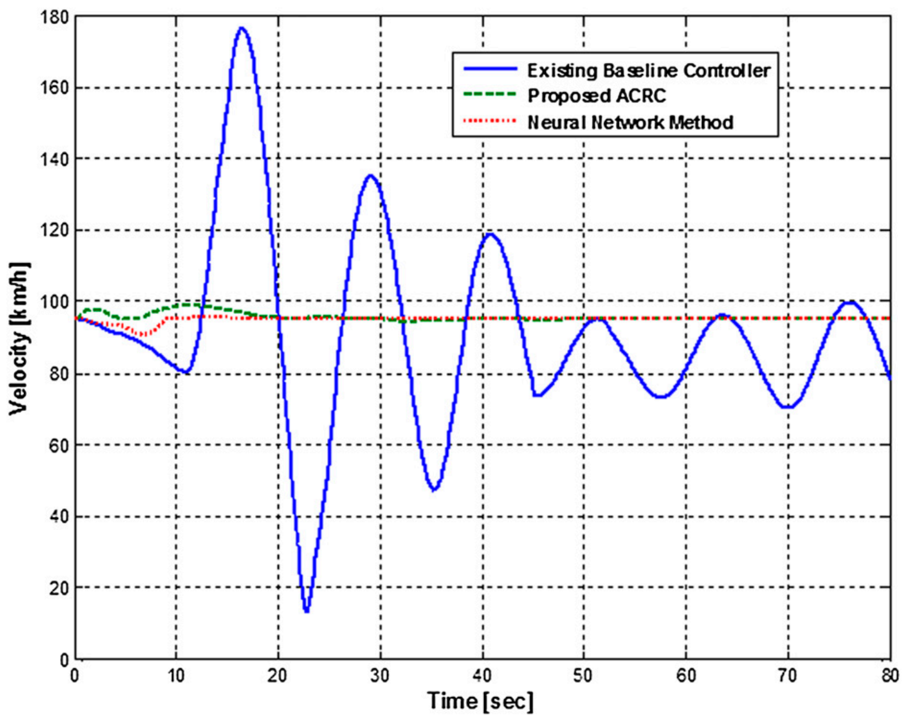

Fig. 15. Velocity response in case 2

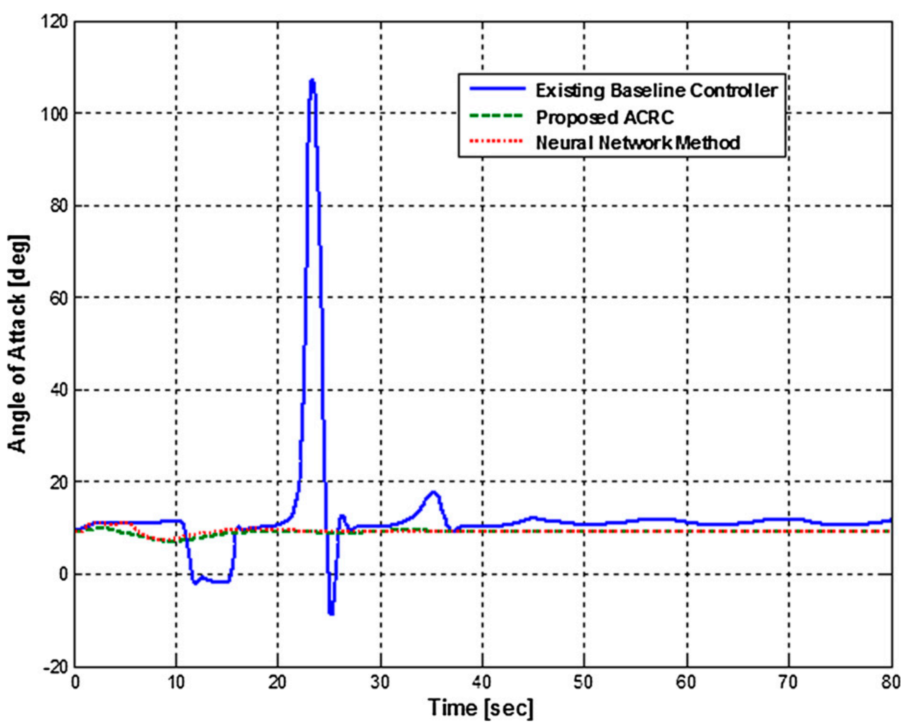

Fig. 16. Response of AoA in case 2

J. Aerosp. Eng., 2013, 26(4): 735-749

deviation. In terms of the AoA response, shown in Fig. 16, the proposed ACRC shows the best performance among the three methods; that is, it has the smallest deviation from the trim condition. The pitch rate and pitch angle responses are represented in Figs. 17 and 18, where it is seen that the performance differences between the proposed ACRC and neural network method are not large, but the baseline controller shows bad performance. In the case of the normalized throttle, presented in Fig. 19, the performances of the ACRC and neural network are relatively equal, but the baseline control shows the worst response. In the case of the elevators, shown in Figs. 20 and 21, the baseline control shows the largest elevator angles. Moreover, from the adaptive signals of

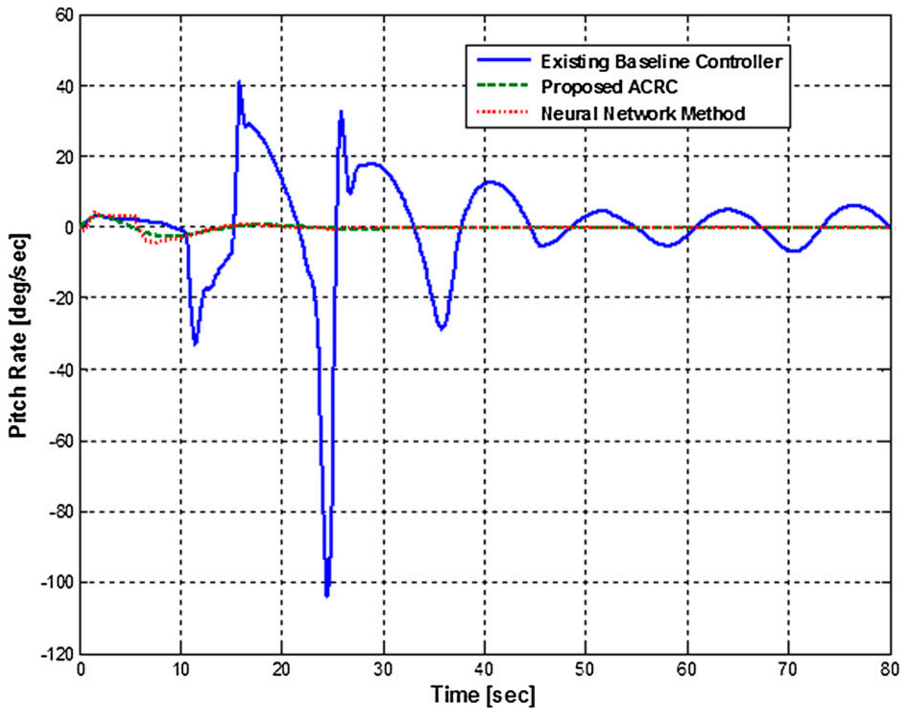

Fig. 17. Pitch rate response in case 2

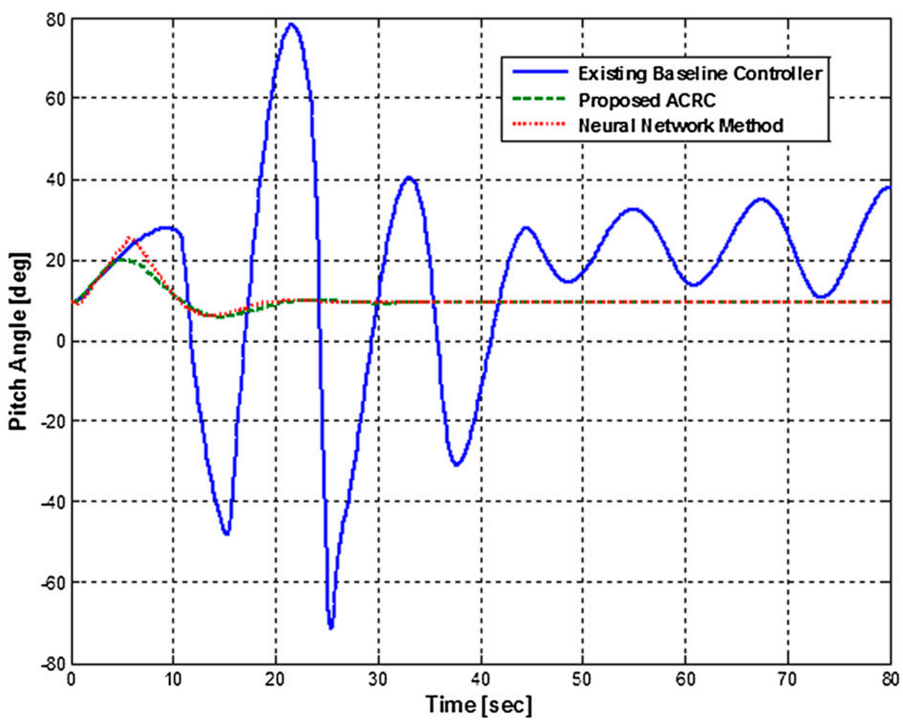

Fig. 18. Pitch angle response in case 2

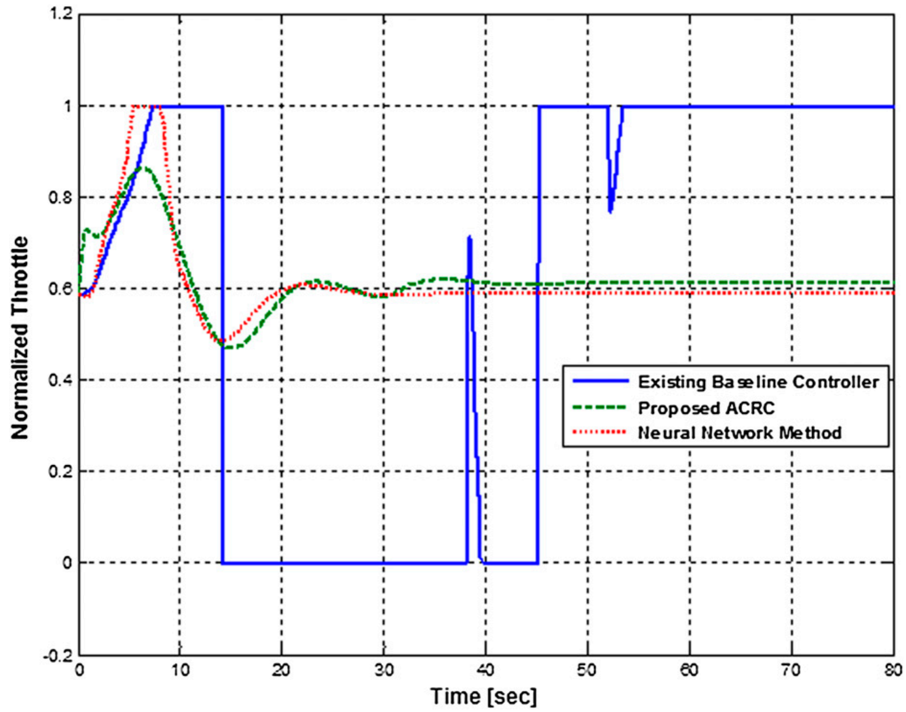

Fig. 19. Normalized throttle input response in case 2

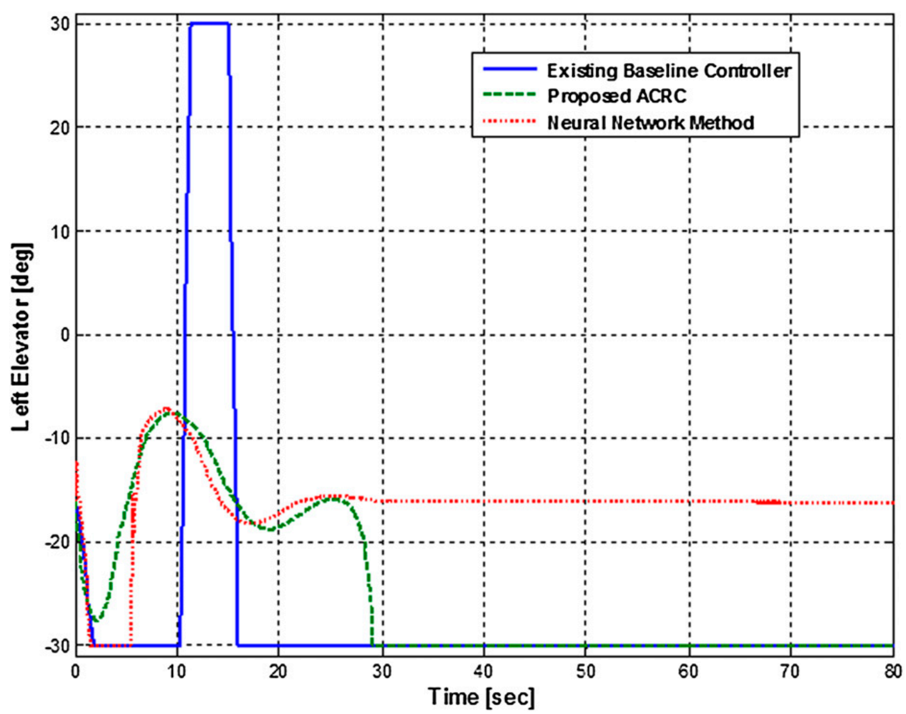

Fig. 20. Left elevator input response in case 2

Fig. 22, the authors find that the proposed ACRC converges to a specific value for approximating the uncertainty function.

Conclusions

In this paper, a novel adaptive control approach, named ACRC, retrofitting an existing baseline controller with an adaptive Chebyshev function approximator is developed and applied to a small UAV flight dynamics model. For verification of the performance, several methods, namely, the proposed ACRC, the neural network method, and baseline control without augmenting an adaptive element, were simulated and compared in the presence of changed flight conditions.

J. Aerosp. Eng., 2013, 26(4): 735-749

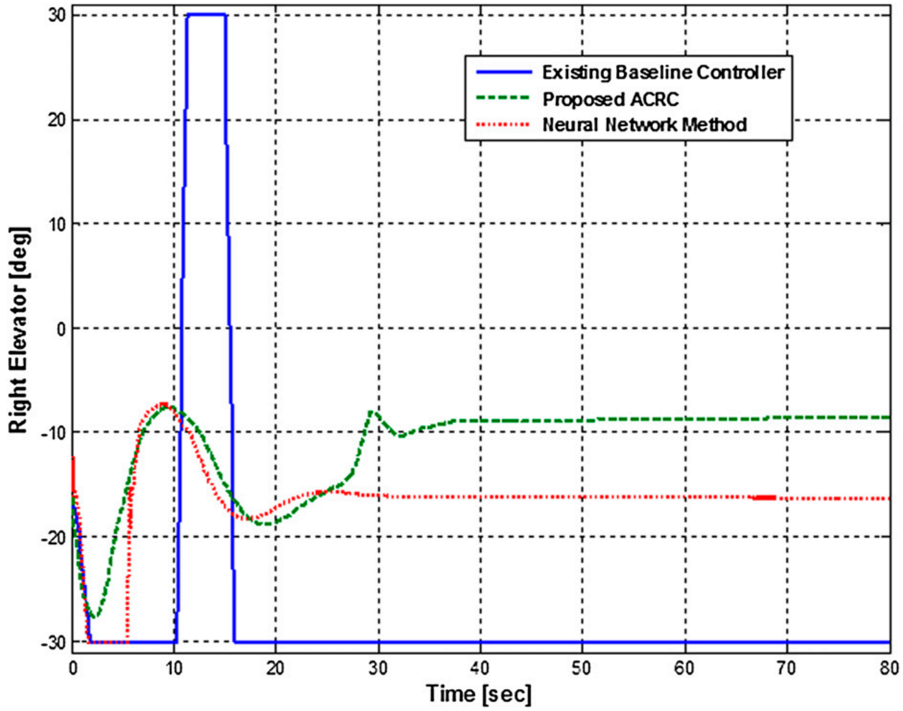

Fig. 21. Right elevator input response in case 2

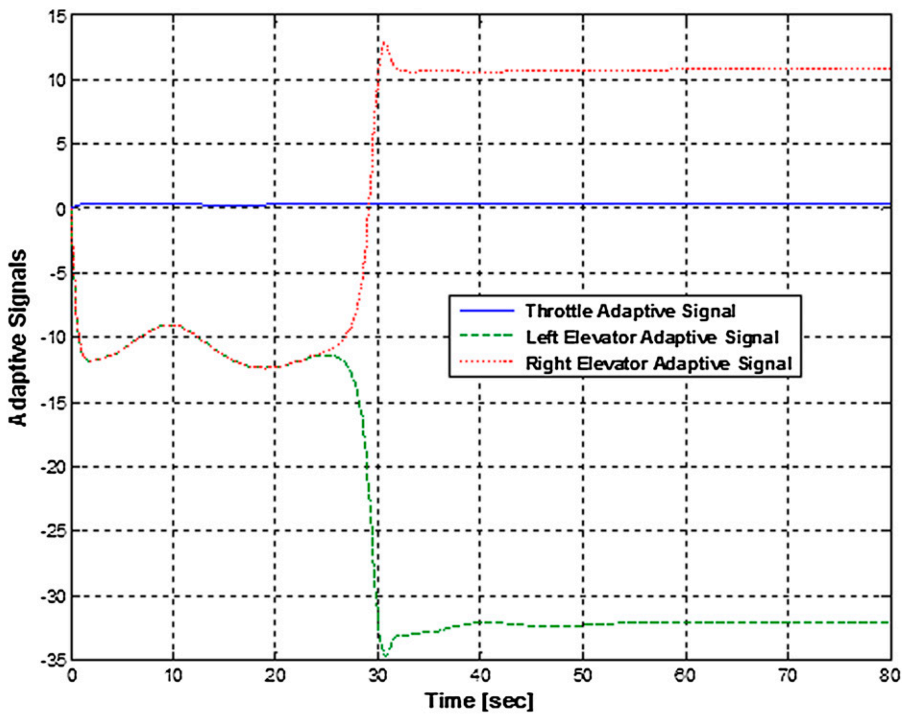

Fig. 22. Adaptive signals of the proposed ACRC in case 2

The simulation results show that the ACRC proposed in this paper is very effective in the presence of uncertainties such as changed flight conditions. This approach has an advantage in that its implementation is less complicated, and consequently, the computational speed is approximately 1.7 times faster than that of the method using the neural network. In particular, it can be usefully applied to small UAVs with restricted on-board hardware performance. It is thus anticipated that the proposed ACRC will replace the neural network approach in flight control systems with constraints on hardware performance.

J. Aerosp. Eng., 2013, 26(4): 735-749

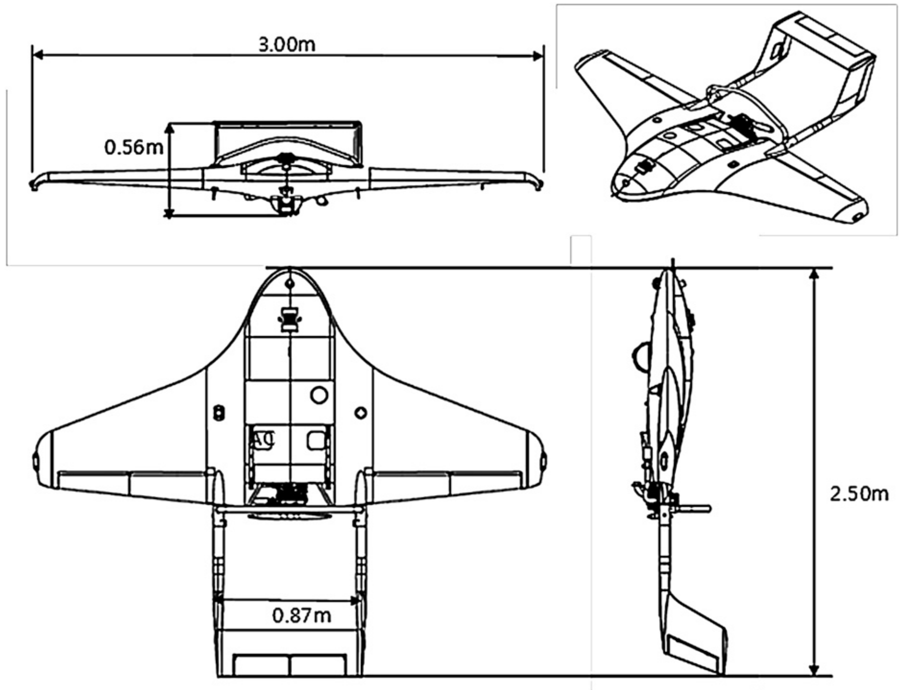

Fig. 23. Configuration of three sides of the UAV used in the simulation

References

Bosković, J. D., Bergstrom, S. E., and Mehra, R. K. (2005). "Robust integrated flight control design under failures, damage, and state-dependent disturbances." *J. Guid. Control Dyn.*, 28(5), 902–917.

Bosković, J. D., and Mehra, R. K. (2002). "An adaptive retrofit reconfigurable flight controller." *Proc., 41st IEEE Conf. on Decision and Control*, Vol. 2, IEEE, New York, 1257–1262.

Bosković, J. D., Prasanth, R., and Mehra, R. K. (2007). "Retrofit fault tolerant flight control design under control effector damage." *J. Guid. Control Dyn.*, 30(3), 703–712.

Calise, A. J., Yang, B. J., and Craig, J. I. (2002). "Augmentation of an existing linear controller with an adaptive element." *Proc., American Control Conf.*, Vol. 2, IEEE, New York, 1549–1554.

Campa, G., Sharma, M., Calise, A. J., and Innocenti, M. (2000). "Neural network augmentation of linear controllers with application to underwater vehicles." *Proc., American Control Conf.*, Vol. 1, IEEE, New York, 75–79.

Dinu, A. D., Botez, R. M., and Cotoi, I. (2006). "Chebyshev polynomials for unsteady aerodynamic calculations in aeroservoelasticity." *J. Aircr.*, 43(1), 165–171.

Farrell, J., and Polycarpou, M. (2006). *Adaptive approximation based control*, Wiley, New York.

Johnson, E. N., Calise, A. J., and El-Shirbiny, H. A. (2000). "Feedback linearization with neural network augmentation applied to X-33 attitude control." *Proc., AIAA Guidance, Navigation, and Control Conf. and Exhibit*, American Institute of Aeronautics and Astronautics, Reston, VA.

Kim, B., and Calise, A. J. (1997). "Nonlinear flight control using neural networks." *J. Guid. Control Dyn.*, 20(1), 26–33.

Kim, N. (2003). "Improved methods in neural network-based adaptive output feedback control with applications to flight control." Ph.D thesis, Georgia Institute of Technology, Atlanta.

Lee, T.-T., and Jeng, J. T. (1998). "The Chebyshev-polynomials-based unified model neural networks for function approximation." *IEEE Trans. Syst. Man Cybern. Part B Cybern.*, 28(6), 925–935.

Lewis, F., Yesildirek, A., and Liu, K. (1996). "Multilayer neural-net robot controller with guaranteed tracking performance." *IEEE Trans. Neural Netw.*, 7(2), 388–399.

Mason, J. C., and Handscomb, D. C. (2003). *Chebyshev polynomials*, CRC Press, New York.

Mathews, J. H., and Fink, K. D. (2004). *Numerical methods using Matlab*, Prentice Hall, New York.

MATLAB 7.7 R2008b [Computer software]. Natick, MA, MathWorks.

Nguyen, N., Krishnakumar, K., Kaneshige, J., and Nespeca, P. (2008). "Flight dynamics and hybrid adaptive control of damaged aircraft." *J. Guid. Control Dyn.*, 31(3), 751–764.

Pashilkar, A. A., Sundararajan, N., and Saratchandran, P. (2006). "Adaptive back-stepping neural controller for reconfigurable flight control systems." *IEEE Trans. Contr. Syst. Technol.*, 14(3), 553–561.

Patra, J. C., and Kot, A. C. (2002). "Nonlinear dynamic system identification using Chebyshev functional link artificial neural networks." *IEEE Trans. Syst. Man Cybern. Part B Cybern.*, 32(4), 505–511.

Rice, J. R. (1964). *The approximation of functions*, Addison-Wesley, Palo Alto, CA.

Sharma, M., and Calise, A. J. (2005). "Neural-network augmentation of existing linear controllers." *J. Guid. Control Dyn.*, 28(1), 12–19.

Simulink R2008b [Computer software]. Natick, MA, MathWorks.

Wohletz, J. M., Paduano, J. D., and Maine, T. (2000). "Retrofit reconfiguration system for a commercial transport." *Proc., AIAA Guidance, Navigation, and Control Conf. and Exhibit*, American Institute of Aeronautics and Astronautics, Reston, VA.

Yesildirek, A., and Lewis, F. (1995). "Feedback linearization using neural networks." *Automatica*, 31(11), 1659–1664.

Zou, A., and Kumar, K. D. (2011). "Adaptive output feedback control of spacecraft formation flying using Chebyshev neural networks." *J. Aerosp. Eng.*, 24(3), 361–372.

J. Aerosp. Eng., 2013, 26(4): 735-749